Resummation in JIMWLK Hamiltonian

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High Energy Scattering

Target (ρ^t) **Projectile (ρ^p)**

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S-matrix:

$$\mathbf{S(Y)} \; = \; \langle \mathbf{T} \langle \mathbf{P}| \; \hat{\mathbf{S}}(\rho^{\mathbf{t}}, \; \rho^{\mathbf{p}}) \; |\mathbf{P}\rangle \, \mathbf{T}\rangle \qquad\qquad \mathbf{Y} \sim \ln(\mathbf{s})$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathbf{t}}, \; \rho^{\mathbf{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \; = \; \langle \mathbf{T} \langle \mathbf{P}| \; \hat{\mathcal{O}}(\rho^{\mathbf{t}}, \; \rho^{\mathbf{p}}) \; |\mathbf{P}\rangle \, \mathbf{T}\rangle$$

How these averages change with increase in energy of the process?

$$\partial_{\mathbf{Y}} \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \; = -\mathcal{H} \, \langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} \qquad\qquad \mathcal{H} \; \rightarrow \; \text{the HE effective Hamiltonian}$$

$\mathcal{H}$ **defines the high energy limit of QCD and is universal**

Expansion in α_s

$$\mathcal{H} = \mathcal{H}_{\mathbf{LO}}(\alpha_{\mathrm{s}}) + \mathcal{H}_{\mathbf{NLO}}(\alpha_{\mathrm{s}}^{2}) + \ldots; \qquad \mathcal{H} = \mathcal{H}[\rho^{\mathrm{t}}, \delta/\delta\rho^{\mathrm{t}}]$$

JIMWLK Hamiltonian is a limit of $\mathcal{H}$ for dilute partonic system ($\rho^{\mathrm{p}} \to \mathbf{0}$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$\mathcal{H}_{\mathbf{LO}}^{\mathbf{JIMWLK}}$ **(1997-2002),** $\mathcal{H}_{\mathbf{NLO}}^{\mathbf{JIMWLK}}$ **with massless quarks (2007-2016),** $\mathcal{H}_{\mathbf{NLO}}^{\mathbf{JIMWLK}}(\mathbf{m_q})$ **(2022)**

Motivation and Objectives

Precise saturation physics phenomenology at NLO is badly needed.

The JIMWLK Hamiltonian at NLO is known for some years, but there are problems there.

- No known recipe for numerical evaluation
- Large transverse logarithms emerge: $\mathcal{H} \sim \alpha_s(\# + \alpha_s(\# + \mathbf{Log}))$,
  If the Log is large, then $\alpha_s \mathbf{Log} \sim \mathbf{1}$ – not a small correction to LO
  There are various types of the large Logs there:
  running coupling effects, (Ioffe) time ordering, DGLAP logs.
  All have to be identified, clearly separated, and independently resummed.

A. Kovner, M. Lublinsky, V. V. Skokov and Z. Zhao,
"DGLAP resummation and the running coupling in NLO JIMWLK," [arXiv:2308.15545 [hep-ph]].

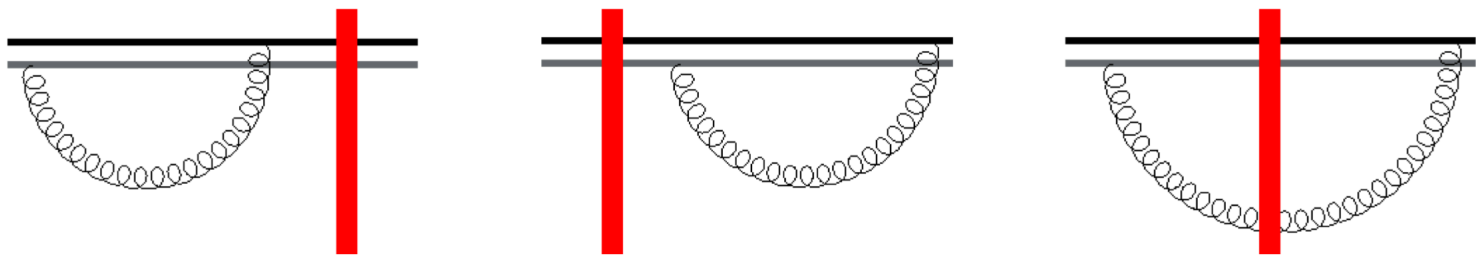
We both resummed the UV divergent Logs into correct form of the running coupling (rcJIMWLK) and derived an RG-improved JIMWLK Hamiltonian.
We have not resummed all large Logs, but only the ones related to DGLAP splittings.

LO JIMWLK Hamiltonian

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

$$\mathcal{H}^{JIMWLK}_{LO} = \int_{x,y,z} K_{LO} \left\{ J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}_A(z) J^b_R(y) \right\}$$

$$K_{LO}(x, y, z) = \frac{\alpha_s}{2\,\pi^2} \frac{(x-z)_i (y-z)_i}{(x-z)^2 (y-z)^2}$$



$$S_A{}^{cd}(z) = \mathcal{P} \exp \left\{ i \int dx^+ \, T^a \, \alpha^a_t(z, x^+) \right\}^{cd} . \qquad \Delta" \alpha_t = \rho_t \quad (YM)$$

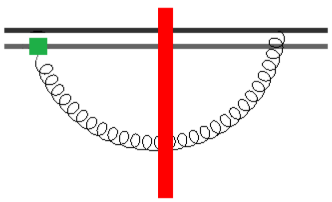
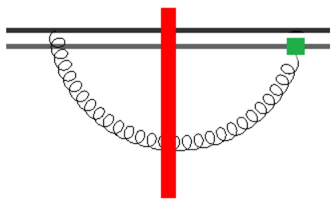
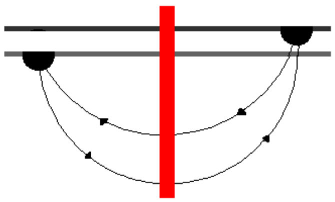
Here $\rho^p \to J_L$ and $\hat{S}\rho^p \to J_R$ are left and right SU(N) generators:

$$J^a_L(x) S^{ij}_A(z) = (T^a S_A(z))^{ij} \, \delta^2(x-z) \qquad J^a_R(x) S^{ij}_A(z) = (S_A(z) T^a)^{ij} \, \delta^2(x-z)$$

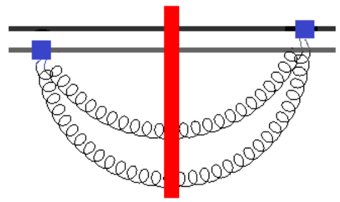
$\mathcal{H}^{JIMWLK}$ contains all the LO BFKL / BKP / TPV physics

JIMWLK Hamiltonian @ NLO

Kovner, ML & Mulian (2013) based on Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)

(a)



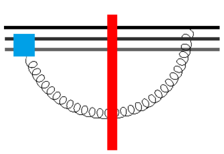
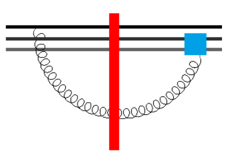
(b)

$$\mathcal{H}^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S_F^\dagger(z)\,t^a\,S_F(z')t^b]\,J_R^b(y)\;-\;J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\left[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\right]$$
$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$
$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w) - J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$

NLO Kernels (Large UV Logs only)

$\mathbf{X} = \mathbf{x} - \mathbf{z}$
$Y = y - z$

$$\mathbf{K_{JSJ}}(\mathrm{b\,terms}) = \frac{\alpha_s^2}{16\pi^3}\left\{-\mathbf{b}\frac{(\mathbf{x}-\mathbf{y})^2}{\mathbf{X}^2\mathbf{Y}^2}\ln(\mathbf{x}-\mathbf{y})^2\mu^2 + \frac{\mathbf{b}}{\mathbf{X}^2}\ln\mathbf{Y}^2\mu^2 + \frac{\mathbf{b}}{\mathbf{Y}^2}\ln\mathbf{X}^2\mu^2\right\} + \cdots$$

Here μ is the normalization point, $\mathbf{b} = \frac{11}{3}\mathbf{N_c} - \frac{2}{3}\mathbf{n_f}$ **,** $\quad \mathbf{b}\,\ln\mathbf{Q}^2/\mu^2 \to \alpha_s(\mathbf{Q}^2)$

Huge ambiguity in identifying Q

Resum large Logs into an effective kernel $\quad \mathbf{K} = \mathbf{K_{LO}} + \mathbf{K_{JSJ}} + \ldots.$

$$\int_{\mathbf{x\,y\,z,z'}} \mathbf{K_{JSSJ}}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{z'})\,\mathbf{J^a_L}(\mathbf{x})\mathbf{J^b_R}(\mathbf{y})\,\left[\mathbf{D^{ab}}(\mathbf{z},\mathbf{z'})\right] \quad \sim \; \mathbf{b}\,\times\,(\mathrm{UV\,divergent\,Log})$$

$$\mathbf{D^{ab}}(\mathbf{z},\mathbf{z'}) \equiv \mathbf{Tr}[\mathbf{T^a}\mathbf{S_A}(\mathbf{z})\mathbf{T^b}\mathbf{S^+_A}(\mathbf{z'})]$$

The UV divergence in $JSSJ$ is trivial: when the two gluons are too close to each other ($z \sim z'$), they cannot be resolved by the target and hence should be counted as a single gluon scattering. We are thus prompted to introduce a "resolution scale" Q

New approach to resummation

1. **Proper definition of the considered observable:**
   Within the finite resolution Q for gluon splitting into two,
   bare gluons $\rightarrow$ dressed gluons: bare Wilson lines $\rightarrow$ *dressed Wilson lines*, $\mathbf{S} \rightarrow \mathbf{S_Q}$

2. **Identification of running coupling correction among higher order terms**
   Not all Logs proportional to b are due to charge renormalization and should be absorbed into some form of α_s running. We get the most intuitive result:

$$\mathbf{K_{LO}} = \frac{\alpha_s}{\mathbf{2}\pi^{\mathbf{2}}}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}} \rightarrow \frac{\sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}}{\mathbf{2}\pi^{\mathbf{2}}}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}}$$

3. **Resummation of additional large Logs.**
   Large Logs associated with the resolution Q are resummed a la DGLAP.

$$\frac{\sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}}{\mathbf{2}\pi^{\mathbf{2}}}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}} \rightarrow \frac{\sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}}{\mathbf{2}\pi^{\mathbf{2}}}\frac{\mathbf{XY}}{\mathbf{X^2Y^2}} \times \delta\mathbf{K}^{\mathbf{DGLAP}}$$

Dressed Wilson line

$$\mathbf{S^{ab}_Q(z) = S^{ab}_A(z)} + \frac{\alpha_s}{2\pi^2}\int_0^1 \mathbf{d}\xi\,\sigma(\xi)\int^{\mathbf{Q^{-1}}}\frac{\mathbf{d^2Z}}{\mathbf{Z^2}}\left(\mathbf{D^{ab}(z+(1-\xi)Z, z-\xi Z)} - \mathbf{N_c\,S^{ab}_A(z)}\right)$$

ξ is the fraction of longitudinal momentum carried by one of the gluons.

$$\sigma(\xi) = \left[\frac{\mathbf{1}}{\xi(\mathbf{1}-\xi)}\left(\xi^2 + (\mathbf{1}-\xi)^2 + \xi^2(\mathbf{1}-\xi)^2\right)\right]_+ \;;$$

This is a P_{gg} splitting function except that we introduce the "+" prescription both for $\xi = 1$ and $\xi = 0$ poles

We go beyond the usual DGLAP: we allow simultaneous scattering of all gluons.

$\mathbf{S_Q} \sim \mathbf{S_A}\,[\mathbf{1} + \alpha_s\,\#\,\mathbf{Log}(\mathbf{Q^2}/\mathbf{Q_s^T})]$. Q_s^T - saturation scale in the target. This large Log has to be resummed via inclusion of multiple consecutive DGLAP splittings:

$$\frac{\partial \mathbf{S_Q(z)}}{\partial \ln \mathbf{Q}} = -\frac{\alpha_s}{2\pi^2}\int_\xi \sigma(\xi)\int_{\phi_\mathbf{Q}}[\mathbf{D_Q(z)} - \mathbf{N_c\,S_Q(z)}]\;; \qquad \mathbf{D_Q(z_1, z_2) \equiv Tr[T^a S_Q(z_1) T^b S^+_Q(z_2)]}$$

RG

Substitute $\mathrm{S_Q}$ into $\mathcal{H}$

The resummed Hamiltonian should be Q-independent:

$$\frac{\mathbf{d}\mathcal{H}}{\mathbf{d}\ln\mathbf{Q}} = \frac{\partial\mathcal{H}}{\partial\ln\mathbf{Q}} + \int_{\mathbf{u}}\left[\frac{\delta\mathbf{H}}{\delta\mathbf{S_Q}(\mathbf{u})}\frac{\partial\mathbf{S_Q}(\mathbf{u})}{\partial\ln\mathbf{Q}}\right] = \mathbf{0}$$

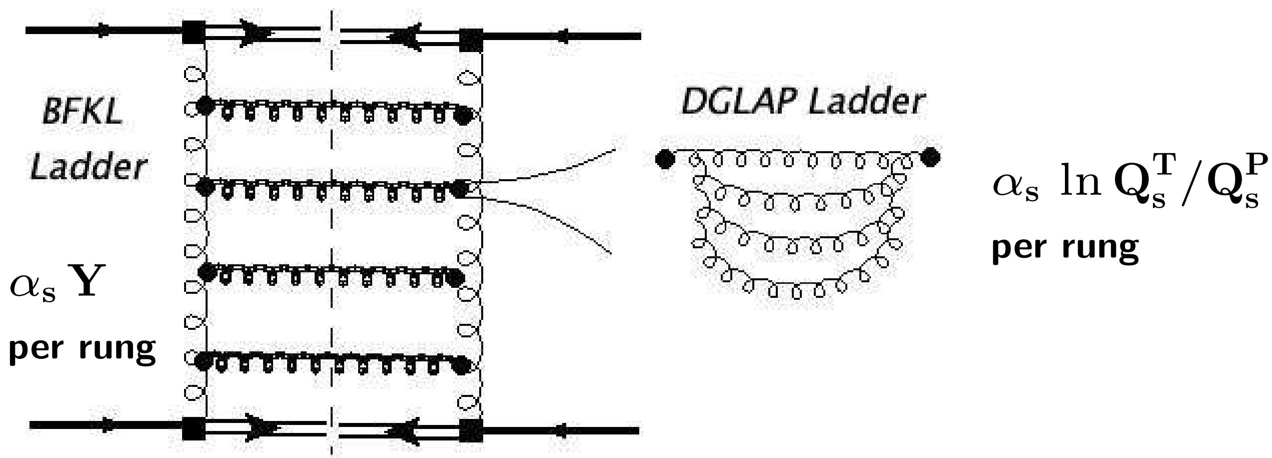
DGLAP-like evolution for the Hamiltonian (evolution in the space of Hamiltonians):

$$\mathcal{H} \;=\; \mathbf{Exp}\left[\int_{\mathbf{Q_s^P}}^{\mathbf{Q_s^T}}\frac{\mathbf{dQ}}{\mathbf{Q}}\mathbf{H_{DGLAP}}\right]\mathcal{H}_{\mathbf{in}}$$

$$\mathbf{H_{DGLAP}} \;=\; \frac{\alpha_{\mathbf{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{u}}\int_{\xi}\sigma(\xi)\int_{\phi_{\mathbf{Q}}}\mathbf{Tr}\left(\left[\mathbf{D_Q}(\mathbf{u}) \;-\; \mathbf{N_c}\,\mathbf{S_Q}(\mathbf{u})\right]\frac{\delta}{\delta\mathbf{S_Q}(\mathbf{u})}\right)$$

Summary/Outlook

- DGLAP-like resummation inside the JIMWLK Hamiltonian has been performed. These DGLAP corrections are large whenever there is a large disparity between the correlation lengths (or saturation momenta) in the projectile and the target. This is precisely JIMWLK's regime of validity.
  The result is a smearing of the WW fields within the $1/Q_s^T$ distance



- rcJIMWLK emerges with the scale choice for the running coupling:
  $\mathbf{K} \sim \sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}$

- Numerical implementations are to follow