A unified description of DGLAP, CSS, and BFKL: TMD factorization bridging large and small x

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initial condition for small-x evolution equation (BFKL) from lattice QCD / global fits ?

- small-x evolution ——> TMD physics

- evolutions of TMDPDF from lattice QCD / global fits: DGLAP+CSS, no BFKL

- different factorization, different IR structures, different evolutions, different nonpertubative TMDPDF — not universal

- need a universal TMDPDF / factorization that contains IR structures of both DGLAP and BFKL in the appropriate limits high-energy limit

BFKL

dipole$(x_B = 0,\ b_\perp)$

all collinear twist

x_B-dep: resum all sub-eikonal

collinear limit

DGLAP, CSS

CSS$(b_\perp) \otimes$ PDF$(x_B,\ b_\perp = 0)$

leading-twist

$b_\perp$-dep: resum all sub-lead twists

not known how

MSTT(-erious) factorization

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Unified description of DGLAP, CSS, and BFKL evolution: TMD factorization bridging large and small *x*

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a TMD factorization unifying IR structures of large and small x

summary

gluon TMDPDF$(x_B, b_\perp)$ operator

NLO (2-gluon) corrections to background fields

MSTT factorization

new general structure: IR & UV div. in trans. mom. + IR & UV in rapidity

$x_B \to 0$

coll. limit

BFKL

DGLAP, CSS

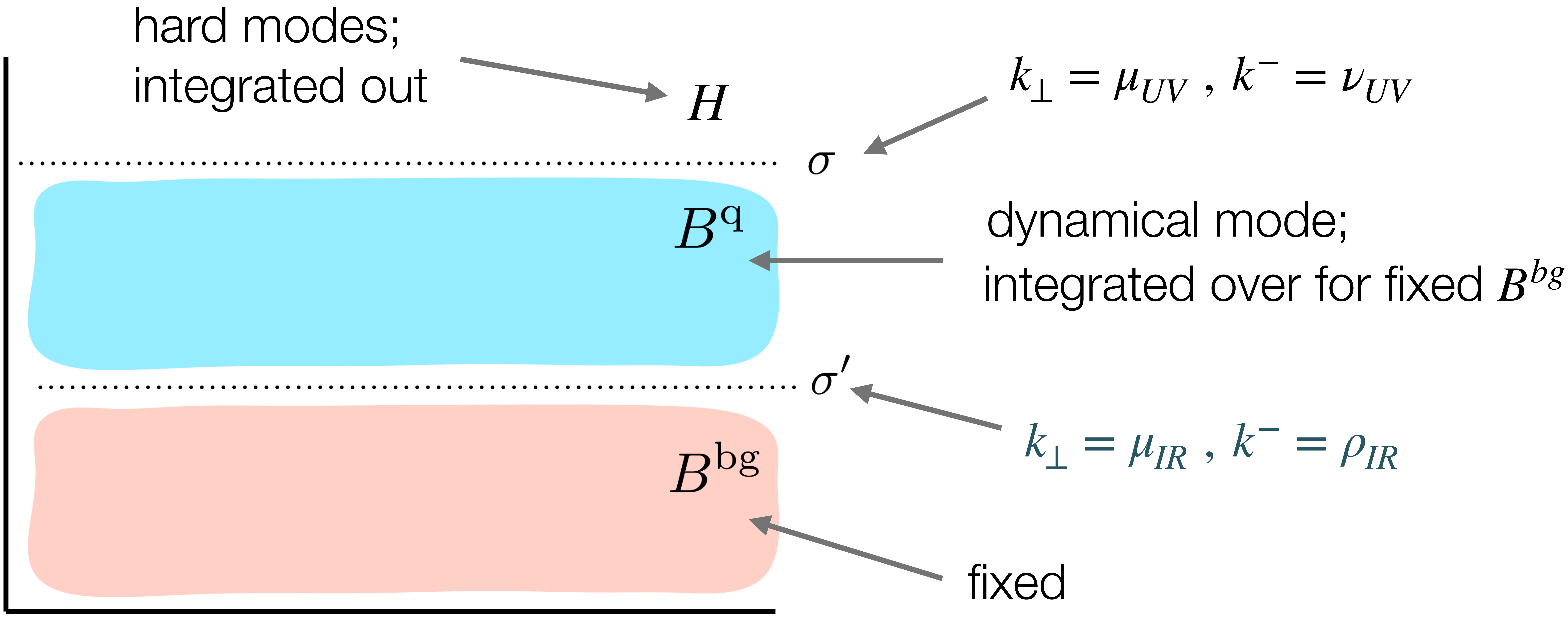
WW$(x_B = 0,\ b_\perp)$

CSS$(b_\perp) \otimes$ PDF$(x_B,\ b_\perp = 0)$

few technicalities — background-field method

$$A \to H + B$$

$$B = B^q + B^{bg}$$



few technicalities — regularization schemes

- divergences in $k_\perp \to \infty\ (\mu_{UV})$ & $k_\perp \to 0\ (\mu_{IR})$: dim-reg

- divergences in $k^- \to \infty\ (\nu_{UV})$ & $k^- \to 0\ (\rho_{IR})$: η-scheme

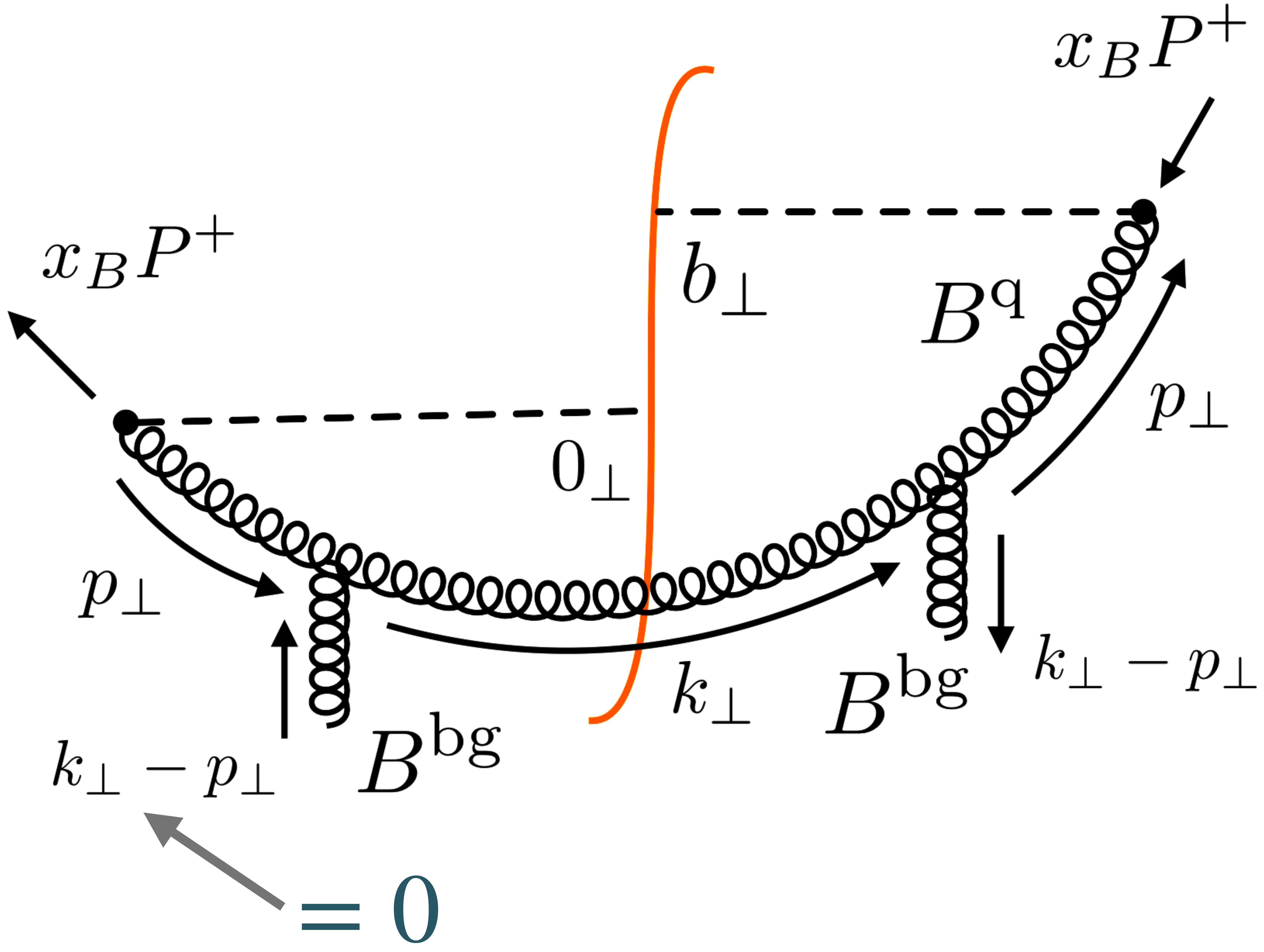
$$\int_0^\infty \frac{dk^-}{k^-} \to \nu^\eta \int_0^\infty \frac{dk^-}{k^-} |k^+|^{-\eta}$$

rapidity divergence in the calculation: $z \equiv \dfrac{x_B}{x_B + \frac{k_\perp^2}{2P^+k^-}}$

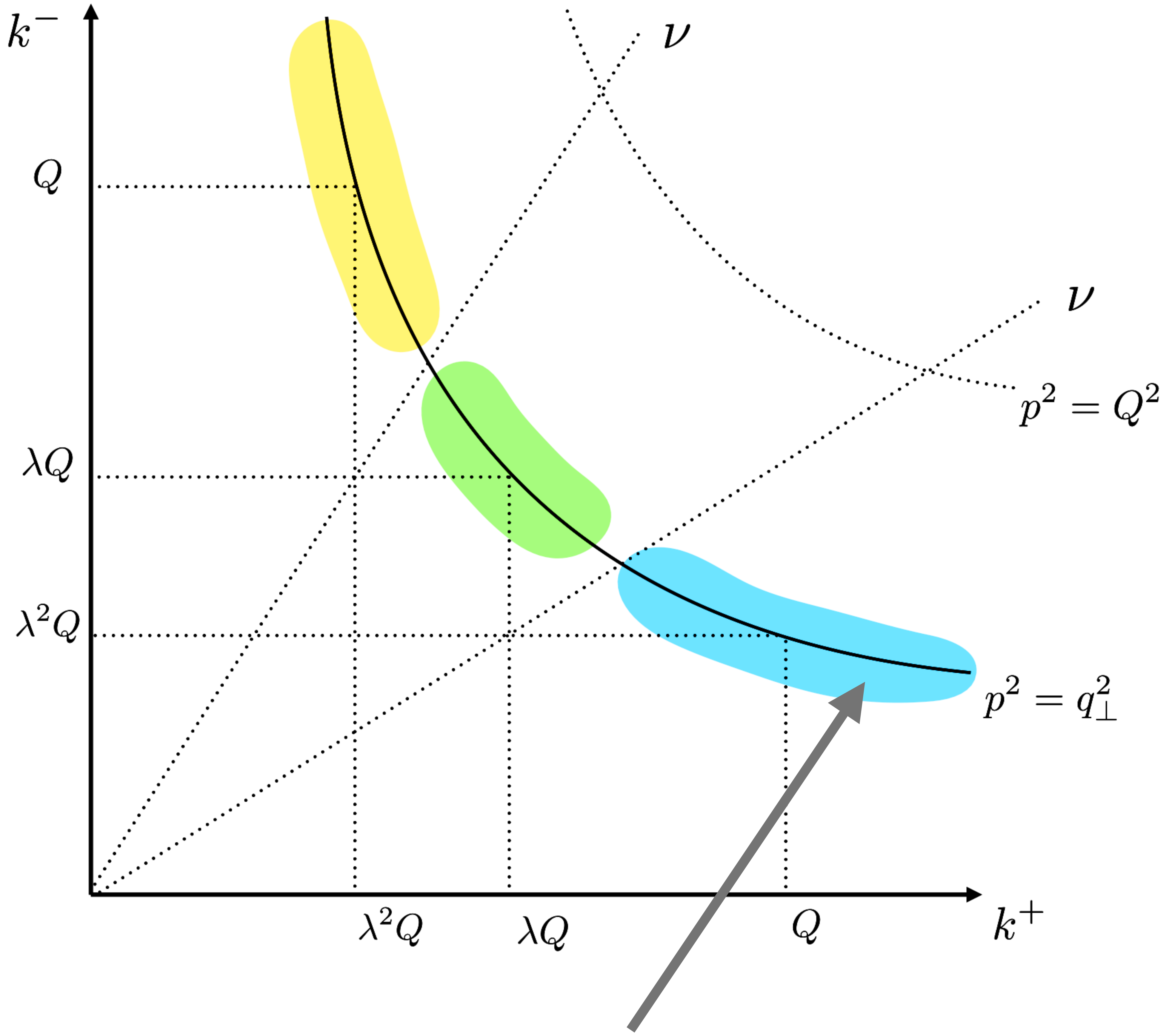
$k^- \to \infty : z \to 1$

$k^- \to 0 : z \to 0$

CSS/SCET: missing ingredients



no trans. mom. supplied by the background field



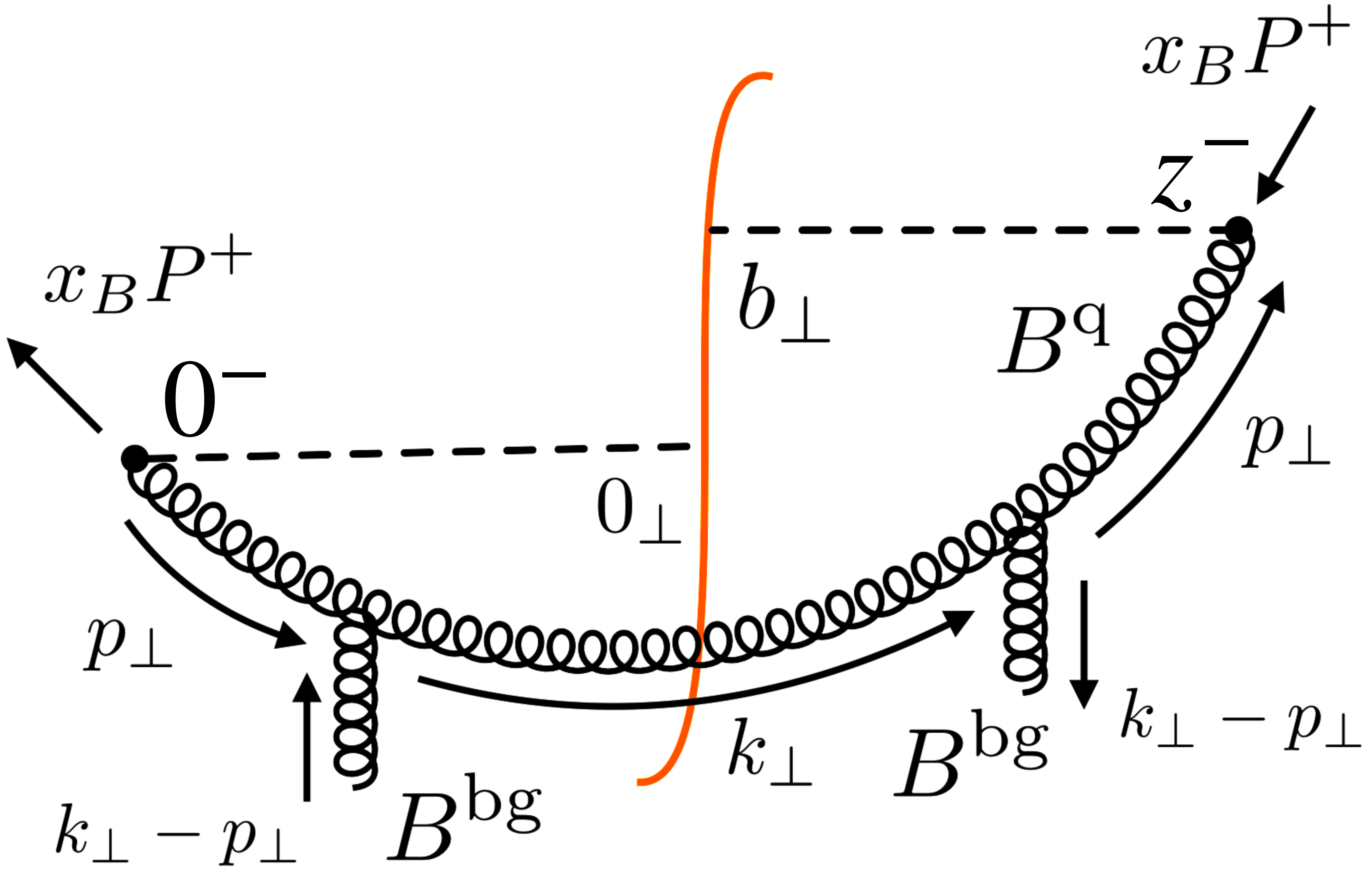
collinear modes are approx. on mass-shell

no virtual corrections

MSTT

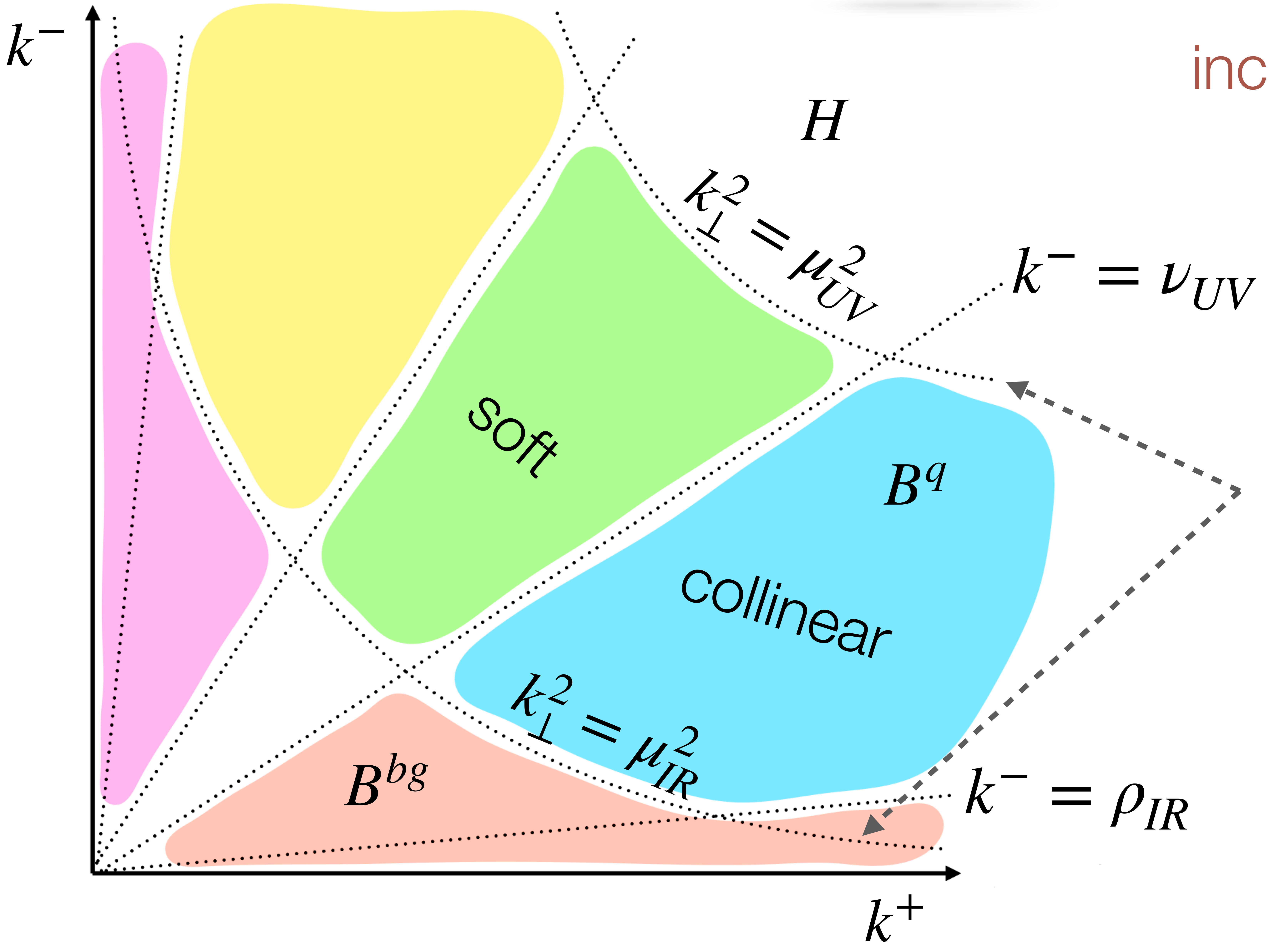
NLO (2-gluon) correction to gluon TMDPDF operator

$$\mathcal{B}_{ij}(x_B, b_\perp) = \int_{-\infty}^{\infty} dz^- e^{-ix_B P^+ z^-} \langle P, S | \bar{T}\{F^m{}_{-i}(z^-, b_\perp)$$
$$\times [z^-, \infty]_b^{ma}\} T\{[\infty, 0^-]_0^{an} F^n{}_{-j}(0^-, 0_\perp)\} | P, S \rangle$$



$k_\perp - p_\perp > 0$

MSTT



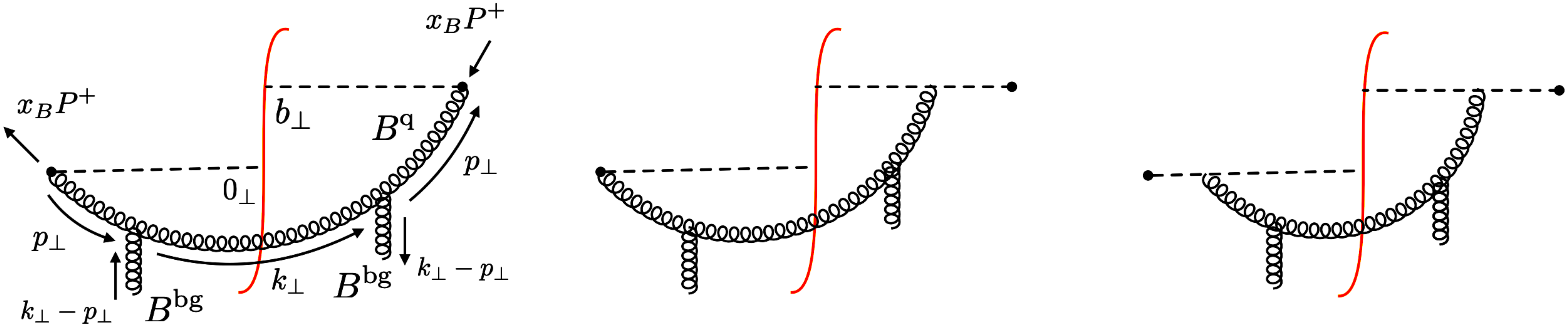
include virtual corrections

$k_\perp^2 > 2k^+k^-$

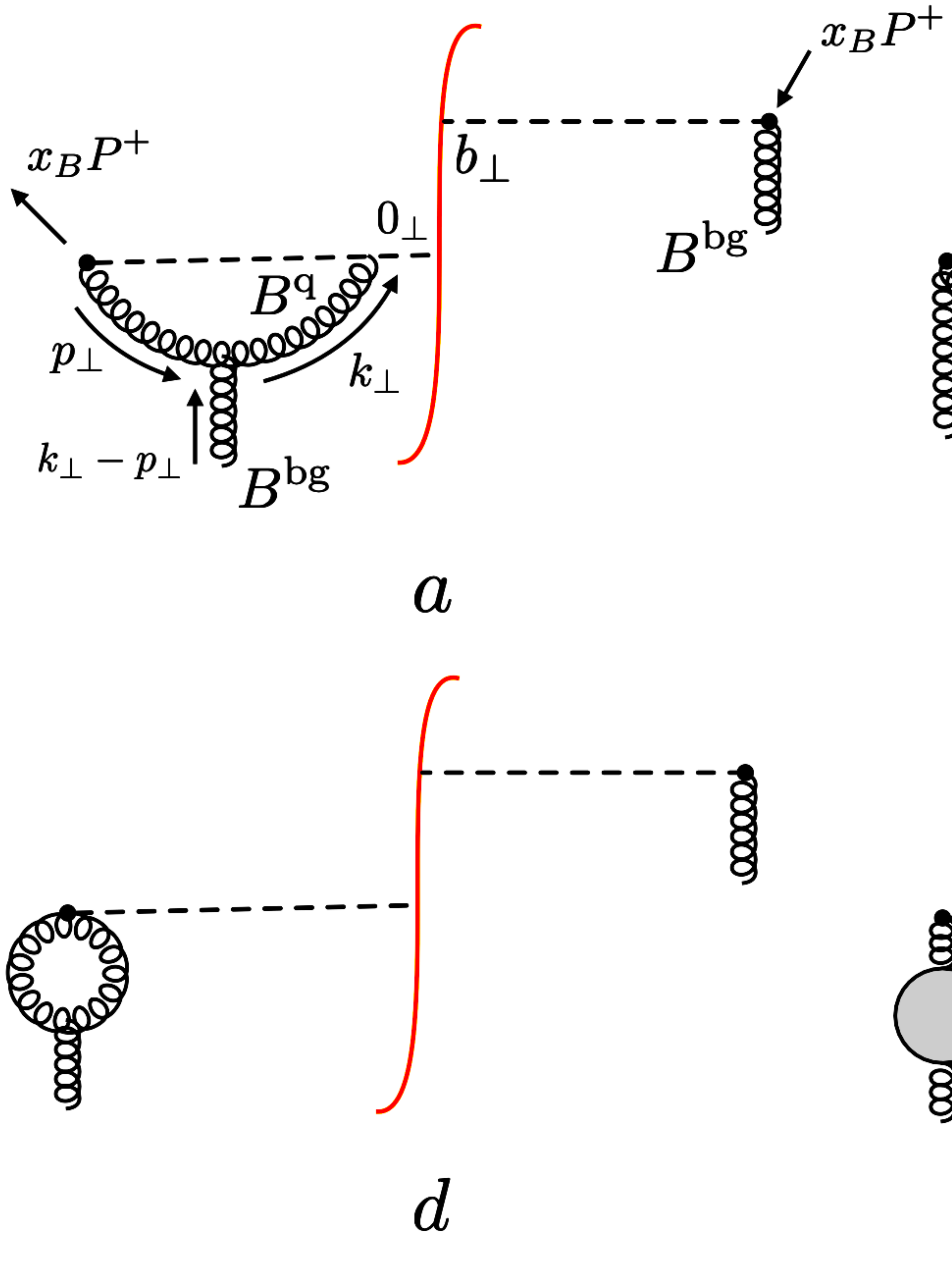
on mass-shell region

$2k^+k^- = k_\perp^2 = \mu^2$

real emissions



virtual emissions



soft factor

MSTT factorization of gluon TMDPDF at NLO

$$f_{ij}(x_B, b_\perp, \mu_{\rm UV}^2, \zeta) = f_{ij}(x_B, b_\perp, \mu_{\rm IR}^2, \rho) - 4\alpha_s N_c \int \bar{d}^2 p_\perp e^{ip_\perp b_\perp} \int_0^1 \frac{dz}{z(1-z)} \int \bar{d}^2 k_\perp \Big[\mathcal{R}^a_{ij;lm}(z, p_\perp, k_\perp)$$

$$+\mathcal{R}^b_{ij;lm}(z, p_\perp, k_\perp)\Big] \int d^2 z_\perp e^{-i(p_\perp - k_\perp) z_\perp} f_{lm}\Big(\frac{x_B}{z}, z_\perp, \mu_{\rm IR}^2, \rho\Big) + \frac{\alpha_s N_c}{2\pi}\Big(-\frac{1}{2}(L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln\frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12}\Big)$$

$$\times f_{ij}(x_B, b_\perp, \mu_{\rm IR}^2, \rho) - \frac{\alpha_s N_c}{\pi} L_b^{\mu_{\rm IR}} \int_0^1 dz \Big[\frac{1}{(1-z)_+} + \frac{1}{z}\Big] f_{ij}\Big(\frac{x_B}{z}, b_\perp, \mu_{\rm IR}^2, \rho\Big) - \frac{\alpha_s N_c}{2\pi} \int d^2 z_\perp \int \bar{d}^2 p_\perp e^{ip_\perp (b-z)_\perp}$$

$$\times\Big(\frac{1}{2}\ln^2\frac{\mu_{\rm IR}^2}{p_\perp^2} + \ln\frac{\mu_{\rm IR}^2}{p_\perp^2}\ln\frac{\rho}{\zeta} - \frac{\pi^2}{12}\Big)\frac{g_{il}p_j p_m + p_i p_l g_{mj}}{p_\perp^2} f_{lm}(x_B, z_\perp, \mu_{\rm IR}^2, \rho)$$

$$+\frac{\alpha_s N_c}{2\pi} \int d^2 z_\perp \int \bar{d}^2 p_\perp e^{ip_\perp (b-z)_\perp}\Big(\frac{\beta_0}{2N_c}\ln\frac{\mu_{\rm UV}^2}{p_\perp^2} + \frac{67}{18} - \frac{5N_f}{9N_c}\Big) f_{ij}(x_B, z_\perp, \mu_{\rm IR}^2, \rho) + O(\alpha_s^2)\,.$$

part of DGLAP

CSS

part of BFKL

$$L_b^\mu \equiv \ln\Big(\frac{b_\perp^2 \mu^2}{4e^{-2\gamma_E}}\Big) \qquad \zeta = x_B P^+$$

gluon TMDPDF parametrization:

$$f_{ij}(x_B, b_\perp) = x_B P^+ \left[-\frac{g_{ij}}{2} f_1(x_B, b_\perp) + \left(\frac{g_{ij}}{2} + \frac{b_i b_j}{b_\perp^2} \right) h_1(x_B, b_\perp) \right]$$

collinear limit

$$f_{ij}(x_b, b_\perp \to 0) = f_1(x_b)$$

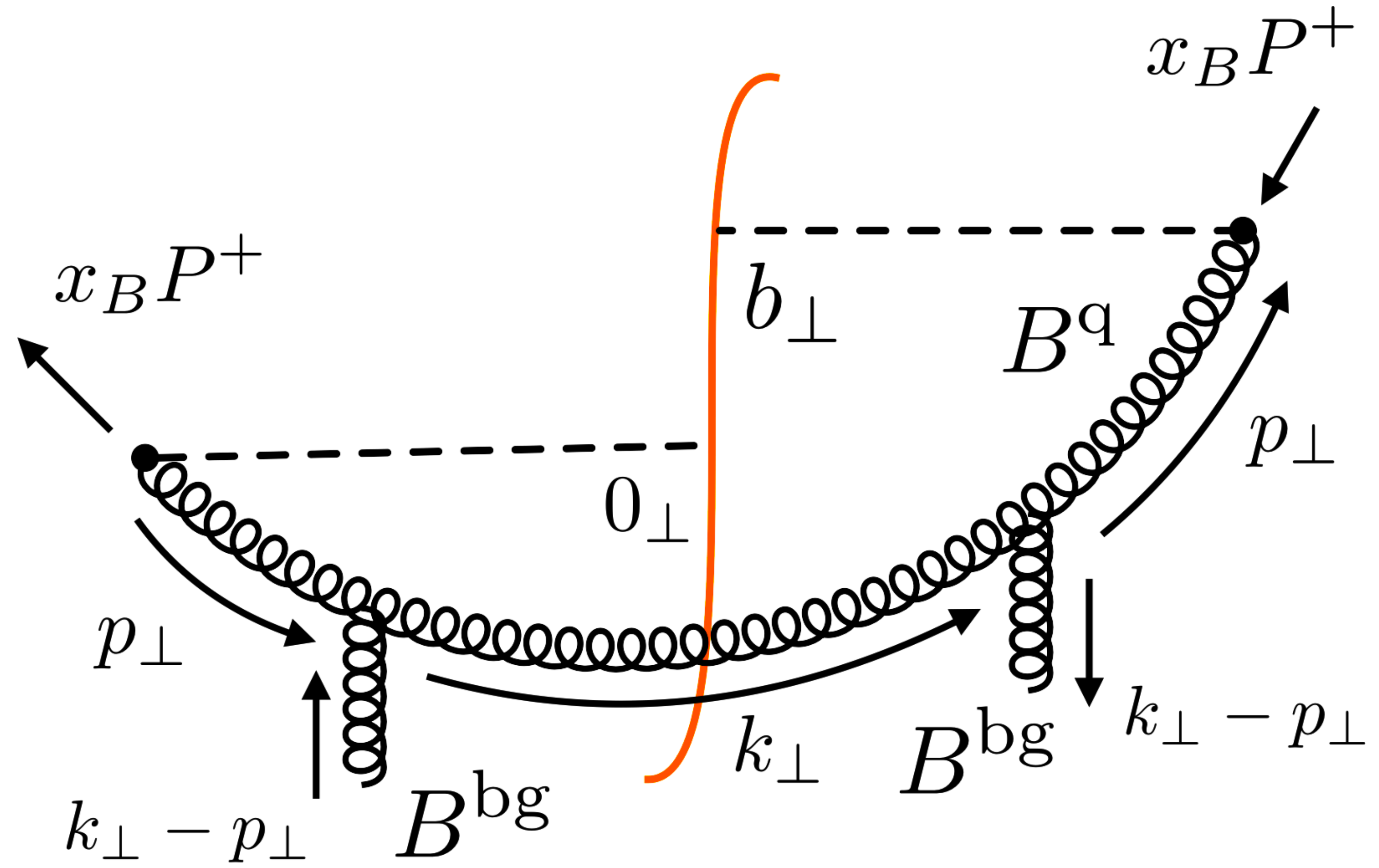
small-x limit

$$f_{ij}(x_b \to 0, b_\perp) = \frac{b_i b_j}{b_\perp^2} \mathscr{H}(b_\perp)$$

collinear limit of MSTT

$$p_\perp \sim b_\perp^{-1} \gg k_\perp - p_\perp \sim \mu_{IR}$$

virtual corrections vanishes,
no rapidity IR div.



$k_\perp - p_\perp = 0$

constant piece —> IR div. $k_\perp$

$$P_{gg}(z) = \frac{1}{(1-z)_+} + \frac{1}{z} - 2 + z - z^2$$

$$f_1(x_B, b_\perp, \mu_{\rm UV}^2, \zeta) = f_1(x_B, 0_\perp, \mu_{\rm IR}^2)$$
$$-\frac{\alpha_s N_c}{\pi} L_b^{\mu_{\rm IR}} \int_0^1 \frac{dz}{z} P_{gg}(z) f_1(\frac{x_B}{z}, 0_\perp, \mu_{\rm IR}^2) + \frac{\alpha_s N_c}{2\pi}\Big(-\frac{1}{2}(L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln\frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12}\Big) f_1(x_B, 0_\perp, \mu_{\rm IR}^2)$$

DGLAP

CSS

$x_B \to 0$ limit of MSTT

$1/z$ of $P_{gg}(z)$ generates additional rapidity IR div.

$$K_{\rm BFKL}(p_\perp, k_\perp) = -\frac{\alpha_s N_c}{\pi} \int d^2 b_\perp e^{ik_\perp b_\perp} L_b^{\mu_{\rm IR}} + \frac{\alpha_s N_c}{\pi} (2\pi)^2 \delta^2(k_\perp) \ln \frac{\mu_{\rm IR}^2}{p_\perp^2}$$

BFKL

CSS

$$f_1(x_B, p_\perp, \mu_{\rm UV}^2, \zeta) \simeq \mathcal{H}_1(p_\perp, \rho) + \ln \frac{\rho}{\zeta} \int đ^2 k_\perp K_{\rm BFKL}(p_\perp, k_\perp) \mathcal{H}_1(p_\perp - k_\perp, \rho)$$
$$+ \frac{\alpha_s N_c}{2\pi} \int d^2 b_\perp \left(-\frac{1}{2} (L_b^{\mu_{\rm UV}})^2 + L_b^{\mu_{\rm UV}} \ln \frac{\mu_{\rm UV}^2}{\zeta^2} - \frac{\pi^2}{12} \right) \int đ^2 k_\perp e^{ik_\perp b_\perp} \mathcal{H}_1(p_\perp - k_\perp, \rho)$$
$$+ \frac{\alpha_s N_c}{2\pi} \left(\frac{\beta_0}{2N_c} \ln \frac{\mu_{\rm UV}^2}{p_\perp^2} + \frac{67}{18} - \frac{5N_f}{9N_c} \right) \mathcal{H}_1(p_\perp, \rho) \,.$$

$x_B \to 0$ limit: MSTT vs Glauber SCET

$$k_\perp^2 \gg 2k^+k^{-1}$$

propagator ~ $1/k_\perp^2$

Glauber SCET: $k_\perp^2 \gg k^+ \sim k^{-1}$ $k^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$ mid rapidity

MSTT: $k^+ \gg k_\perp^2 \gg k^{-1}$ $k^\mu \sim Q(1, \lambda^4, \lambda)$ forward rapidity

$(+, -, \perp)$, $\lambda \ll 1$

summary

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new general evolution

$x_B \to 0$

coll. limit

BFKL

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WW$(x_B = 0,\ b_\perp)$

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