Diffractive processes at the NLO in a saturation framework

Michael Fucilla

Laboratoire de Physique des 2 Infinis Irène Joliot-Curie

in collaboration with

Renaud Boussarie, Andrey V. Grabovsky, Emilie Li,
Lech Szymanowski, Samuel Wallon

based on

[JHEP 11 (2016) 149], **[JHEP 03 (2023) 159]**,
[JHEP 02 (2024) 165] + [work in preparation]

DIS2024, Grenoble, France, 08-12 April 2024

Outline

Introduction

- Saturation physics
- Shockwave formalism

Diffractive dijet/hadron production

- Diffractive dijet/hadron production
- Deeply virtual meson production

Outline

The high-energy limit of QCD

- **Semi-hard** collision process → stringent *scale hierarchy*

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}, \qquad Q^2 \text{ a hard scale},$$

↗

Regge kinematic region

$$\alpha_s(Q^2)\,\ln\left(\frac{s}{Q^2}\right) \sim 1 \implies \text{all-order } \textbf{resummation} \text{ needed}$$

- **Linear regime** of high-energy QCD

The **BFKL** (Balitsky-Fadin-Kuraev-Lipatov) approach

i. Leading-Logarithmic-Approximation (**LLA**): $(\alpha_s \ln s)^n$

ii. Next-to-Leading-Logarithmic-Approximation (**NLLA**): $\alpha_s(\alpha_s \ln s)^n$

iii. Progress on **next-to-NLLA**

[Falcioni, Gardi, Maher, Milloy, Vernazza (2022)]
[Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi (2022)]

- **Non-linear** (saturation) **regime**

B-JIMWLK (Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov) evolution equations

Motivation

- NLL corrections → precision era in small-x physics
- **Evolution kernels** in the saturation regime are known at NLO
  [Balitsky, Chirilli (2007)], [Kovner, Lublinsky, Mulian (2013)]
- **Non-perturbative models** for the description of the target
  [McLerran, Venugopalan (1994)]
  [Golec-Biernat, Wusthof (1998)]
- Full NLL predictions requires **NLO impact factors** (still challenging to compute) → complex analytical results
  [Balitsky, Chirilli (2011)], [Beuf (2016)]
  [Chirilli, Xiao, Yuan (2012)]
  [Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016-17)]
  [Roy, Venugopalan (2020)]
  [Caucal, Salazar, Schenke, Stebel, Venugopalan (2021-24)]
  [Beuf, Lappi, Mulian, Paatelainen, Penttala (2022-24)] [Tuomas's talk]
  [Lappi, Mäntysaari, Pentalla (2020-22)]
  [Bergabo, Jalilian-Marian (2022-24)], [Jamal's talk]
  [Taels, Altinoluk, Beuf, Marquet (2020-22)] [P. Taels (2023)] [Yair's talk]
- Long series of works devoted to **subeikonal** corrections
  [WG2's talks on thursday morning]
- Precise observables to reveal without ambiguity the saturation of gluons in nucleons and nuclei, and to study the **Color Glass Condensate (CGC)**

Saturation physics

- DIS total cross-section

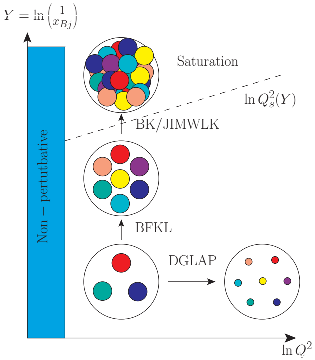
$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}(x,\vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0} = \left(\frac{1}{x}\right)^{\omega_0}$$

- ***Martin-Froissart bound***

$$\sigma_{tot} \lesssim c \ln^2 s$$



- ***Saturation effects***
  i. Very dense system $\Longrightarrow$ Recombination effects
  ii. In large nuclei $\Longrightarrow$ Multiple re-scattering ($\alpha_s^2 A^{1/3}$ resummation)

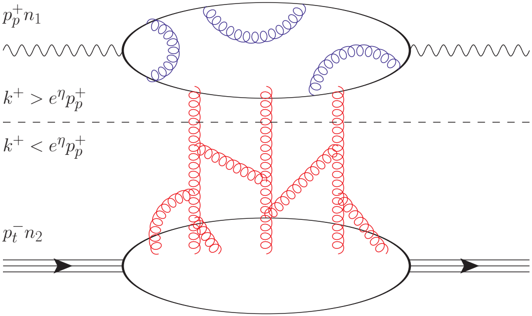
- Characteristic ***Saturation scale***

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\text{QCD}} \qquad \alpha_s(Q_s^2) \ll 1 \implies \textit{Weakly coupled QCD}$$

[McLerran, Venugopalan (1994)]

Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



- Separation of the gluonic field into "fast" (quantum) part and "slow" (classical) part through a rapidity parameter $\eta < 0$

[Balitsky (2001)]

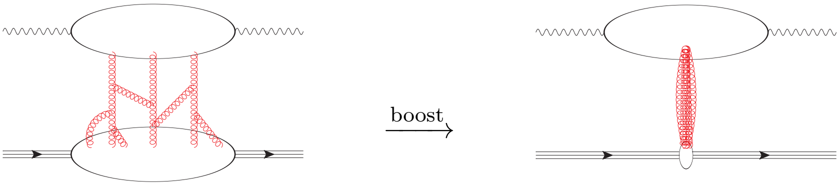
$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k})$$

$e^\eta \ll 1$

Shockwave approach

- Large longitudinal Boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+,x^-,\vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+,x^-,\vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+,x^-,\vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$b^\mu(x^+,x^-,\vec{x}) = \delta(x^+)\mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

Shockwave approximation

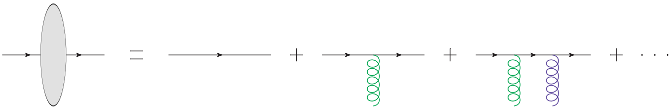
- Light-cone gauge $A \cdot n_2 = 0$

$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

Shockwave approach

- Multiple interactions with the target → ***path-ordered Wilson lines***

$$U^{\eta}_{\vec{z}_i} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^{+} b_{\eta}^{-}\left(z_i^{+},\vec{z}_i\right)\right]$$



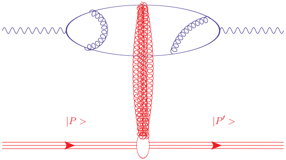
$$U_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^{+} b_{\eta}^{-}\left(z_i^{+},\vec{z}_i\right) + (ig)^2\int_{-\infty}^{+\infty} dz_i^{+} dz_j^{+} b_{\eta}^{-}\left(z_i^{+},\vec{z}_i\right) b_{\eta}^{-}\left(z_j^{+},\vec{z}_i\right)\theta\left(z_{ij}^{+}\right) + \cdots$$

- Factorization in the Shockwave approximation

$$\mathcal{M}^{\eta} = N_c\int d^d z_{1\perp} d^d z_{2\perp}\,\Phi^{\eta}(z_{1\perp},z_{2\perp})\left\langle P'\left|\left[\frac{1}{N_c}\mathrm{Tr}\left(U^{\eta}_{\vec{z}_1}U^{\eta\dagger}_{\vec{z}_2}\right)-1\right](\vec{z}_1,\vec{z}_2)\right|P\right\rangle$$

- ***Dipole operator***

$$\mathcal{U}^{\eta}_{ij} = \frac{1}{N_c}\mathrm{Tr}\left(U^{\eta}_{\vec{z}_i}U^{\eta\dagger}_{\vec{z}_j}\right) - 1$$



Balitsky-JIMWLK evolution equations

- ***Balitsky-JIMWLK evolution equations*** for the dipole

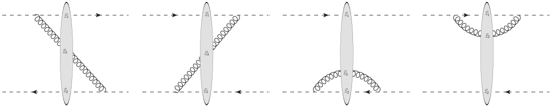
[Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2 \vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2} \vec{z}_{31}^{\,2}} \right) \left[\underbrace{\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12}}_{\text{BFKL}} - \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32} \right]$$

$$\frac{\partial \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32}}{\partial \eta} = \cdots \qquad \longleftarrow \qquad \textit{Balitsky hierarchy}$$

$$\vdots$$

- Double dipole contribution and Dipole contribution



- Dipole contribution

Outline

Introduction

- Saturation physics
- Shockwave formalism

Diffractive dijet/hadron production

- Diffractive dijet/hadron production
- Deeply virtual meson production

Diffractive dijet/hadron production

- Precise predictions to detect **saturation** effects at both the EIC or LHC

  [Iancu, Mueller, Triantafyllopoulos (2022)]
- Possibility of studying multi-dimensional **gluon tomography**

  [Hatta, Xiao, Yuan (2022)]

  [Hauksson, Iancu, Mueller, Triantafyllopoulos (2024)] [Sigtryggur's talk]
- ***Diffractive dijet/hadron(s) production at NLO***

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to j_1(p_{h1}) + j_2(p_{h2}) + P(p_0')$$

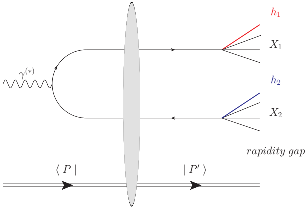
[Boussarie, Grabovsky, Szymanowski, Wallon (2016)]

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + h_2(p_{h2}) + X + P(p_0') \qquad (X = X_1 + X_2)$$

[M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + X + P(p_0')$$

[M. F., Grabovsky, Li, Szymanowski, Wallon (2024)]



i. General kinematics (t, Q^2) and photon polarization

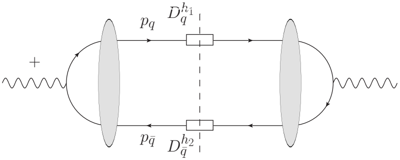
ii. Rapidity gap between $(h_1 h_2 X)$ and P'

iii. $\vec{p}_{12}^{\,2} \gg \vec{p}_{h_1}^{\,2}, \vec{p}_{h_2}^{\,2} \gg \Lambda_{QCD}^2$

LO cross-section

- Sudakov decomposition for the momenta: $p_i^\mu = x_i p_\gamma^+ n_1^\mu + \frac{\vec{p}^{\,2}}{2x_i p_\gamma^+} n_2^\mu + p_\perp^\mu$

$$\begin{array}{cccc} & + & - & \perp \\ p_\gamma = \Big(& p_\gamma^+ \,, & -\dfrac{Q^2}{2p_\gamma^+} \,, & \vec{0} \Big) \end{array}$$

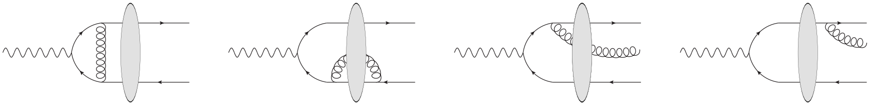


- Collinearity $(p_q^+, \vec{p}_q) = (x_q/x_{h_1})(p_{h_1}^+, \vec{p}_{h_1})$ and $(p_{\bar{q}}^+, \vec{p}_{\bar{q}}) = (x_{\bar{q}}/x_{h_2})(p_{h_2}^+, \vec{p}_{h_2})$

$$\frac{d\sigma_{0JI}^{h_1 h_2}}{dx_{h_1} dx_{h_2} d^d\vec{p}_{h_1} d^d\vec{p}_{h2}} = \sum_q \int_{x_{h_1}}^1 \frac{dx_q}{x_q} \int_{x_{h_2}}^1 \frac{dx_{\bar{q}}}{x_{\bar{q}}} \left(\frac{x_q}{x_{h1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h2}}\right)^d$$
$$D_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}\right) \frac{d\hat{\sigma}_{JI}}{dx_q dx_{\bar{q}} d^d\vec{p}_q d^d\vec{p}_{\bar{q}}} + (h_1 \leftrightarrow h_2)$$

$J, I \rightarrow$ photon polarization for respectively the complex conjugated amplitude and the amplitude.

Treatment of UV and rapidity divergences



Rapidity divergences $(x_g \to 0)$

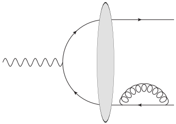
- Coming from Φ_{V_2} (double dipole part of the virtual contribution)
- Regularized by **longitudinal cut-off**: $|p_g^+| = |x_g| p_\gamma^+ > \alpha p_\gamma^+ \implies \ln\alpha$ term
- B-JIMWLK evolution from the non-physical cutoff α to the rapidity e^η

$$U_{\vec{x}}^{\alpha} = U_{\vec{x}}^{e^\eta} + \int_{e^\eta}^{\alpha} d\rho \left(\frac{\partial U_{\vec{x}}^{\rho}}{\partial \rho} \right) \implies \Phi_{V_2} \longrightarrow \tilde{\Phi}_{V_2} = \Phi_{V_2} - \Phi_0 \otimes \mathcal{K}_{\mathrm{B-JIMWLK}}$$

UV-divergences $(\vec{p}_g^{\,2} \to \infty)$

- Just dressing of the external quark lines

$$\Phi_{\mathrm{dress}} \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

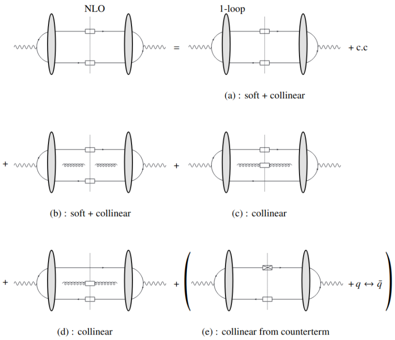


- $\epsilon_{IR} = \epsilon_{UV}$ turns **UV** into **IR** divergences

NLO cross-section in a nutshell

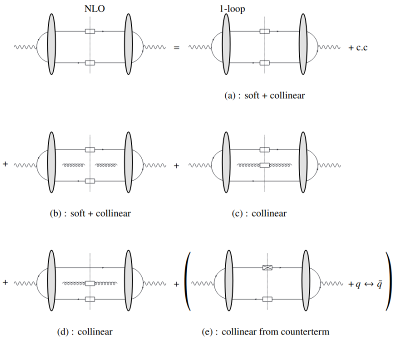
- Different *fragmentation mechanisms*

  i. Quark/anti-quark fragmentation
  ii. Quark/gluon fragmentation
  iii. Anti-quark/gluon fragmentation

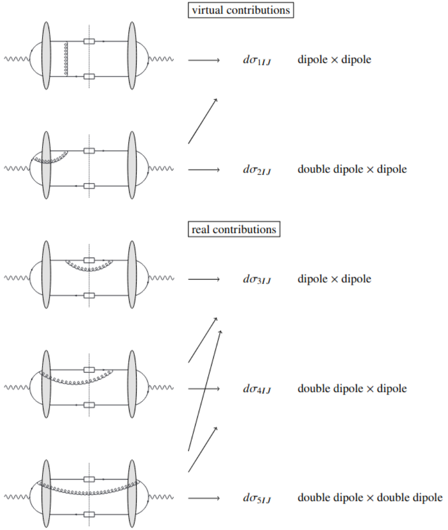


- Different *fragmentation mechanisms*

  i. Quark/anti-quark fragmentation
  ii. Quark/gluon fragmentation
  iii. Anti-quark/gluon fragmentation

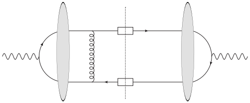


- Operator structure classification

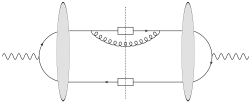


IR singularities: Quark/anti-quark fragmentation

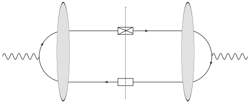
- Divergent contributions



$d\sigma_{1IJ}$

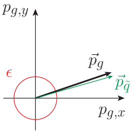


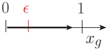
$d\sigma_{3IJ}$



$d\sigma_{\text{counter}}$

- Collinear divergence

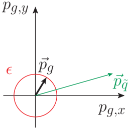


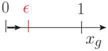


i. $\vec{p}_g \to \vec{p}_{\tilde{q}} = \frac{x_g}{x_q}\vec{p}_q$

ii. x_g generic

- Soft divergence





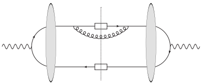
i. $\vec{p}_g \equiv x_g \vec{u}$

ii. $x_g \to 0$ and $\vec{u}$ generic

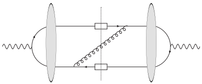
- Soft and collinear divergence ($x_g \to 0$ and $\vec{u} \to \frac{\vec{p}_q}{x_q}$)

IR singularities

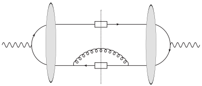
- Divergences: $q\bar{q}$-fragmentation



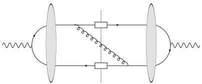
(1): soft + collinear (qg)



(2): soft



(3): soft + collinear ($\bar{q}g$)

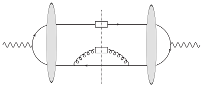


(4): soft

- Treatment of divergences in a nutshell

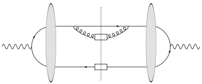
$$d\sigma_1 + d\sigma_{3,\mathrm{soft}} + \underbrace{(d\sigma_3^{(1)} - d\sigma_{3,\mathrm{soft}}^{(1)})}_{d\sigma_{3,\mathrm{collinear}}^{(1)}} + (d\sigma_3^{(2)} - d\sigma_{3,\mathrm{soft}}^{(2)}) + \underbrace{(d\sigma_3^{(3)} - d\sigma_{3,\mathrm{soft}}^{(3)})}_{d\sigma_{3,\mathrm{collinear}}^{(3)}} + ((d\sigma_3^{(4)} - d\sigma_{3,\mathrm{soft}}^{(4)})) + d\sigma_{\mathrm{counter}}$$

- Divergences: qg-fragmentation



(5): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{\bar{q}g(5)}$

- Divergences: $\bar{q}g$-fragmentation



(6): collinear $\longrightarrow d\sigma_{3,\mathrm{collinear}}^{\bar{q}g(6)}$

Renormalization of FFs and gluon fragmentation

- Renormalized quark FFs (similar for the anti-quark)

$$\tilde{D}_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) = D_q^{h_1}\left(\frac{x_{h_1}}{x_q}, \mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left[\left[P_{qq}\otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right) + \left[P_{gq}\otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)\right]$$

Renormalization of FFs and gluon fragmentation

- Renormalized quark FFs (similar for the anti-quark)

$$\tilde{D}_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) = D_q^{h_1}\left(\frac{x_{h_1}}{x_q},\mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left[[P_{qq}\otimes D_q^{h_1}]\left(\frac{x_{h_1}}{x_q},\mu_F\right) + [P_{gq}\otimes D_g^{h_1}]\left(\frac{x_{h_1}}{x_q},\mu_F\right)\right]$$

$$\downarrow$$

$$d\sigma_{LL}^{h_1 h_2}\Big|_{\mathrm{ct}} = \frac{4\alpha_{\mathrm{em}}Q^2}{(2\pi)^{4(d-1)}N_c}\sum_q Q_q^2 \int_{x_{h_1}}^1 dx_q \int_{x_{h_2}}^1 dx_{\bar{q}}\, x_q x_{\bar{q}} \left(\frac{x_q}{x_{h_1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h_2}}\right)^d \delta(1-x_q-x_{\bar{q}})$$

$$\times\mathcal{F}_{LL}\left(-\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Bigg\{\underbrace{[P_{qq}\otimes D_q^{h_1}]\left(\frac{x_{h_1}}{x_q},\mu_F\right)}_{(1)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_F\right)$$

$$+\underbrace{[P_{gq}\otimes D_g^{h_1}]\left(\frac{x_{h_1}}{x_q},\mu_F\right)}_{(6)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}},\mu_F\right) + \left[\left(q,x_q,x_{h_1}\right)\leftrightarrow\left(\bar{q},x_{\bar{q}},x_{h_2}\right)\right]\Bigg\} + (h_1\leftrightarrow h_2)$$

Renormalization of FFs and gluon fragmentation

- Renormalized quark FFs (similar for the anti-quark)

$$\tilde{D}_q^{h_1}\left(\frac{x_{h_1}}{x_q}\right) = D_q^{h_1}\left(\frac{x_{h_1}}{x_q}, \mu_F\right) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\left[\left[P_{qq} \otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right) + \left[P_{gq} \otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)\right]$$

$$\downarrow$$

$$d\sigma_{LL}^{h_1 h_2}\Big|_{\rm ct} = \frac{4\alpha_{\rm em} Q^2}{(2\pi)^{4(d-1)} N_c} \sum_q Q_q^2 \int_{x_{h_1}}^1 dx_q \int_{x_{h_2}}^1 dx_{\bar{q}} x_q x_{\bar{q}} \left(\frac{x_q}{x_{h_1}}\right)^d \left(\frac{x_{\bar{q}}}{x_{h_2}}\right)^d \delta(1 - x_q - x_{\bar{q}})$$

$$\times \mathcal{F}_{LL}\left(-\frac{\alpha_s}{2\pi}\right)\left(\frac{1}{\hat{\epsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\Bigg\{\underbrace{\left[P_{qq} \otimes D_q^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)}_{(1)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}, \mu_F\right)$$

$$+\underbrace{\left[P_{gq} \otimes D_g^{h_1}\right]\left(\frac{x_{h_1}}{x_q}, \mu_F\right)}_{(6)} D_{\bar{q}}^{h_2}\left(\frac{x_{h_2}}{x_{\bar{q}}}, \mu_F\right) + \left[\left(q, x_q, x_{h_1}\right) \leftrightarrow \left(\bar{q}, x_{\bar{q}}, x_{h_2}\right)\right]\Bigg\} + (h_1 \leftrightarrow h_2)$$

- Finite part of the cross sections

$$d\sigma_{h_1,h_2} = \sum_{(a,b)} D_a^{h_1} \otimes D_b^{h_2} \otimes d\hat{\sigma}_{ab} \qquad (a,b) = \{(q,\bar{q}), (q,g), (g,\bar{q})\}$$

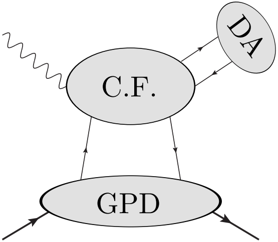
- Extension to the **semi-inclusive diffractive DIS (SIDDIS)** at the NLO **[M.F., Grabovsky, Li, Szymanowski, Wallon (2024)]**

Deeply virtual meson production (DVMP)

- Exclusive ρ-meson leptoproduction

  $\gamma^{(*)}(p_\gamma) + P(p_0) \to \rho(p_\rho) + P(p_0')$

- Extensively studied at HERA



- NLO corrections to the production of a longitudinally polarized ρ-meson in the saturation regime

  [Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)]

  [Mäntysaari, Pentalla (2022)]

- Transversally polarized ρ-meson start at the **twist-3**
- Collinear factorization at the next-to-leading power leads to end point singularities

  [Anikin, Teryaev (2002)]

- Exclusive light-meson production at the twist-3 within the BFKL approach (forward case)

  [Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]

Transversely polarized light-meson production

- Exclusive light-meson production at the **twist-3** within the Shockwave approach

  [Boussarie, M.F. , Szymanowski, Wallon (to appear)]

  i. Saturation corrections to DVMP in the transversely polarized case
  ii. Both forward and non-forward result
  iii. Coordinates and momentum space representation
  iiii. Linearization **[Caron-Hout (2013)]** $\Longrightarrow$ BFKL results

- Effective background field operators

$$\left[\psi_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \psi\left(z_0\right) - \int \mathrm{d}^D z_2 G_0\left(z_{02}\right)\left(V_{\boldsymbol{z}_2}^{\dagger} - 1\right)\gamma^+\psi\left(z_2\right)\delta(z_2^+)$$

$$\left[\bar{\psi}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int \mathrm{d}^D z_1\bar{\psi}\left(z_1\right)\gamma^+\left(V_{\boldsymbol{z}_1} - 1\right)G_0\left(z_{10}\right)\delta(z_1^+)$$

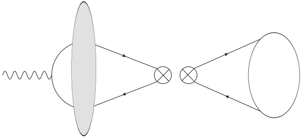
$$\left[A_{\text{eff}}^{\mu a}\ (z_0)\right]_{z_0^+<0} = A^{\mu a}\left(z_0\right) + 2i\int \mathrm{d}^D z_3\delta(z_3^+)F_{-\sigma}^{b}\left(z_3\right)G^{\mu\sigma_\perp}\left(z_{30}\right)\left(U_{\boldsymbol{z}_3}^{ab} - \delta^{ab}\right)$$

- Higher-twist formalisms

  Covariant collinear factorization **[Ball, Braun, Koike, Tanaka (1998)]**
  Light-cone collinear factorization **[Anikin, Teryaev (2002)]**

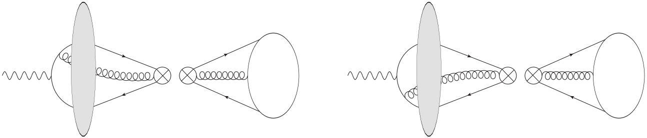
Higher-twist formalism

- 2-body contribution (**kinematic twist**)



$$\mathcal{A}_2 = -ie_f \int \mathrm{d}^D z_0 \theta\left(-z_0^+\right) \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\text{eff}}\left(z_0\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\text{eff}}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

- 3-body contribution (**genuine twist**)



$$\mathcal{A}_{3,q} = \left(-ie_q\right)\left(ig\right) \int \mathrm{d}^D z_4 \mathrm{d}^D z_0 \theta\left(-z_4^+\right) \theta(-z_0^+)$$

$$\times \left\langle P\left(p'\right) M\left(p_M\right) \middle| \overline{\psi}_{\text{eff}}\left(z_4\right) \gamma_\mu A_{\text{eff}}^{\mu a}\left(z_4\right) t^a G\left(z_{40}\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\text{eff}}\left(z_0\right) \middle| P\left(p\right) \right\rangle$$

Summary and outlook

Summary

- **Full NLO computation of diffractive di-hadron production and SIDDIS**

  [M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]

  [M. F., Grabovsky, Li, Szymanowski, Wallon (2024)]
- Full cancellation of divergences has been observed between real, virtual corrections and counterterms from renormalized FFs.
- *General kinematics* (Q^2, t) and *arbitrary photon polarization* means either photo or electro-production.
- Results are also applicable to ultra-peripheral collisions at the LHC.
- **Deeply virtual light-meson production at the twist-3**

  [Boussarie, M.F., Szymanowski, Wallon (to appear)]

Summary and outlook

Summary

- **Full NLO computation of diffractive di-hadron production and SIDDIS**

  [M. F., Grabovsky, Li, Szymanowski, Wallon (2023)]

  [M. F., Grabovsky, Li, Szymanowski, Wallon (2024)]
- Full cancellation of divergences has been observed between real, virtual corrections and counterterms from renormalized FFs.
- *General kinematics* (Q^2, t) and *arbitrary photon polarization* means either photo or electro-production.
- Results are also applicable to ultra-peripheral collisions at the LHC.
- **Deeply virtual light-meson production at the twist-3**

  [Boussarie, M.F., Szymanowski, Wallon (to appear)]

Outlook

- Looking for special kinematic configuration → diffractive dijet production in the back-to-back limit, TMD factorization in SIDDIS
- Resummation of large logarithms in the impact factors
- **Phenomenological analyisis!**

Thanks for your attention

Backup

Saturation physics

- DIS total cross-section

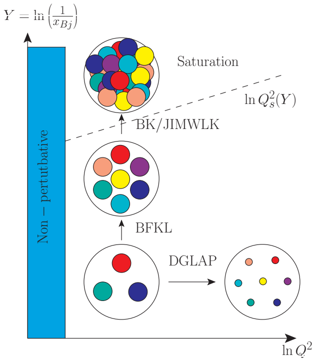
$$\sigma_{\gamma^* P}(x) = \Phi_{\gamma^*\gamma^*}(\vec{k}\,) \otimes_{\vec{k}} \mathcal{F}(x,\vec{k})$$

$$\downarrow$$

$$\sigma_{\gamma^* P}(x) \sim \left(\frac{s}{Q^2}\right)^{\omega_0} = \left(\frac{1}{x}\right)^{\omega_0}$$

- ***Martin-Froissart bound***

$$\sigma_{tot} \lesssim c \ln^2 s$$



- The violation of Martin-Froissart bound means a breakdown of the unitarity
- The violation is physically interpretable as an *infinite growth* of the unintegrated gluon density at small value of the Bjorken-x

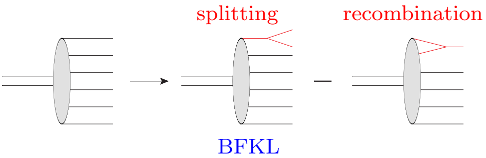
$$\boxed{\Delta x_\perp = \frac{1}{Q}}$$

Saturation physics

- ***Saturation effects***

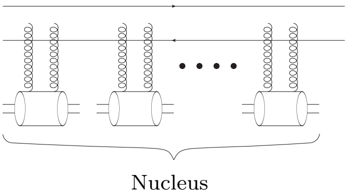
  i. Very dense system $\Longrightarrow$ *Recombination effects*

  [Gribov, Levin, Ryskin — Mueller and Qui (1980-1983)]



  ii. In large nuclei $\Longrightarrow$ *Multiple re-scattering* ($\alpha_s^2 A^{1/3}$ resummation)

  [Glauber (1959)—Gribov (1969)], [Kovchegov (1999)]



Color glass condensate

- Characteristic ***Saturation scale***

$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda^2_{\mathrm{QCD}} \qquad \alpha_s(Q_s^2) \ll 1 \implies \textit{Weakly coupled QCD}$$

  Saturation window: $Q^2 < Q_s^2$

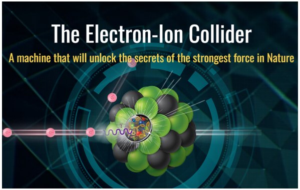
- The small-x gluon field can be obtained by solving the classical Yang-Mills equation (MV-model)

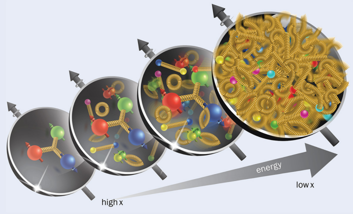
  [McLerran, Venugopalan (1994)]

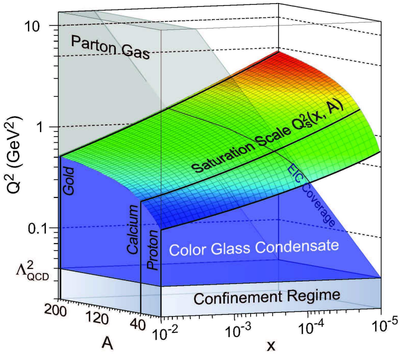
- The solution that is obtained has three main properties:

  i. **Color** $\rightarrow$ dominated by colored particle (gluons)
  ii. **Condensate** $\rightarrow$ very high-density of gluons
  iii. **Glass** $\rightarrow$ well-separated time scales between small-x and large-x, with this latter appearing as "frozen"

- *Quantum corrections* to MV model $\rightarrow$ non-linear small-x evolution







- At the **EIC**, the saturation scale Q_s will be in the perturbative range

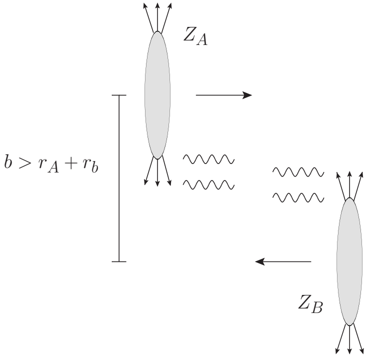
$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3} \Lambda_{\rm QCD}^2$$

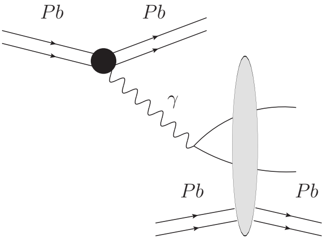
- **Perturbative control on gluonic saturation**

$$\Lambda_{QCD}^2 \ll Q^2 \ll Q_s^2$$

Ultra-Peripheral collisions at the LHC

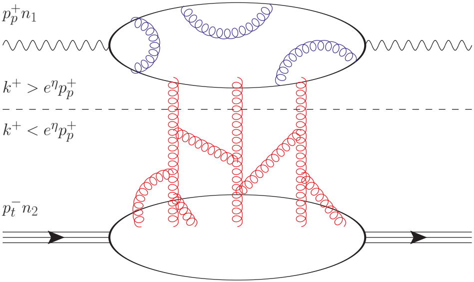
- **Ultra-peripheral collisions (UPCs)** → two projectiles with radii r_A and r_B interact with an impact parameter $b > R_A + R_B$





- UPCs are mediated by electromagnetic interactions
- Quasi-real photons cloud → **Equivalent photon approximation (EPA)**

Shockwave approach: kinematics



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

- Light-cone **Sudakov** vectors

$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1) \; , \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1) \; , \quad (n_1 \cdot n_2) = 1$$

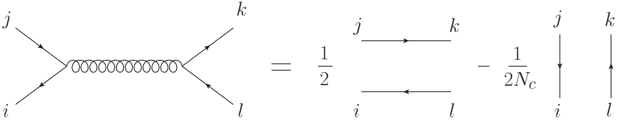
- Light-cone coordinates

$$x = (x^0, x^1, x^2, x^3) \to (x^+, x^-, \vec{x})$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

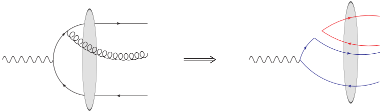
Balitsky-Kovchegov evolution equation

- Large-N_c limit **['t Hooft (1974)]**



$$t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$

- Double dipole → Dipole × dipole



$$\langle \mathcal{U}^\eta_{13}\mathcal{U}^\eta_{32}\rangle \to \langle \mathcal{U}^\eta_{13}\rangle \, \langle \mathcal{U}^\eta_{32}\rangle$$

- Hierarchy of equations broken → closed non-linear BK equation **[Balitsky (1995)]**, **[Kovchegov (1999)]**

$$\frac{\partial \langle \mathcal{U}^\eta_{12}\rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\vec{z}_{31}^{\,2}}\right)\left[\langle \mathcal{U}^\eta_{13}\rangle + \langle \mathcal{U}^\eta_{32}\rangle - \langle \mathcal{U}^\eta_{12}\rangle - \langle \mathcal{U}^\eta_{13}\rangle\langle \mathcal{U}^\eta_{32}\rangle\right]$$

with $\langle \mathcal{U}^\eta_{12}\rangle \equiv \langle P'|\mathcal{U}^\eta_{12}|P\rangle$

Dilute Regime

- Reggeon definition

$$R^a(\boldsymbol{z}) \equiv \frac{f^{abc}}{gC_A} \ln\left(U_{\boldsymbol{z}}^{bc}\right)$$

- Expansion in Reggeon

$$V_{\boldsymbol{z}_1} = 1 + ig\boldsymbol{t}^a R^a(\boldsymbol{z}_1) - \frac{1}{2} g^2 \boldsymbol{t}^a \boldsymbol{t}^b R^a(\boldsymbol{z}_1) R^b(\boldsymbol{z}_1) + O\left(g^3\right)$$

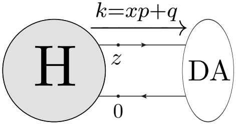
- BFKL factorization

$$\mathcal{A} = -\frac{g^2}{4N_c}(2\pi)^d \delta^d(\boldsymbol{q}-\boldsymbol{\Delta}) \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathcal{U}(\ell) \int_0^1 \mathrm{d}x$$
$$\times \underbrace{\left[\Phi\left(x, \ell - \frac{x-\bar{x}}{2}\boldsymbol{\Delta}\right) + \Phi\left(x, -\boldsymbol{\ell} - \frac{x-\bar{x}}{2}\boldsymbol{\Delta}\right) - \Phi(x, \bar{x}\boldsymbol{\Delta}) - \Phi(x, -x\boldsymbol{\Delta})\right]}_{\Phi_{BFKL}(x, \boldsymbol{l}, \boldsymbol{\Delta})}$$

- $\mathcal{U}(\boldsymbol{l}) \rightarrow$ unintegrated gluon density in the BFKL sense

$$\mathcal{U}(\boldsymbol{\ell}) \equiv \int \mathrm{d}^d \boldsymbol{v} \mathrm{e}^{-i(\boldsymbol{\ell}\cdot\boldsymbol{v})} \left\langle P\left(p'\right) \middle| R^a\left(\frac{\boldsymbol{v}}{2}\right) R^a\left(-\frac{\boldsymbol{v}}{2}\right) \middle| P(p) \right\rangle ,$$

Light-cone collinear factorization



- 2-body amplitude

$$\mathcal{A}_2 = \int \frac{d^4k}{(2\pi)^4} \int d^4z e^{-ik\cdot z} \langle M(p)|\overline{\psi}_{\alpha}^{\,i}(z)\,\psi_{\beta}^{\,j}(0)\,|0\rangle H_{2,\alpha\beta}^{ij}$$

- 2-body amplitude after Fierz decomposition

$$\mathcal{A}_2 = \frac{1}{4N_c} p^+ \int \frac{\mathrm{d}x}{2\pi} \int \frac{\mathrm{d}q^-}{2\pi} \int \frac{\mathrm{d}^d \boldsymbol{q}}{(2\pi)^d} \int \mathrm{d}^D z \ \mathrm{e}^{-ixp^+z^- - iq^-z^+ + i(\boldsymbol{q}\cdot\boldsymbol{z})}$$
$$\times \left\langle M(p) \middle| \overline{\psi}(z)\,\Gamma_\lambda \psi(0) \middle| 0 \right\rangle \mathrm{tr}\left[H_2(xp+q)\,\Gamma^\lambda\right]$$

- Taylor expansion of the hard part

$$H_2(xp+q) = H_2(xp) + q_{\perp\mu}\left[\frac{\partial}{\partial q_{\perp\mu}} H_2(xp+q)\right]_{k=xp} + \mathrm{h.t.}$$

Light-cone collinear factorization

- 2-body factorized form up to twist-3

$$\mathcal{A}_2 = \frac{1}{4N_c}\int \mathrm{d}x\, p^+ \int \frac{\mathrm{d}z^-}{2\pi} \mathrm{e}^{-ixp^+z^-}$$
$$\times \left\{ \left\langle M(p) \left| \overline{\psi}(z^-)\, \Gamma_\lambda \psi(0) \right| 0 \right\rangle \mathrm{tr}\left[H_2(xp)\, \Gamma^\lambda \right] \right.$$
$$\left. + i \left\langle M(p) \left| \overline{\psi}(z^-)\, \overleftrightarrow{\partial}_{\perp\mu} \Gamma_\lambda \psi(0) \right| 0 \right\rangle \mathrm{tr}\left[\partial_\perp^\mu H_2(xp)\, \Gamma^\lambda \right] \right\}$$

- 3-body contribution $\to$ gauge invariant result

$$\mathcal{A}_3 = \int \frac{\mathrm{d}^D k_q}{(2\pi)^D} \frac{\mathrm{d}^D k_g}{(2\pi)^D} \int \mathrm{d}^D z_q \mathrm{d}^D z_g \mathrm{e}^{-i(k_q \cdot z_q) - i(k_g \cdot z_g)}$$
$$\times \left\langle M(p) \left| \overline{\psi}^{\,i}_\alpha(z_q)\, \Gamma_\lambda\; g A^a_\mu(z_g)\, \psi^j_\beta(0) \right| 0 \right\rangle \mathrm{tr}\left[H^{ija,\mu}_{3,\alpha\beta}(k_q, k_g)\, \Gamma^\lambda \right]$$

- 3-body contribution factorized

$$\mathcal{A}_3 = \frac{1}{2(N_c^2-1)} \int \mathrm{d}x_q \mathrm{d}x_g \left(p^+\right)^2 \int \frac{\mathrm{d}z_q^-}{2\pi} \frac{\mathrm{d}z_g^-}{2\pi} \mathrm{e}^{-ix_q p^+ z_q^- - ix_g p^+ z_g^-}$$
$$\times \left\langle M(p) \left| \overline{\psi}(z_q^-)\, \Gamma_\lambda g A_\mu(z_g^-)\, \psi(0) \right| 0 \right\rangle \mathrm{tr}\left[t^b H_3^{\mu,b}(x_q p, x_g p)\, \Gamma^\lambda \right] .$$

Covariant collinear factorization

- 2- and 3-body operators in gauge invariant form

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,F^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

where

$$[z,0]=\mathcal{P}_{\exp}\left[ig\int_0^1 dt A^\mu\,(tz)\,z_\mu\right]$$

- Expansion of the amplitude in powers of $x^2=0$ (deviation from light-cone)
  [Balitsky, Braun (1989)]

- Correlators without gauge links can be easily related to the fully gauge invariant one at the **twist-3**

2-body contribution

- Dipole amplitude

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\Psi\left(x,\boldsymbol{r}\right) \int \mathrm{d}^d\boldsymbol{b}\, \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{b}} \left\langle P\left(p'\right) \middle| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \middle| P\left(p\right) \right\rangle \, .$$

- Wavefunction overlap

$$\Psi_2\left(x,\boldsymbol{r}\right) = e_f\delta\left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+}q_\mu\right)$$

$$\times \left[\phi_{\gamma^+}(x,\boldsymbol{r})\left(2x\bar{x}q^\mu - i(x-\bar{x})\frac{\partial}{\partial r_{\perp\mu}}\right) + \epsilon^{\mu\nu+-}\phi_{\gamma^+\gamma^5}(x,\boldsymbol{r})\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- 2-body correlators

$$\phi_{\Gamma^\lambda}(x,\boldsymbol{r}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M\left(p_M\right) \middle| \mathrm{tr}_c\overline{\psi}\left(r\right)\Gamma^\lambda\psi\left(0\right) \middle| 0 \right\rangle_{r^+=0}$$

3-body contribution

- 3-body amplitude

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3}\int dx_i \theta(x_i)\right)\delta(1-x_1-x_2-x_3)\int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1+x_2\boldsymbol{z}_2+x_3\boldsymbol{z}_3)}$$

$$\times\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)\left\langle P\left(p'\right)\left|\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}-\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}-\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}+\frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2}\right|P\left(p\right)\right\rangle$$

- Wavefunction overlap

$$\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right) = \frac{e_q q^+}{2(2\pi)}\frac{N_c^2}{N_c^2-1}\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)$$

$$\times\left\{\chi_{\gamma^+\sigma}\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]\right.$$

$$\left.+\chi_{\gamma^+\gamma^5\sigma}\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]\right\}$$

- 3-body correlators

$$\chi_{\Gamma^\lambda,\sigma}\equiv\chi_{\Gamma^\lambda,\sigma}(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3)$$

$$=\int_{-\infty}^{\infty}\frac{\mathrm{d}z_1^-}{2\pi}\frac{\mathrm{d}z_2^-}{2\pi}\frac{\mathrm{d}z_3^-}{2\pi}\mathrm{e}^{-ix_1q^+z_1^--ix_2q^+z_2^--ix_3q^+z_3^-}\left\langle M\left(p_M\right)\left|\overline{\psi}\left(z_1\right)\Gamma^\lambda F_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\right|0\right\rangle_{z^+_{1,2,3}}$$