

Inferring the Initial Condition for the BK Equation

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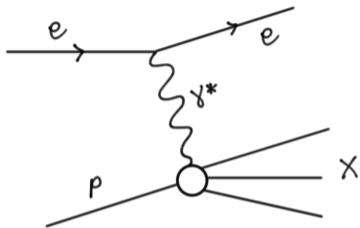
C. Casuga, M. Karhunen, H. Mäntysaari

Center of Excellence in Quark Matter, University of Jyväskylä

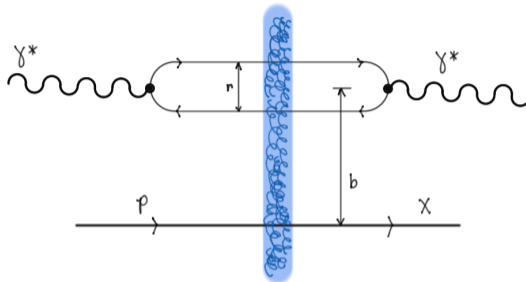
April 10, DIS 2024



Deep Inelastic Scattering in the Dipole Picture

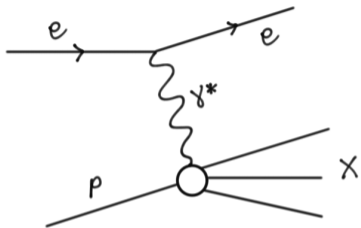


IMF picture

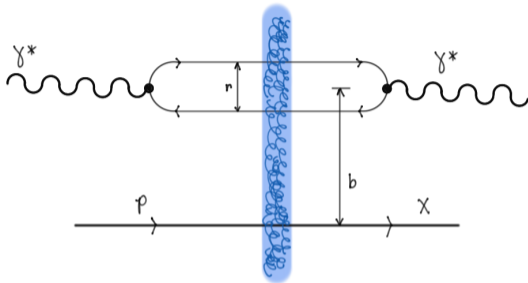


Dipole Picture

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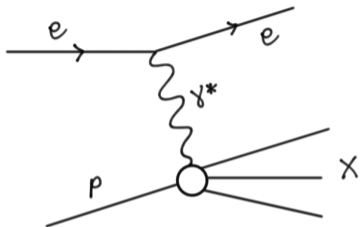
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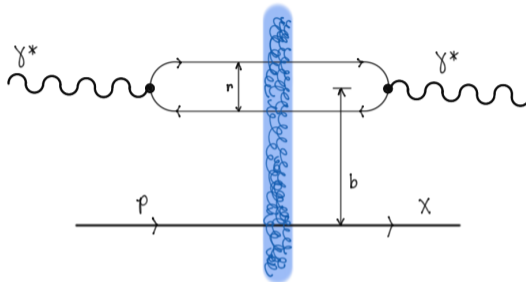
Dipole Picture

$$T_{T;L}^p(x; Q^2) \quad \frac{0}{2} \quad N(r; x) \quad \text{LCWF}$$

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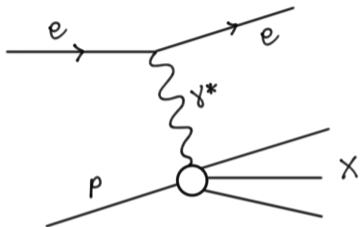


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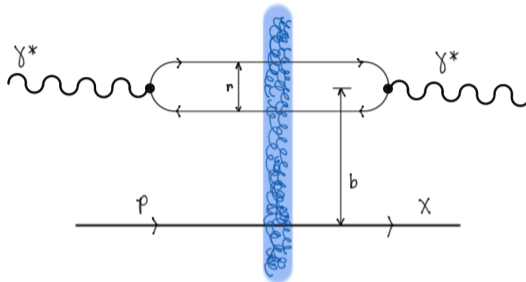
proton transverse area

$$T_{T;L}^p(x; Q^2) \stackrel{0}{=} \frac{2}{2} N(r; x) \quad \text{LCWF}$$

Deep Inelastic Scattering in the Dipole Picture



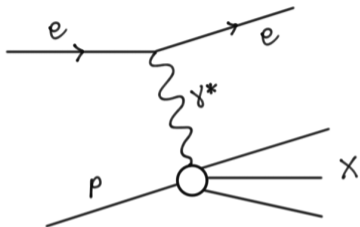
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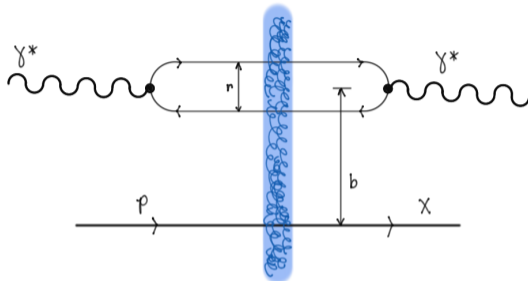
Dipole Picture

$$T_{T;L}^p(x; Q^2) = \frac{0}{2} \overset{\text{dipole-target scattering amplitude}}{N(r; x)} \quad \text{LCWF}$$

Deep Inelastic Scattering in the Dipole Picture



IMF picture



Dipole Picture

$$T_{T;L}^p(x; Q^2) \sim \frac{0}{2} N(r; x) \quad \text{LCWF}$$

$$\text{rcBK: } N(r; x = x_0; Q_{s0}^2; e_c) \sim \mathcal{F}^2 N(r; x)$$

Objectives

- Constrain model parameters, $[Q_{s0}^2; \dots; e_c; C^2; \dots]_{0=2}$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

- 3 H.Mantysaari, T. Lappi (2013): 1309.6963
- 3 AAMQS Collaboration (2010) arXiv:1012.4408
- 3 H.Hanninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.
- ! This setup: Leading order acc. + light quarks only

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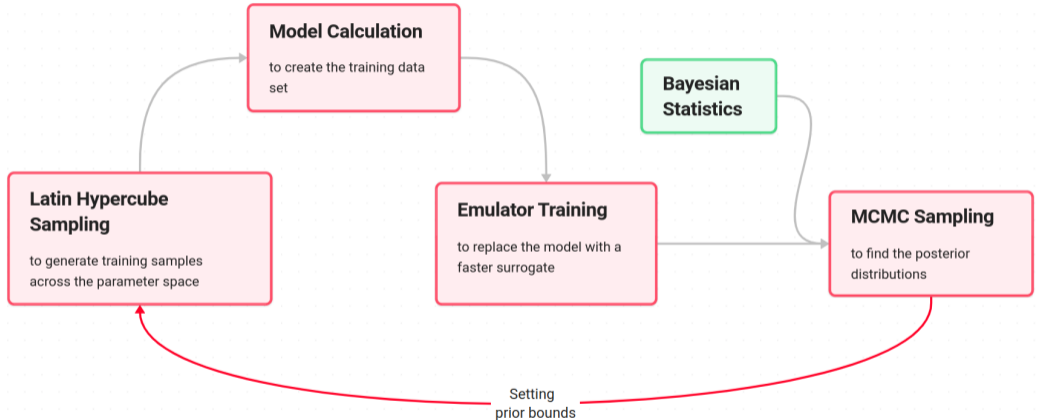
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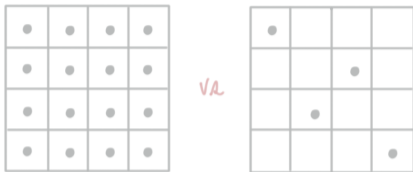
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Typical Bayesian Workflow



Latin Hypercube Sampling

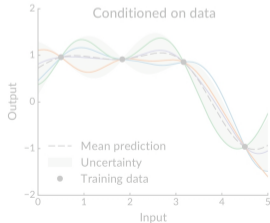


Principal Component Analysis

*Image: J. Bernhard PhD Thesis (2018)

Gaussian Process Emulator

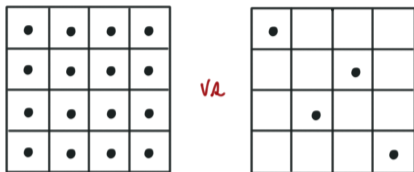
GPs learn the parameter dependence of the model!



*Image: J. Bernhard PhD Thesis (2018)

Validation

Latin Hypercube Sampling

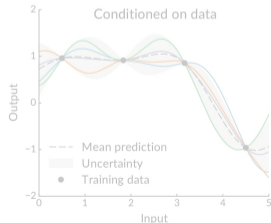


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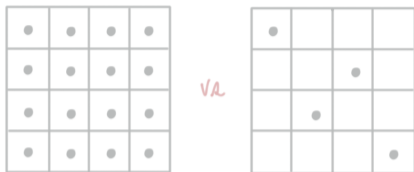
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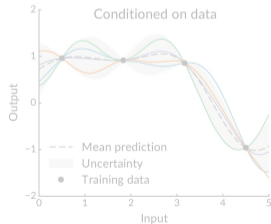


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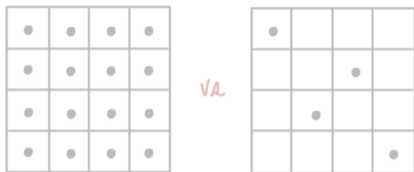
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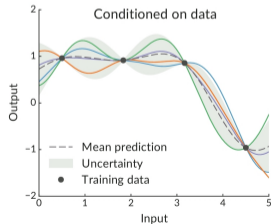


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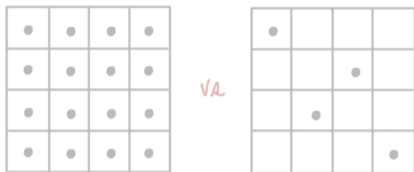
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Validation

Acceptance probability:

$$= \frac{P(x_{+1})}{P(x)}$$


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Bayesian Statistics

$$P(\theta) = \text{posterior} = \text{likelihood} \times \text{prior}$$

- Likelihood: how well data matches the model at
- Prior: bounds of the parameter space

no title

Results: Posterior Distribution

$$N(\mathbf{r}; x_0) = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} (\mathbf{r} - x_0)^T \Sigma^{-1} (\mathbf{r} - x_0) \right\}$$

$$s(\mathbf{r}) = \frac{12}{(33 - N_f) \log \frac{4C^2}{r^2 - r_{QCD}^2}}$$

- 1 HERA data prefers a α_s ≈ 0.12
- 2 With α_s as a free parameter, we obtain a wider posterior distribution
- 3 O -diagonal plots = correlations

| $[Q_{s0}^2; e_c; C^2; \alpha_s=2]$ | $\alpha_s = 1$ | MV^e [H.M. & T.L. (2013)]

no title

Results: Posterior Distribution

$$N(\mathbf{r}; x_0) = \frac{1}{\sqrt{|j\mathbf{r}j|}} \exp \left[-\frac{(\mathbf{r}^2 Q_{s;0}^2)}{4} \ln \frac{1}{Q_{CD}} + e_c \right]$$

$$s(\mathbf{r}) = \frac{12}{(33 - N_f) \log \frac{4C^2}{r^2 Q_{CD}^2}}$$

- 1 HERA data prefers a $\mu = 1$
- 2 With μ as a free parameter, we obtain a wider posterior distribution
- 3 O -diagonal plots = correlations

| $[Q_{s0}^2; e_c; C^2; \mu=2]$ | $\mu = 1$ | MV^e [H.M. & T.L. (2013)]

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Results: Posterior Distribution

$$N(\mathbf{r}; x_0) = \frac{1}{(2\pi)^{12}} \exp \left\{ -\frac{(\mathbf{r}^2 Q_{s;0}^2)}{4} \ln \frac{1}{|\mathbf{r}|} + e_c \right\}$$

$$s(\mathbf{r}) = \frac{12}{(33 - N_f) \log \frac{4C^2}{\mathbf{r}^2 Q_{OCD}^2}}$$

- 1 HERA data prefers a $\mathbf{r}^2 = 1$
- 2 With \mathbf{r}^2 as a free parameter, we obtain a wider posterior distribution
- 3 O -diagonal plots = correlations

| $[Q_{s0}^2; e_c; C^2; \mathbf{r}^2 = 1]$ | MV^e [H.M. & T.L. (2013)]

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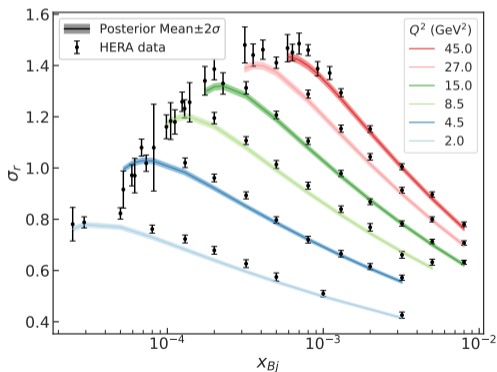
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Posterior Samples, Median and MAP

5 - parameter	$Q_{s0}^2 [GeV^2]$		e_c	C^2	$\sigma_0=2[mb]$	χ^2_{dof}	Q_s^2
median	0.067	1.01	27.5	4.72	14.0	1.016	0.288
MAP	0.077	1.01	15.6	4.47	13.9	1.012	0.289



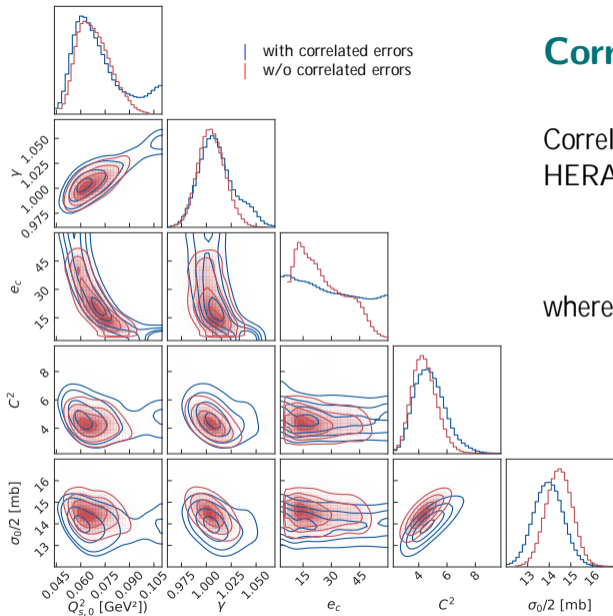
! Good agreement with HERA data

Correlated systematic errors

Correlated systematic uncertainties from the HERA data are input to:

$$P(\mathbf{y}) = \mathbf{y}(\mathbf{\theta})^T \mathbf{C}^{-1}(\mathbf{\theta}) \mathbf{y}(\mathbf{\theta})$$

where $\mathbf{C} = \mathbf{C}_{\text{emu}} + \mathbf{C}_{\text{exp}}$.



- Wider posterior distributions
- Moderate change to parameter correlations

Inclusive quark production

$$d \sigma_{q+A \rightarrow q+X} = xq(x; \mathbf{k}^2) \mathcal{S}_p(\mathbf{k})$$

$\mathcal{S}_p(\mathbf{k})$! 2DFT of Dipole amplitude

$$= \int d^2\mathbf{r} e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$[1 - N(\mathbf{r}; x = x_0)]$$

- > 1 result to negative 2DFT values [B. Giraud et. al. (2016)]

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Nuclear Modification factor at the EIC and beyond

$$R_{eA} = \frac{F_{2;A}}{AF_{2;p}}$$

where $Q_{s;A}^2 = Q_{s;p}^2 \quad 0=2 \quad AT_A(b)$.

- Evolution speed uncertainty (C^2) become more significant towards smaller x_{Bj}

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Summary & Outlook

- Posterior distribution for $[Q_{s0}^2; e_c; C^2; \alpha_s=2]$ BK IC
- Method for propagating the uncertainties of non-perturbative BK IC (first time!)
- Accounted for correlated errors in HERA data (first time!)

Further work

- Fit of the next-to-leading accuracy + heavy quarks

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Back up Slides

Parameters

- $\alpha=2$, half of normalization to the cross section, proton transverse area, $2 \int d^2b | \dots |^2$
- $Q_{s,0}^2$, **related** to the saturation scale at initial x . Previous fits used GBW parametrization:

$$N(\mathbf{r}; x_0) = 1 - \exp \left(- \frac{(\mathbf{r}^2 Q_{s,0}^2)^\alpha}{4} \right)$$

- C^2 , connects the running coupling in \mathbf{r} to its Fourier transform
- β , anomalous dimension, controlling the steepness of the cross section related to its fall-off for small dipoles
- e_c , infrared cut-off in the MV model

F_2 Structure Function for Nucleus

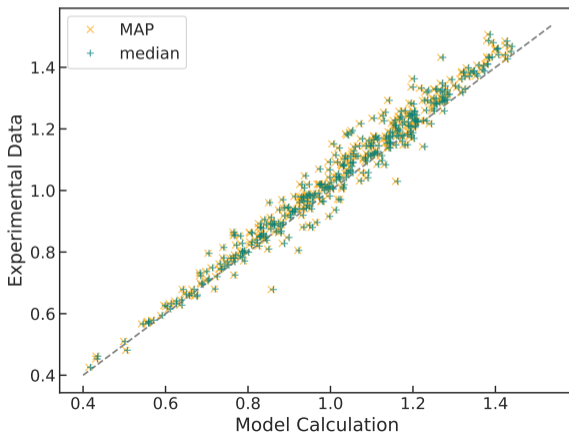
$$N_A(\mathbf{r}; \mathbf{b}; x = x_0) = 1 - \exp\left[-\frac{2}{A} T_A(\mathbf{b}) \frac{r^2 Q_{s,0}^2}{4} \ln \frac{1}{|j\mathbf{r}|_{\text{QCD}}} + e_c\right] e^{\dots}$$

where $T_A(\mathbf{b})$ is the transverse thickness function of the nucleus of mass number A , obtained through integrating the Woods-Saxon distribution

$$T_A(\mathbf{b}; Z) = \frac{h \rho \frac{n}{d}}{1 + \exp\left[\frac{\mathbf{b}^2 + Z^2}{R_A}\right]}$$

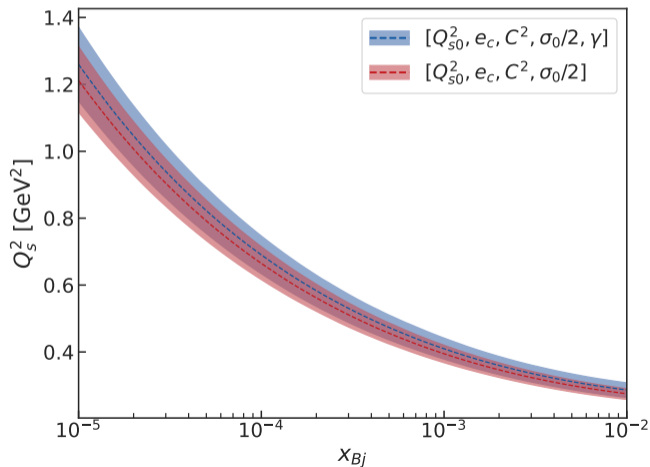
Emulator vs Model

$$\frac{2}{\text{d.o.f}} = \frac{1}{N-p} \mathbf{y}(\cdot)^T \text{exp}^{-1} \mathbf{y}(\cdot);$$

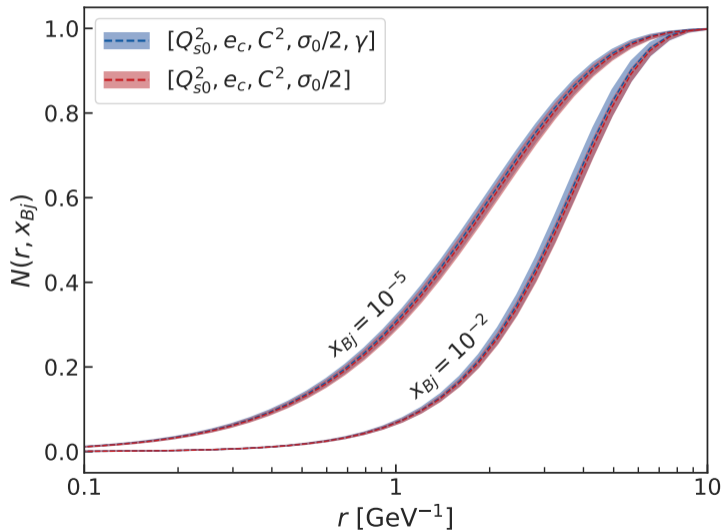


Initial Saturation Scale

$$N(r^2 = 2=Q_s^2) = 1 \quad e^{-1=2}$$



Initial and Evolved $N(r; y)$



Nuclear Modification Ratio

