The Balitsky-Kovchegov equation and dipole orientation

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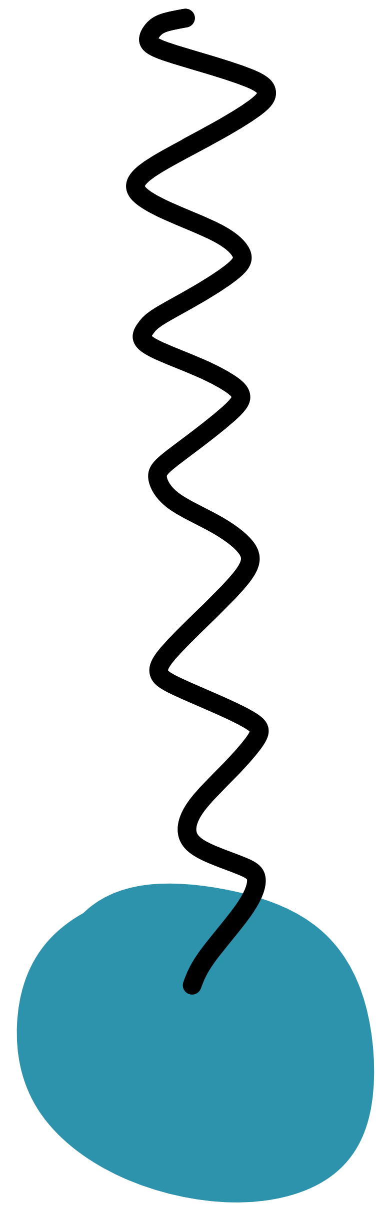
J. G. Contreras

Matěj Vaculčiak, 10. 4. 2024 **https://doi.org/10.1016/j.physletb.2023.138360** **CTU in Prague**

Intro

- low-x hadron structure, gluon saturation
- probing hadron (target) with photon (projectile)
  - ep (HERA), eI (EIC), pp, pPb, PbPb (LHC)
- Balitsky-Kovchegov equation
  - gluon evolution ~ dipole evolution

↑ large N_c



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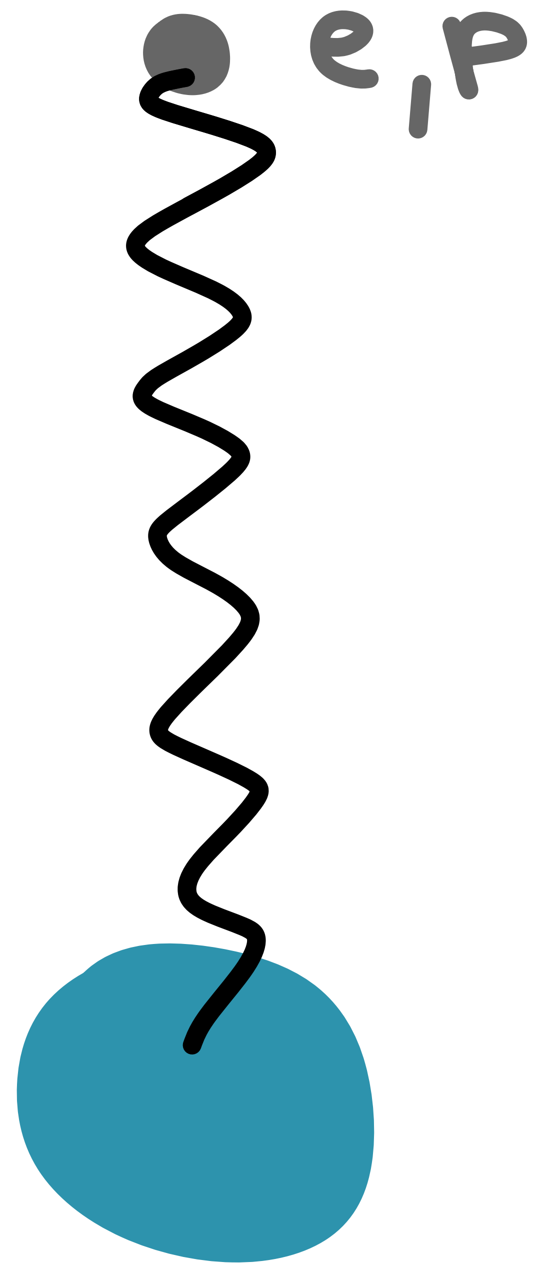

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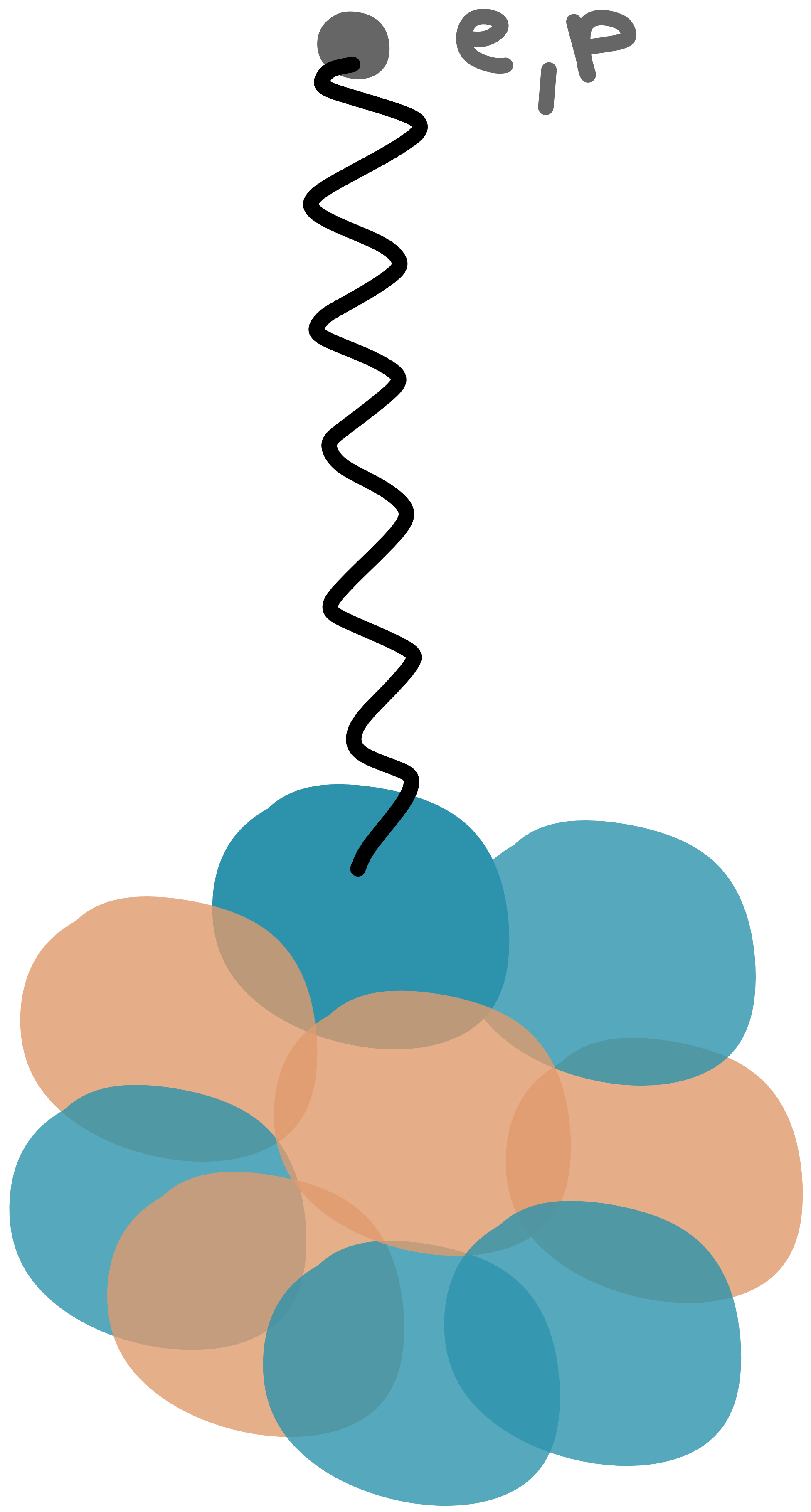

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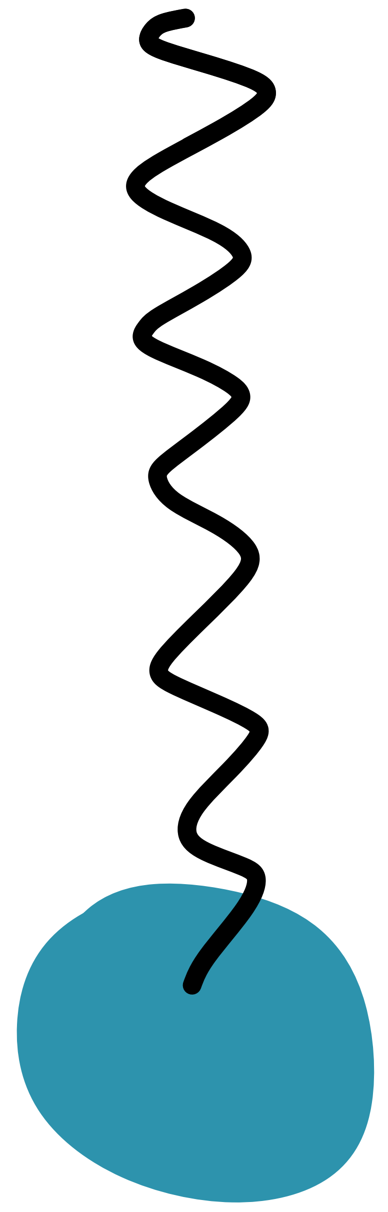

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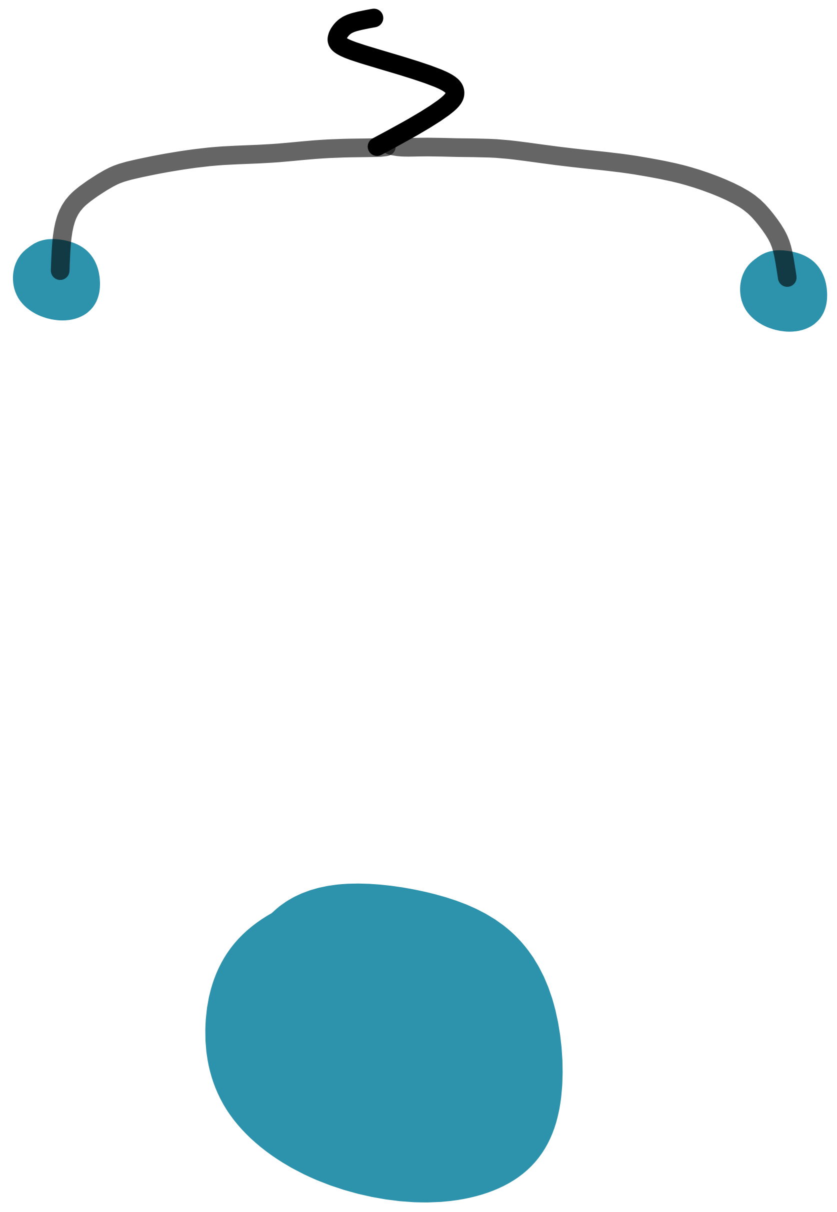

Intro

- dipole amplitude

$\ln \frac{x_0}{x}$

$$N(\eta, \underline{x}, \underline{y}) \to N(\eta, r, b, \theta, \varphi)$$

- collinearly improved kernel

$$K = \frac{\bar{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min\{r_1^2, r_2^2\}} \right]^{\pm \bar{\alpha}_s A_1}$$

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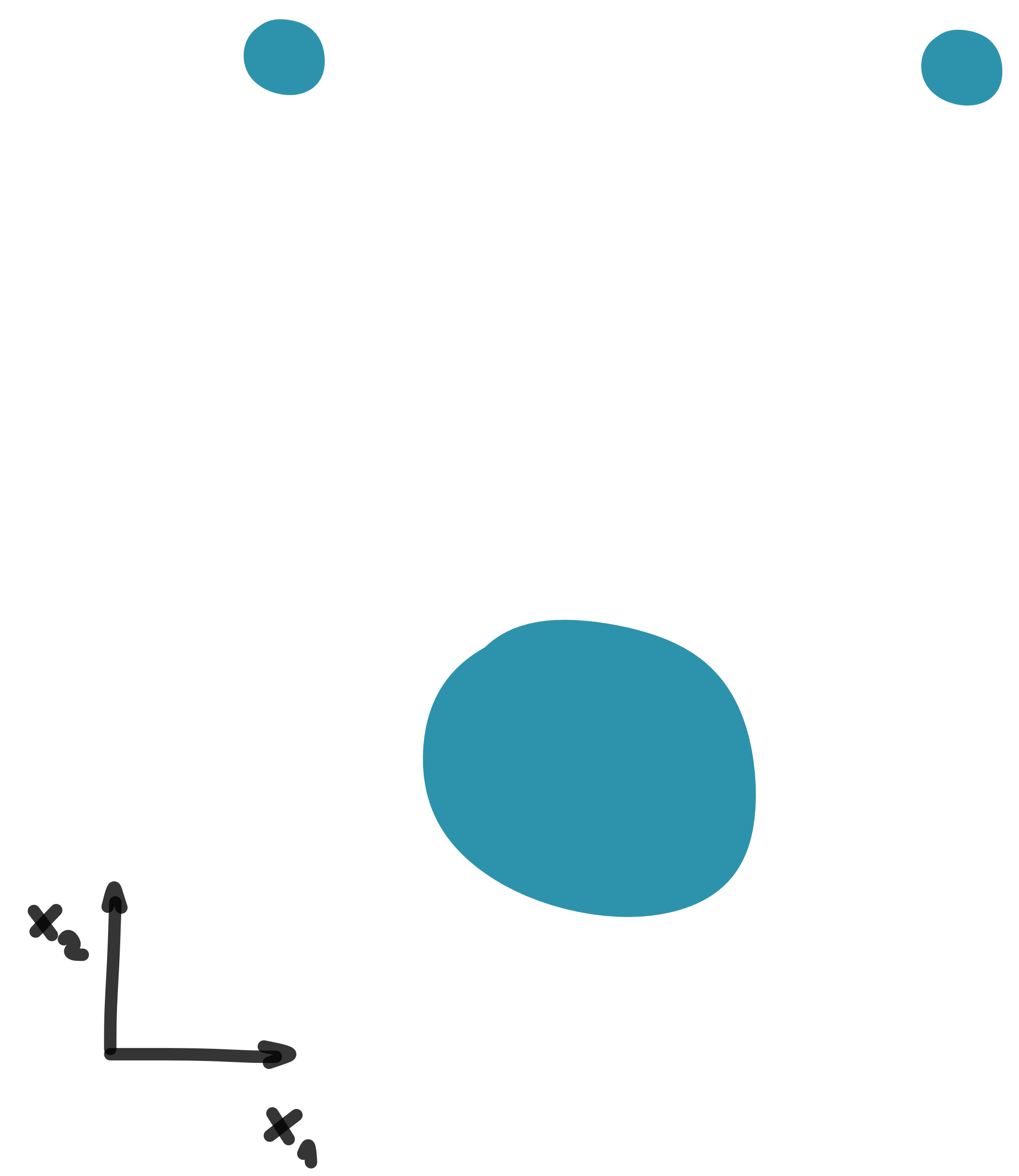

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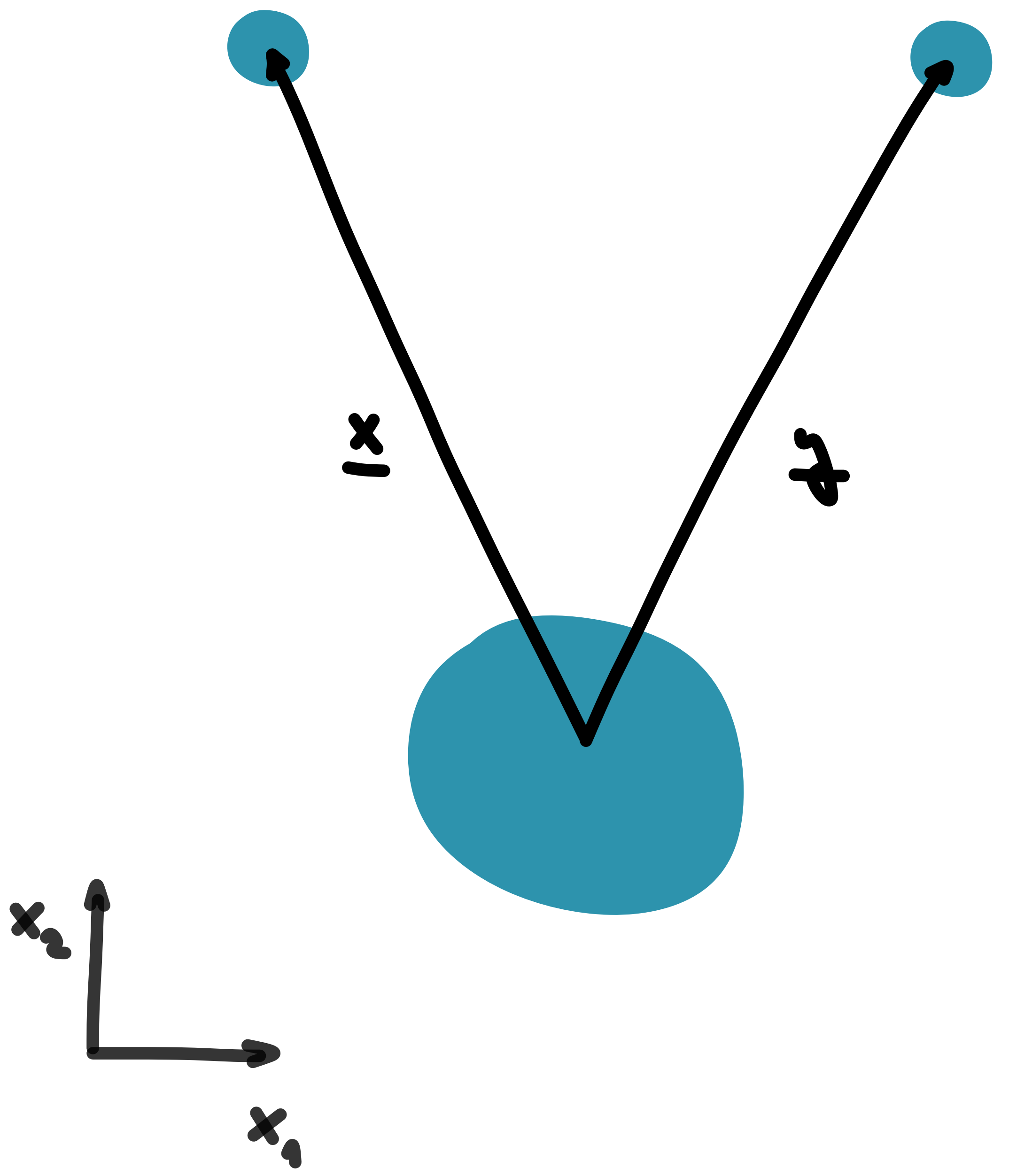

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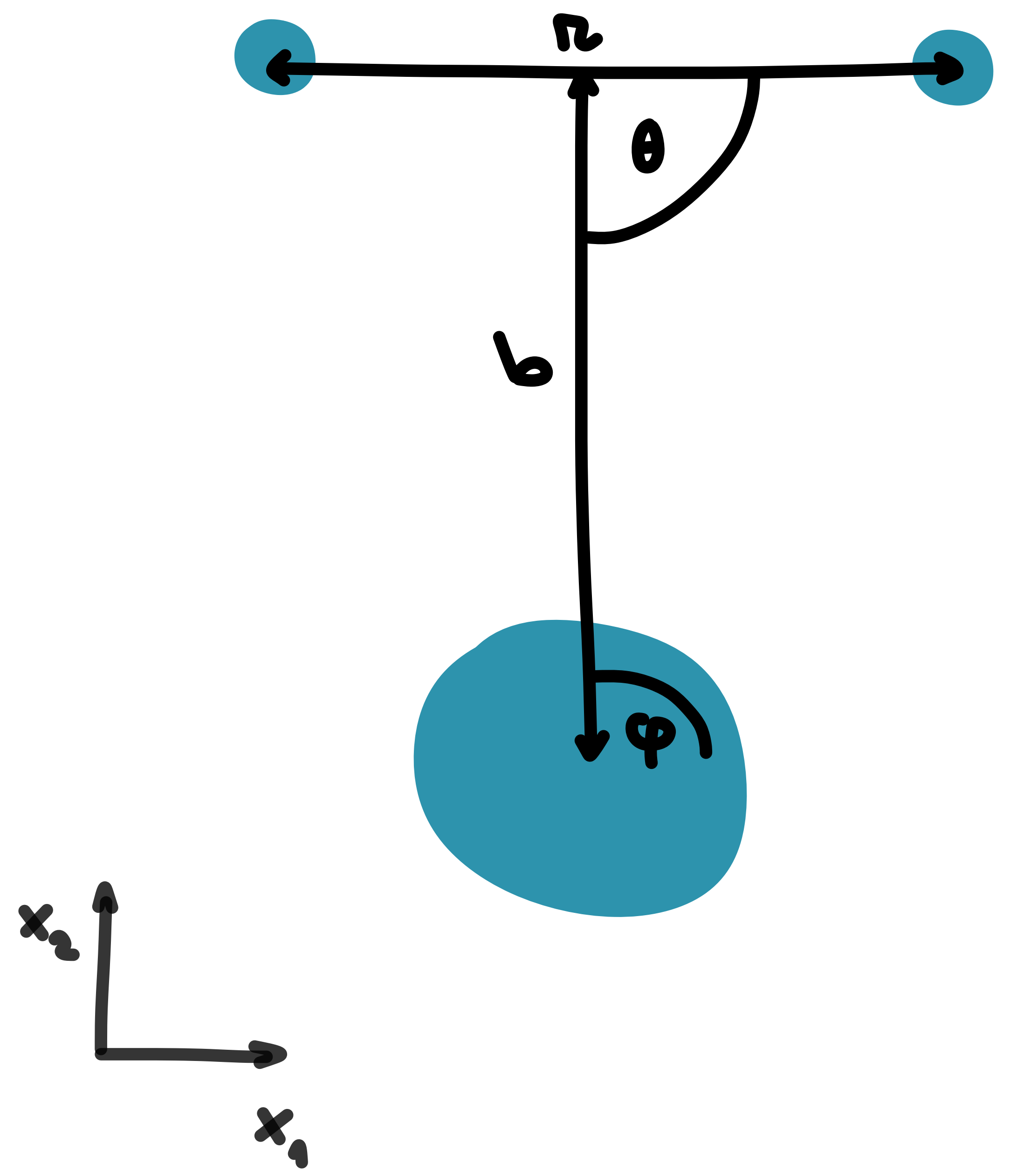

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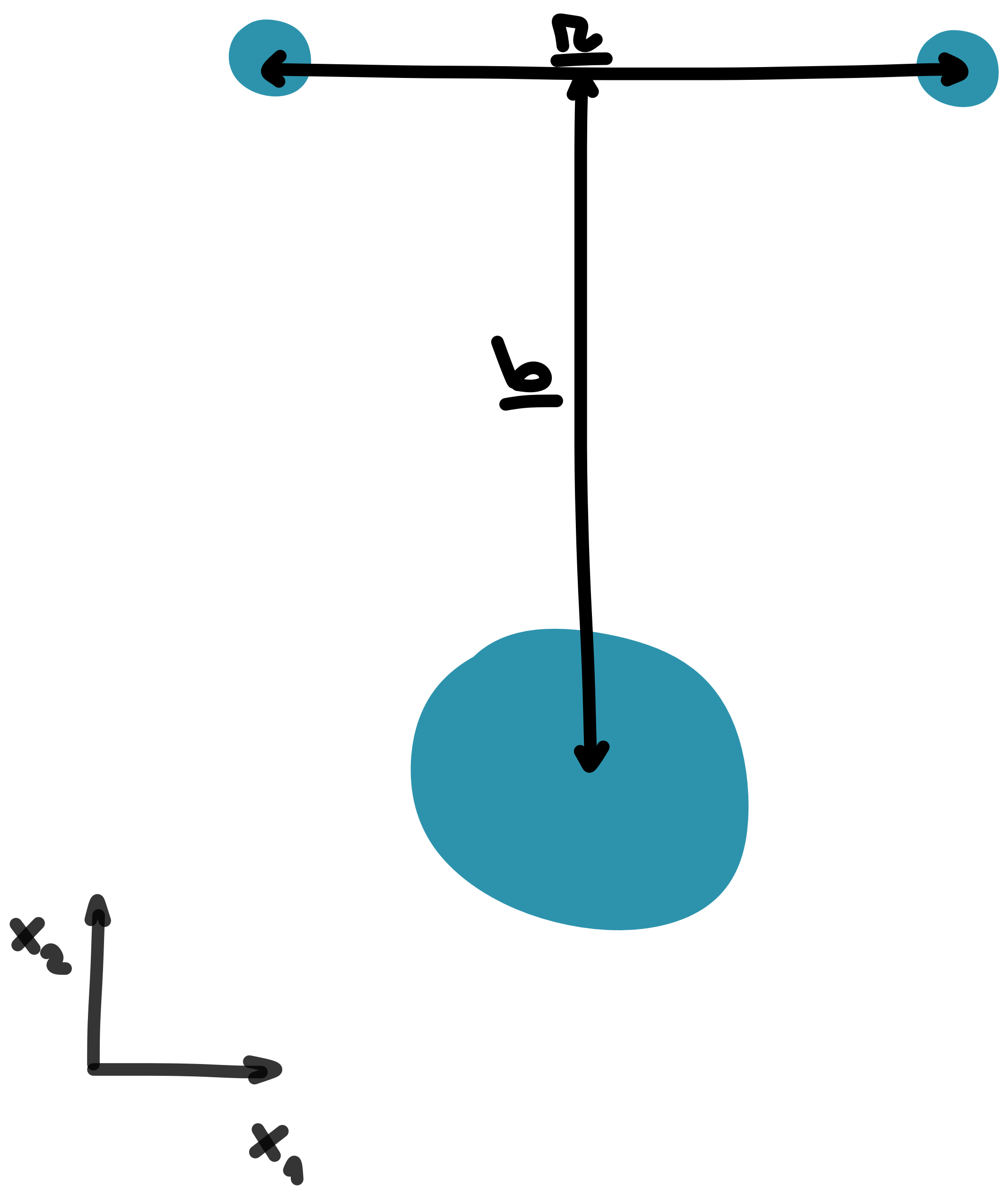

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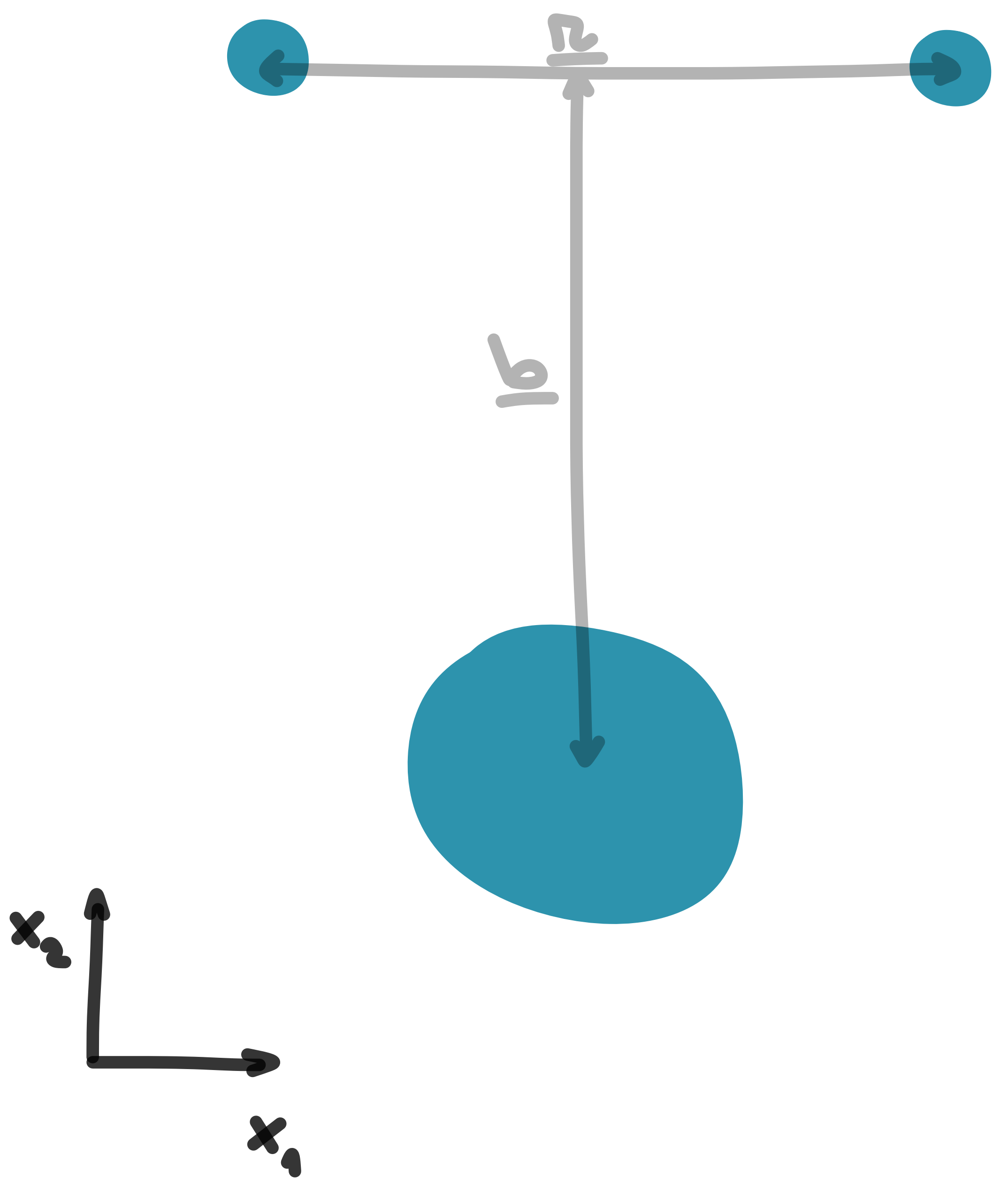

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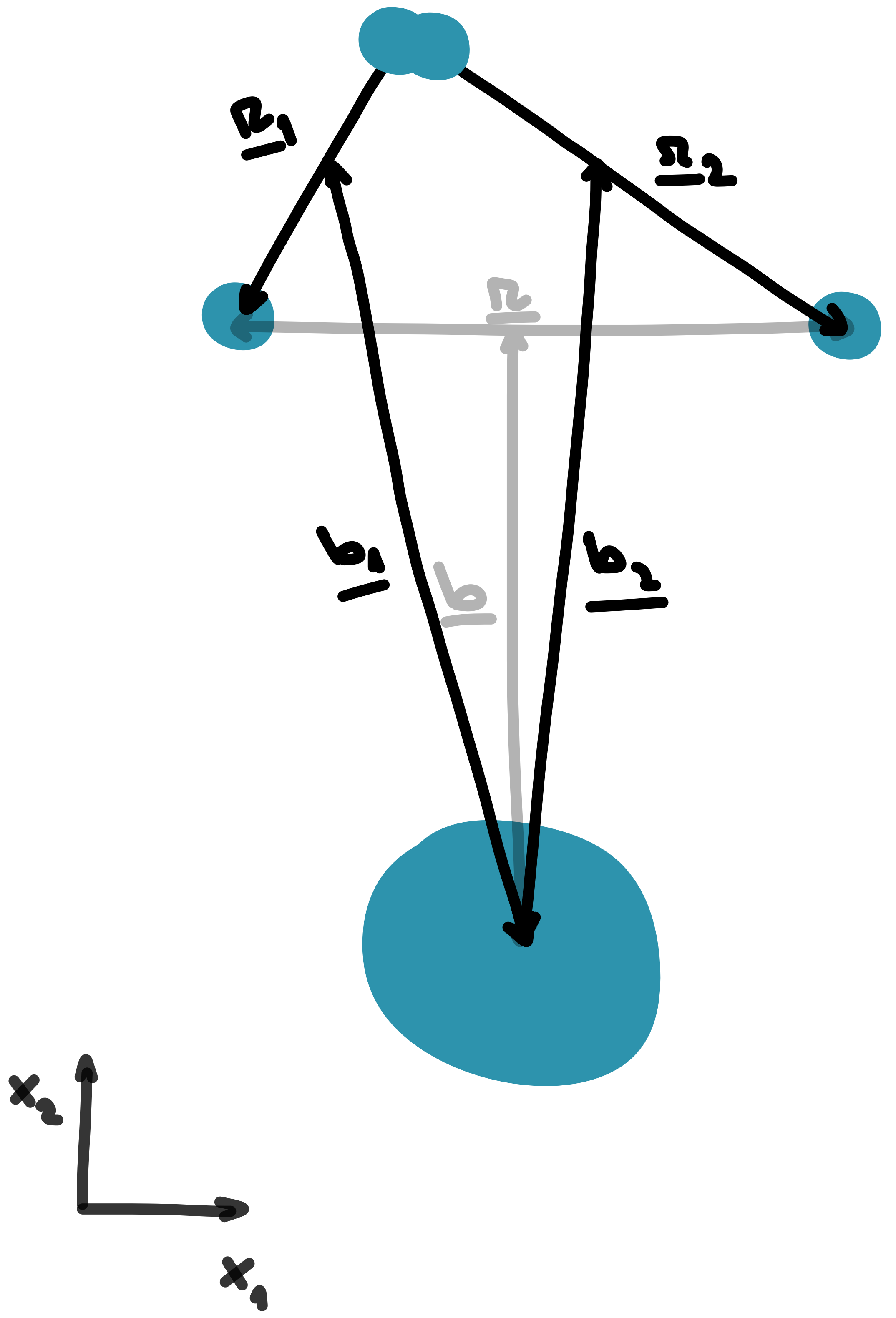

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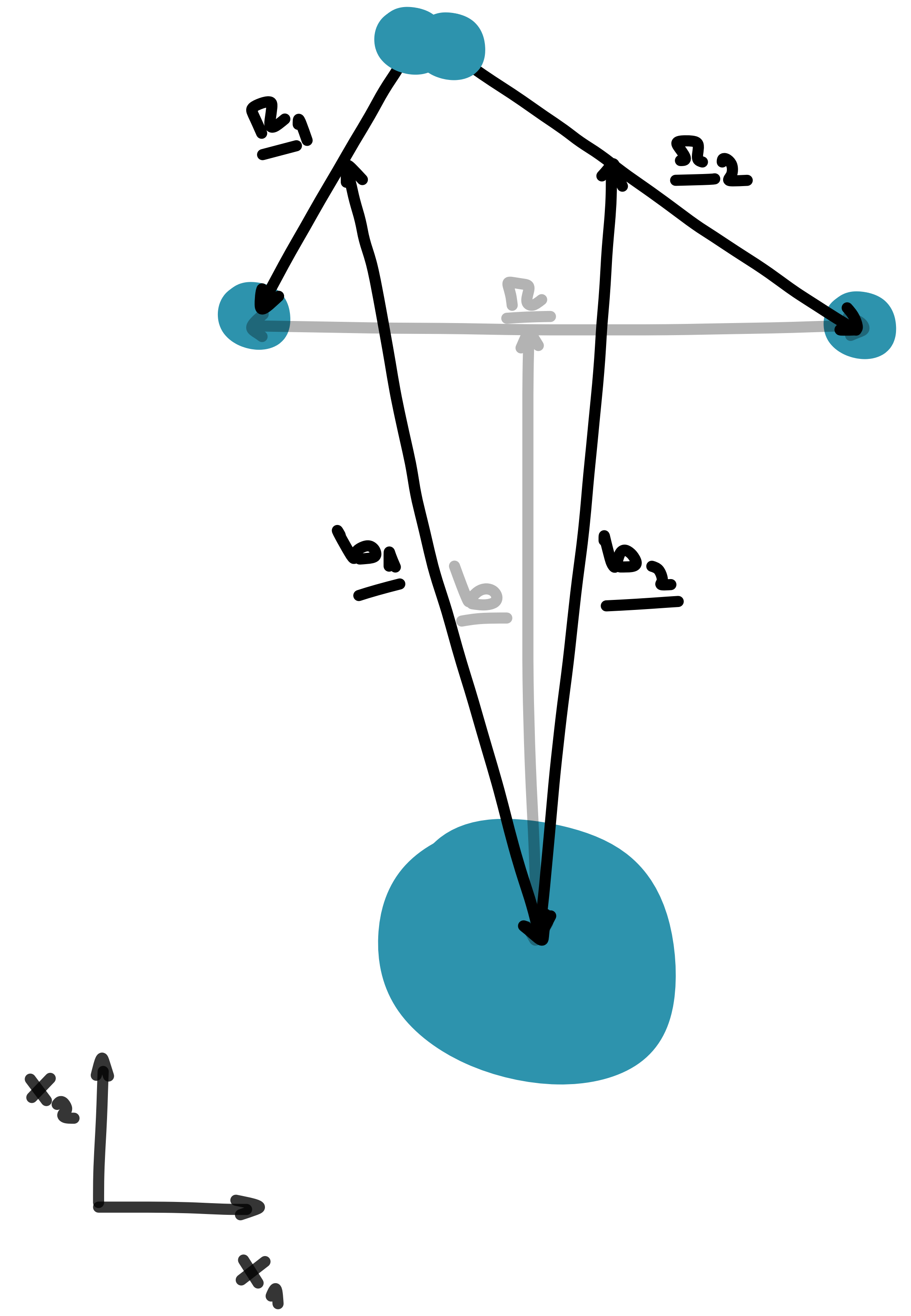

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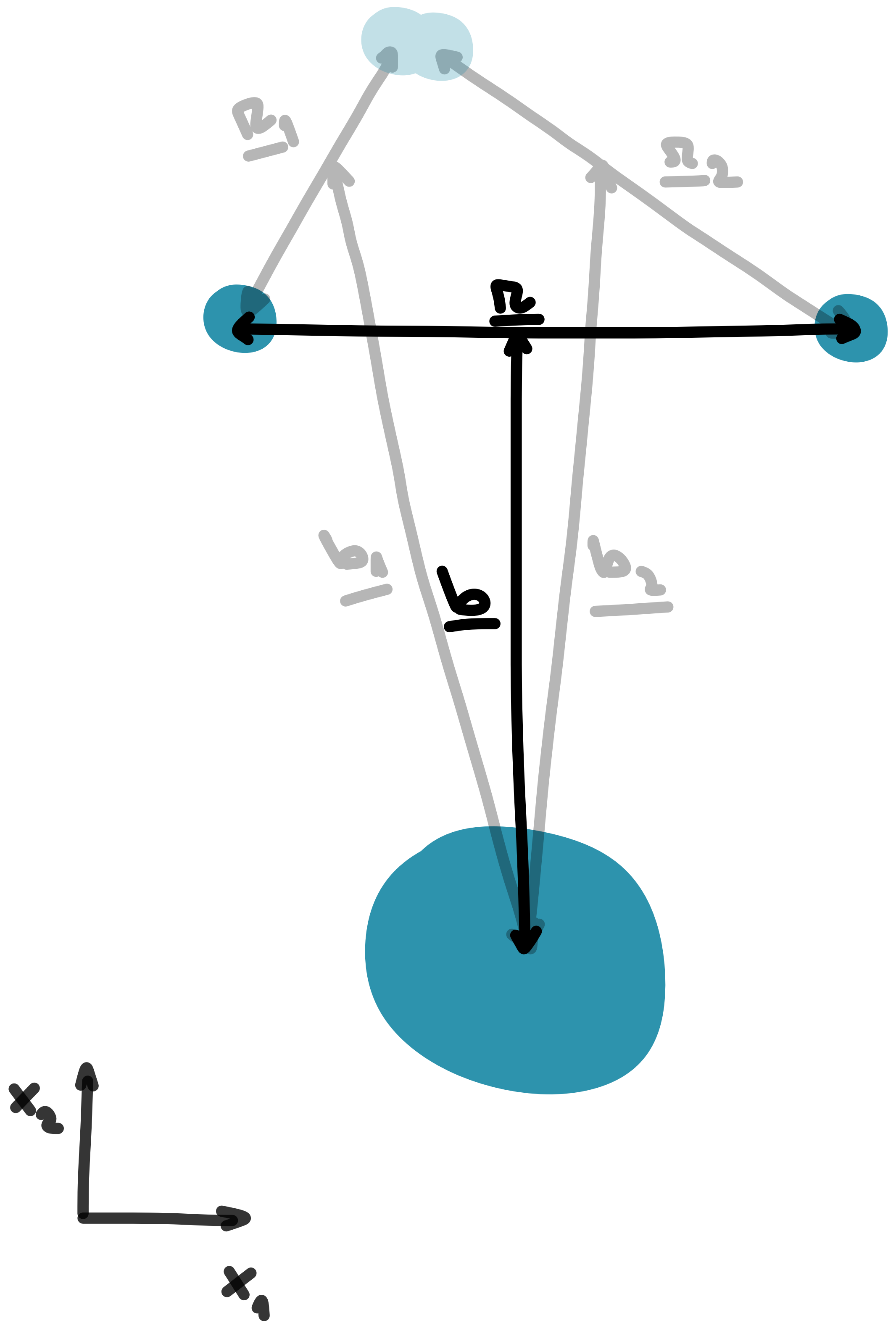

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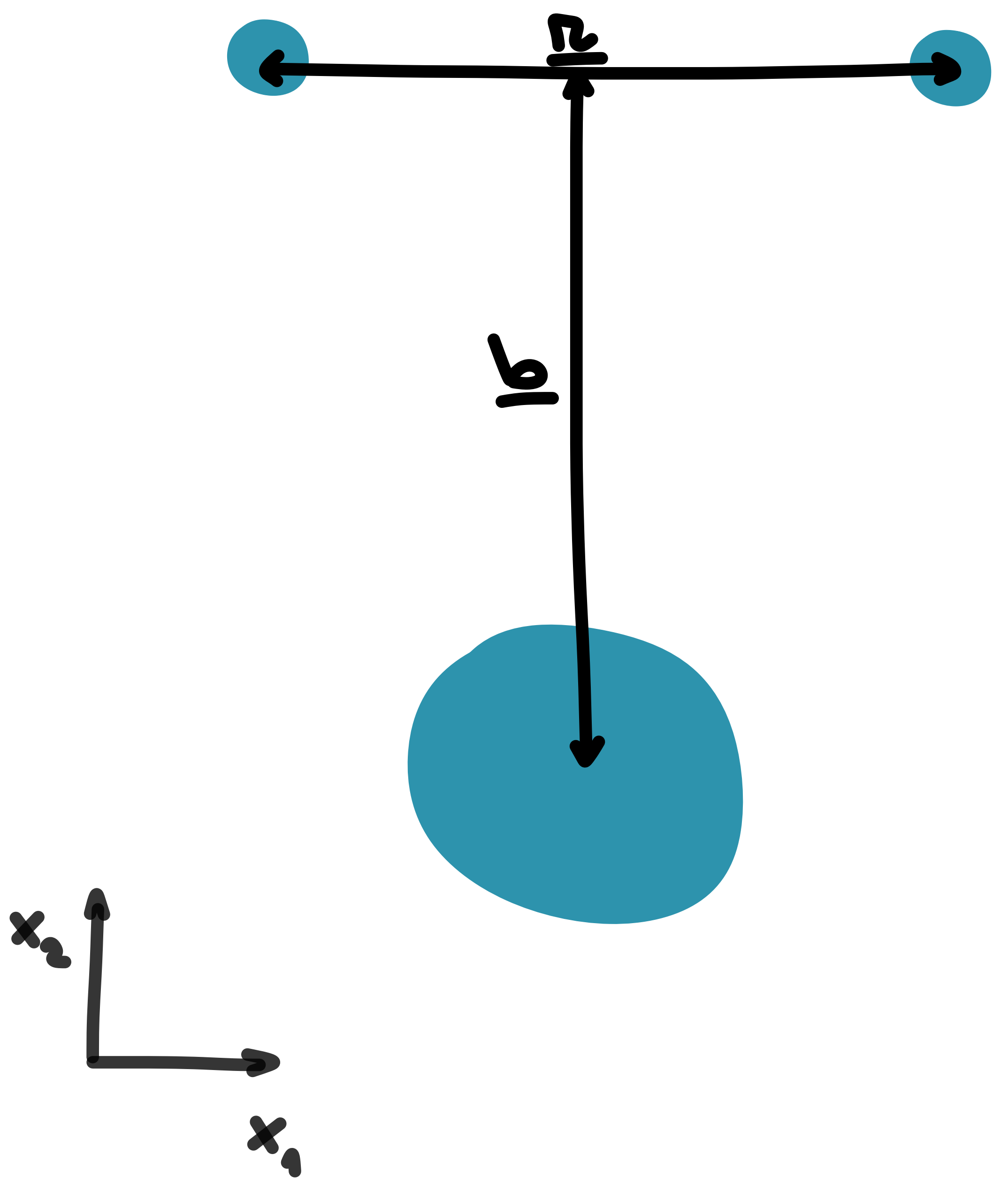

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1D BK - amplitude

- infinite target approximation

$$2\int \mathrm{d}\underline{b} N(\eta, \underline{r}, \underline{b}) \approx \sigma_0 \, N(\eta, r)$$

- MV initial condition

$$N(\eta \leq 0,\ r) = 1 - e^{-\frac{1}{4}(r^2 Q_{s0}^2)^{\gamma} \ln(\frac{1}{r\Lambda} + e)}$$

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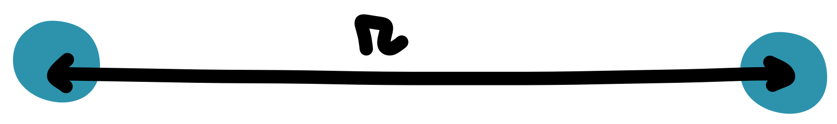

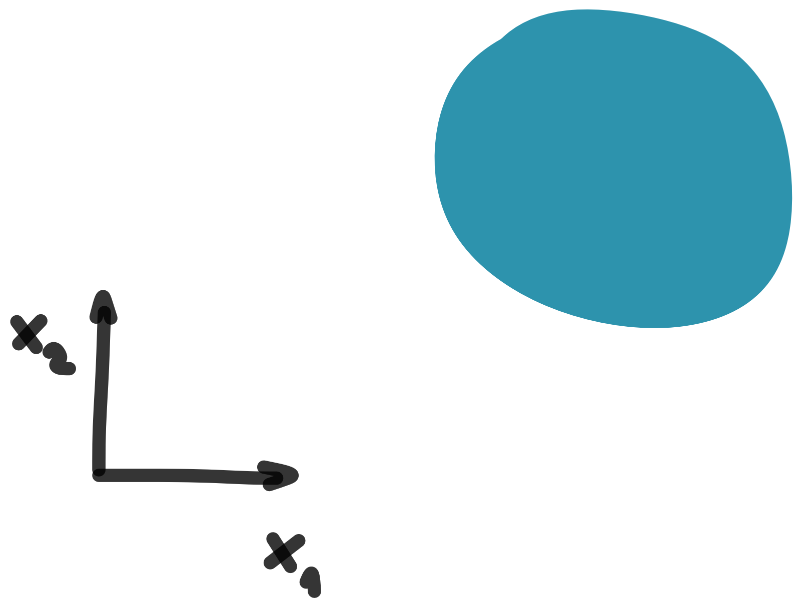

[McLerran, Venugopalan, 1998]

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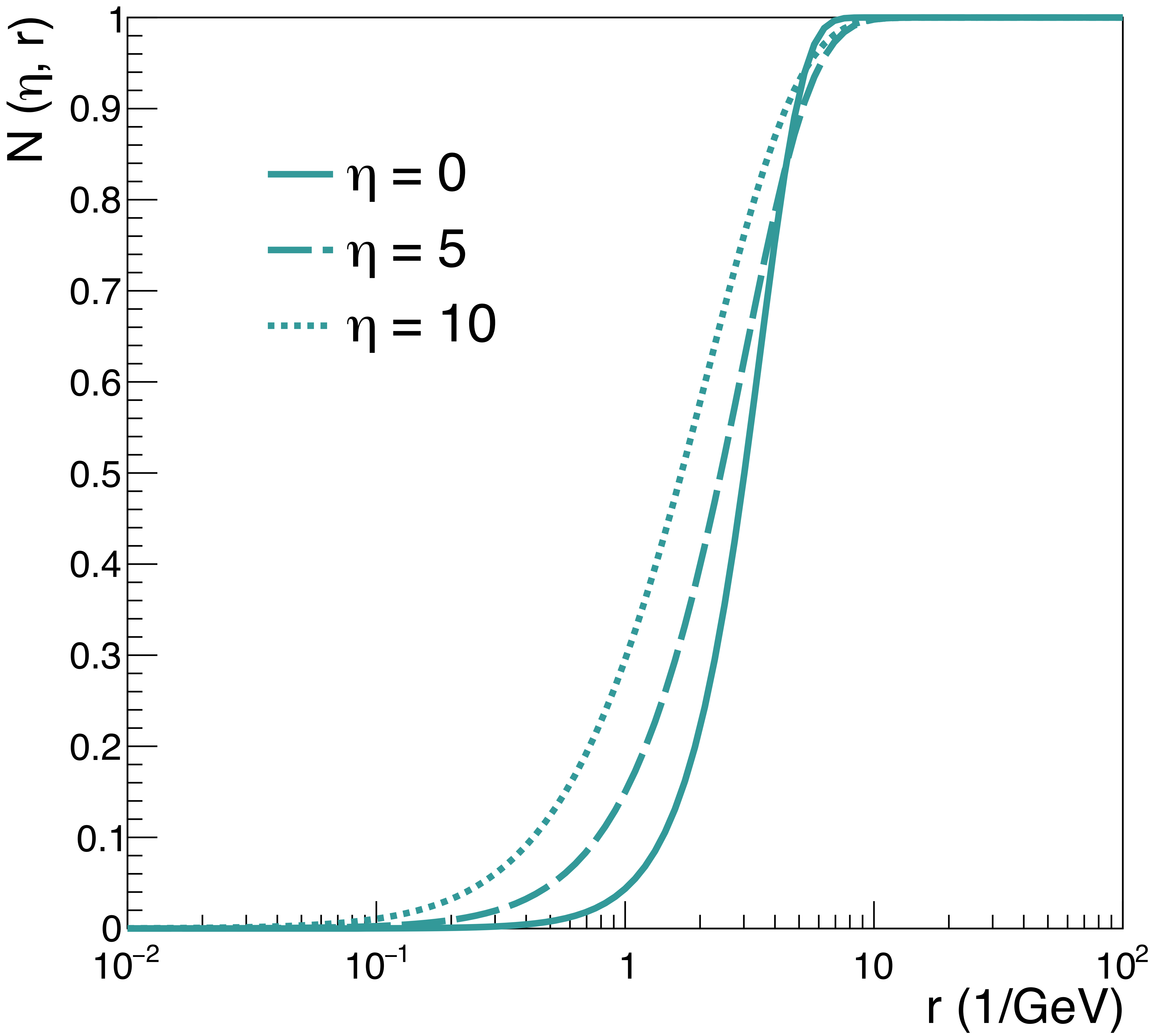


1D BK - data

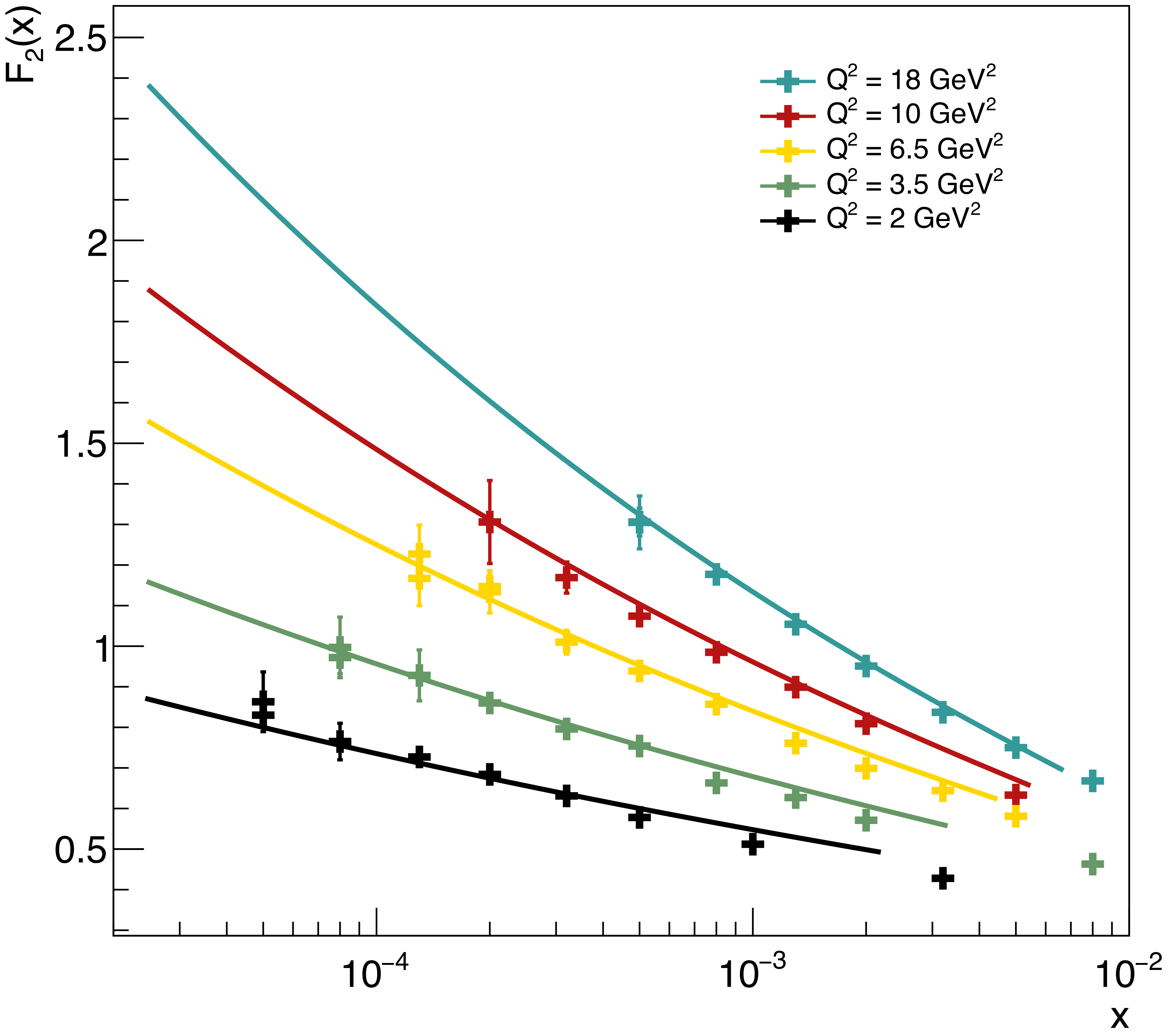
- proton structure functions

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_L^{\gamma^* p}\left(x, Q^2\right) + \sigma_T^{\gamma^* p}\left(x, Q^2\right)\right)$$

$$\sigma_{L,T}^{\gamma^* p}(x, Q^2) = \sum_f \int \mathrm{d}^2\underline{r} \int_0^1 \mathrm{d}z \, |\psi_{T,L}^{(f)}(\underline{r}, Q^2, z)|^2 \, 2 \int \mathrm{d}^2\underline{b} N(x_f, \underline{r}, \underline{b})$$

[Golec-Biernat, Wüsthoff, 1998]

1D BK - data



- HERA data described

[HERA, 2010]

2D BK - amplitude

- impact parameter dependence

$$2\int \mathrm{d}\underline{b}N(\eta, \underline{r}, \underline{b}) \approx 4\pi \int \mathrm{d}bN(\eta, r, b)$$

- initial condition
  - GBW (*r*)
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$$N(\eta \leq 0,\ r,\ b) = 1 - e^{-\frac{Q_s^2}{4}r^2 e^{-\frac{b^2}{2B} - \frac{r^2}{8B}}}$$

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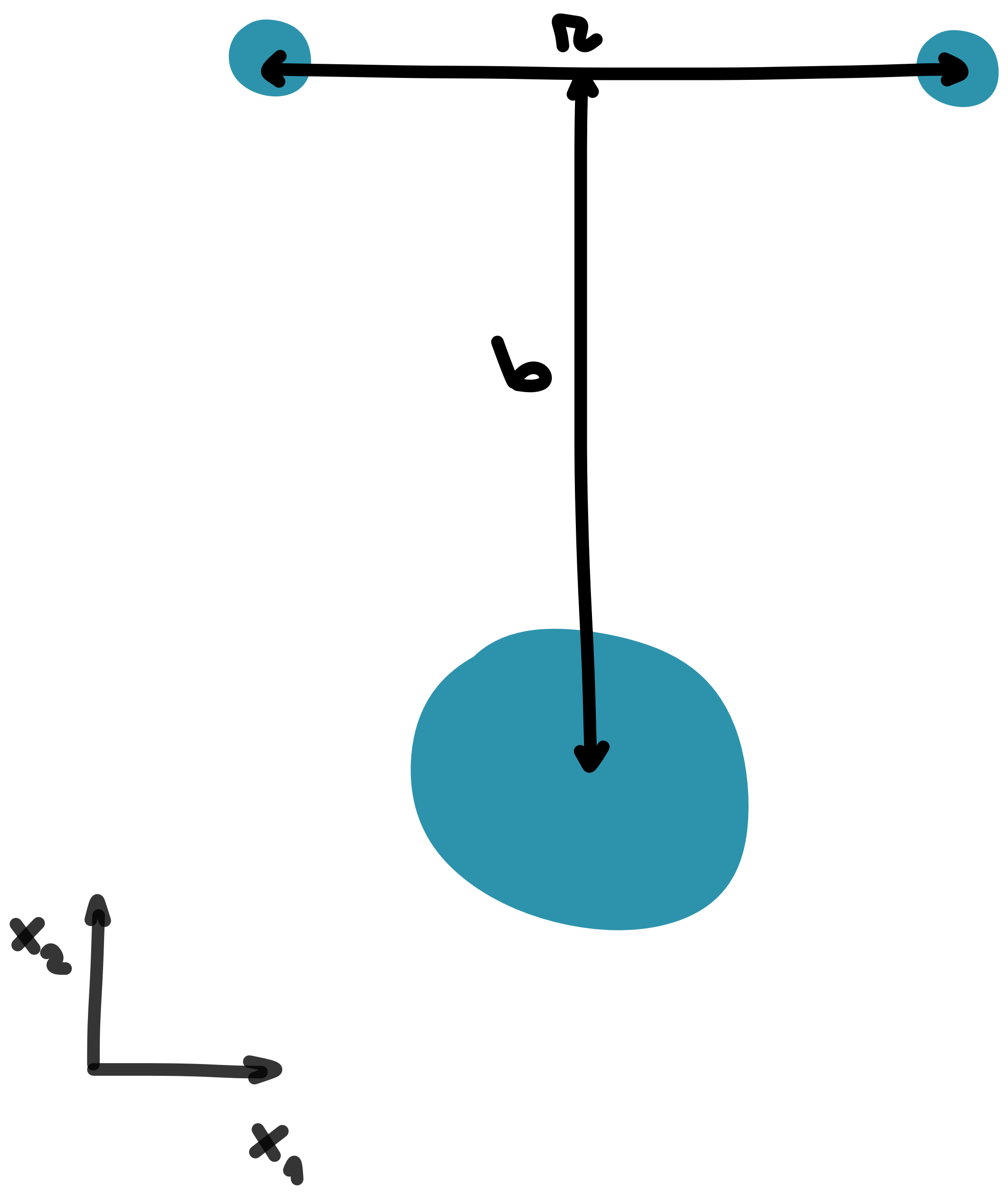

[Cepila, Contreras, Matas, 2018]

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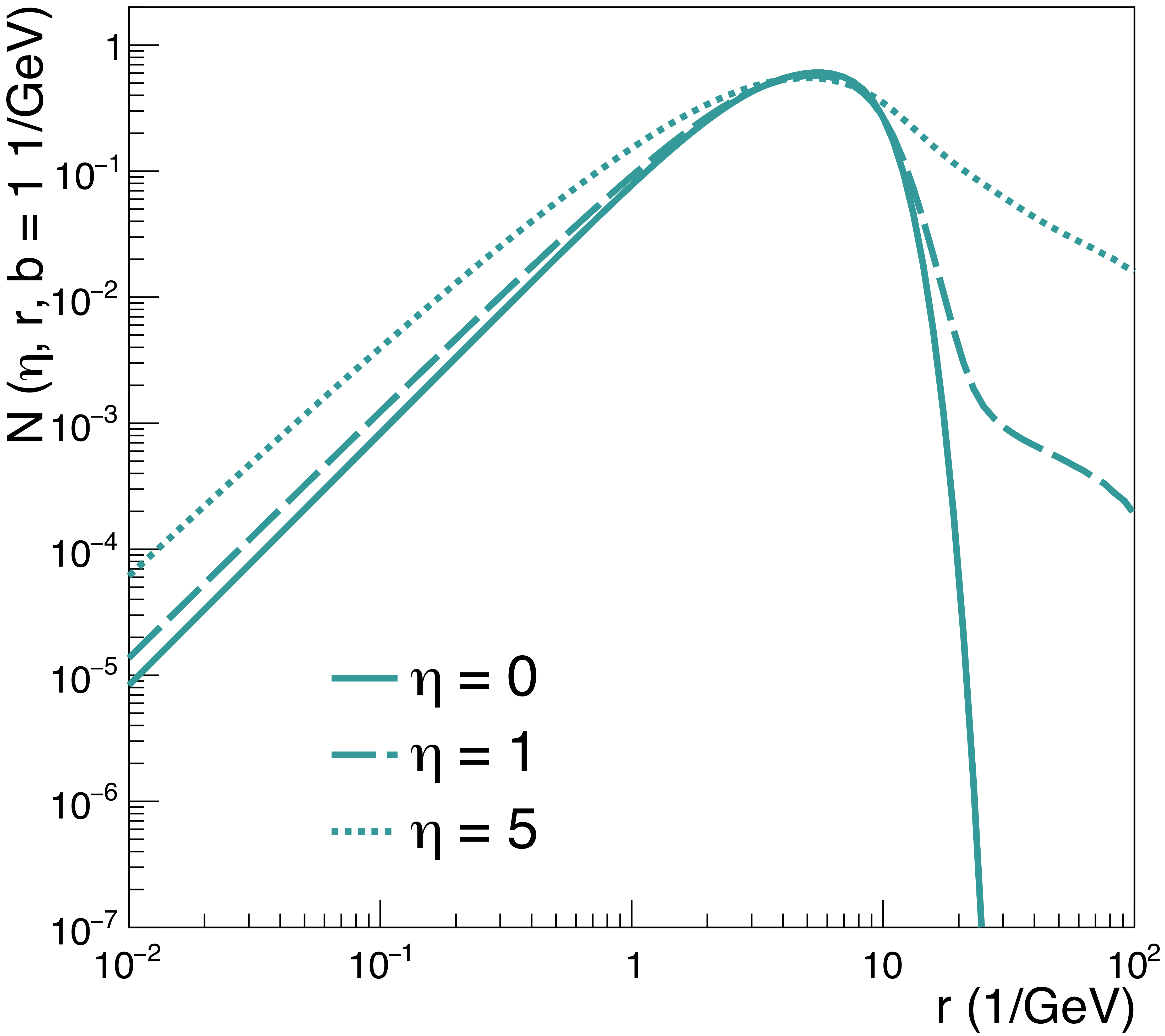

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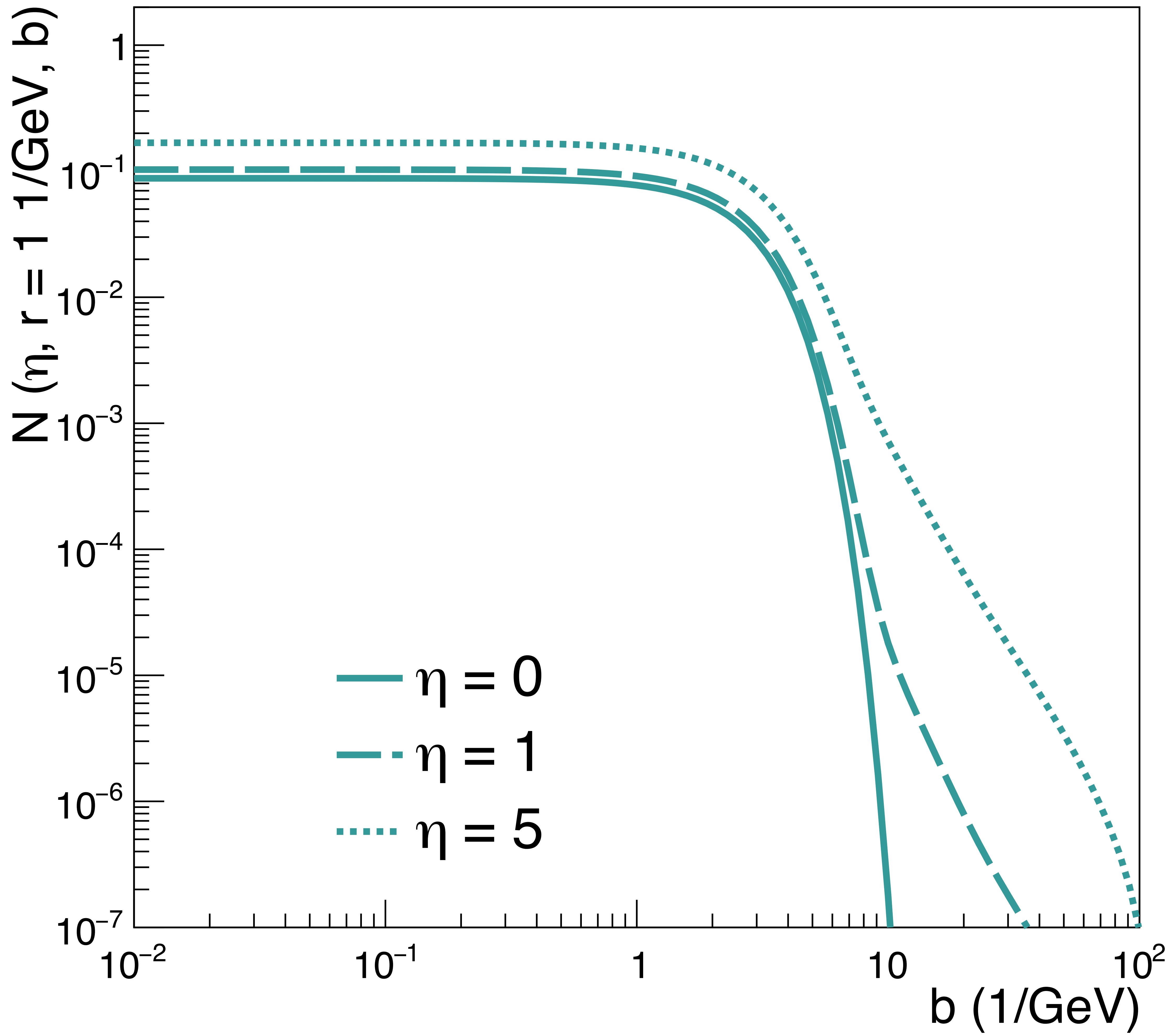

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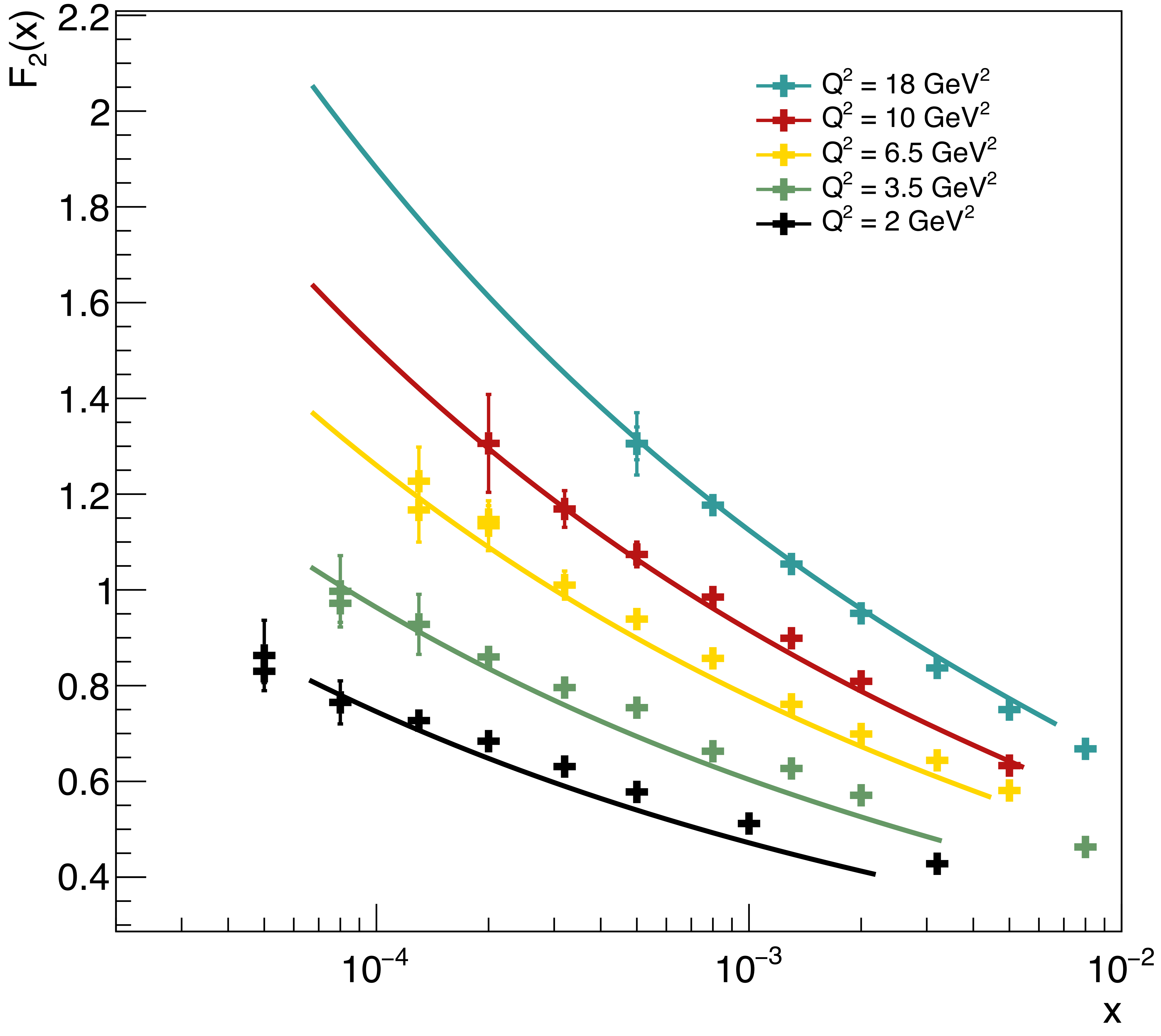

- HERA data still described

2D BK - data

- coherent vector meson production

$$\frac{\mathrm{d}\sigma_{\mathrm{T,L}}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\int \mathrm{d}\underline{r}\int_0^1 \frac{\mathrm{d}z}{4\pi}\int \mathrm{d}^2\underline{b}\left(\Psi_E^\dagger\Psi\right)_{T,L}(Q^2, z, r)e^{-i[\underline{b}-(\frac{1}{2}-z)\underline{r}]\underline{\Delta}}2N(\eta, r, \underline{b})\right|^2$$

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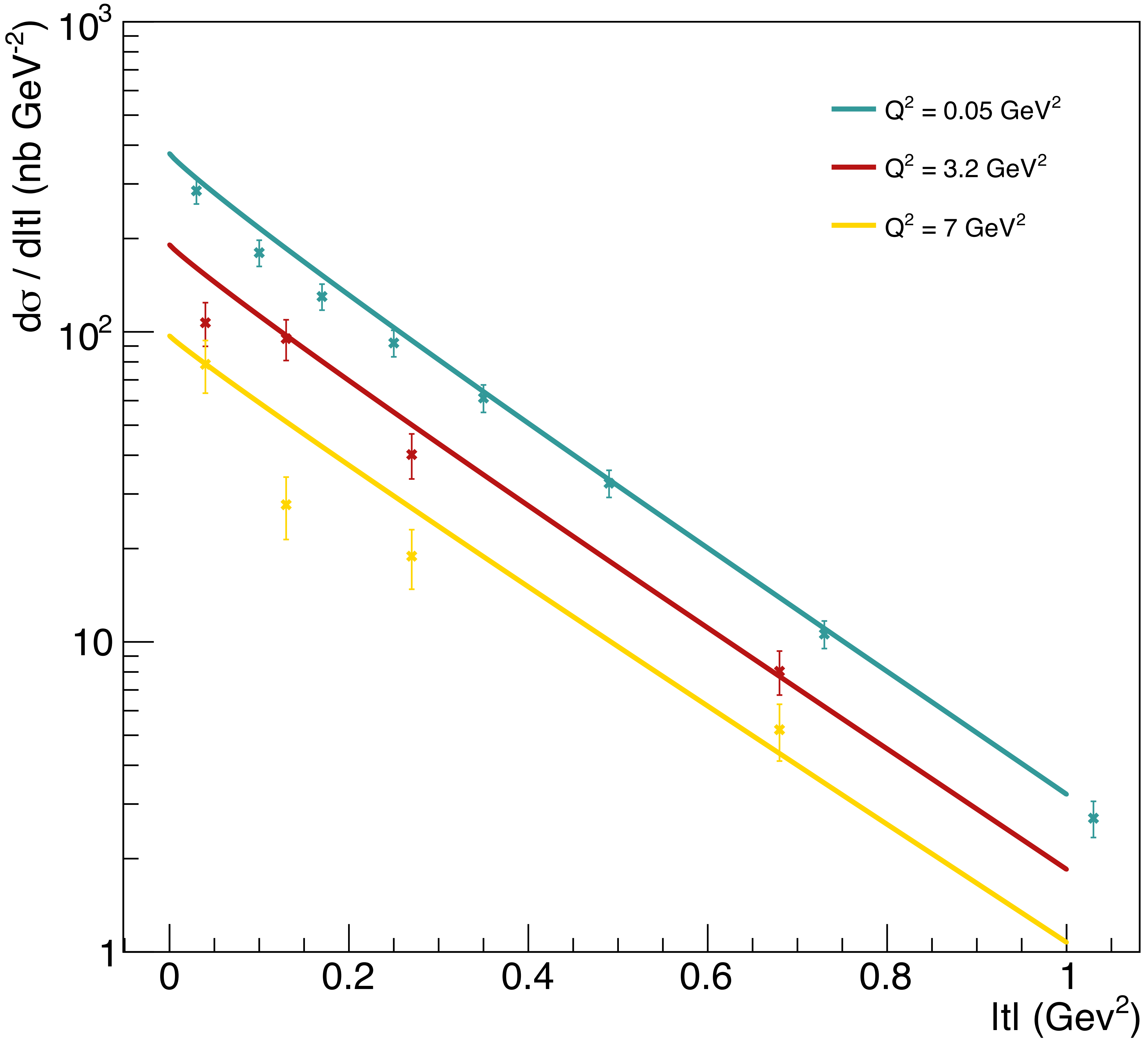

- HERA data described
- J/ψ, W=100 GeV

[HERA, 2006]

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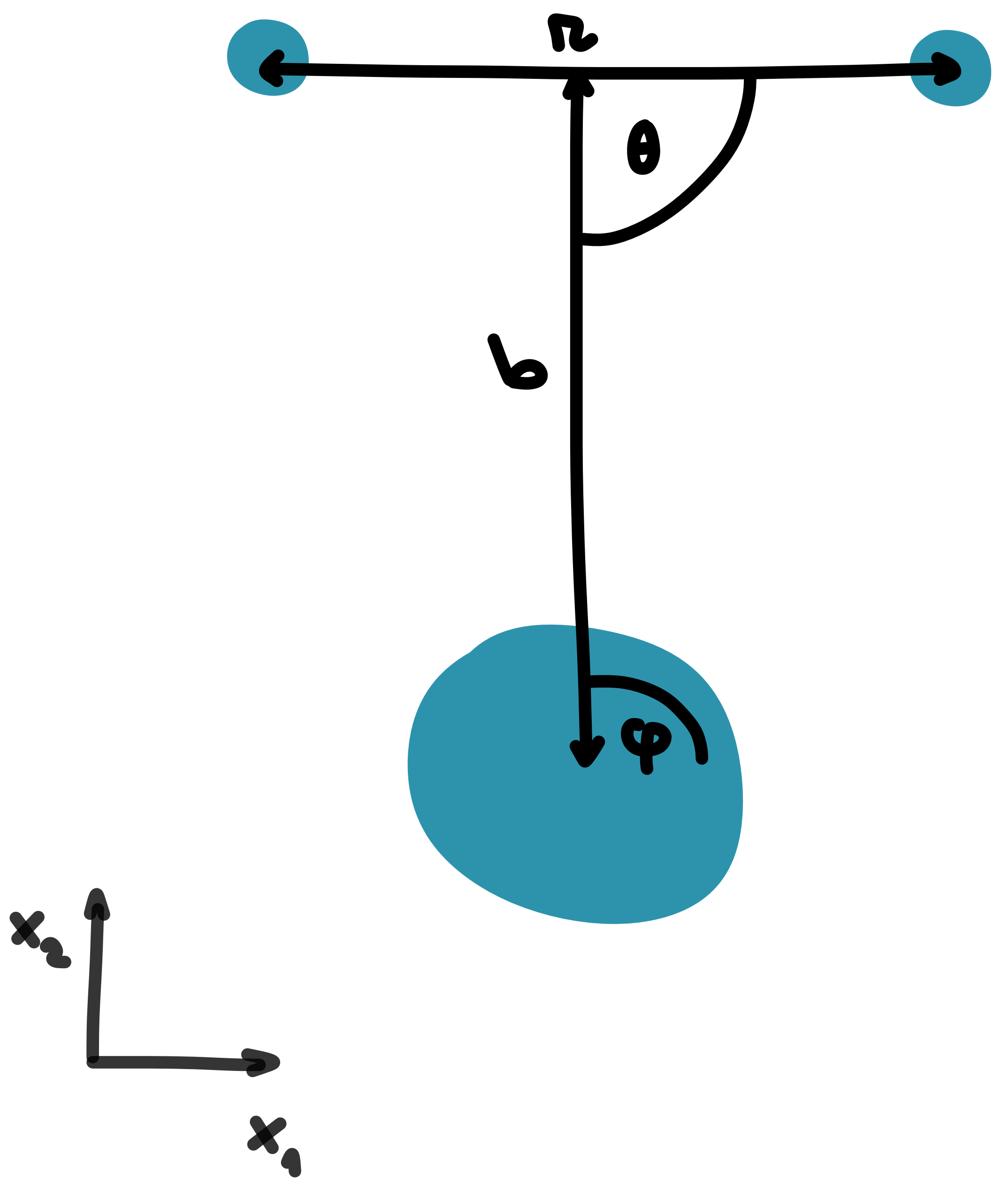

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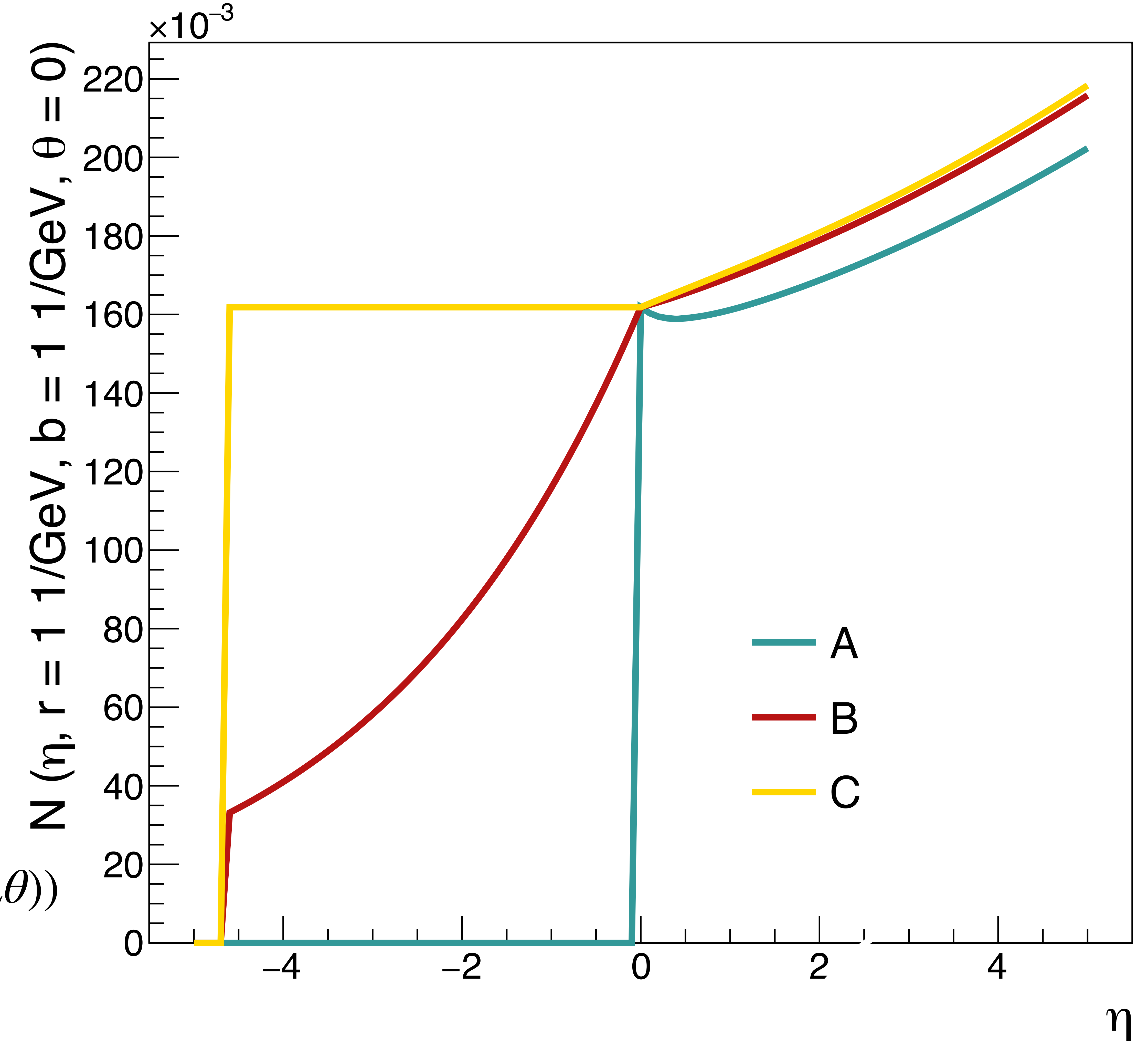

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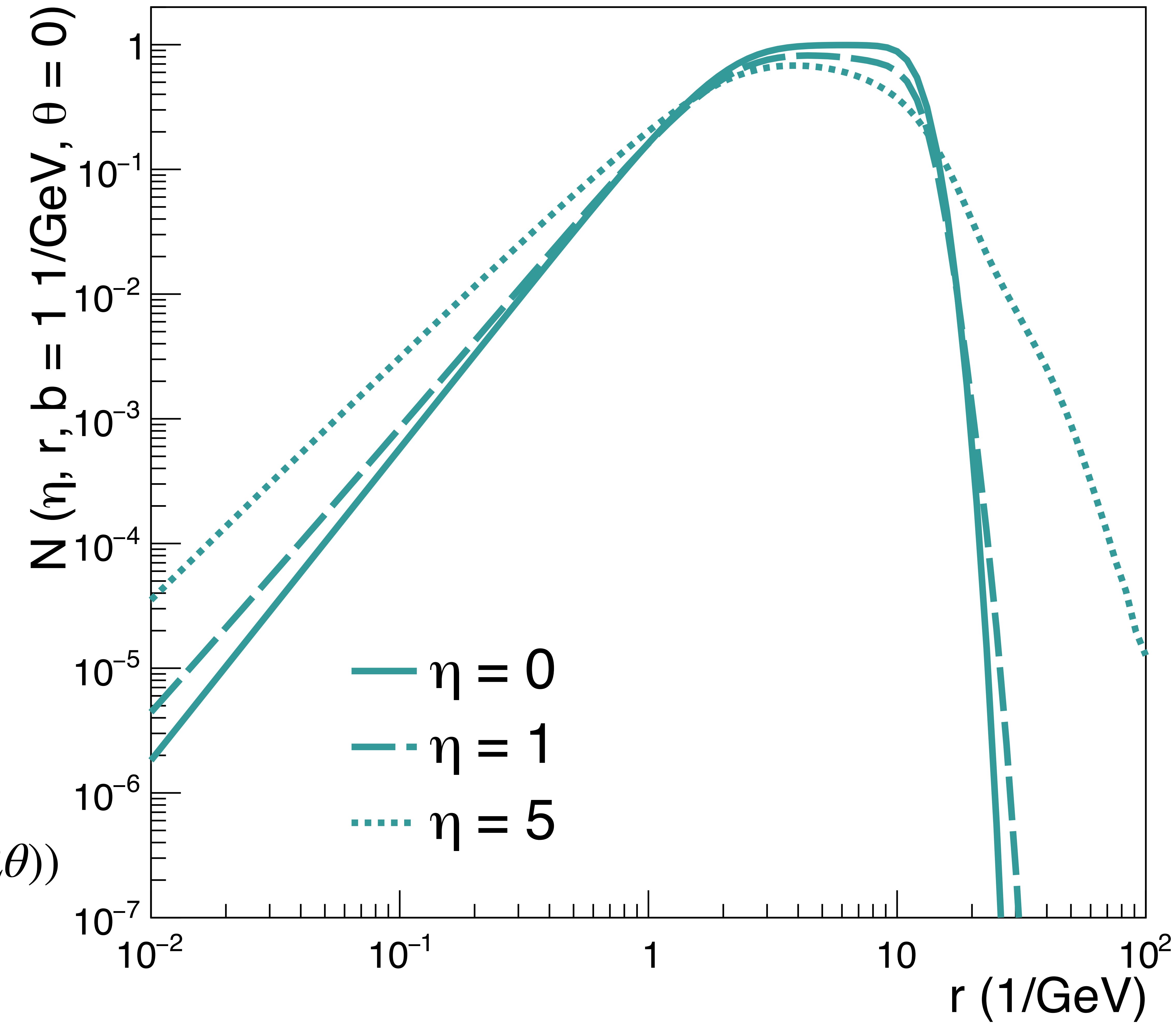

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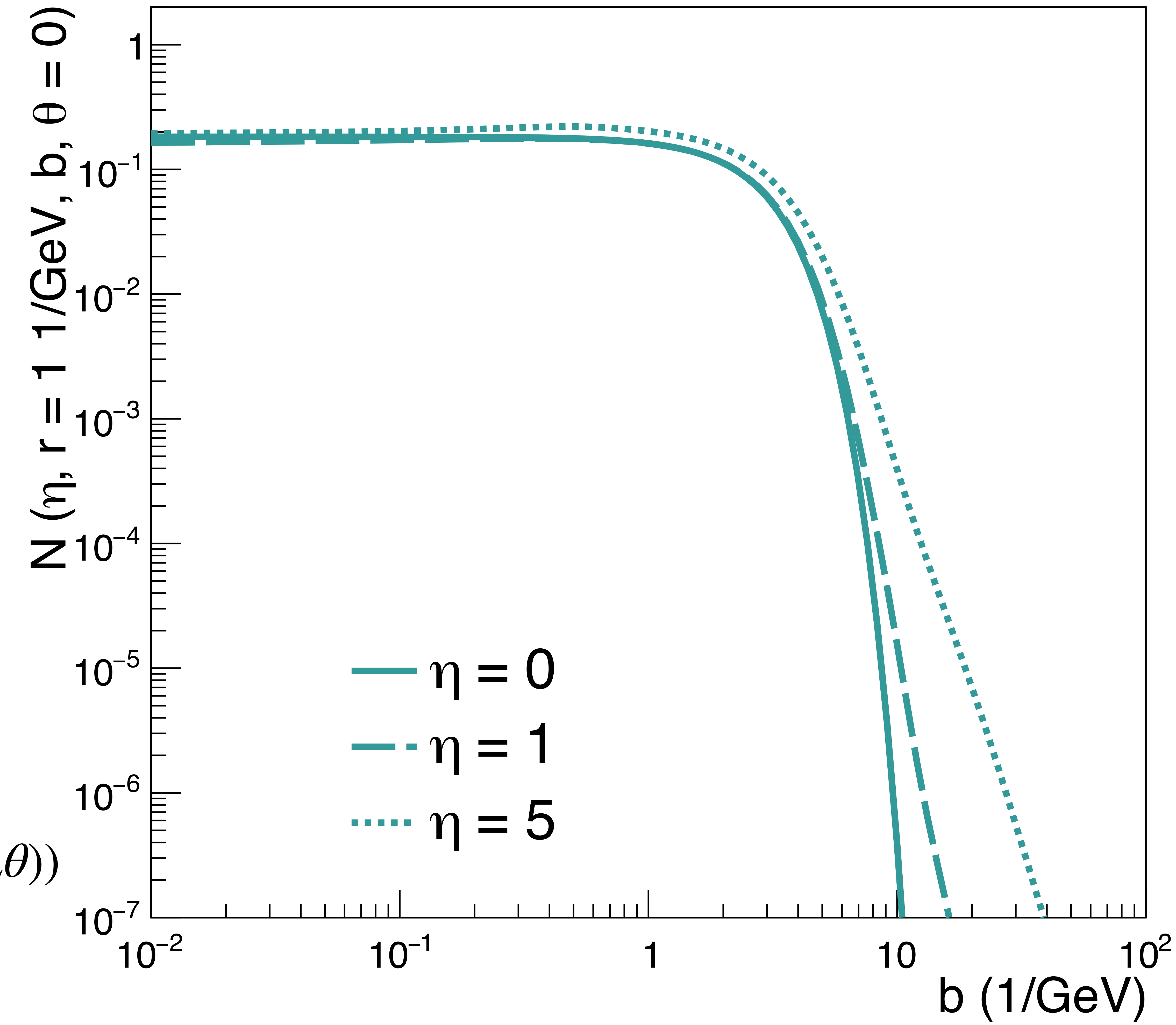

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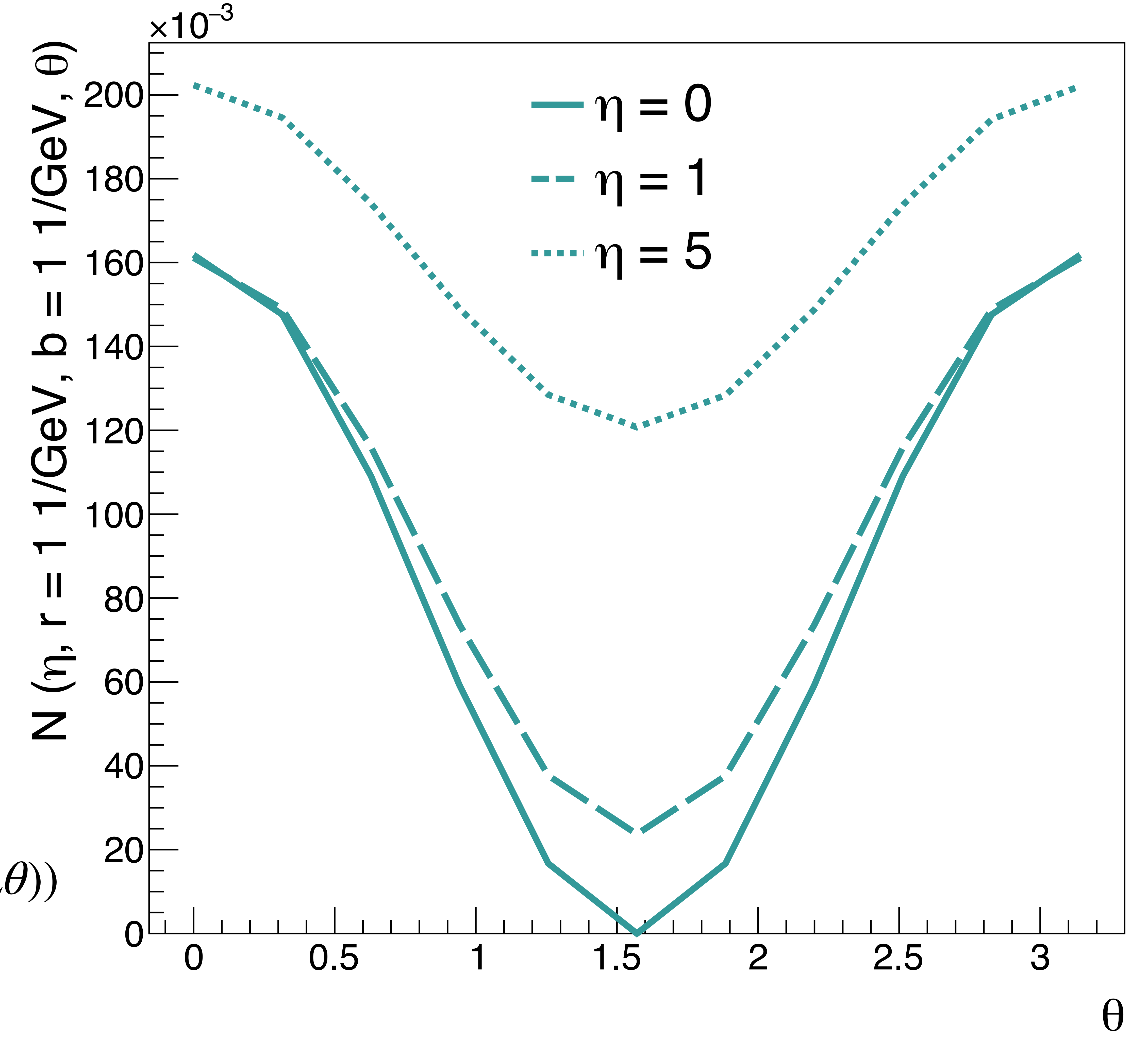

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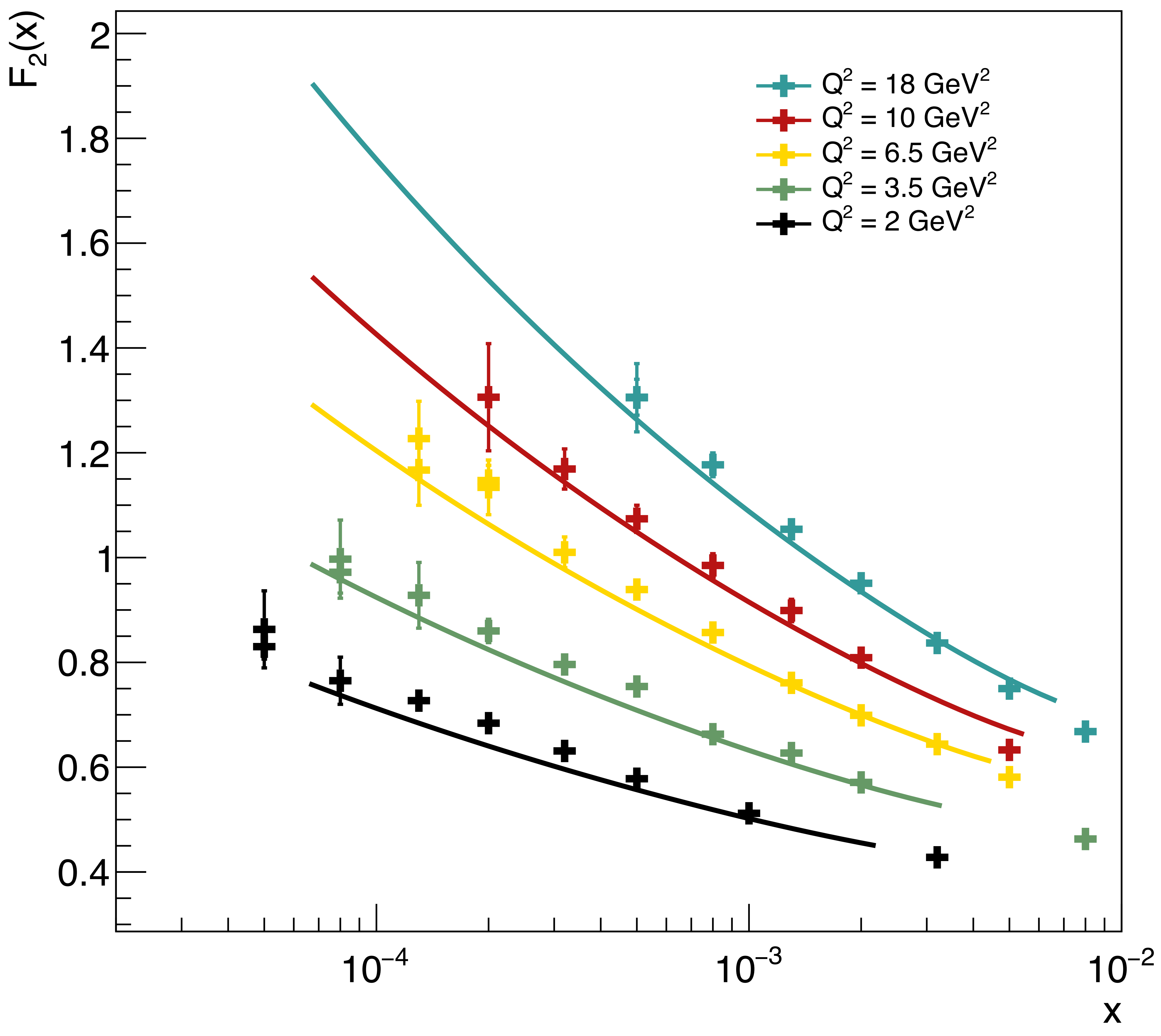
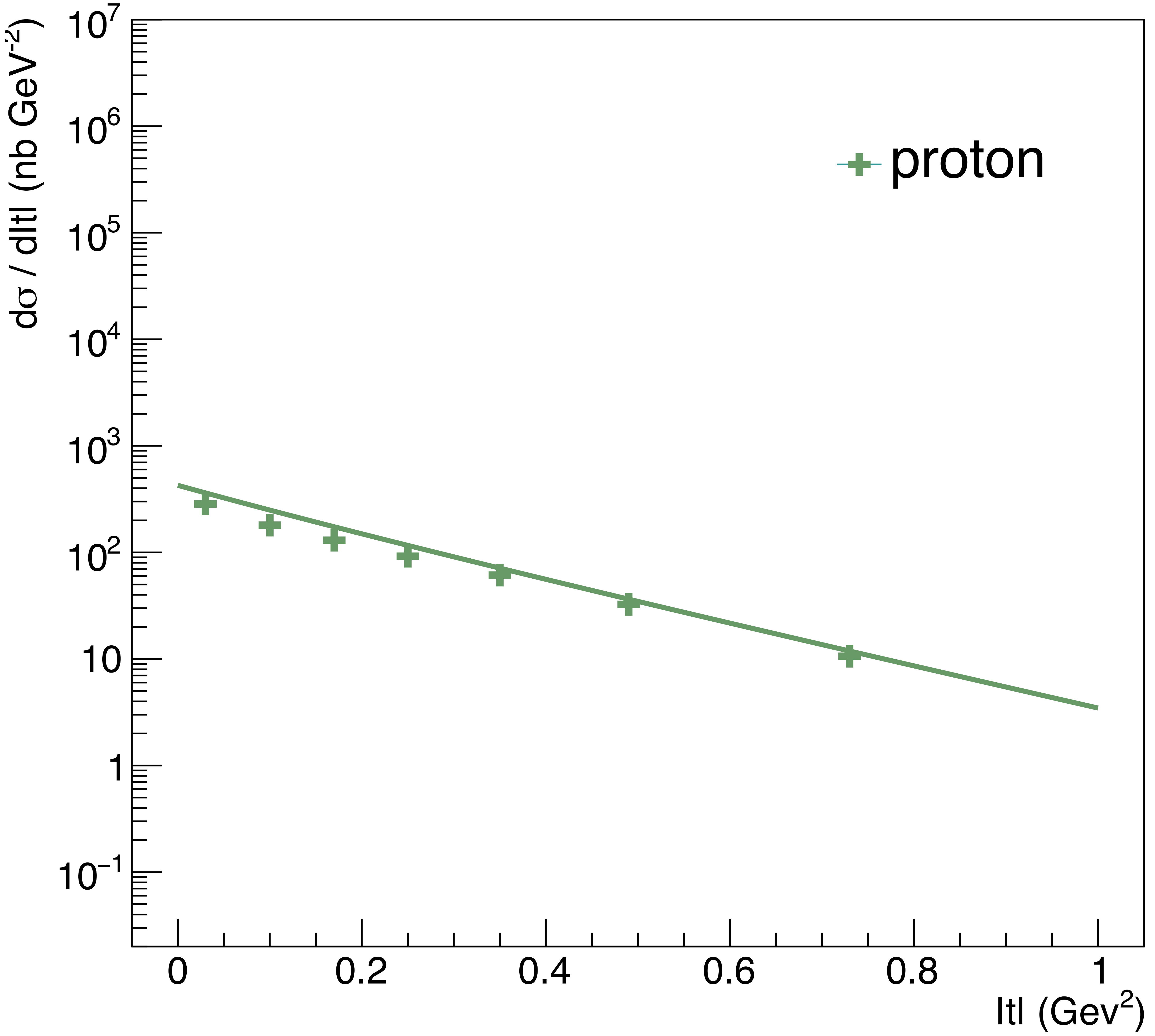


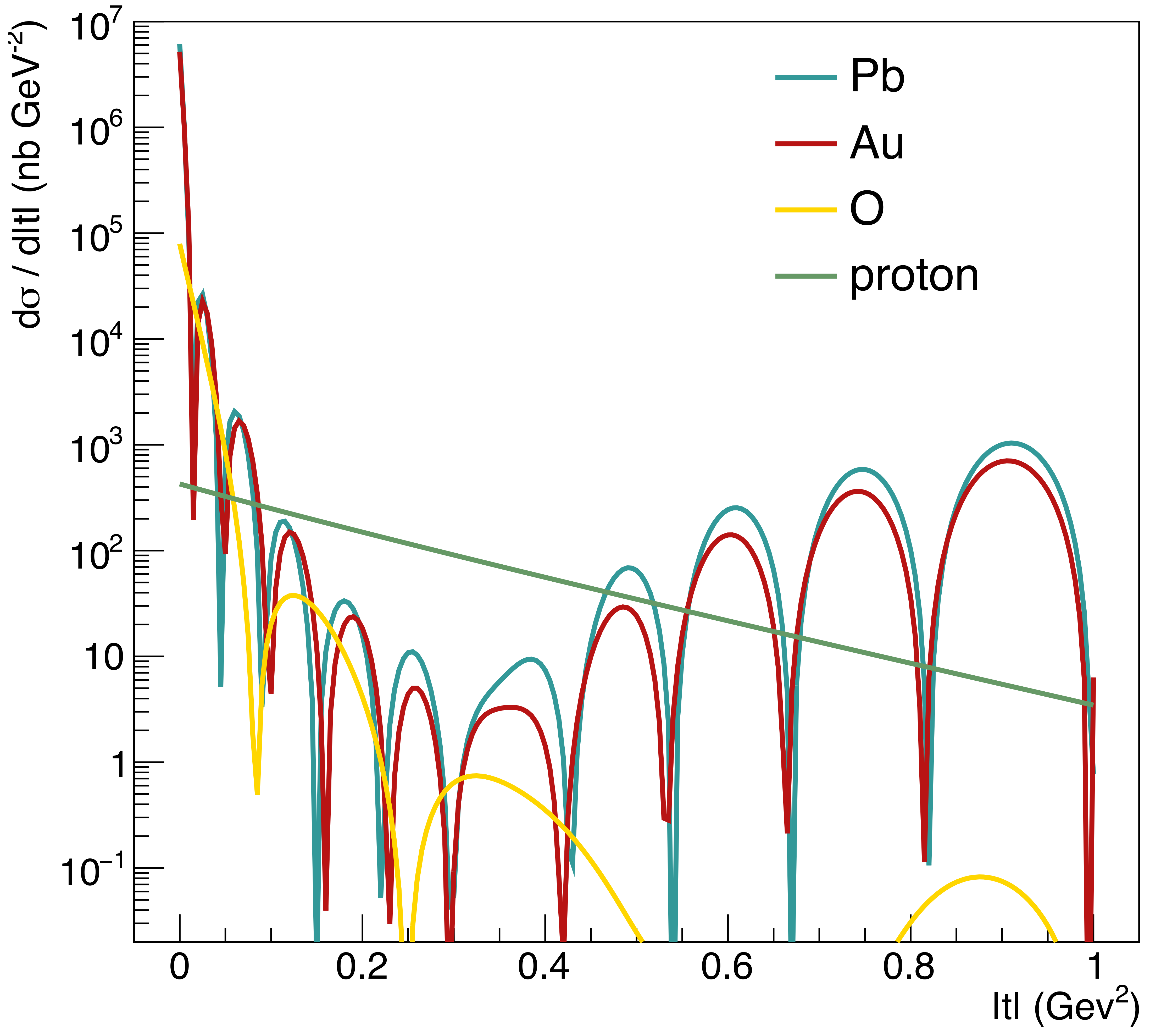
$F_2(x)$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
10^{-4}
10^{-3}
10^{-2}
x
Q^2 = 18 GeV^2
Q^2 = 10 GeV^2
Q^2 = 6.5 GeV^2
Q^2 = 3.5 GeV^2
Q^2 = 2 GeV^2



$d\sigma$ / d|t| (nb GeV^{-2})
10^7
10^6
10^5
10^4
10^3
10^2
10
1
10^{-1}
0
0.2
0.4
0.6
0.8
1
|t| (Gev^2)
proton

3D BK - data

- EIC predictions
- coherent nuclear J/ψ production
- nuclear initial condition
    - Gaussian to Woods-Saxon



3D BK - summary

- successfull reconstruction of former data description
- EIC predictions for vector meson production
- tool ready for potential
  - modeling TMDs, GTMDs, ...
  - calculating DVCS, dijets, ...

thank you