Transverse energy-energy correlators in the Color-Glass Condensate

Zhongbo Kang, Jani Penttala, Fanyi Zhao, Yiyu Zhou

arXiv:2311.17142 [hep-ph]

Accepted for *Physical Review D*

University of California, Los Angeles

SURGE collaboration

DIS 2024

Energy-energy correlators (EEC)

- Two-point energy correlator
- Particles weighted by their energy

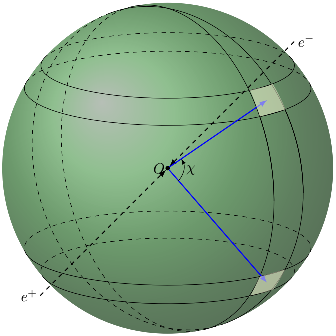
  $\Rightarrow$ Less sensitive to the nonperturbative IR region

  - One of the first infrared-safe event shapes in QCD

    Basham, Brown, Ellis, Love, Phys.Rev.Lett. 41 (1978) 1585, Phys.Lett.B 85 (1979) 297-299

- $e^+ + e^- \to X$:

$$\frac{\mathrm{d}\Sigma_{e^+e^-}}{\mathrm{d}\cos\chi} = \sum_{i,j} \int \mathrm{d}\sigma \, \frac{E_i E_j}{Q^2} \delta(\cos\theta_{ij} - \cos\chi)$$



Moult, Zhu, 1801.02627

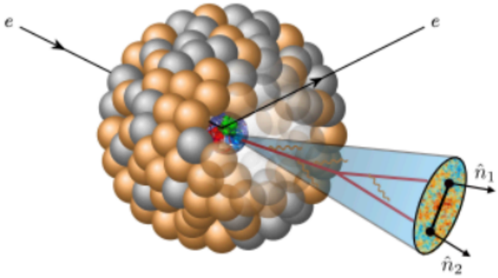
See also Haitao Li's talk yesterday

Different limits of EEC

- Collinear limit: $\chi \approx 0$
  - Probes jet substructure

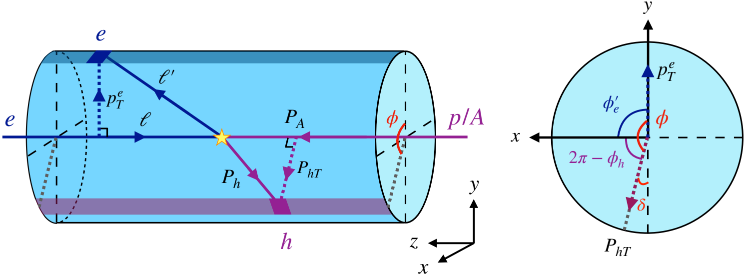
    Dixon, Moult, Zhu, 1905.01310
- Back-to-back limit: $\chi \approx \pi$
  - Probes TMD physics

    Moult, Zhu, 1801.02627



Transverse energy-energy correlators (TEEC)

$e + p/A \to e' + h$



$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sum_h \int \mathrm{d}\sigma_{\mathrm{DIS}} \frac{E_{T,l}E_{T,h}}{E_{T,l}\sum_i E_{T,i}} \delta\left(\tau - \frac{1+\cos\phi}{2}\right), \quad \tau = \frac{1+\cos\phi}{2}$$

- Generalization of the EEC Ali, Pietarinen, Stirling, Phys.Lett.B 141 (1984) 447-454
  - More suitable for hadronic colliders
- Back-to-back region in the azimuthal plane: $\phi \approx \pi \Leftrightarrow \tau \ll 1$

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu)\sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}}\int_0^\infty \frac{\mathrm{d}b}{\pi}\cos\left(2b\sqrt{\tau}p_T^e\right) f_q^{(u)}\left(x,b,\mu,\zeta/\nu^2\right) S_{nn_h}(b,\mu,\nu) J_q^{(u)}(b,\mu,\zeta'/\nu^2)$$

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos\left(2b\sqrt{\tau}p_T^e\right) f_q^{(u)}\left(x,b,\mu,\zeta/\nu^2\right) S_{nn_h}(b,\mu,\nu) J_q^{(u)}\left(b,\mu,\zeta'/\nu^2\right)$$

$$\sigma_0 = \frac{2\alpha_{\mathrm{em}}^2}{sQ^2}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \quad \text{(the leading-order partonic cross section)}$$

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos\left(2b\sqrt{\tau}p_T^e\right) f_q^{(u)}\left(x,b,\mu,\zeta/\nu^2\right) S_{nn_h}(b,\mu,\nu) J_q^{(u)}(b,\mu,\zeta'/\nu^2)$$

$$H(Q,\mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left[-\ln^2\left(\frac{\mu^2}{Q^2}\right) - 3\ln\left(\frac{\mu^2}{Q^2}\right) - 8 + \frac{\pi^2}{6}\right] \text{ is the hard function}$$

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos\left(2b\sqrt{\tau}p_T^e\right) f_q^{(u)}\left(x,b,\mu,\zeta/\nu^2\right) S_{nn_h}(b,\mu,\nu) J_q^{(u)}(b,\mu,\zeta'/\nu^2)$$

$f_q^{(u)}\left(x,b,\mu,\zeta/\nu^2\right)$ is the unsubtracted quark TMD

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos\left(2b\sqrt{\tau}p_T^e\right) f_q^{(u)}\left(x, b, \mu, \zeta/\nu^2\right) S_{nn_h}(b,\mu,\nu) J_q^{(u)}(b,\mu,\zeta'/\nu^2)$$

$S_{nn_h}(b,\mu,\nu)$ is the soft function for soft gluon radiation

TEEC for lepton-hadron production in DIS at small x

- Factorization proven for similar processes
  - Lepton-jet production Liu et al., 1812.08077; Arratia et al., 2007.07281
  - Combined TMD and small-x framework for dijets at NLO

    Caucal et al, 2108.06347, 2208.13872; Taels et al, 2204.11650

$\Rightarrow$ Strong motivation for factorization in this process

- Using SCET we can write a factorization formula for TEEC

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau\,\mathrm{d}y_e\,\mathrm{d}^2\boldsymbol{p}_T^e} = \sigma_0 H(Q,\mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos(2b\sqrt{\tau} p_T^e) f_q^{(u)}(x, b, \mu, \zeta/\nu^2) S_{nn_h}(b,\mu,\nu) J_q^{(u)}(b,\mu,\zeta'/\nu^2)$$

$J_q^{(u)}(b,\mu,\zeta'/\nu^2)$ is the unsubtracted TEEC jet function

Renormalization

The unsubtracted quantities can be renormalized using the standard TMD soft functions

$$f_q^{(u)}(x, b, \mu, \zeta/\nu^2) = \frac{1}{\sqrt{S_{n\bar{n}}(b, \mu, \nu)}} f_q(x, b, \mu, \zeta), \qquad J_q^{(u)}(b, \mu, \zeta'/\nu^2) = \frac{\sqrt{S_{n\bar{n}}(b, \mu, \nu)}}{S_{nn_h}(b, \mu, \nu)} J_q\left(b, \mu, \hat{\zeta}\right)$$

$$f_q^{(u)}(x, b, \mu, \zeta/\nu^2) S_{nn_h}(b, \mu, \nu) J_q^{(u)}(b, \mu, \zeta'/\nu^2) = f_q(x, b, \mu, \zeta) J_q\left(b, \mu, \hat{\zeta}\right)$$

Our equation for TEEC can then be written as:

$$\textbf{TEEC} = \frac{\mathrm{d}\sigma}{\mathrm{d}\tau \, \mathrm{d}y_e \, \mathrm{d}^2 \boldsymbol{p}_T^e} = \sigma_0 H(Q, \mu) \sum_q e_q^2 \frac{p_T^e}{\sqrt{\tau}} \int_0^\infty \frac{\mathrm{d}b}{\pi} \cos(2b\sqrt{\tau} p_T^e) f_q(x, b, \mu, \zeta) J_q\left(b, \mu, \hat{\zeta}\right)$$

- The dependence on the rapidity renormalization scale ν cancels
- For the renormalization and Collins–Soper scales we set $\mu^2 = \zeta = \hat{\zeta} = Q^2$

Quark distribution – TMD evolution

- $f_q(x, b, \mu, \zeta)$ satisfies the TMD evolution equations for the scales μ and ζ

$$\frac{\mathrm{d}}{\mathrm{d}\ln\sqrt{\zeta}} \ln f_q(x, b, \mu, \zeta) = K(b, \mu) \qquad \frac{\mathrm{d}}{\mathrm{d}\ln\mu} \ln f_q(x, b, \mu, \zeta) = \gamma_\mu^q[\alpha_s(\mu), \zeta/\mu^2]$$

- We can write: $f_q(x, b, \mu, \zeta) = f_q\left(x, b, \mu_{b_*}, \mu_{b_*}^2\right) \exp[-S_{\mathrm{pert}}(\mu, \mu_{b_*}, \zeta)]$

  where the perturbative Sudakov factor

$$S_{\mathrm{pert}}(\mu, \mu_{b_*}, \zeta) = -K(b_*, \mu_{b_*}) \ln\left(\frac{\sqrt{\zeta}}{\mu_{b_*}}\right) - \int_{\mu_{b_*}}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \gamma_{\mu'}^q \left[\alpha_s(\mu'), \frac{\zeta}{\mu'^2}\right]$$

  evolves the quark TMD from the initial scale μ_{b_*} to the scales $\mu = \sqrt{\zeta} = Q$

- Note: nonperturbative Sudakov factor assumed to be part of the initial condition

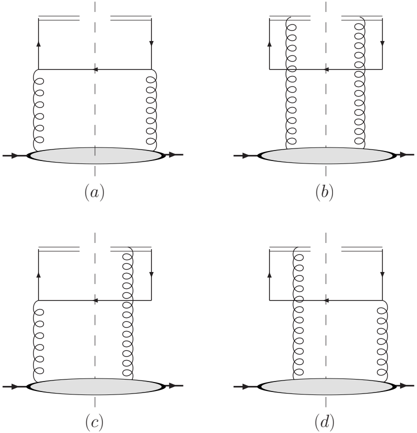
Quark distribution – relation to the small-x dipole amplitudes

Initial condition for the TMD evolution:

Quark TMD at small x

$$xf_q\left(x, |\mathbf{b}|, \mu_{b_*}, \mu_{b_*}^2\right) = \frac{N_c S_\perp}{8\pi^4} \int \mathrm{d}\epsilon_f^2 \, \mathrm{d}^2\boldsymbol{r} \, \frac{(\mathbf{b}+\mathbf{r})\cdot\mathbf{r}}{|\mathbf{b}+\mathbf{r}||\mathbf{r}|} \epsilon_f^2$$
$$\times K_1(\epsilon_f|\mathbf{r}|)\Big[1 + \mathcal{S}_x(|\mathbf{b}|) - \mathcal{S}_x(|\mathbf{b}+\mathbf{r}|) - \mathcal{S}_x(|\mathbf{r}|)\Big]$$

- where $N(|r|) = 1 - \mathcal{S}_x(|\mathbf{r}|) = 1 - \frac{1}{N_c} \mathrm{tr}\left[V(\frac{\mathbf{r}}{2})V^\dagger(-\frac{\mathbf{r}}{2})\right]$ is the dipole amplitude
- Initial scale chosen as $\mu_{b_*} = 2e^{-\gamma_E}/b_*$ where $b_* = b/\sqrt{1 + b^2/b_{\max}^2}$ and $b_{\max} = 1.5\ \mathrm{GeV}^{-1}$



Marquet, Xiao, Yuan, 0906.1454

Models for the dipole amplitude

1. The Golec-Biernat–Wüsthoff (GBW) model:

$$\mathcal{S}_x(r) = \exp\left(-\frac{r^2 Q_\mathrm{s}^2(x)}{4}\right)$$

   where the saturation scale Q_s reads: $Q_\mathrm{s}^2(x) = 1\ \mathrm{GeV}^2 \times \left(\frac{x_0}{x}\right)^\lambda$

2. McLerran–Venugopalan model at x_0

$$\mathcal{S}_{x_0}(r) = \exp\left[-\frac{r^2 Q_{s,0}^2}{4} \ln\left(\frac{1}{r\Lambda_\mathrm{QCD}} + e_c \cdot e\right)\right]$$

   with the running-coupling Balitsky–Kovchegov (rcBK) evolution equation

   $\frac{\partial}{\partial \ln(1/x)} \mathcal{S}_x(|\mathbf{r}|) = \int \mathrm{d}^2\mathbf{r}'\, \mathcal{K}(\mathbf{r},\mathbf{r}')\left[\mathcal{S}_x(|\mathbf{r}'|)\mathcal{S}_x(|\mathbf{r}-\mathbf{r}'|) - \mathcal{S}_x(|\mathbf{r}|)\right]$ for the x dependence

Parameters from Golec-Biernat–Wüsthoff, hep-ph/9807513 (GBW) and Lappi, Mäntysaari, 1309.6963 (rcBK)

TEEC jet function

- The TEEC jet function is given in terms of the TMD fragmentation functions $\widetilde{D}_{1,h/q}$:

$$J_q\left(b, \mu, \hat{\zeta}\right) \equiv \sum_h \int_0^1 \mathrm{d}z\, z \widetilde{D}_{1,h/q}\left(z, b, \mu, \hat{\zeta}\right)$$

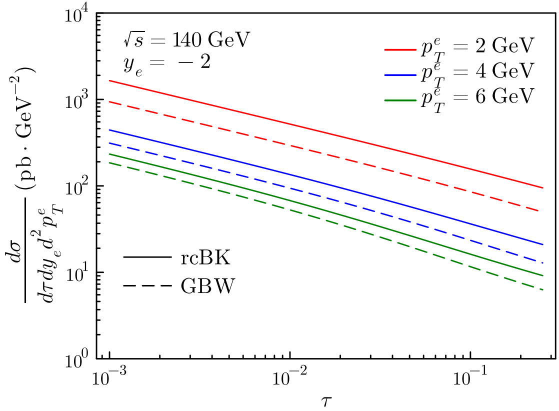
- The TMD FFs can be further written in terms of the collinear FFs as

$$\widetilde{D}_{1,h/q}\left(z, b, \mu, \hat{\zeta}\right) = \sum_i \int_z^1 \frac{\mathrm{d}y}{y} C_{i\leftarrow q}\left(\frac{z}{y}, b\right) D_{h/i}(y, \mu_{b_*}) \times \exp\left[-S_{\mathrm{pert}}\left(\mu, \mu_{b_*}, \hat{\zeta}\right)\right] \times \exp\left[-S_{\mathrm{NP}}\left(z, b, Q_0, \hat{\zeta}\right)\right]$$

  - $C_{i\leftarrow q}\left(\frac{z}{y}, b\right) = \delta_{iq}\delta(1-z)$ are the matching coefficients at LO
  - Sudakov factors S_{pert} and S_{NP} handle the TMD evolution of the TMD FFs
  - Perturbative Sudakov the same as for the quark TMD
  - Nonperturbative Sudakov modeled using Echevarria, Kang, Terry, 2009.10710; Sun et al, 1406.3073

Numerical results – proton targets

- Studied in the EIC kinematics
- Sensitivity to the dipole amplitude: Up to a factor 2 difference between rcBK and GBW at $p_T^e = 2\,\mathrm{GeV}$
- Low-momentum region most sensitive to gluon saturation
  $\Rightarrow$ Potentially a good process to study saturation effects



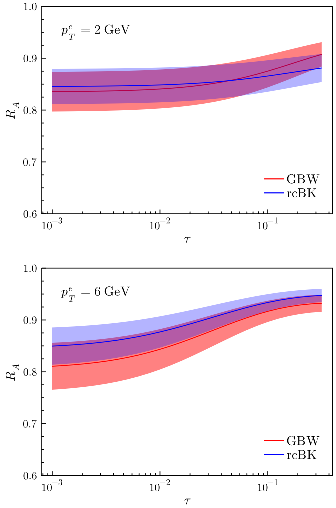
$x(p_T^e = 2\,\mathrm{GeV}) = 0.002$

$x(p_T^e = 6\,\mathrm{GeV}) = 0.008$

Numerical results – nuclear suppression

- We model nuclei by changing the saturation scale $Q^2_{s,A} = c\, A^{1/3}\, Q^2_s$ where $c \in [0.5, 1]$ to estimate uncertainty in the nuclear geometry
- To study nuclear suppression we consider the quantity
  $$R_A = \frac{1}{A} \frac{\mathrm{d}\sigma_{eA}}{\mathrm{d}\tau \mathrm{d}y_e \mathrm{d}^2 \boldsymbol{p}^e_T} \Big/ \frac{\mathrm{d}\sigma_{ep}}{\mathrm{d}\tau \mathrm{d}y_e \mathrm{d}^2 \boldsymbol{p}^e_T}$$
- Without saturation $R_A \to 1$
  - Smaller dipoles probed when $\sqrt{\tau} p^e_T$ large

    $\Rightarrow$ Saturation effects smaller in this region
- Nuclear modification of $15 - 20\%$ can be expected for $\tau \ll 1$



Summary

- TEEC is an infrared-safe event-shape observable
  - Probes TMD physics in the back-to-back region
- We have considered TEEC for DIS using the combined TMD and small-x framework
  - The quark TMD and the jet function evolved with the TMD evolution
  - The initial condition for the quark TMD modeled using the small-x dipole amplitude
- TEEC found to be sensitive to saturation in the back-to-back limit $\tau \ll 1$

$\Rightarrow$ An interesting process to measure at the EIC