Precision studies of gluon saturation in DIS at small x:

NLO corrections to SIDIS and double inclusive hadron production

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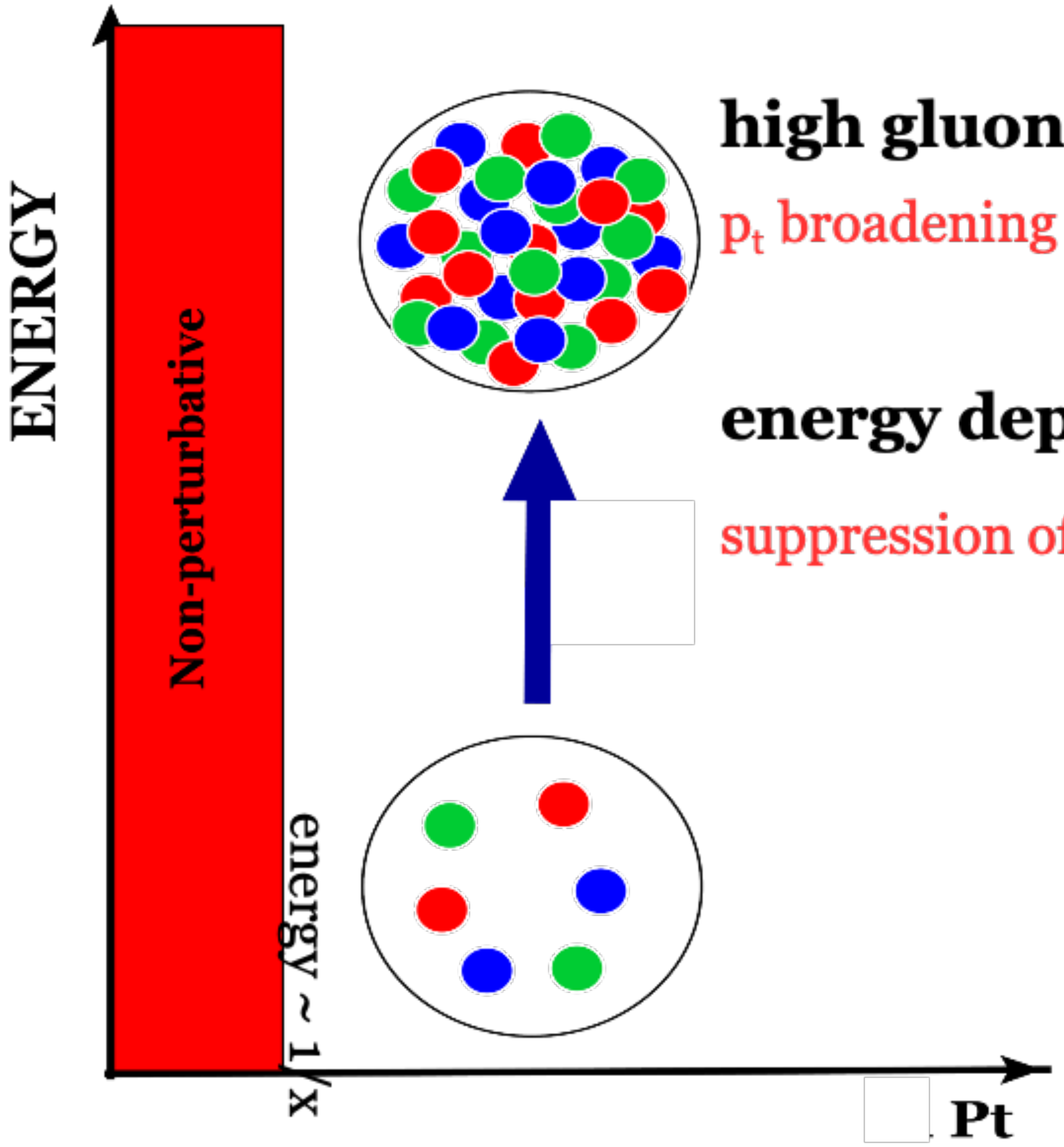
Grenoble*, France, 2024*

QCD at high energy: gluon saturation



high gluon density: Eikonal multiple scattering

p_t broadening

energy dependence: x-evolution via JIMWLK/BK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

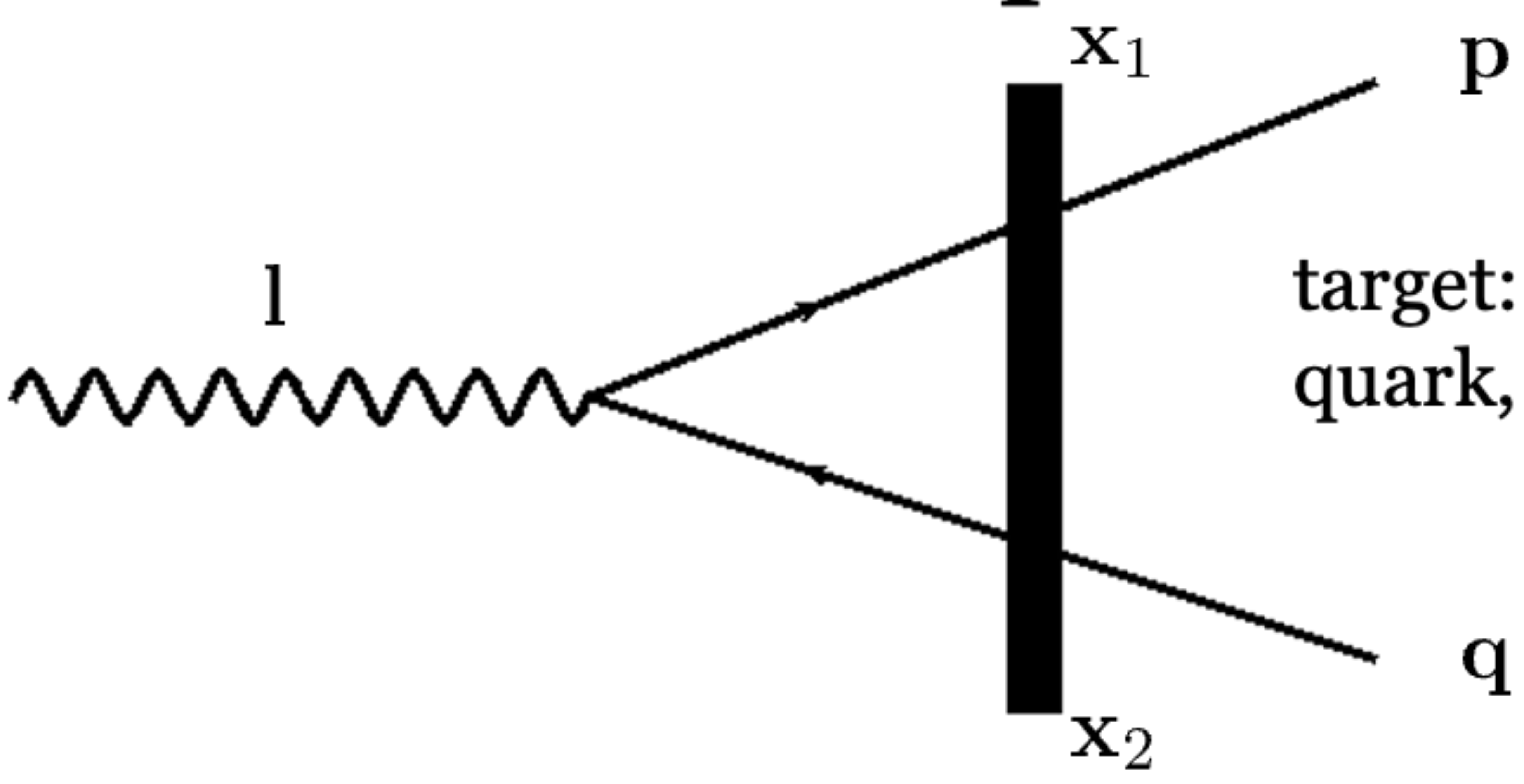
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

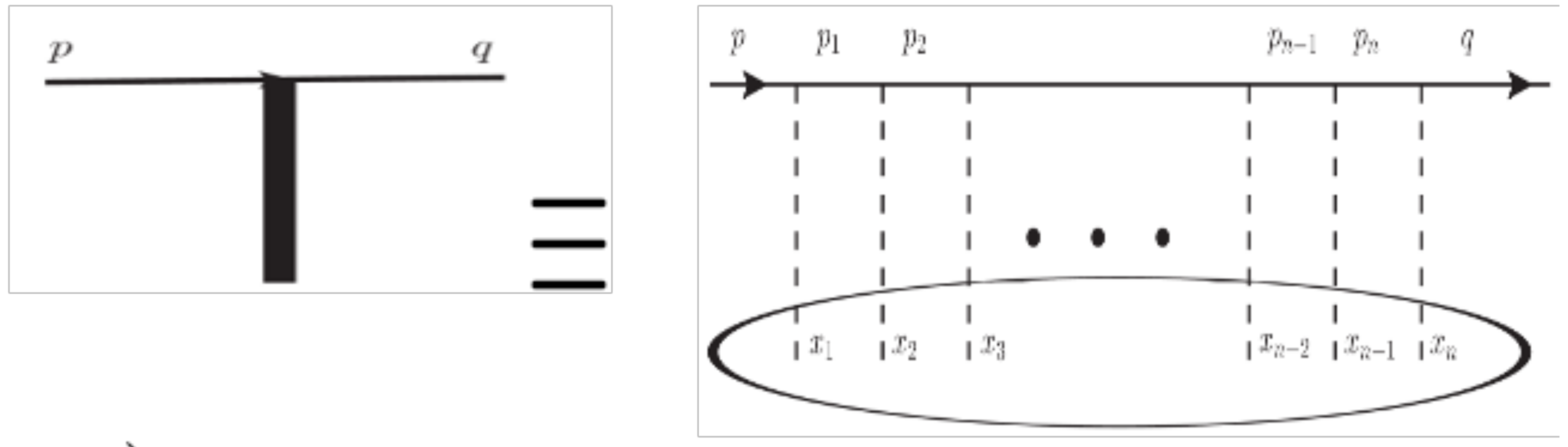
Shadowing/Nuclear modification factor
Azimuthal angular correlations (dihadrons/dijets,...)
Long range rapidity correlations (ridge,...)
Connections to TMDs,...

$$\mathbf{x \leq 0.01}$$

Inclusive dihadron production in <u>forward rapidity</u>: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8 x_\perp e^{ip\cdot(x_1'-x_1)} e^{iq\cdot(x_2'-x_2)} \left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4 z_1 z_2 K_0(|x_{12}|Q_1) K_0(|x_{1'2'}|Q_1) +$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\, K_1(|x_{12}|Q_1) K_1(|x_{1'2'}|Q_1)\Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

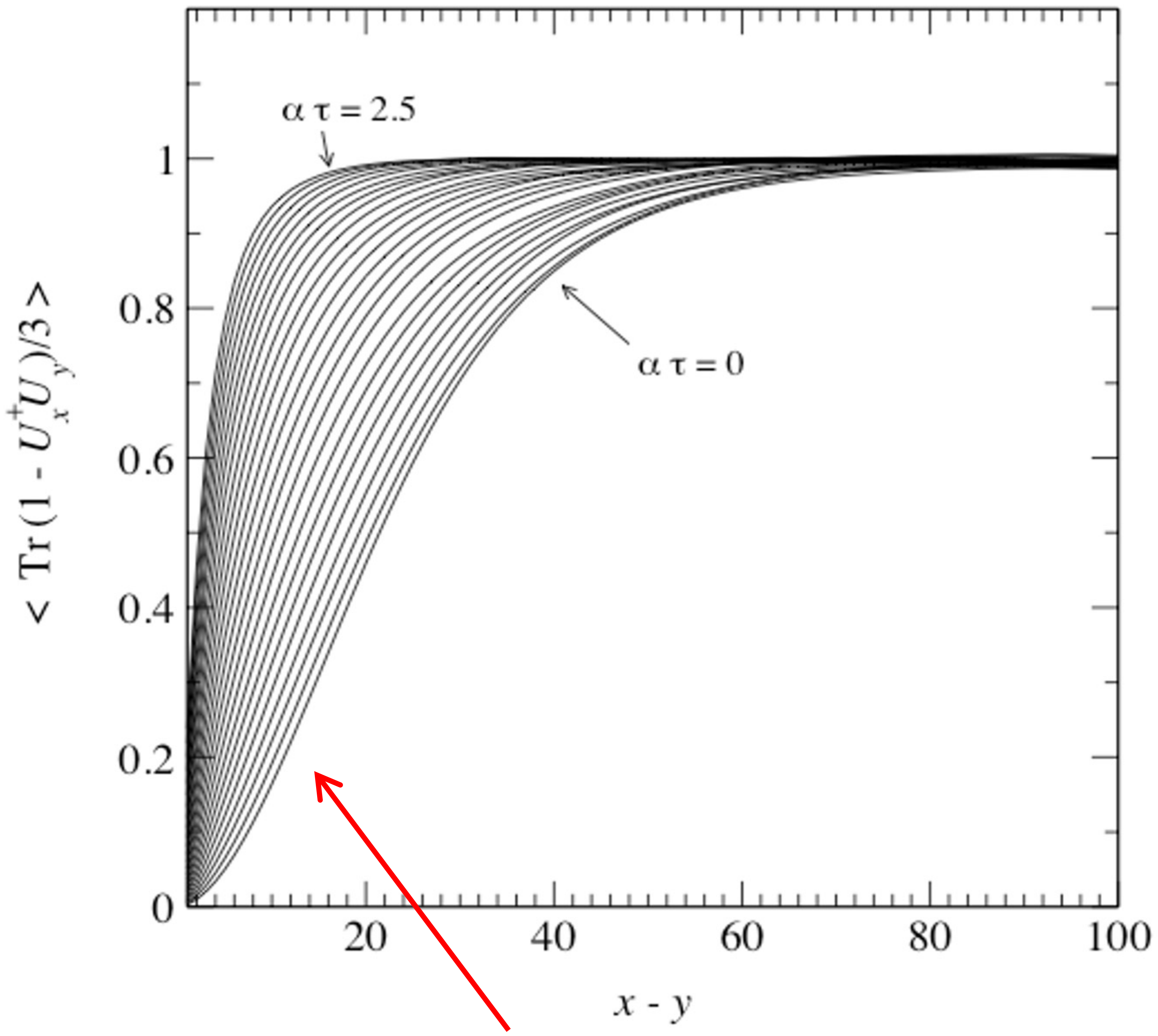
$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$

quadrupole

$S_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$

Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

Solution of BK evolution equation



$\sim \mathbf{r_t^2\, xG(x, 1/r_t^2)}$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

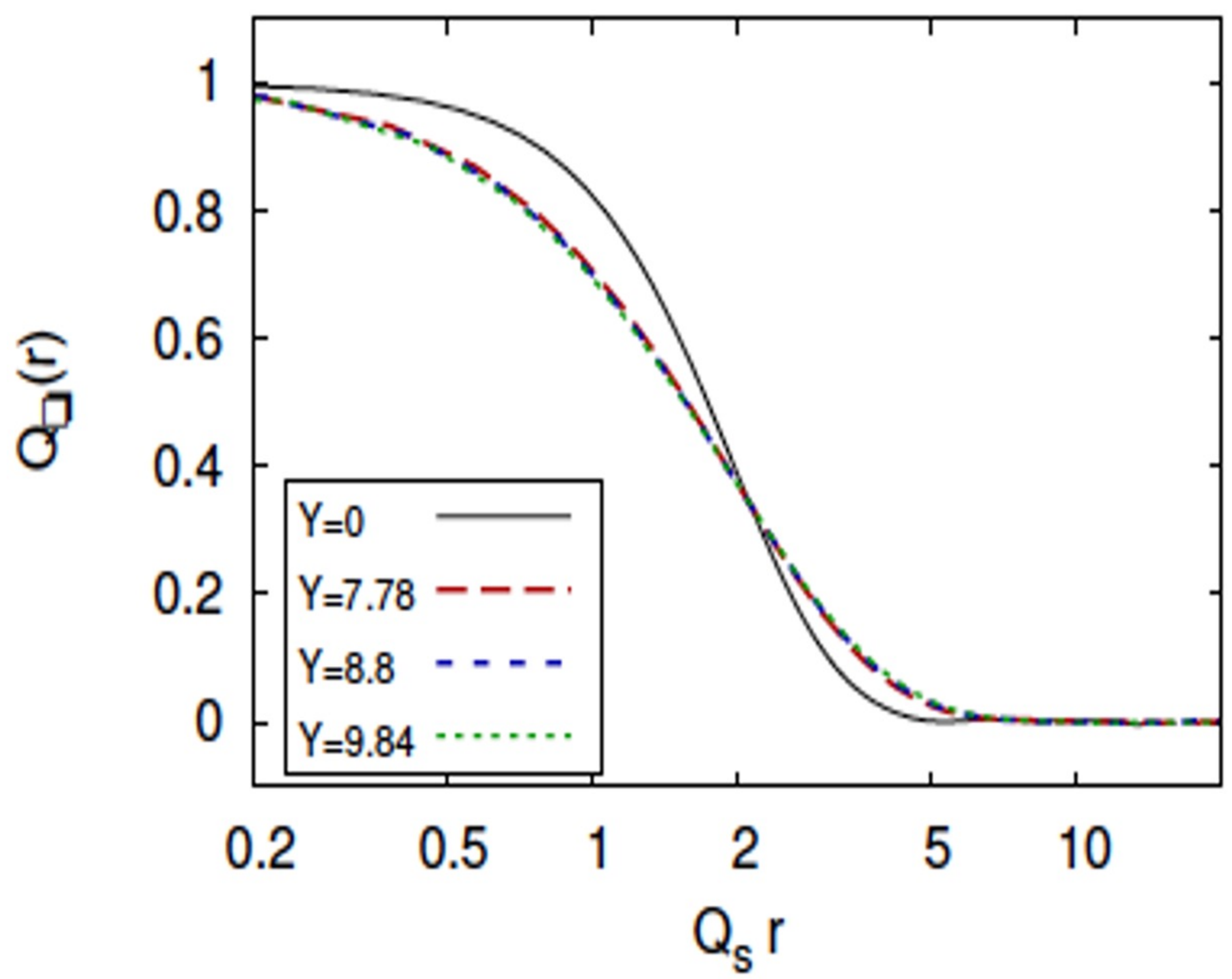
$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma \qquad Q_s^2 < p_t^2$$

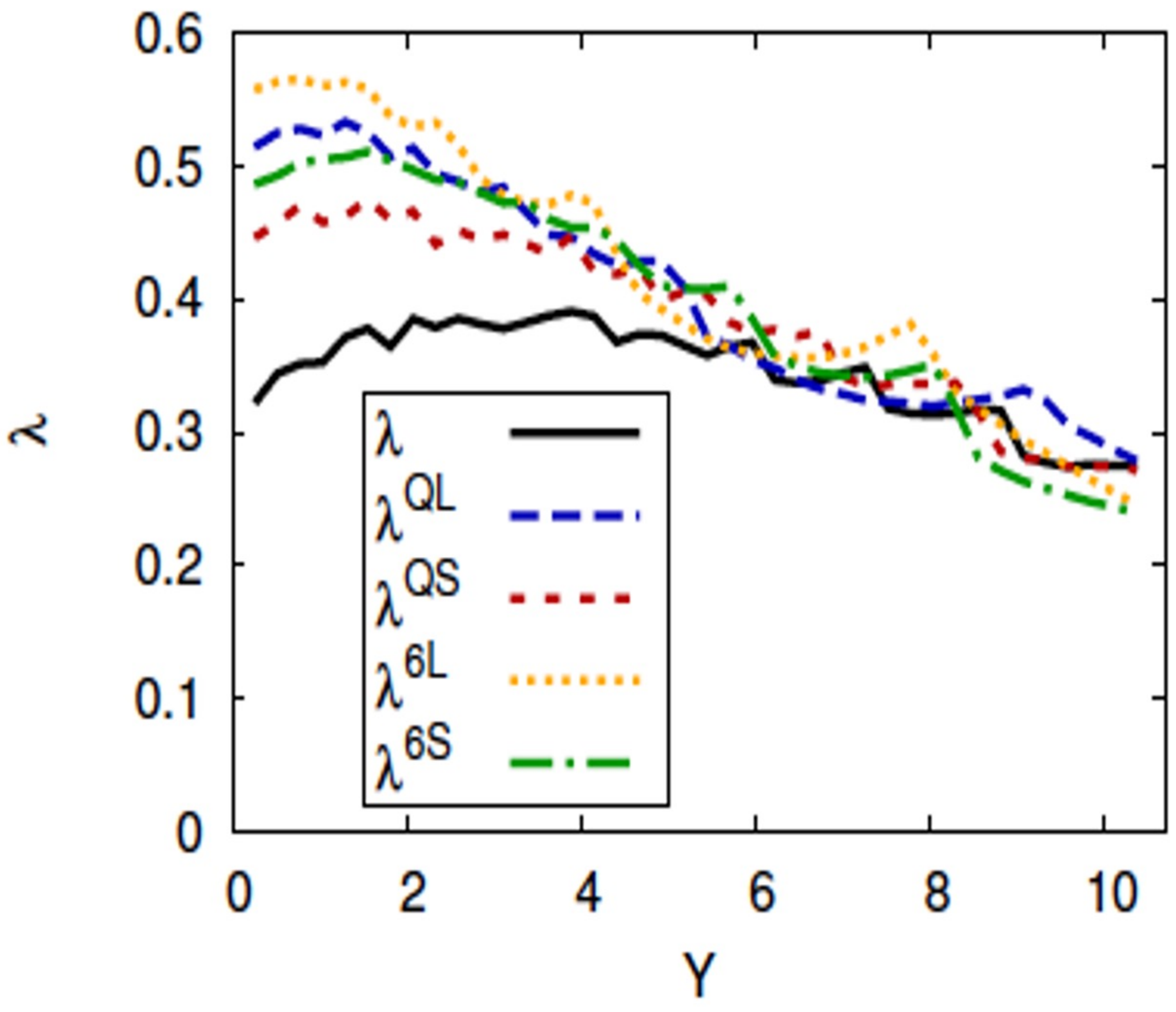
Rummukainen-Weigert, NPA739 (2004) 183

Quadrupole: JIMWLK evolution

Dumitru-JJM-Lappi-Schenke-Venugopalan: PLB706 (2011) 219

$$< Q(r, \bar{r}, \bar{s}, s) > \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$





scaling

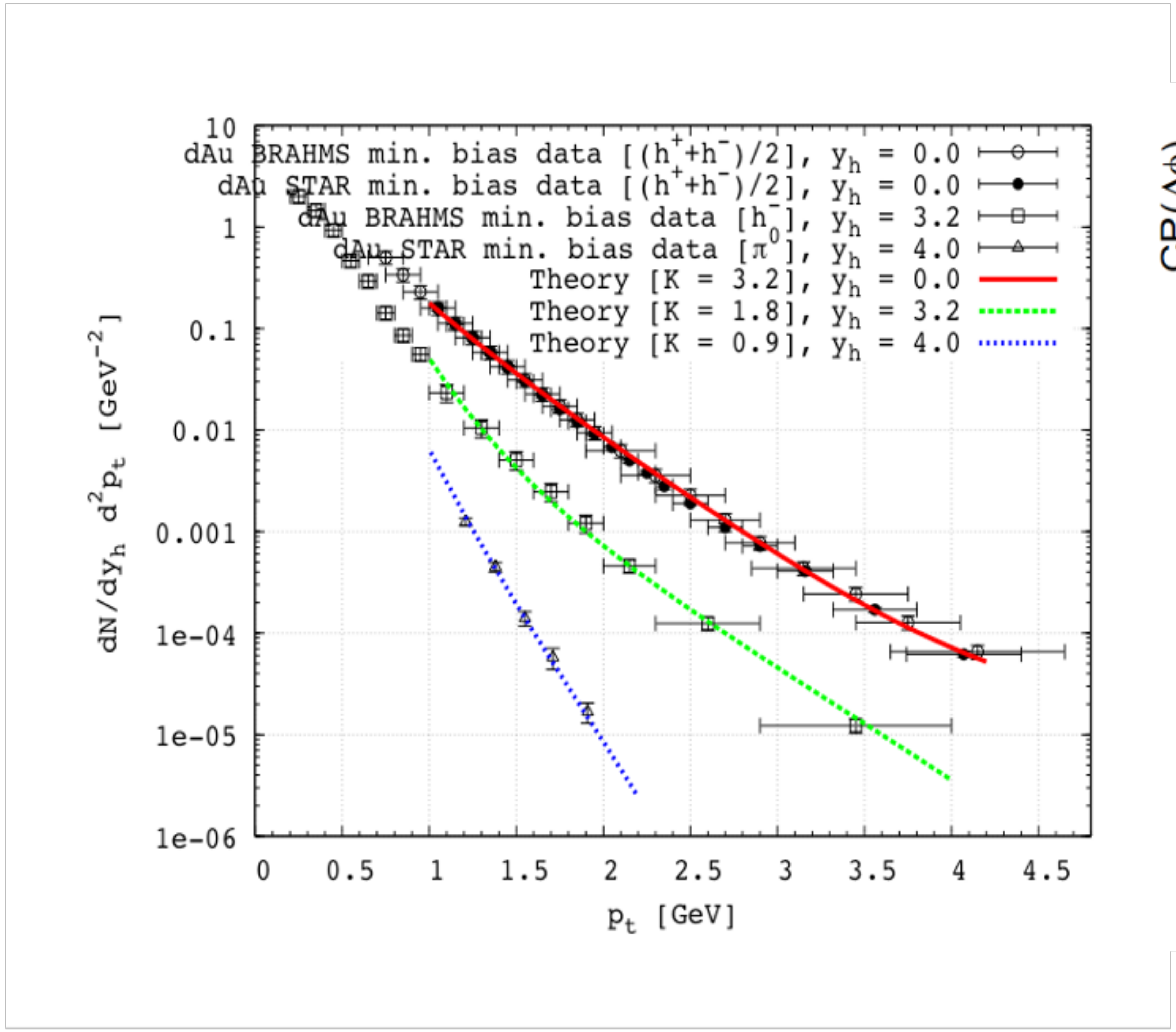
energy dependence

connection to/understanding from statistical physics: S. Munier,....

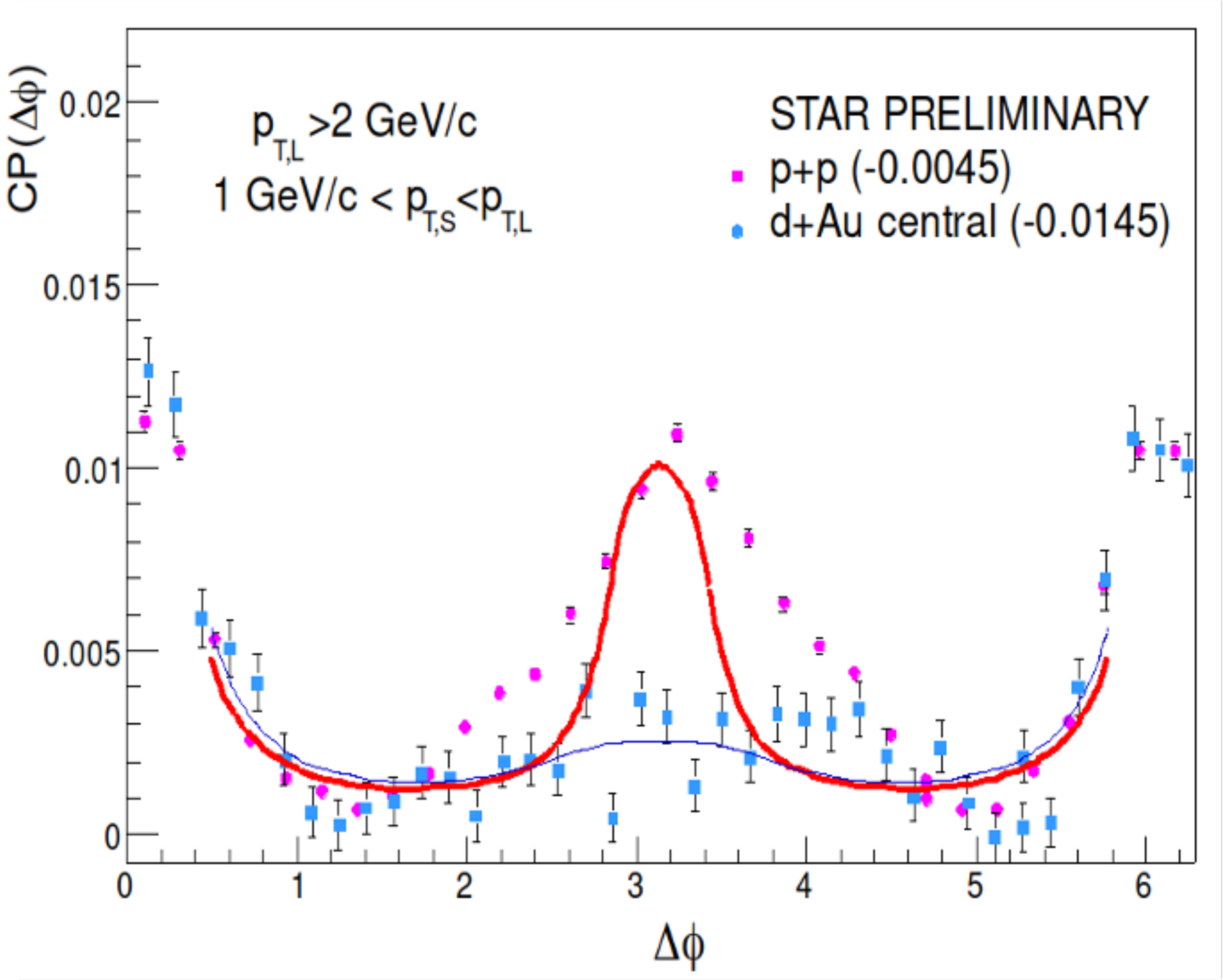
Fisher-Kolgomorov-Petrovsky-Piscounov (FKPP) equation

CGC at RHIC

Single and double inclusive hadron production in dA collisions



Dumitru,Hayashigaki, JJM, NPA770 (2006) 57



Albacete, Marquet, PRL105 (2010) 162301

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

NLO corrections to SIDIS

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),…………

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

……………………

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

……………………

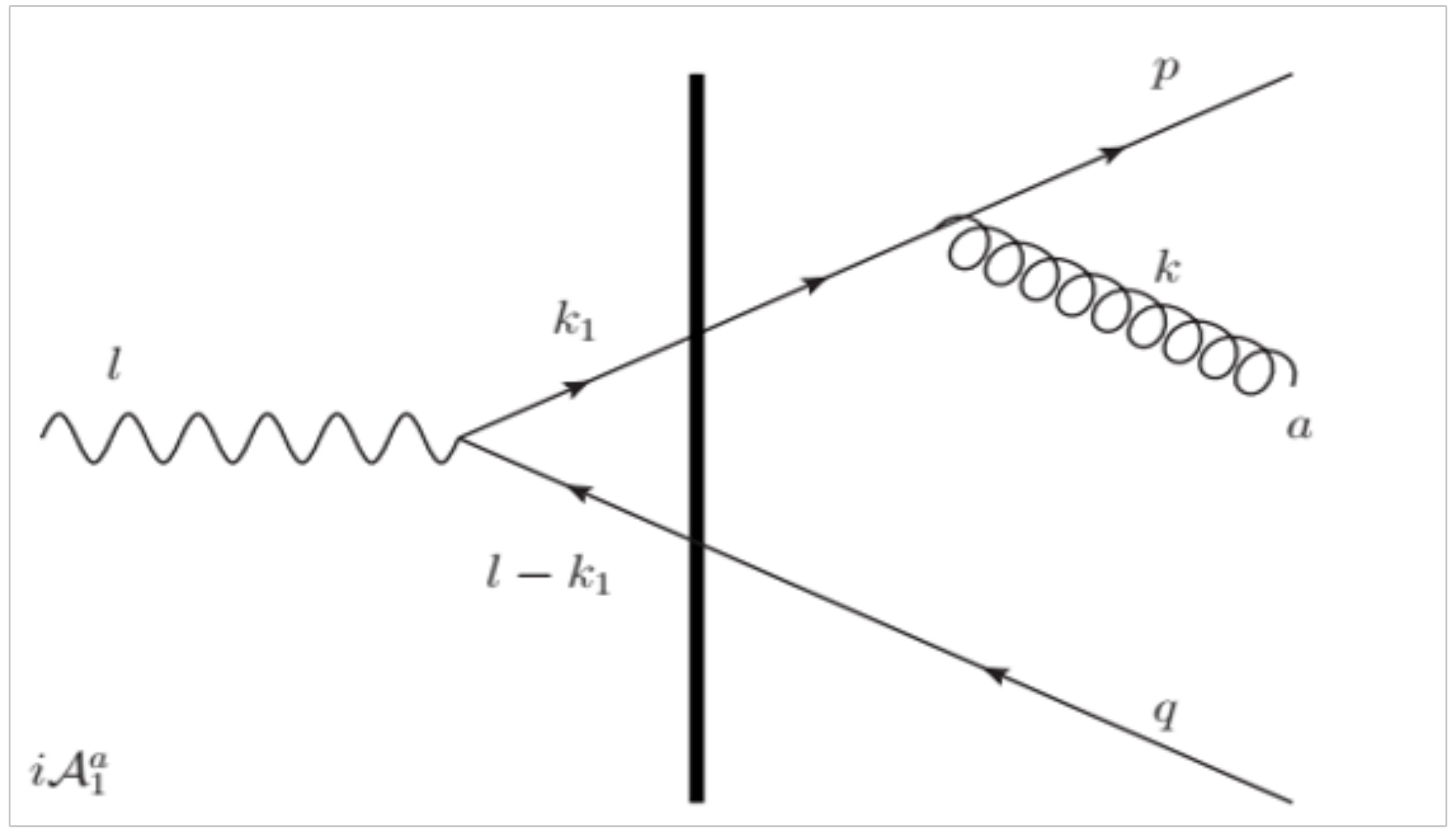
NLO corrections to exclusive light/heavy vector meson production (+)

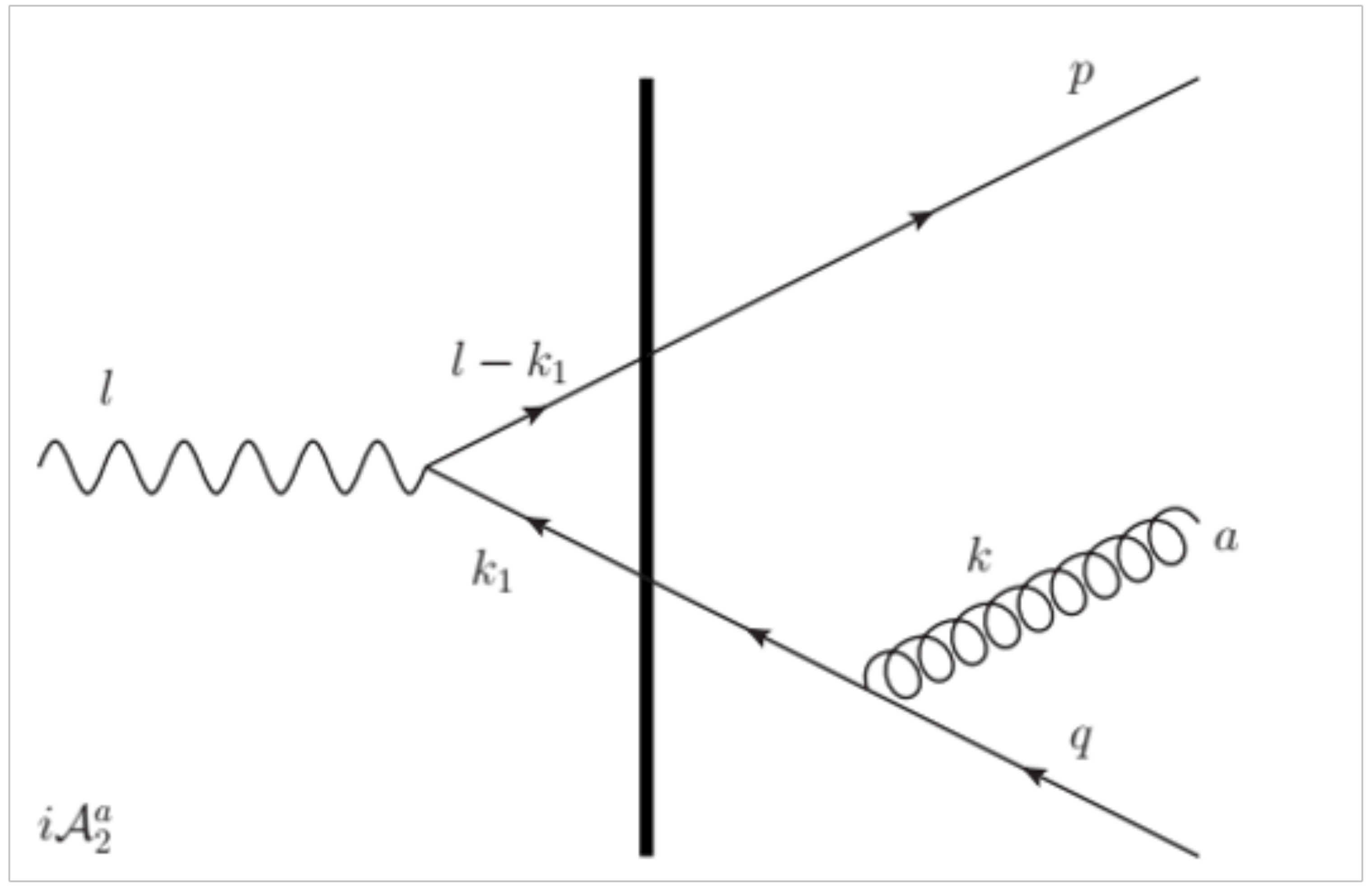
Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

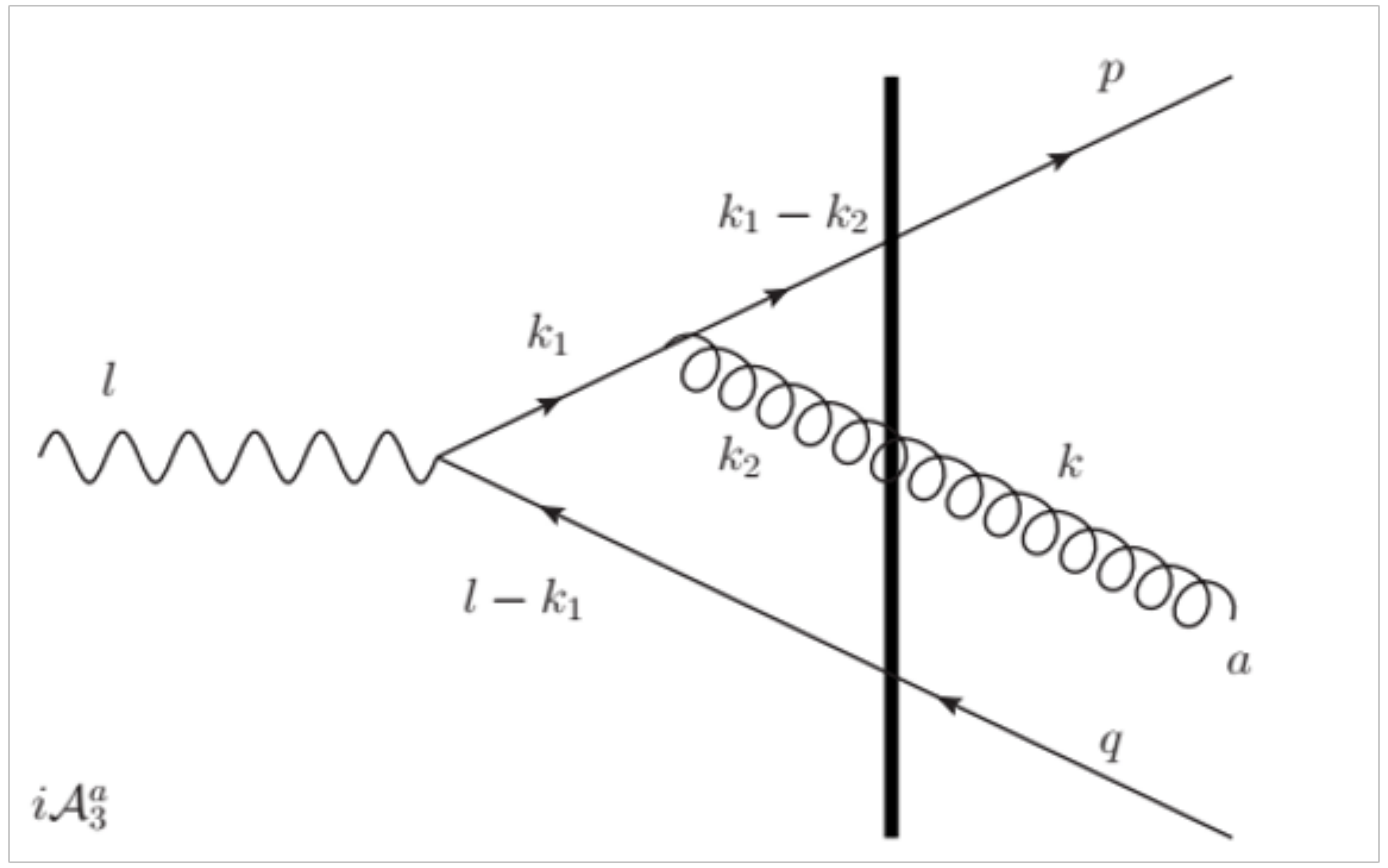
Mantyssari, Penttala (2021, 2022)

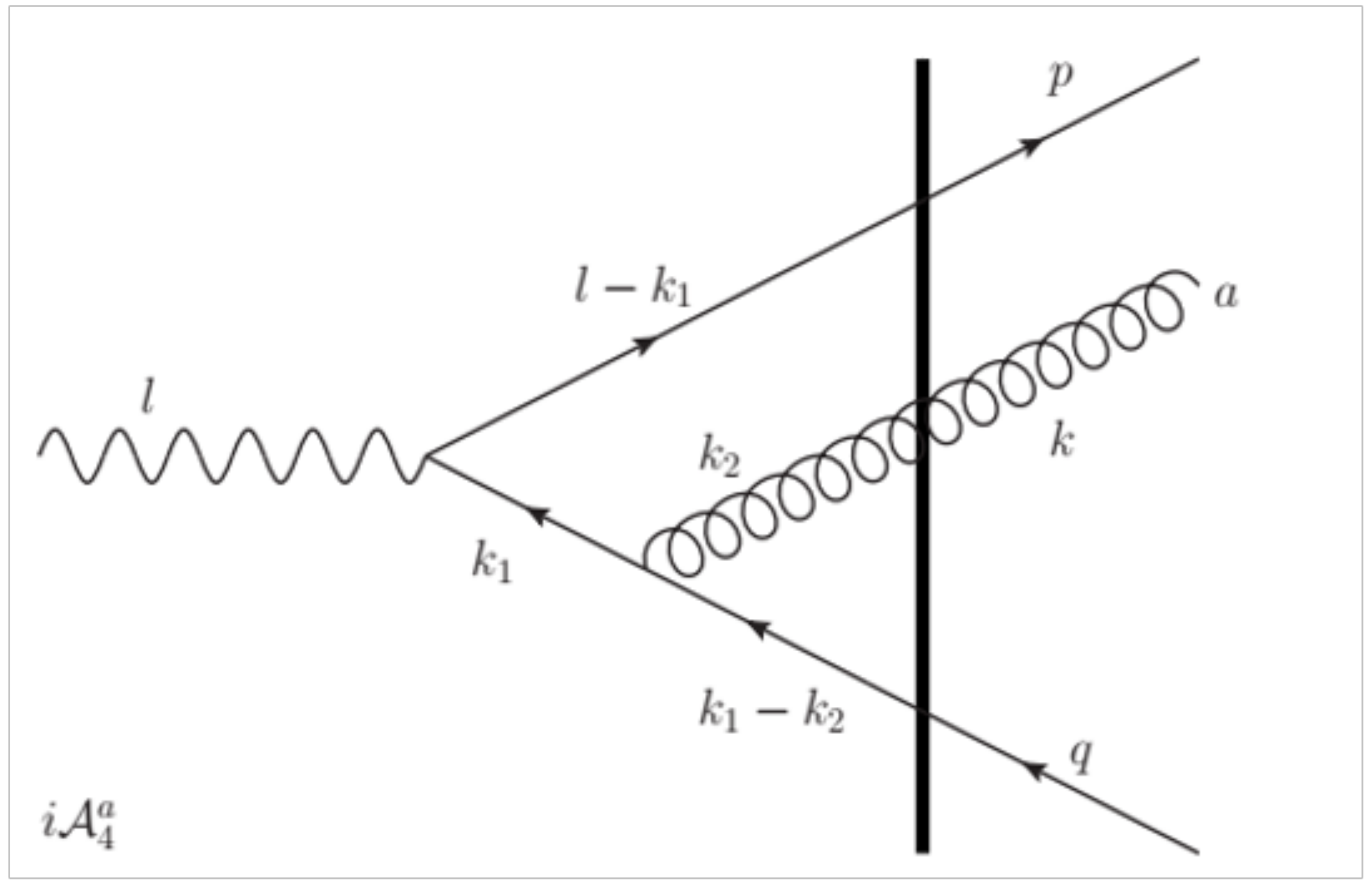
……………………

One loop corrections - real diagrams



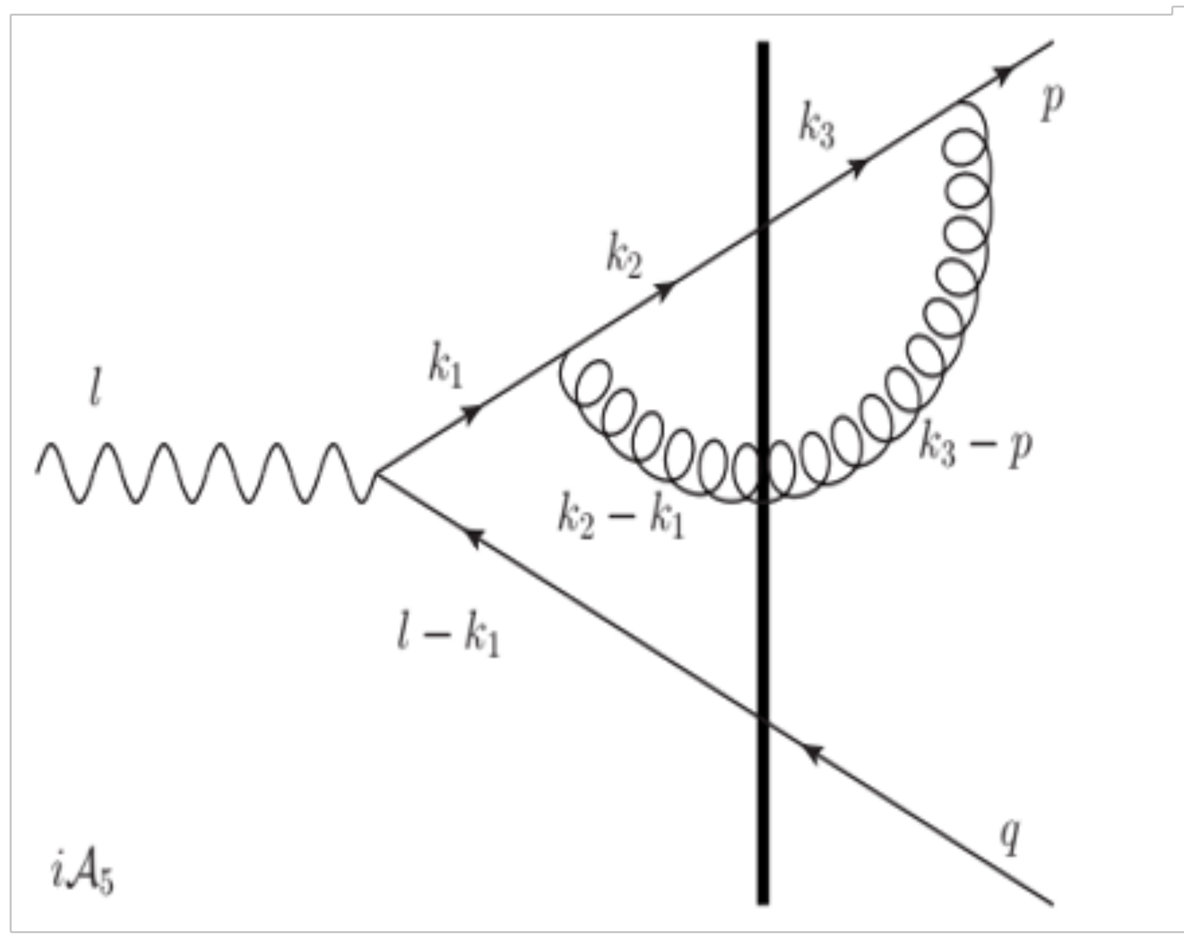


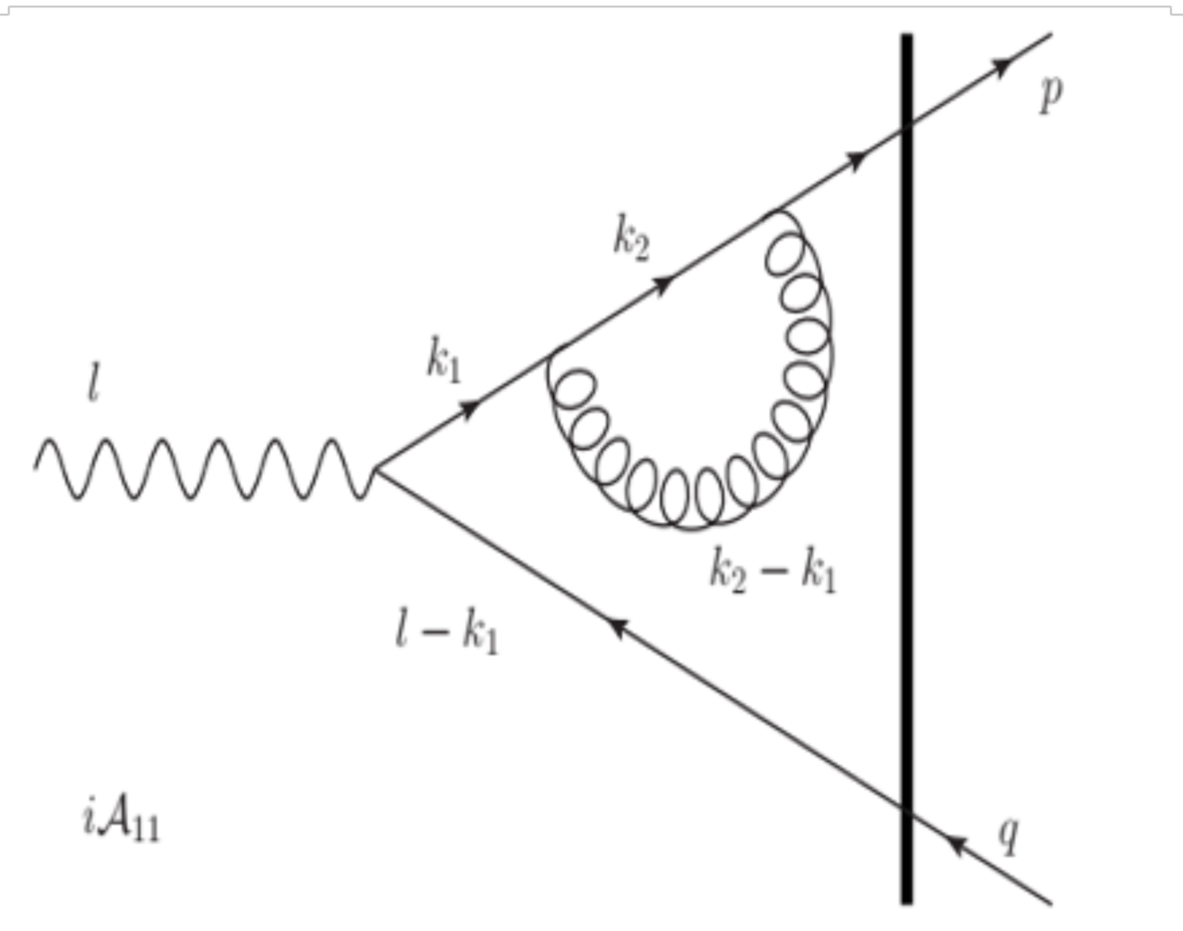


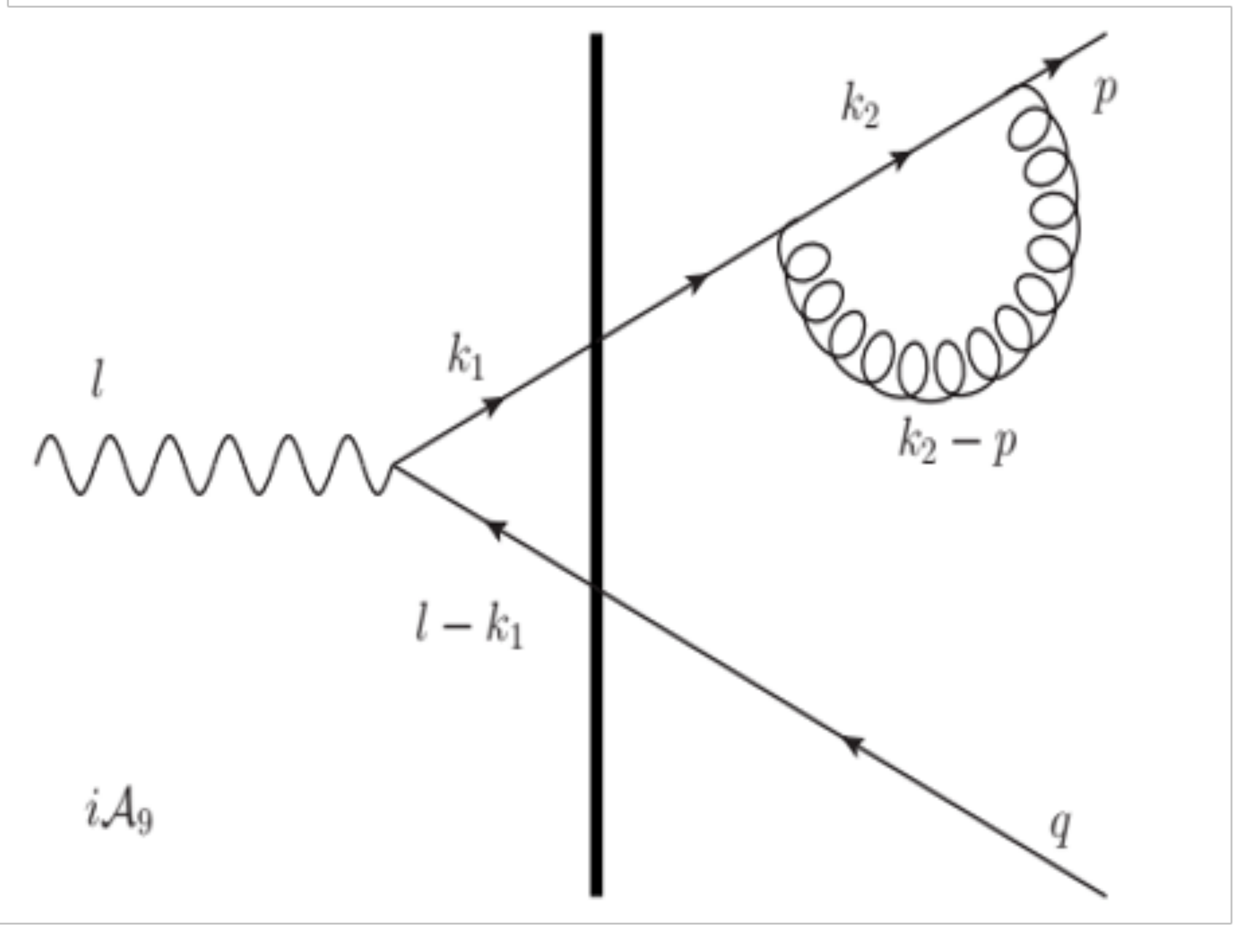


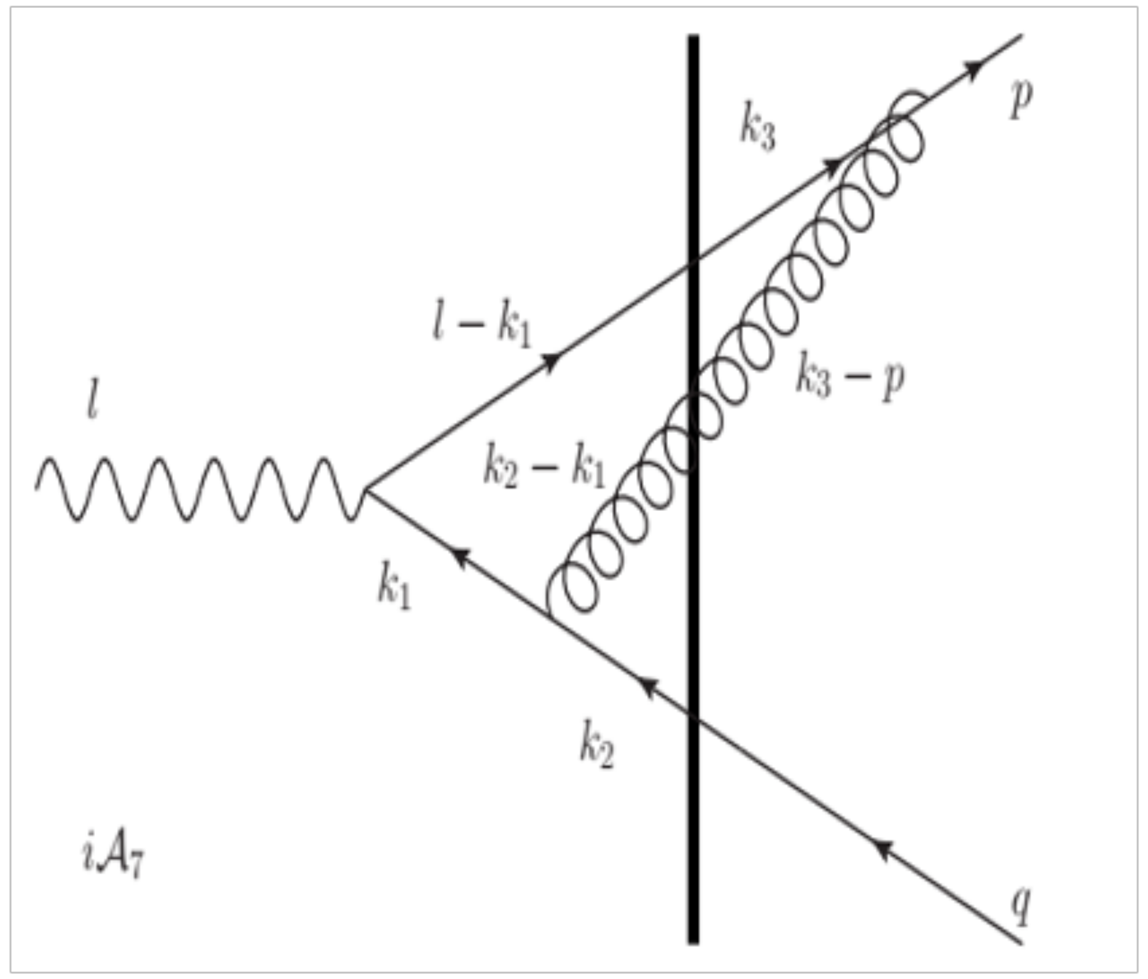
3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

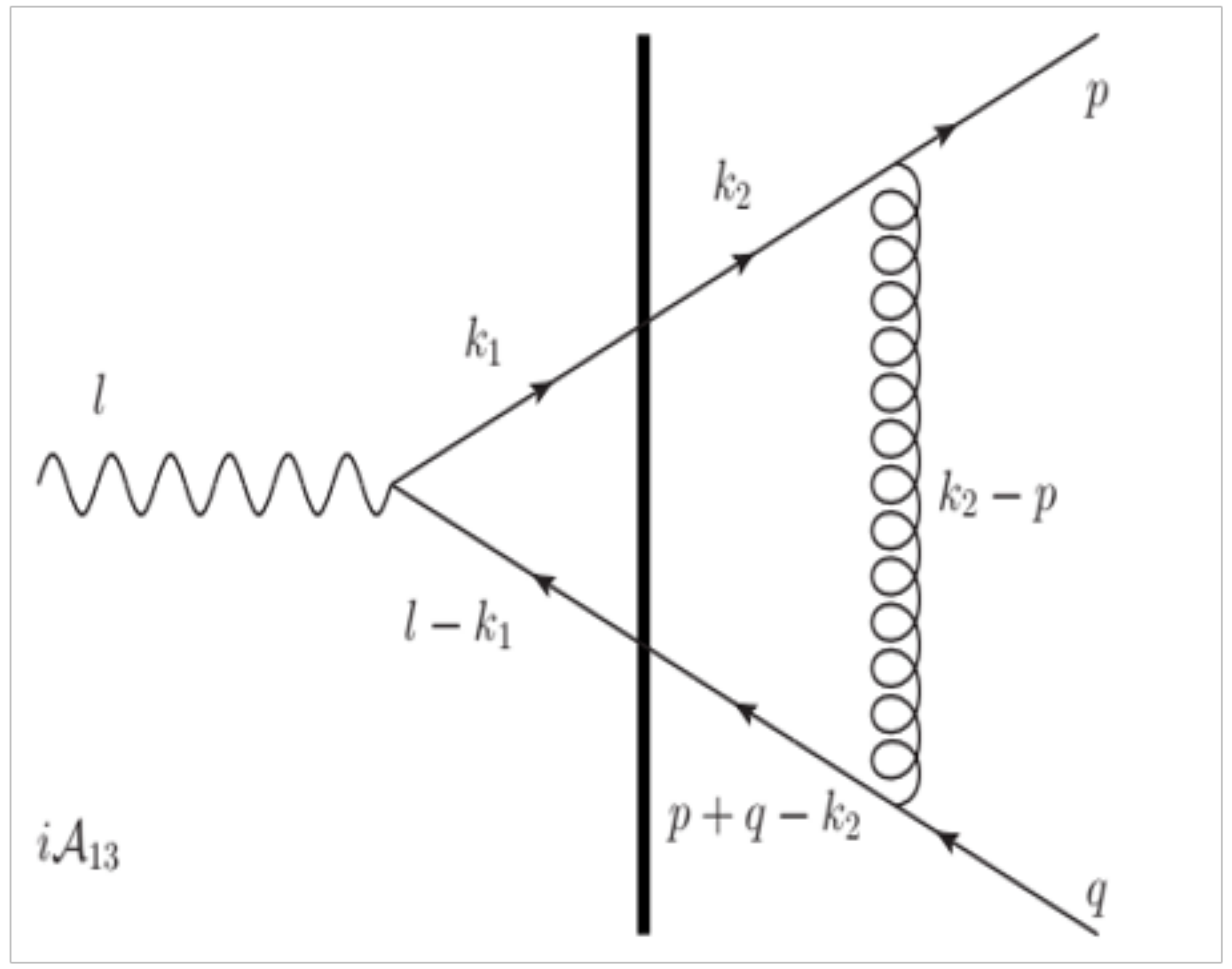
One loop corrections – virtual diagrams

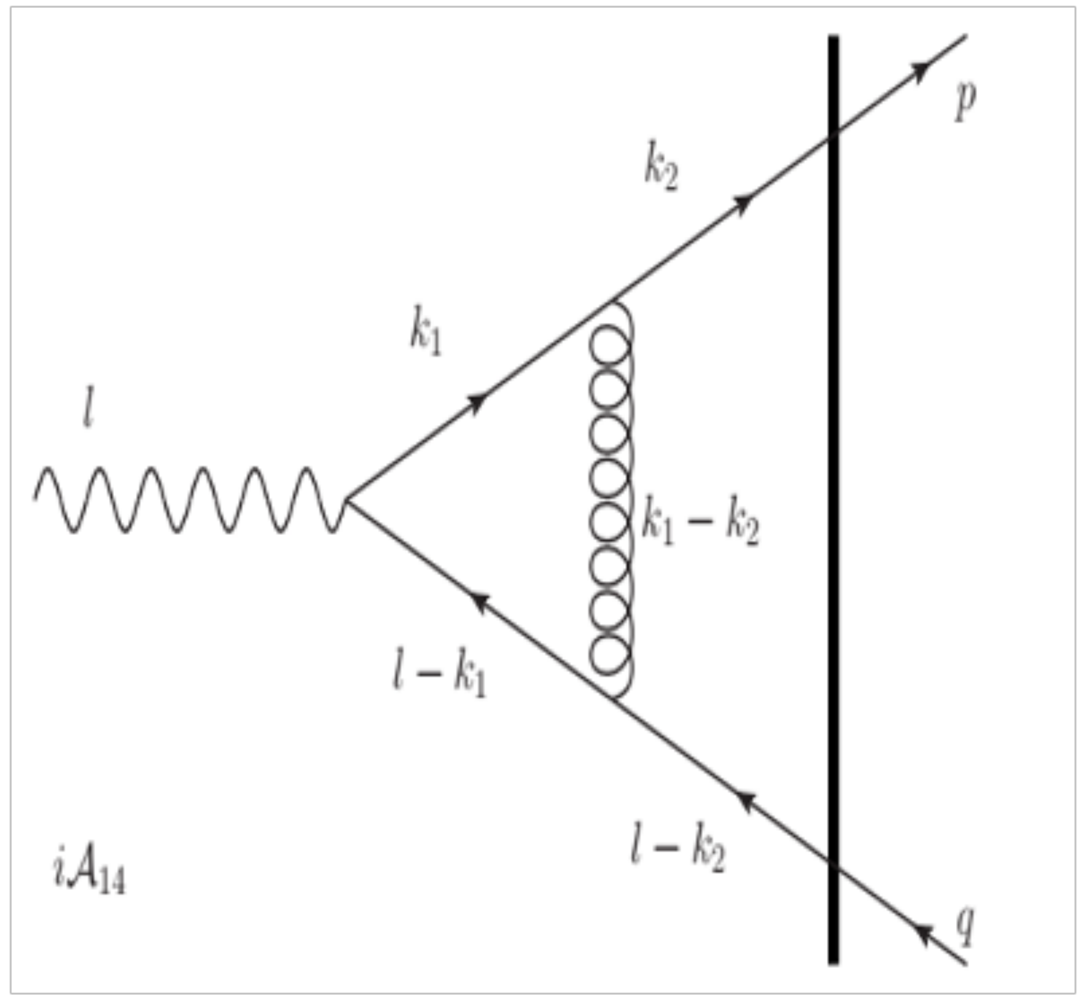












F. Bergabo and JJM, dihadrons, 2207.03606
P. Taels et al., dijets, 2204.11650
P. Caucal et al., dijets, 2108.06347

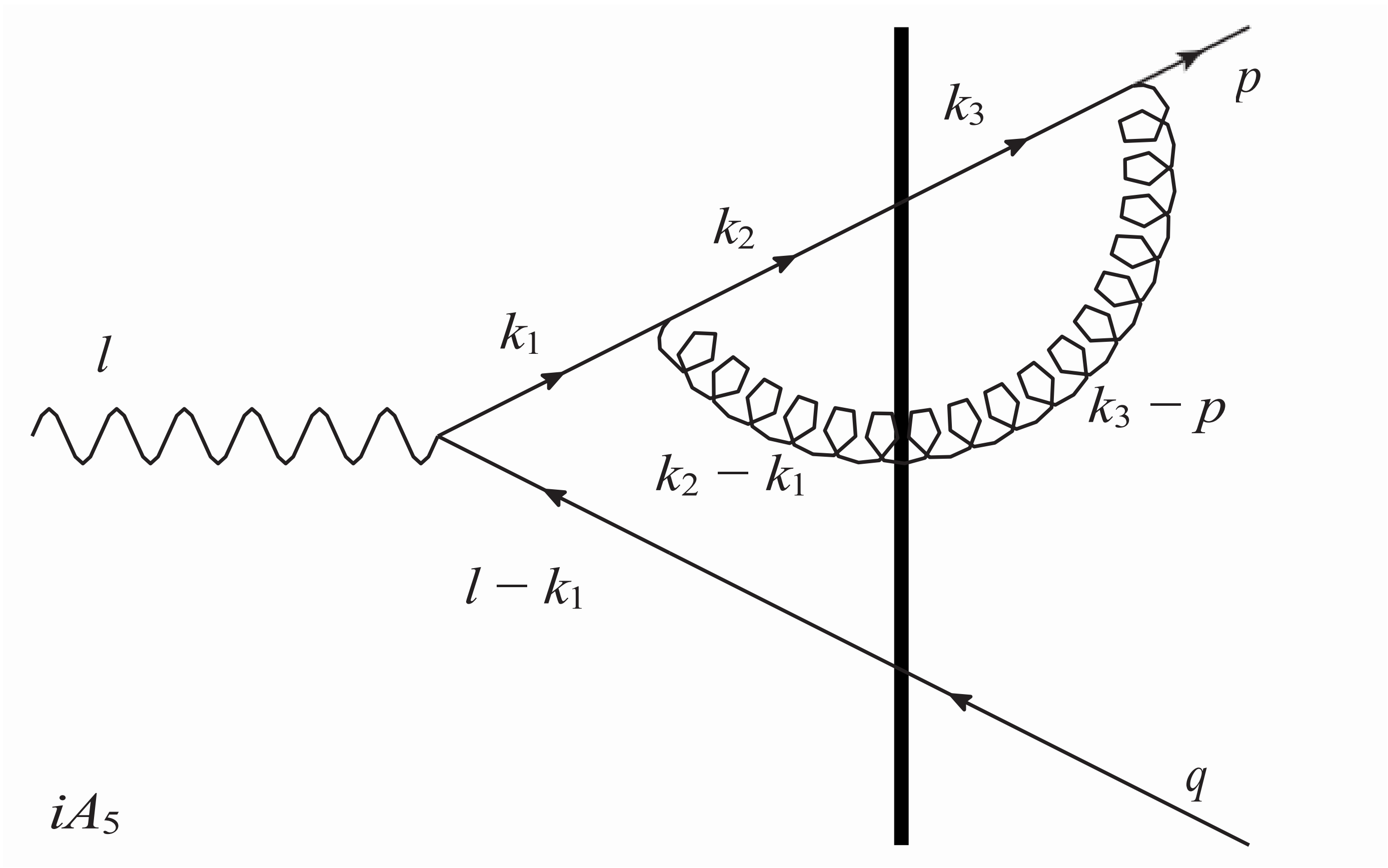
divergences

- ***Ultraviolet:***

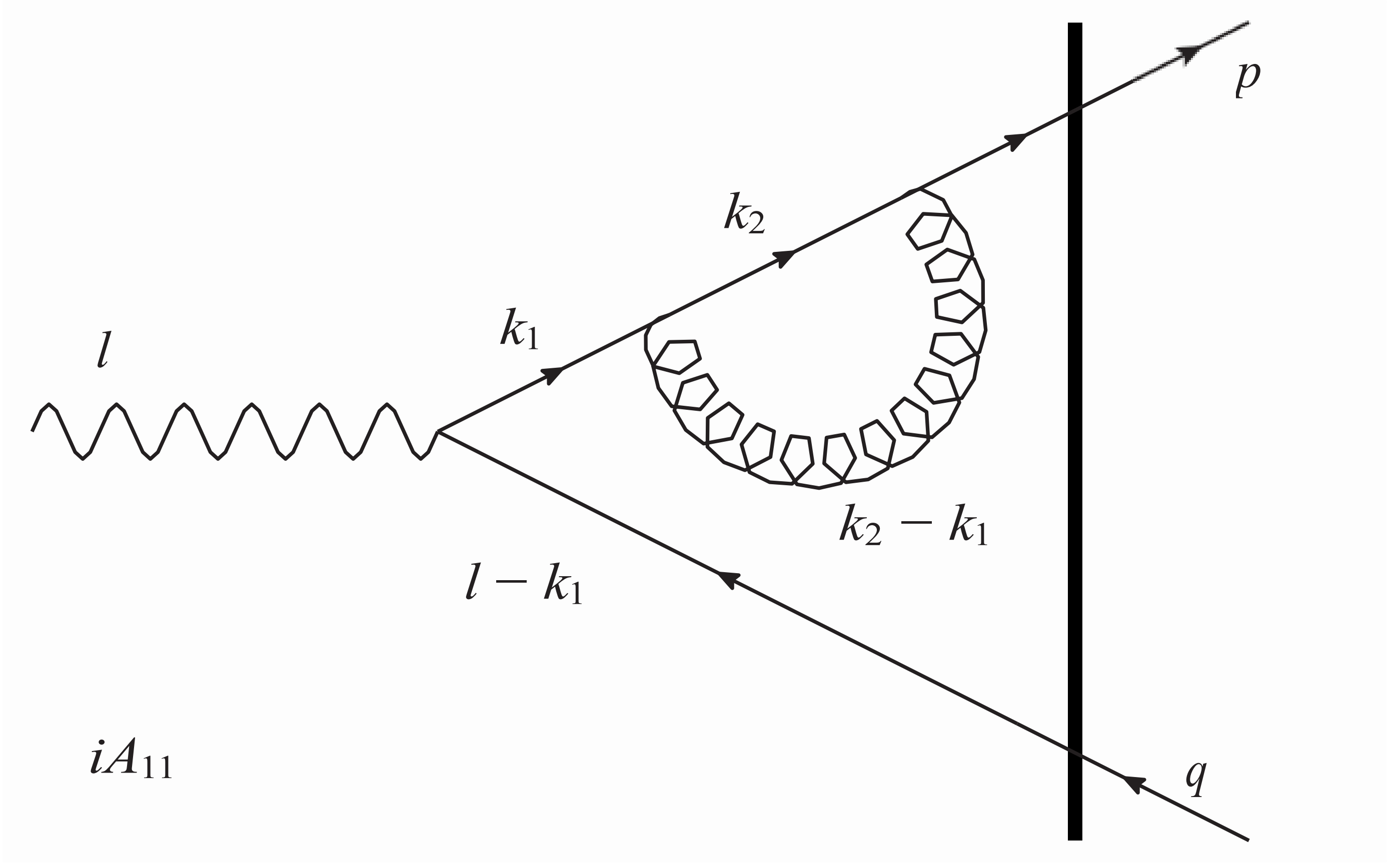
Real corrections are UV finite

UV divergences cancel among virtual corrections $\mathbf{k} \to \infty$ **or** $\mathbf{x_3} \to \mathbf{x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



$+$



$= 0$

divergences

- ***Soft:***

$$\mathbf{k}^{\mu} \to \mathbf{0} \;\; (\mathbf{x_3} \to \infty \;\;\; \mathbf{AND} \;\;\; \mathbf{z} \to \mathbf{0})$$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_{9})_{soft} = 0,$$

$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$

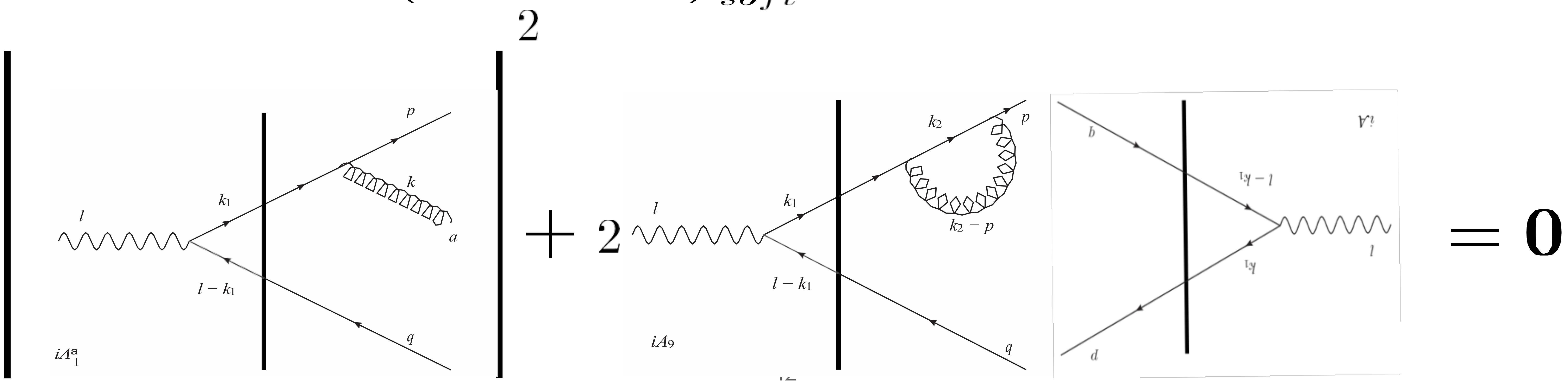
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$

$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$

$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$

$$(d\sigma_{5} + d\sigma_{7})_{soft} = 0$$

$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

- ***Rapidity:*** $\mathbf{z \to 0}$ ***, but finite*** $\boldsymbol{k_t}$

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}}\, \delta(1-z_1-z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde{\Delta}_{12} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'}\right) S_{132'1'} S_{23} + \left(\tilde{\Delta}_{1'2'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde{\Delta}_{12} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{21'}\right) S_{322'1'} S_{13} + \left(\tilde{\Delta}_{1'2'} + \tilde{\Delta}_{11'} - \tilde{\Delta}_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} + \tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'}\right) S_{122'1'} - \left(\tilde{\Delta}_{12} + \tilde{\Delta}_{1'2'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde{\Delta}_{11'} + \tilde{\Delta}_{22'} - \tilde{\Delta}_{12'} - \tilde{\Delta}_{21'}\right) S_{11'} S_{22'} - 2\tilde{\Delta}_{12}\left(S_{13}S_{23} - S_{12}\right) - 2\tilde{\Delta}_{1'2'}\left(S_{1'3}S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

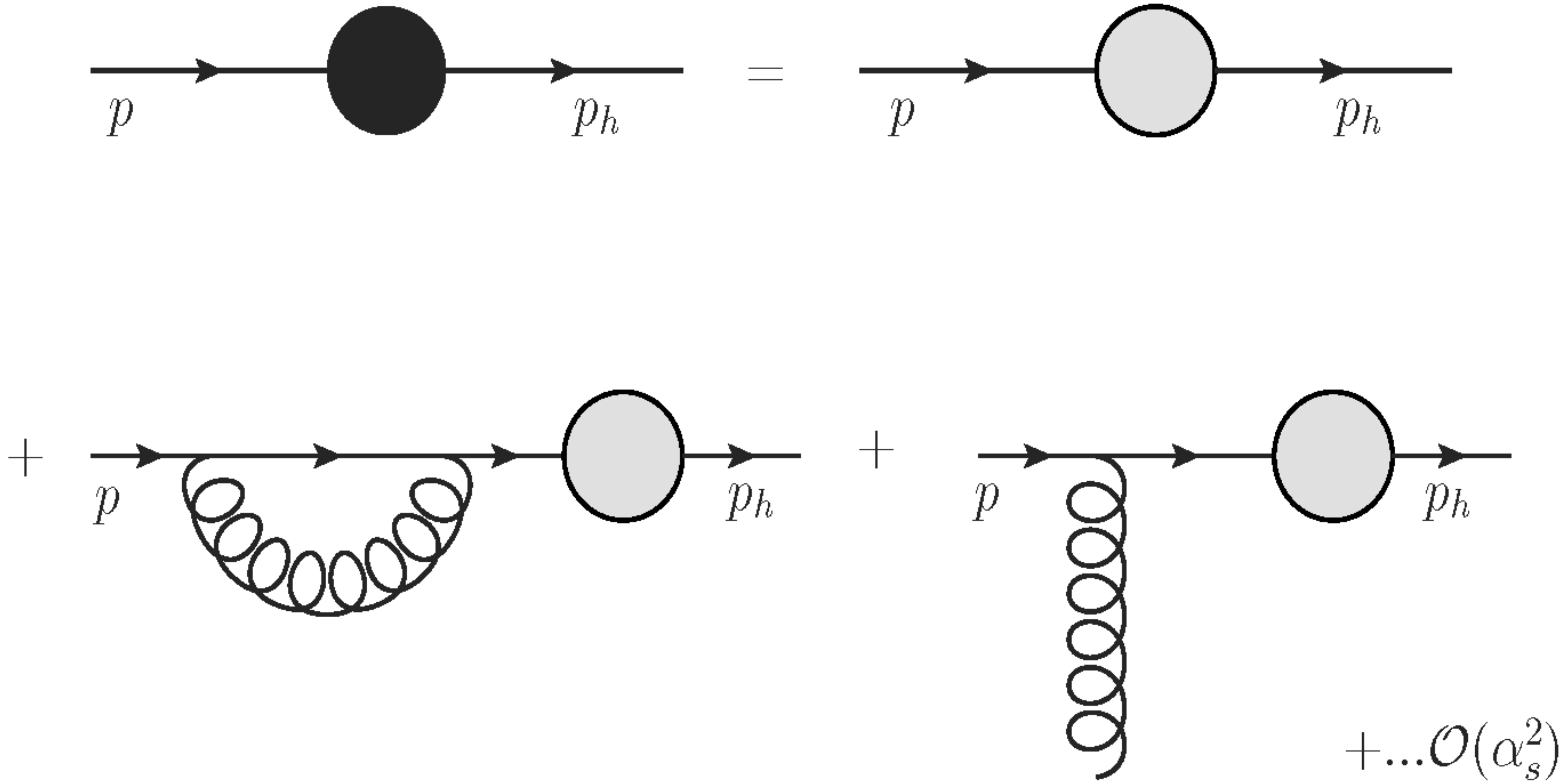
$$\tilde{\boldsymbol{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$

divergences

- ***Collinear:***

$$\frac{1}{(p+k)^2} = \frac{1}{|\vec{p}||\vec{k}|(1-cos\,\theta)} \longrightarrow \infty \quad as \quad \theta \to 0$$

Collinear divergences are absorbed into evolution of parton-hadron fragmentation functions



collinear divergences

real corrections

$$\frac{d\sigma_{LO+1-1}^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = \int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 N_c (z_1 z_2)^3}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H\left(\mathbf{p},\mathbf{q},z_2\right) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\int \frac{d\xi_1}{\xi_1^3}\delta(1-z_2-z_1/\xi_1)\left[\delta(1-\xi_1) + 2\alpha_s\, P_{qq}(\xi_1)\int d^2\mathbf{k}\frac{e^{i\mathbf{k}\cdot(\mathbf{x}_1'-\mathbf{x}_1)}}{\left(\xi_1\mathbf{k}-(1-\xi_1)\mathbf{p}\right)^2}\right]$$

with $$P_{qq}(\xi_1) = C_F\frac{(1+\xi_1^2)}{(1-\xi_1)}$$

virtual corrections

$$\frac{d\sigma_{9}^{\gamma^* A\to h_1 h_2 X}}{d^2\mathbf{p}_h\, d^2\mathbf{q}_h\, dy_1\, dy_2} = -\int_0^1 dz_{h_1} \int_0^1 dz_{h_2} \frac{4e^2 Q^2 (z_1 z_2)^3 N_c}{(2\pi)^7 (z_{h_1} z_{h_2})^2} H(\mathbf{p},\mathbf{q},z_2) D^0_{h_1/q}(z_{h_1}) D^0_{h_2/\bar{q}}(z_{h_2})$$

$$\times\, \alpha_s \int_0^1 d\xi\, P_{qq}(\xi) \int d^2\mathbf{k}\frac{1}{\left(\mathbf{k}-(1-\xi)\mathbf{p}\right)^2}\delta(1-z_1-z_2)$$

these are combined into DGLAP evolution of fragmentation functions

$$D_{h_1/q}(z_{h1},\mu^2) = \int_{z_{h1}}^1 \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h1}}{\xi}\right)\left[\delta(1-\xi) + \frac{\alpha_s}{2\pi}P_{qq}(\xi)\,\log\left(\frac{\mu^2}{\Lambda^2}\right)\right]$$

Divergences

•Ultraviolet

real corrections are UV finite

UV divergences cancel among virtual diagrams

•Soft

soft divergences cancel between real and virtual diagrams real and virtual diagrams

•Collinear

collinear divergences are absorbed into fragmentation functions

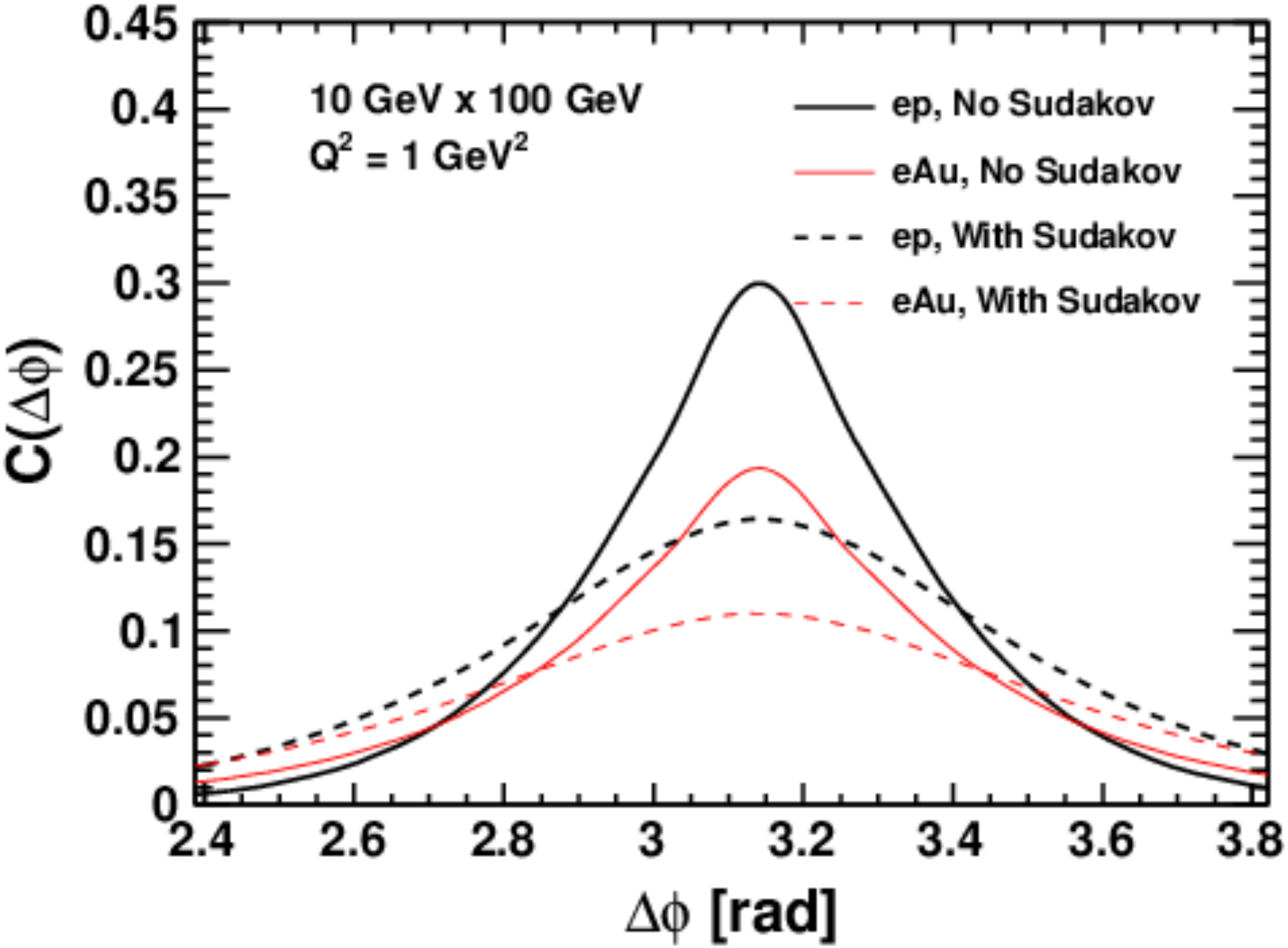
•Rapidity

Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A\to h_1 h_2 X} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\text{finite}}_{NLO}$$

Back to back limit: deep connections to physics of TMDs, Sudakov effect,....

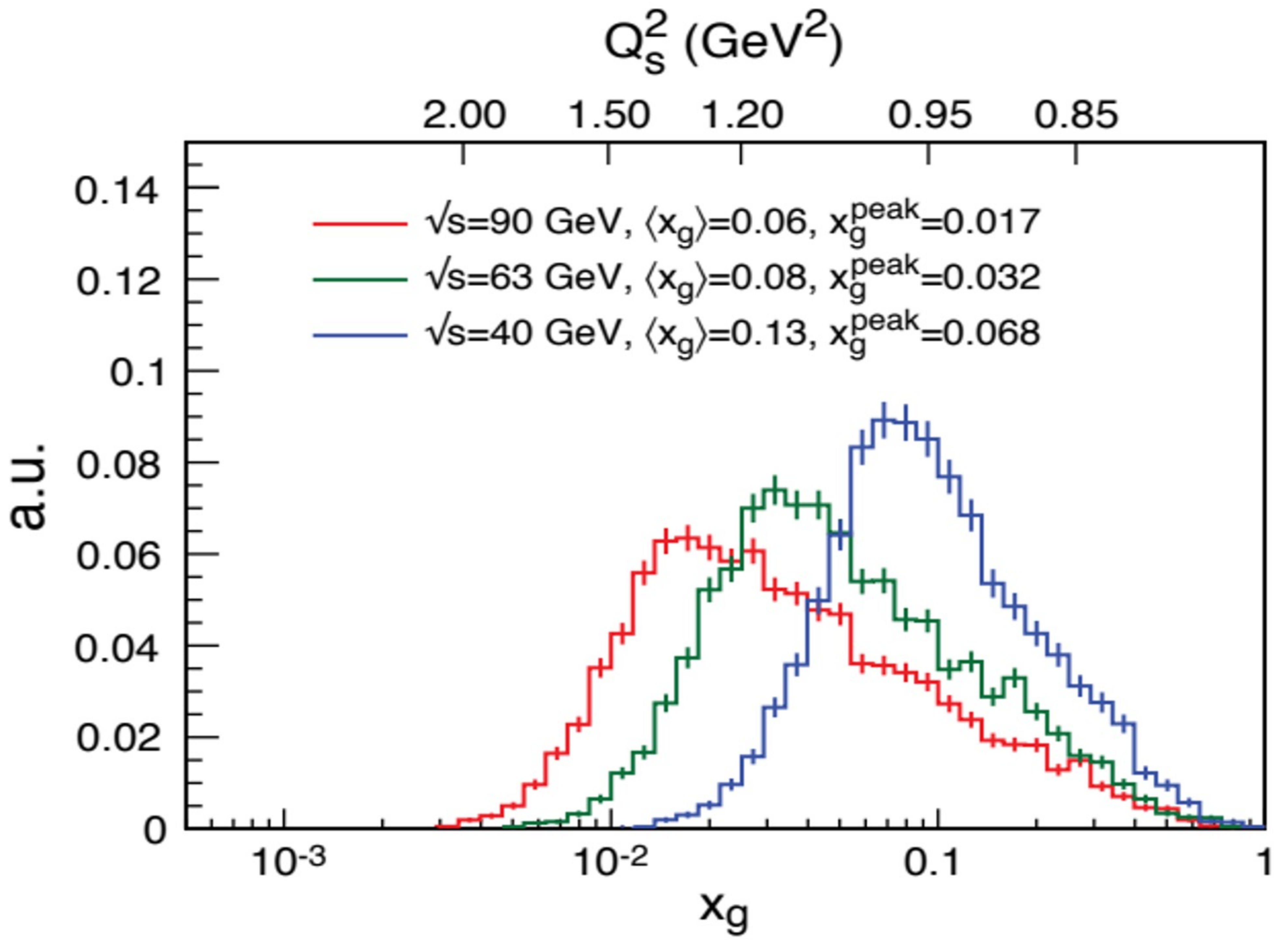
UPC ,.....



Zheng, Aschenauer, Lee, Xiao, PRD89 (2014) 074037

EIC

kinematics of inclusive dihadron production



Aschenauer et al. arXiv:1708.01527

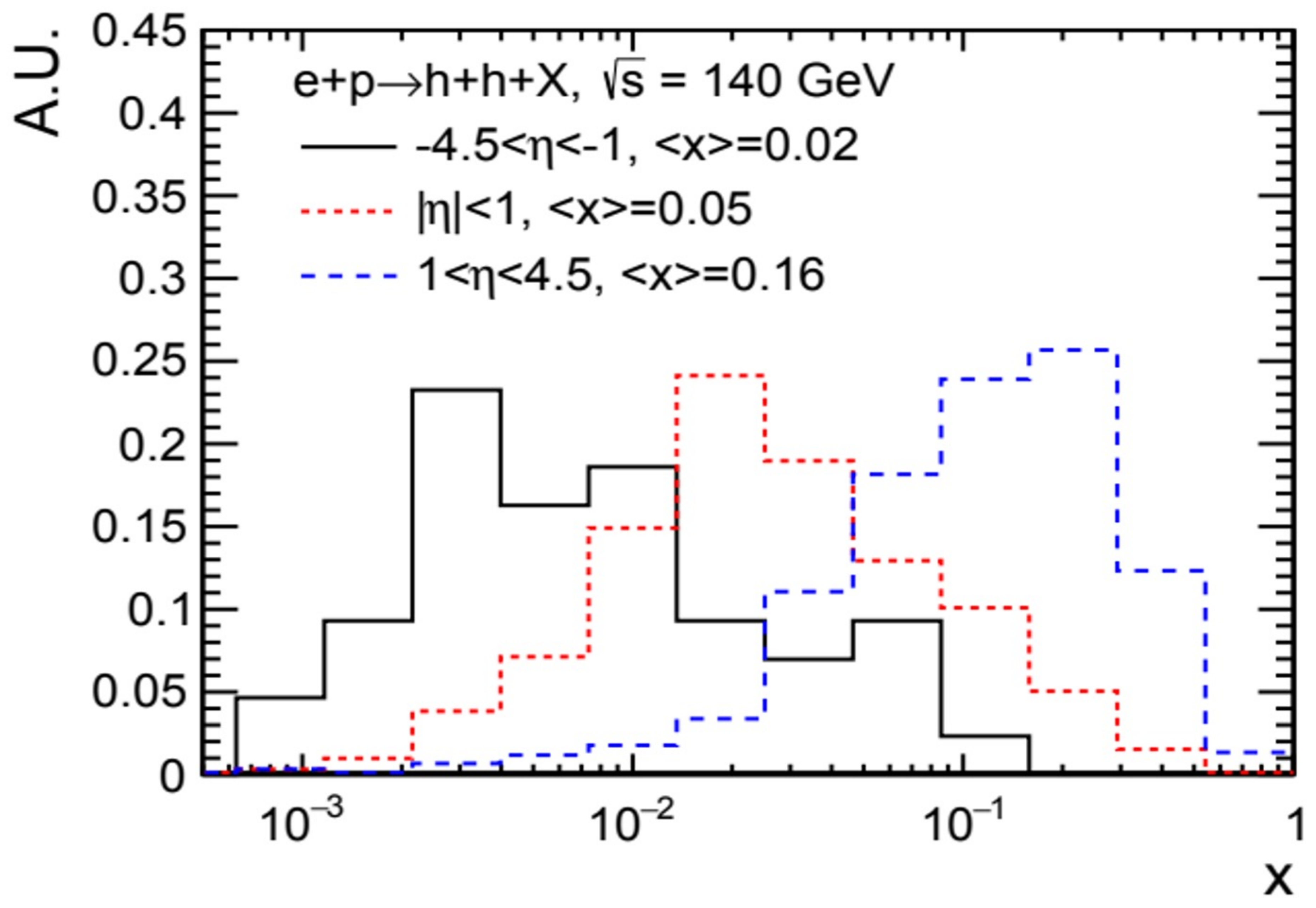


Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x

Single inclusive hadron production in DIS at small x: NLO

F. Bergabo, JJM, JHEP 01 (2023) 095 (longitudinal photon)

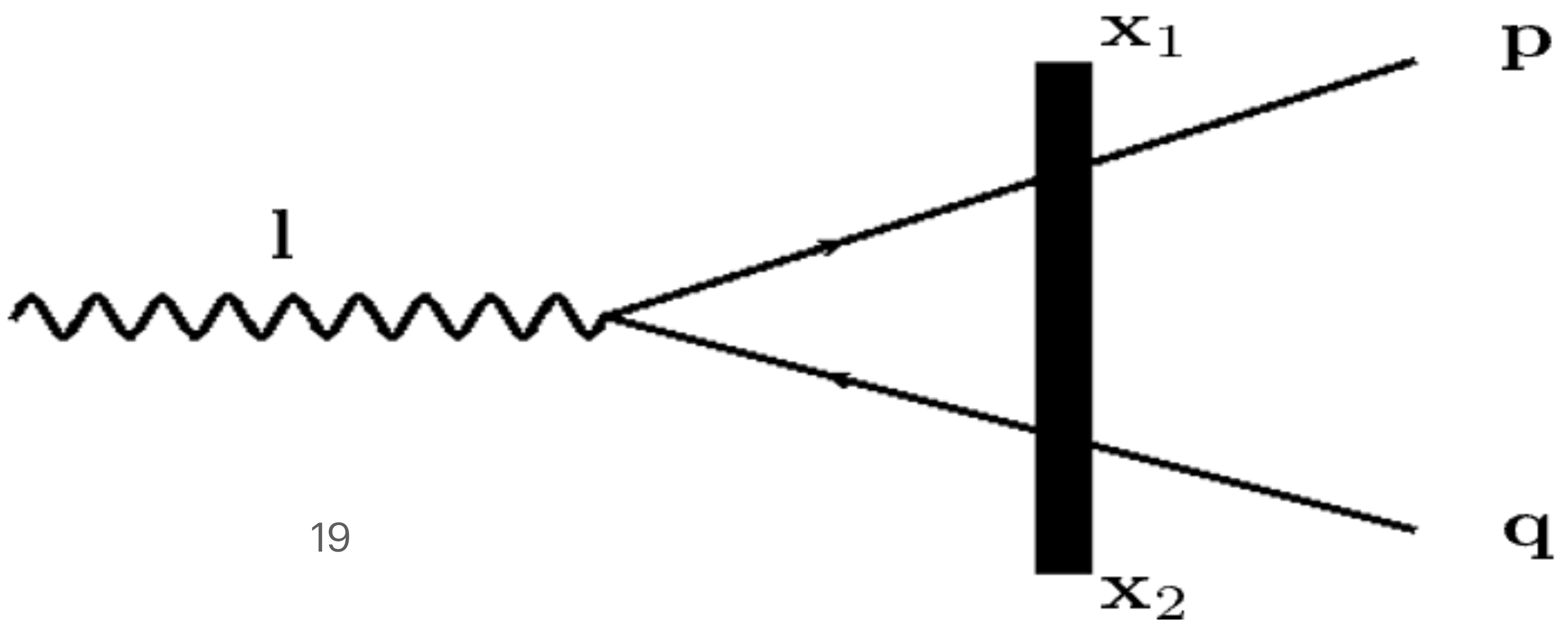
Larger kinematic phase space at EIC

Sudakov (can it be avoided?)

Dipoles only

Forward rapidity: quark or antiquark production

LO: integrate out quark



Single inclusive hadron production in DIS at small x: NLO

all terms with quadrupoles cancel; only dipoles contribute to the cross section

cancellations of divergences as before

$$\sigma^{\gamma^* A \to hX} = \sigma_{LO} \otimes \text{JIMWLK} + \sigma_{LO} \otimes D_{h/\bar{q}}(z_h, \mu^2) + \sigma_{NLO}^{\text{finite}}$$

phenomenology: need to consider hadronization of any of the 3 partons

consider hadronization of the gluon

Gluon hadronizing

$$k_h^+ \frac{\mathrm{d}\sigma}{\mathrm{d}^2\mathbf{k}_h\,\mathrm{d}k_h^+} = \frac{8e^2Q^2N_c}{(2\pi)^5} \int \frac{\mathrm{d}z_h}{z_h^2}\, z^3(1-z)^2 \int \mathrm{d}^6\,\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_2) K_0(|\mathbf{x}_{12'}|Q_2)\; [S_{22'} - S_{12} - S_{12'} + 1]$$

$$e^{i\frac{\mathbf{k}_h}{z_h}\cdot\mathbf{x}_{22'}} \int \frac{\mathrm{d}\xi}{\xi}\, D^{(0)}_{h/g}(\frac{z_h}{\xi})\, C_F\, \frac{\alpha_s}{\pi}\, P_{gq}(\xi)\, \log\frac{\mu^2}{\Lambda^2}$$

$$+ \quad \cdots\cdots$$

with $z \equiv \frac{k_h^+}{z_h l^+}$ and $P_{gq}(\xi) = \frac{1+(1-\xi)}{\xi}$

this can be combined with quark hadronizing contributions

$$\mathrm{d}\sigma^{\gamma^* A \to hX} = \mathrm{d}\sigma_{LO} \otimes \mathrm{JIMWLK} + \mathrm{d}\sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) + \mathrm{d}\sigma^{\mathrm{finite}}_{NLO}$$

so far: SIDIS for longitudinal photons

work on transverse photons is almost complete

Summary

QCD at high energy

dense hadron: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,...

to be probed precisely at EIC

toward precision: NLO, sub-eikonal corrections, ...

CGC is limited to small x (low p_t)

EIC has a large arm in x

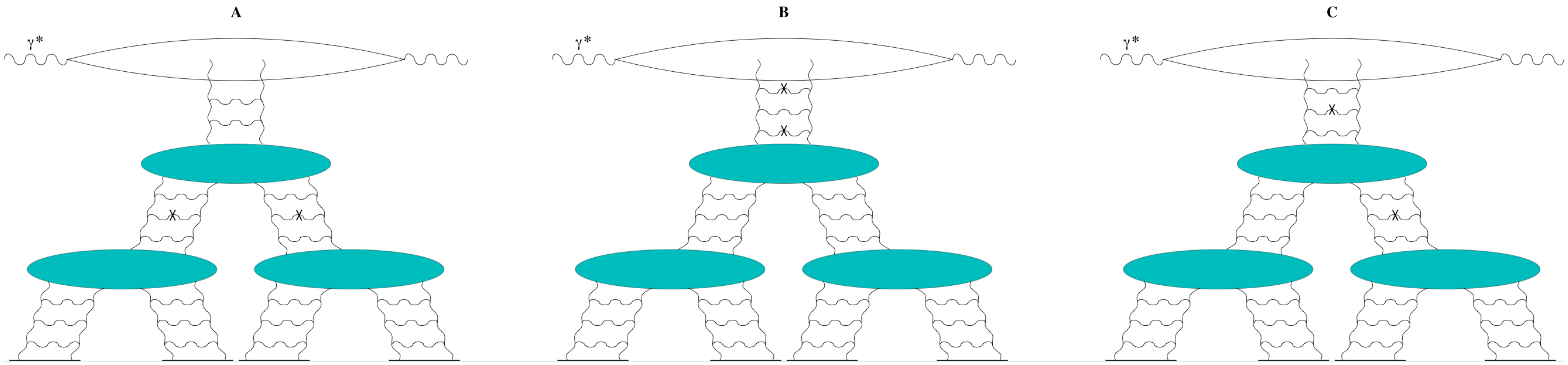
attempts to generalize CGC to include large Q^2 physics

Inclusive dihadron production in DIS at small x:

central vs forward rapidity

Inclusive dihadron production in midrapidity: LO

JJM, Yu. Kovchegov
PRD70 (2004) 114017



need a very large rapidity window

target is treated as a classical color field $\mathbf{A}_a^{\mu} = \delta^{\mu -}\, n^{\mu}\, S_a(x^+, \mathbf{x})$

scatterings of gluons on the target encoded in Wilson lines $\mathbf{U}\ (\mathbf{x}_1), U^{\dagger}(\mathbf{x}_2)$

leading log evolution included