Inferring the Initial Condition for the BK Equation

Phys.Rev.D 109 (2024) 054018 arXiv:2311.10491

C. Casuga, M. Karhunen, H. Mäntysaari

Center of Excellence in Quark Matter, University of Jyväskylä

April 10, DIS 2024

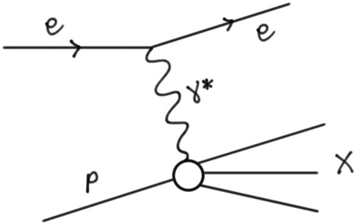
Centre of Excellence in Quark Matter

Research Council of Finland

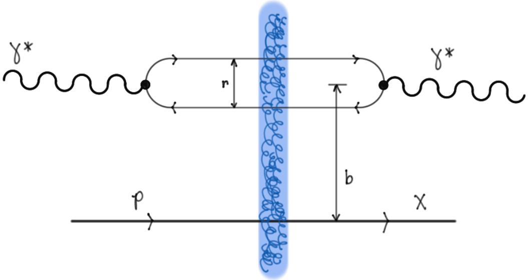
JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

HELSINKI INSTITUTE OF PHYSICS

Deep Inelastic Scattering in the Dipole Picture

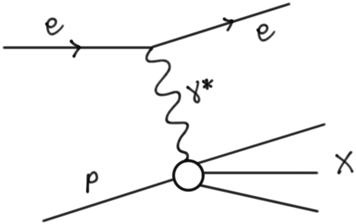


IMF picture

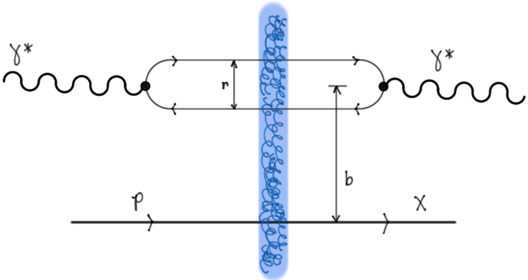


Dipole Picture

Deep Inelastic Scattering in the Dipole Picture



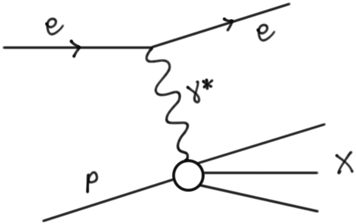
IMF picture



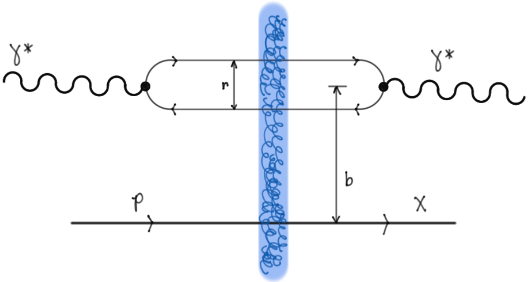
Dipole Picture

$$\sigma_{T,L}^{\gamma^* p}(x, Q^2) \sim \frac{\sigma_0}{2} \otimes \mathcal{N}(r, x) \otimes \{\mathrm{LCWF}\}$$

Deep Inelastic Scattering in the Dipole Picture



IMF picture

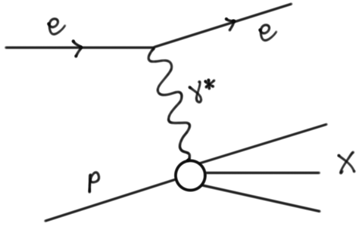


Dipole Picture

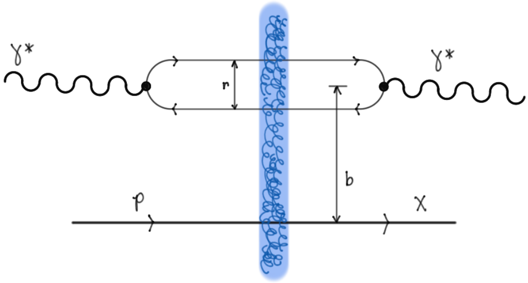
proton transverse area

$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \otimes \mathcal{N}(r, x) \otimes \{\mathrm{LCWF}\}$$

Deep Inelastic Scattering in the Dipole Picture



IMF picture

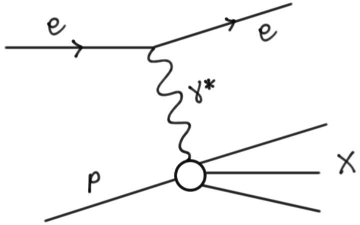


Dipole Picture

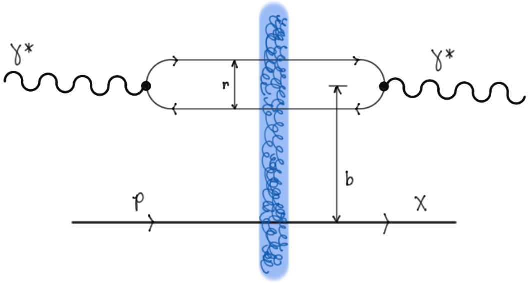
dipole-target
scattering amplitude

$$\sigma^{\gamma^* p}_{T,L}(x, Q^2) \sim \frac{\sigma_0}{2} \otimes \mathcal{N}(r, x) \otimes \{\mathrm{LCWF}\}$$

Deep Inelastic Scattering in the Dipole Picture



IMF picture



Dipole Picture

$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \otimes \mathcal{N}(r, x) \otimes \{\mathrm{LCWF}\}$$

$$\textbf{rcBK:}\, \mathcal{N}(r, x = x_0; Q_{s0}^2, \gamma, e_c) \xrightarrow{C^2} \mathcal{N}(r, x)$$

Objectives

- Constrain model parameters, $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

✓ H.Mäntysaari, T. Lappi (2013): 1309.6963

✓ AAMQS Collaboration (2010) arXiv:1012.4408

✓ H.Hänninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Objectives

- Constrain model parameters, $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

✓ H.Mäntysaari, T. Lappi (2013): 1309.6963

✓ AAMQS Collaboration (2010) arXiv:1012.4408

✓ H.Hänninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Objectives

- Constrain model parameters, $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

- ✓ H.Mäntysaari, T. Lappi (2013): 1309.6963
- ✓ AAMQS Collaboration (2010) arXiv:1012.4408
- ✓ H.Hänninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Objectives

- Constrain model parameters, $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

  Some previous fits to HERA data:
  - ✓ H.Mäntysaari, T. Lappi (2013): 1309.6963
  - ✓ AAMQS Collaboration (2010) arXiv:1012.4408
  - ✓ H.Hänninen et al. (2020) arXiv:2007.01645

  ... this work: provides uncertainty for the BK initial condition!
- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Objectives

- Constrain model parameters, $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

✓ H.Mäntysaari, T. Lappi (2013): 1309.6963

✓ AAMQS Collaboration (2010) arXiv:1012.4408

✓ H.Hänninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Objectives

- Constrain model parameters, $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2]$ against combined HERA reduced cross section data ...

Some previous fits to HERA data:

✓ H.Mäntysaari, T. Lappi (2013): 1309.6963

✓ AAMQS Collaboration (2010) arXiv:1012.4408

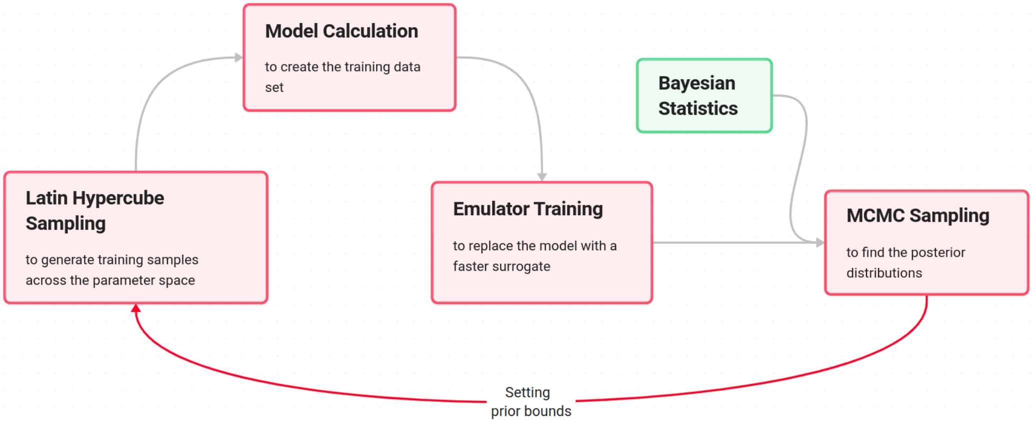
✓ H.Hänninen et al. (2020) arXiv:2007.01645

... this work: provides uncertainty for the BK initial condition!

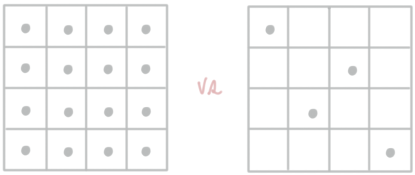
- Tool: Bayesian inference to extract posterior distribution.
- Account for correlated experimental uncertainties in HERA data.

! This setup: Leading order acc. + light quarks only

Typical Bayesian Workflow



Latin Hypercube Sampling



Principal Component Analysis

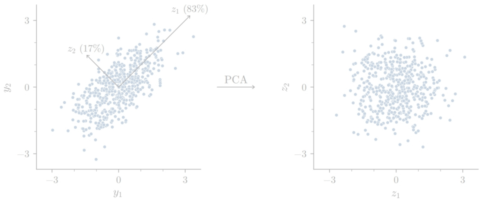


*Image: J.Bernhard PhD Thesis (2018)

Gaussian Process Emulator

GPs learn the parameter dependence of the model!

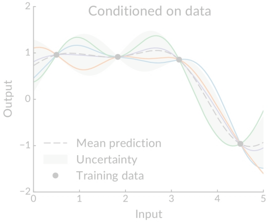
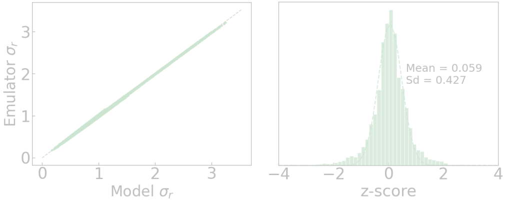
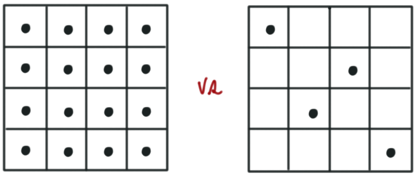


*Image: J.Bernhard PhD Thesis (2018)

Validation



Latin Hypercube Sampling



Principal Component Analysis

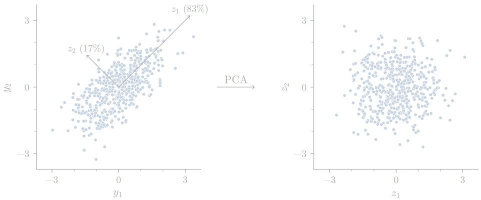


*Image: J.Bernhard PhD Thesis (2018)

Gaussian Process Emulator

GPs learn the parameter dependence of the model!

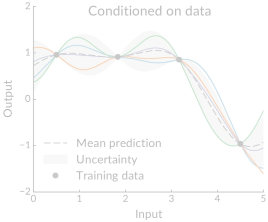
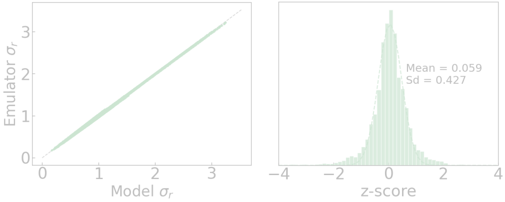
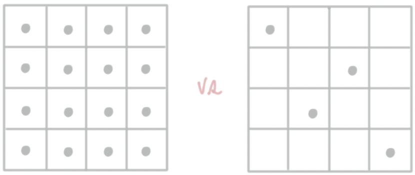


*Image: J.Bernhard PhD Thesis (2018)

Validation



Latin Hypercube Sampling



Principal Component Analysis

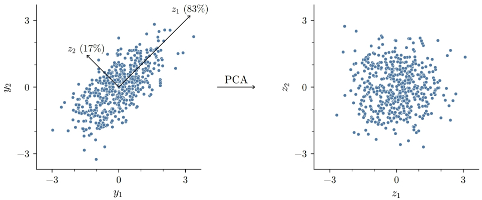


*Image: J.Bernhard PhD Thesis (2018)

Gaussian Process Emulator

GPs learn the parameter dependence of the model!

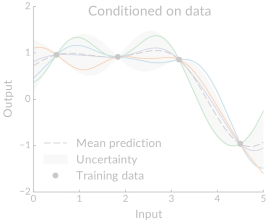
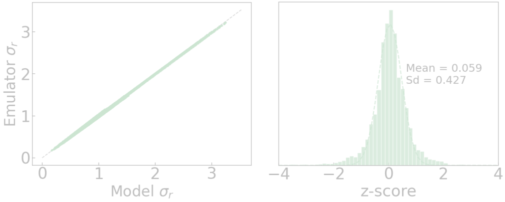
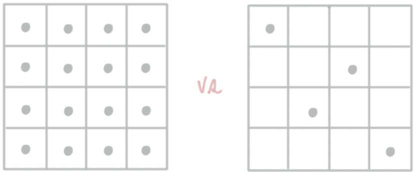


*Image: J.Bernhard PhD Thesis (2018)

Validation



Latin Hypercube Sampling



Principal Component Analysis

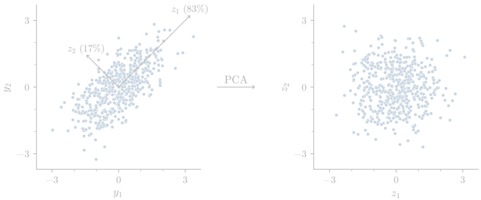


*Image: J.Bernhard PhD Thesis (2018)

Gaussian Process Emulator

GPs learn the parameter dependence of the model!

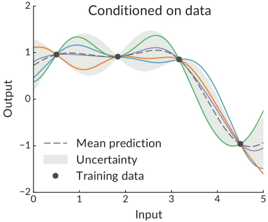
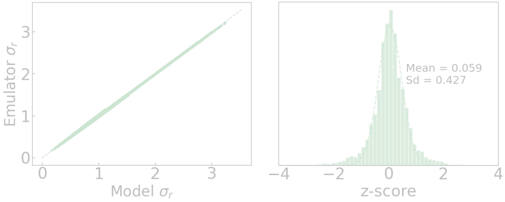
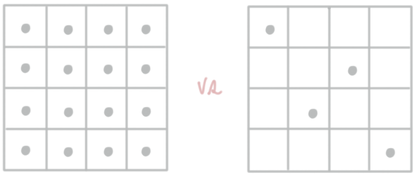


*Image: J.Bernhard PhD Thesis (2018)

Validation



Latin Hypercube Sampling



Principal Component Analysis

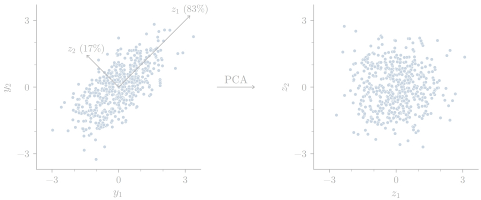


*Image: J.Bernhard PhD Thesis (2018)

Gaussian Process Emulator

GPs learn the parameter dependence of the model!

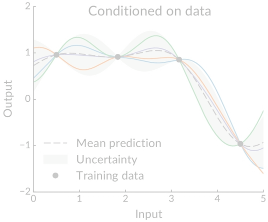
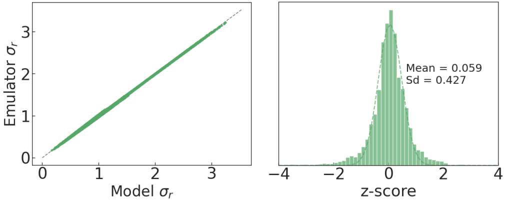


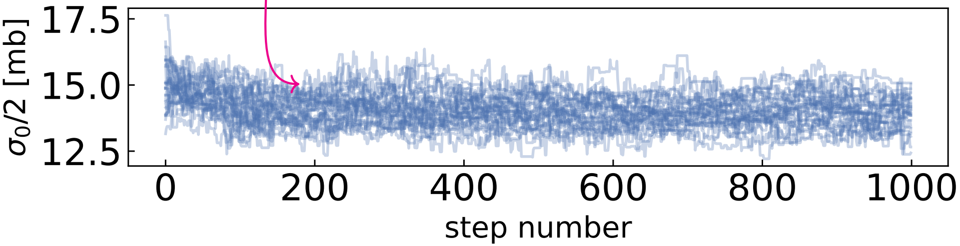
*Image: J.Bernhard PhD Thesis (2018)

Validation



Acceptance probability:

$$\alpha = \frac{P(\theta_{X+1})}{P(\theta_X)}$$



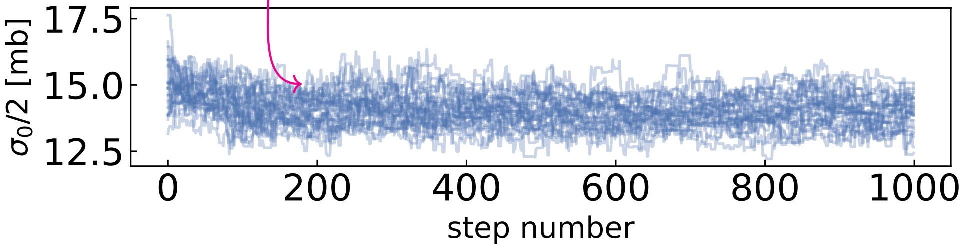
Acceptance probability:

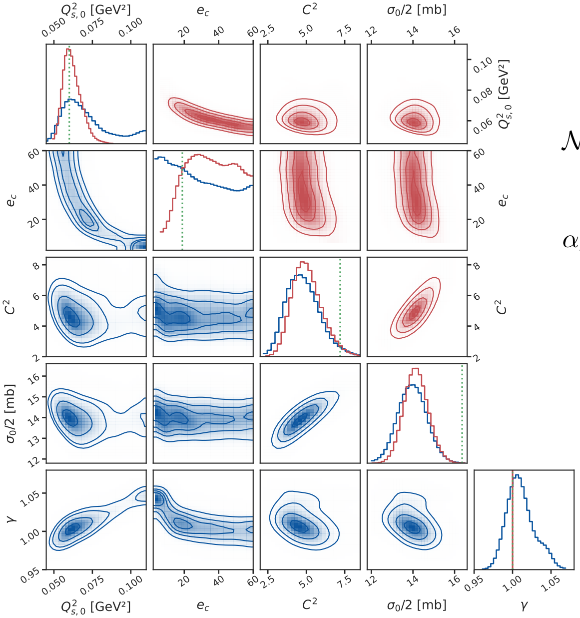
$$\alpha = \frac{P(\theta_{X+1})}{P(\theta_X)}$$

Bayesian Statistics

$$P(\theta) = \text{posterior} = \text{likelihood} \times \text{prior}$$

- Likelihood: how well data matches the model at θ
- Prior: bounds of the parameter space





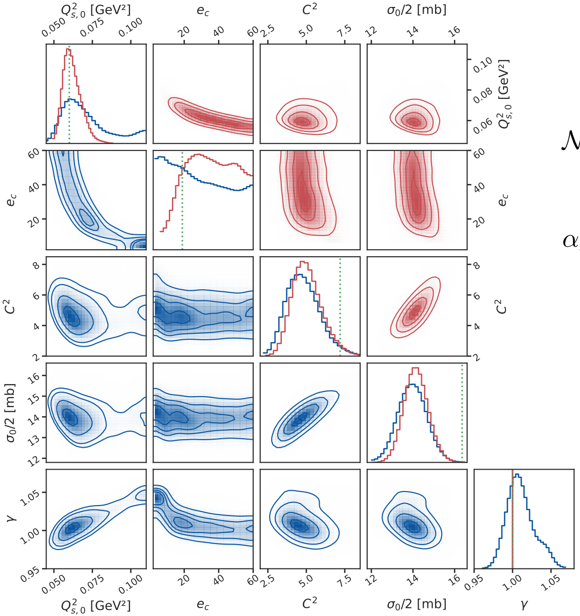
Results: Posterior Distribution

$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q^2_{s,0})^\gamma}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

$$\alpha_s(\mathbf{r}) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{\mathbf{r}^2\Lambda^2_{QCD}}\right)}$$

1. HERA data prefers a $\gamma \approx 1$
2. With γ as a free parameter, we obtain a wider posterior distribution
3. Off-diagonal plots = correlations

— $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ — $\gamma = 1$ — MV^e [H.M.& T.L. (2013)]



Results: Posterior Distribution

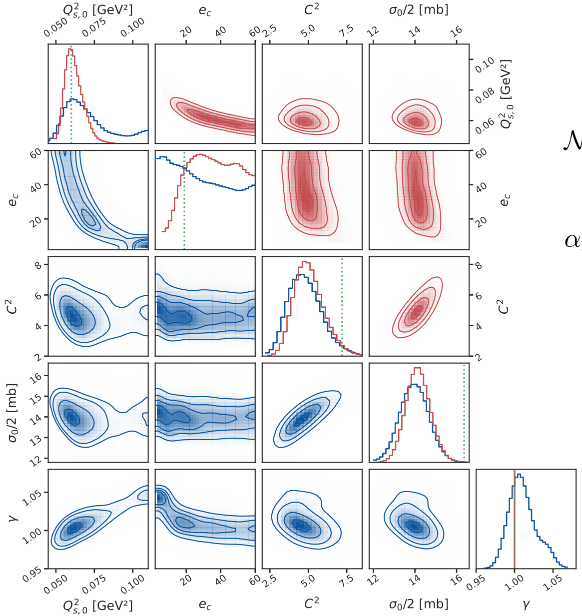
$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q_{s,0}^2)^{\gamma}}{4}\ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

$$\alpha_s(\mathbf{r}) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{\mathbf{r}^2\Lambda_{QCD}^2}\right)}$$

1. HERA data prefers a $\gamma \approx 1$
2. With γ as a free parameter, we obtain a wider posterior distribution
3. Off-diagonal plots = correlations

— $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ — $\gamma = 1$ — MV^e [H.M.& T.L. (2013)]

Results: Posterior Distribution

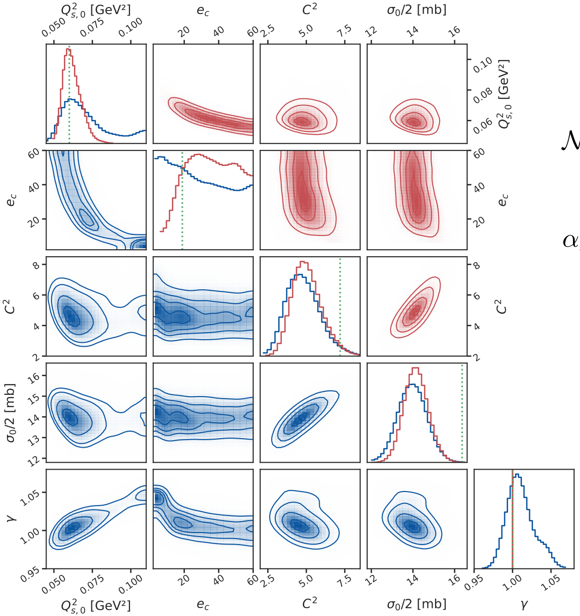


$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q^2_{s,0})^{\gamma}}{4}\ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

$$\alpha_s(\mathbf{r}) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{\mathbf{r}^2\Lambda^2_{QCD}}\right)}$$

1. HERA data prefers a $\gamma \approx 1$
2. With γ as a free parameter, we obtain a wider posterior distribution
3. Off-diagonal plots = correlations

— $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ — $\gamma = 1$ — MV^e [H.M.& T.L. (2013)]



Results: Posterior Distribution

$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q_{s,0}^2)^\gamma}{4}\ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

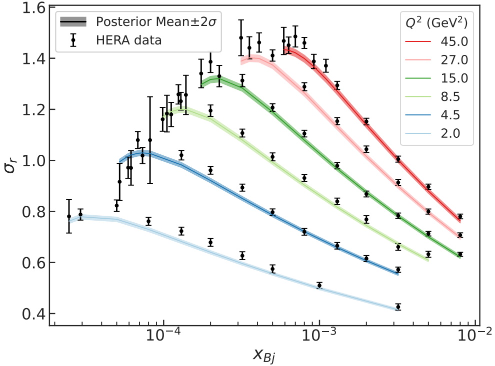
$$\alpha_s(\mathbf{r}) = \frac{12\pi}{(33 - N_f)\log\left(\frac{4C^2}{\mathbf{r}^2\Lambda_{QCD}^2}\right)}$$

1. HERA data prefers a $\gamma \approx 1$
2. With γ as a free parameter, we obtain a wider posterior distribution
3. Off-diagonal plots = correlations

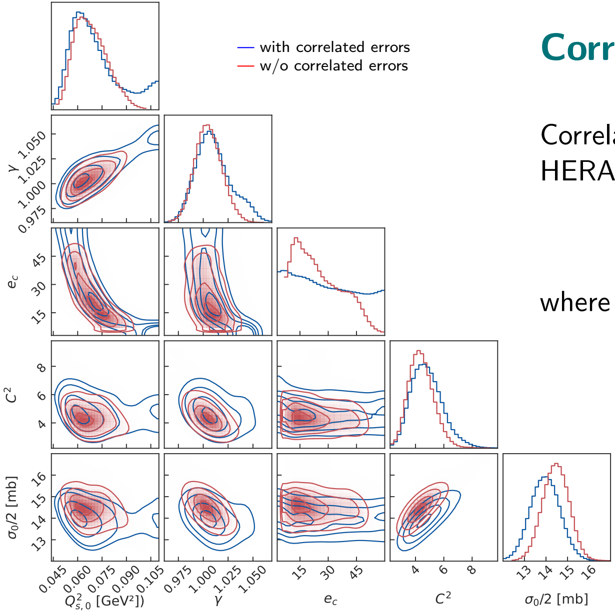
— $[Q^2_{s0}, \gamma, e_c, C^2, \sigma_0/2]$ — $\gamma = 1$ — MV^e [H.M.& T.L. (2013)]

Posterior Samples, Median and MAP

5 - parameter	$Q_{s0}^2[GeV^2]$	γ	e_c	C^2	$\sigma_0/2[mb]$	χ^2/dof	Q_s^2
median	0.067	1.01	27.5	4.72	14.0	1.016	0.288
MAP	0.077	1.01	15.6	4.47	13.9	1.012	0.289



! Good agreement with HERA data



Correlated systematic errors

Correlated systematic uncertainties from the HERA data are input to:

$$P(\boldsymbol{\theta}) \sim -\Delta\mathbf{y}(\boldsymbol{\theta})^T \Sigma^{-1}(\boldsymbol{\theta}) \Delta\mathbf{y}(\boldsymbol{\theta})$$

where $\Sigma = \Sigma_{\rm emu} + \Sigma_{\rm exp}$.

- Wider posterior distributions
- Moderate change to parameter correlations

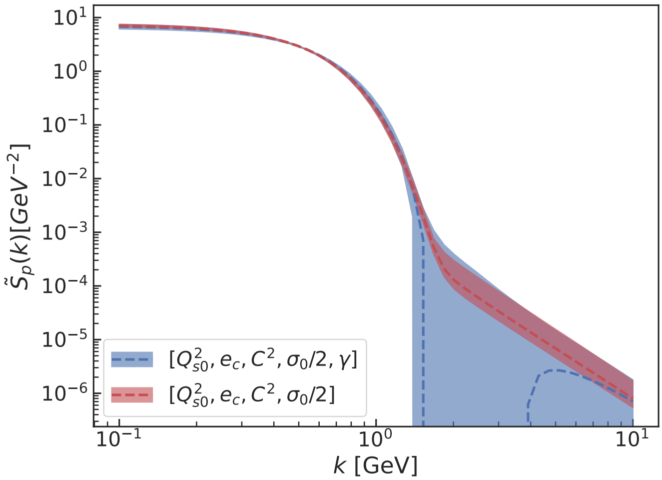
Inclusive quark production

$$\mathrm{d}\sigma^{q+A\to q+X} = xq(x,\mathbf{k}^2)\tilde{S}_p(\mathbf{k})$$

$\tilde{S}_p(\mathbf{k}) \to$ 2DFT of Dipole amplitude

$$= \int \mathrm{d}^2\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}} \times [1 - \mathcal{N}(\mathbf{r}, x = x_0)]$$

- $\gamma > 1$ result to negative 2DFT values [B. Giraud et. al. (2016)]

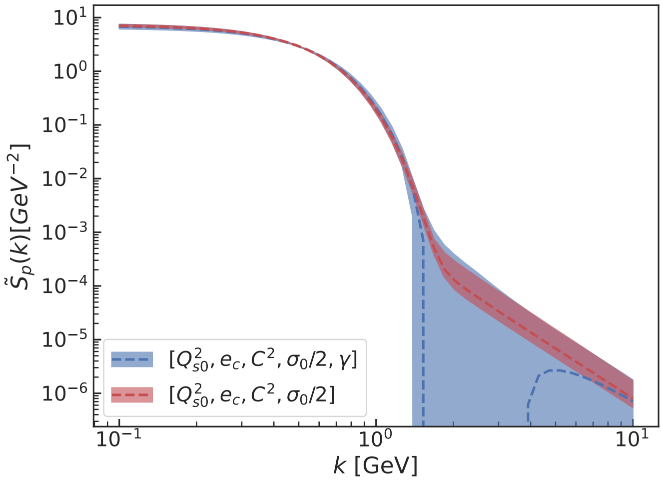


Inclusive quark production

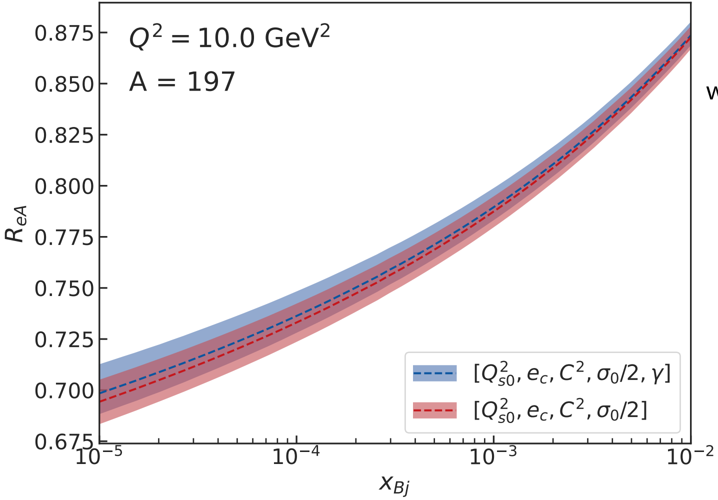
$$\mathrm{d}\sigma^{q+A\to q+X} = xq(x, \mathbf{k}^2)\tilde{S}_p(\mathbf{k})$$

$$\tilde{S}_p(\mathbf{k}) \to \text{2DFT of Dipole amplitude}$$
$$= \int \mathrm{d}^2\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}} \times [1 - \mathcal{N}(\mathbf{r}, x = x_0)]$$

- $\gamma > 1$ result to negative 2DFT values [B. Giraud et. al. (2016)]



Nuclear Modification factor at the EIC and beyond



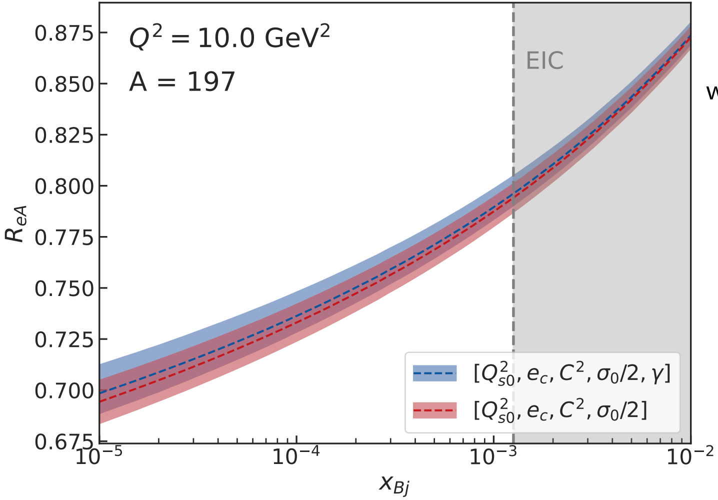
$$R_{eA} = \frac{F_{2,A}}{AF_{2,p}}$$

where $Q^2_{s,A} = Q^2_{s,p} \cdot \sigma_0/2 \cdot AT_A(b)$.

- Evolution speed uncertainty ($\sim C^2$) become more significant towards smaller x_{Bj}

! These measurements could provide further constraint for the initial condition

Nuclear Modification factor at the EIC and beyond



$$R_{eA} = \frac{F_{2,A}}{AF_{2,p}}$$

where $Q^2_{s,A} = Q^2_{s,p} \cdot \sigma_0/2 \cdot AT_A(b)$.

- Evolution speed uncertainty ($\sim C^2$) become more significant towards smaller x_{Bj}

! These measurements could provide further constraint for the initial condition

Summary & Outlook

- Posterior distribution for $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2] \sim$ BK IC
- Method for propagating the uncertainties of non-perturbative BK IC (first time!)
- Accounted for correlated errors in HERA data (first time!)

Further work

- Fit of the next-to-leading accuracy + heavy quarks

Summary & Outlook

- Posterior distribution for $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2] \sim$ BK IC
- Method for propagating the uncertainties of non-perturbative BK IC (first time!)
- Accounted for correlated errors in HERA data (first time!)

Further work

- Fit of the next-to-leading accuracy + heavy quarks

Summary & Outlook

- Posterior distribution for $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2] \sim$ BK IC
- Method for propagating the uncertainties of non-perturbative BK IC (first time!)
- Accounted for correlated errors in HERA data (first time!)

Further work

- Fit of the next-to-leading accuracy + heavy quarks

Summary & Outlook

- Posterior distribution for $[Q_{s0}^2, \gamma, e_c, C^2, \sigma_0/2] \sim$ BK IC
- Method for propagating the uncertainties of non-perturbative BK IC (first time!)
- Accounted for correlated errors in HERA data (first time!)

Further work

- Fit of the next-to-leading accuracy + heavy quarks

Back up Slides

Parameters

- $\sigma_0/2$, half of normalization to the cross section, proton transverse area, $2\int \mathrm{d}^2 b \to \sigma_0$
- $Q^2_{s,0}$, **related** to the saturation scale at initial x. Previous fits used GBW parametrization:

$$\mathcal{N}(\mathbf{r}, x_0) = 1 - \exp\left[-\frac{(\mathbf{r}^2 Q^2_{s,0})^\gamma}{4}\right]$$

- C^2, connects the running coupling in $\mathbf{r}$ to its Fourier transform
- γ, anomalous dimension, controlling the steepness of the cross section related to its fall-off for small dipoles
- e_c, infrared cut-off in the MV model

F_2 Structure Function for Nucleus

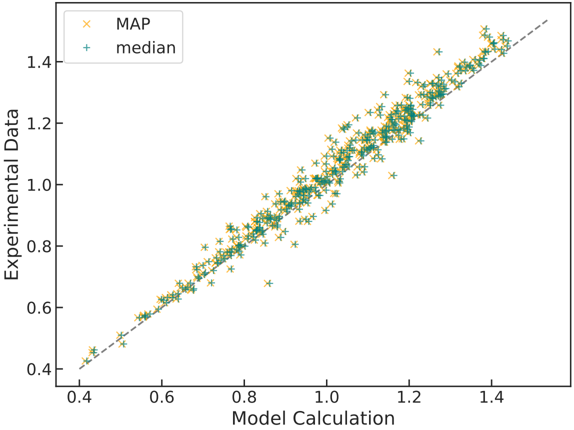
$$N_A(\mathbf{r}, \mathbf{b}, x = x_0) = 1 - \exp\left[-AT_A(\mathbf{b})\frac{\sigma_0}{2}\frac{\left(\mathbf{r}^2 Q_{\mathrm{s},0}^2\right)^{\gamma}}{4}\ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

where $T_A(\mathbf{b})$ is the transverse thickness function of the nucleus of mass number A, obtained through integrating the Woods-Saxon distribution

$$\rho_A(\mathbf{b}, z) = \frac{n}{1 + \exp\left[\frac{\sqrt{\mathbf{b}^2 + z^2} - R_A}{d}\right]}$$

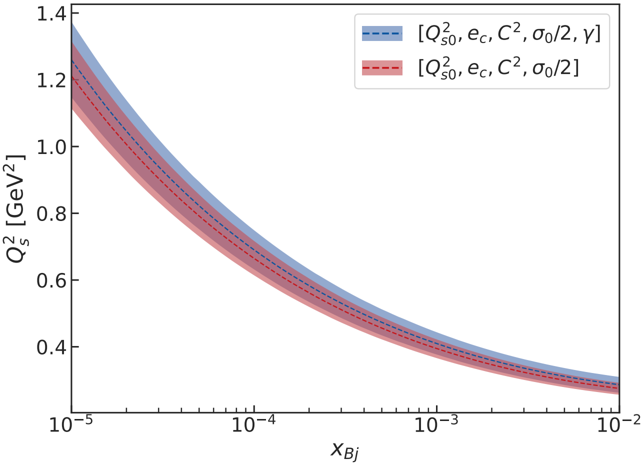
Emulator vs Model

$$\frac{\chi^2}{\mathrm{d.o.f}} = \frac{1}{N-p}\Delta\mathbf{y}(\boldsymbol{\theta})^T\Sigma_{\mathrm{exp}}^{-1}\Delta\mathbf{y}(\boldsymbol{\theta}),$$

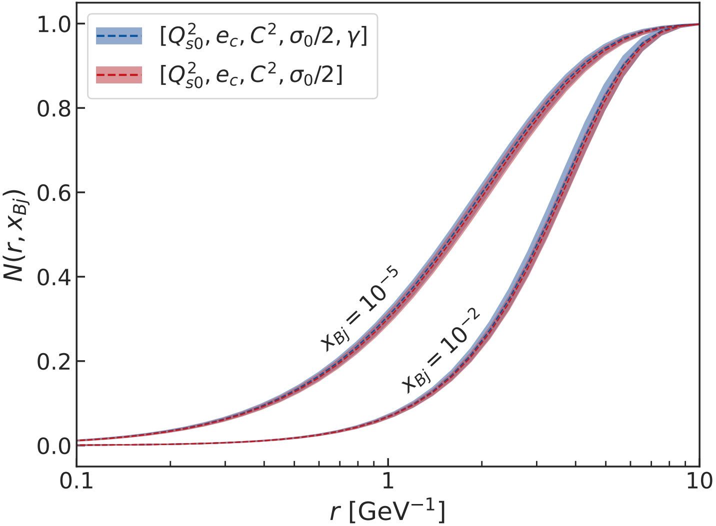


Initial Saturation Scale

$$N(\mathbf{r}^2 = 2/Q_s^2) = 1 - e^{-1/2}$$



Initial and Evolved $\mathcal{N}(r, y)$



Nuclear Modification Ratio