NON-EIKONAL EFFECTS IN DIJET PRODUCTION AT THE ELECTRON-ION COLLIDER

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HIGH-ENERGY SCATTERING

- At high-energy, the medium is composed by a dense ensemble of gluons: **semi-classical approximation**:
  - Classical gluon background field (small-x): $A^\mu(x)$
  - Quantum source (large-x): $J^\mu(x)$

- At high-energy we perform the **eikonal approximation**:
  - The rapidity difference between the source and the probe is infinity
  - Power suppressed corrections of the CoM energy are suppressed

THE EIKONAL APPROXIMATION

- **Only gluons** contribute to the low-x (classical) regime of the medium
- The classical field is **infinitely boosted** with respect to the (right-moving) probe:

$$A^\mu(x) = \Lambda^\mu_\nu A_0^\nu(\Lambda^{-1}x) = (\gamma A_0^-, \gamma^{-1}A_0^+, \mathbf{A}_0^\perp)$$

  - Only the longitudinal (-) component is probed

$$A^-(x) = \gamma A_0^-(\Lambda^{-1}x) = \gamma A_0^-(\gamma x^+, \gamma^{-1}x^-, \mathbf{x}^\perp)$$

  - The field can only be probed at $x^+ = 0$ (***shockwave approximation***)
  - The probe is not sensitive to the x^- dependence of the field (***frozen gluons***)

$$A^\mu(x) = \delta^{\mu -}\delta(x^+)a^-(\mathbf{x}^\perp)$$

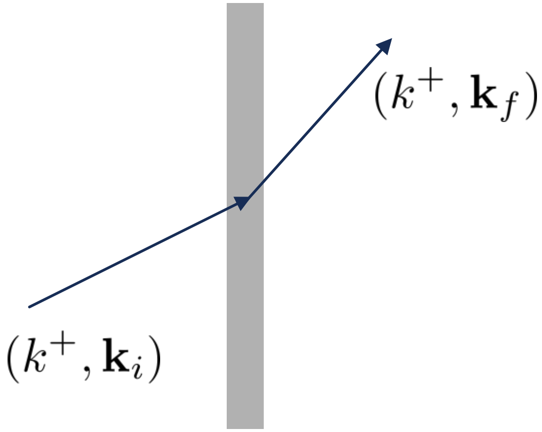
EIKONAL SCATTERING

- Color particle - background field scattering:
  - The projectile propagator can be solved exactly:

$$\Delta(x,y) \propto \delta^{(2)}(\mathbf{x}-\mathbf{y})U_{[x^+,y^+]}(\mathbf{x})$$

  - It is **diagonal** in the transverse coordinates
  - There is **no exchange of longitudinal** (+) **momentum** in the interaction
  - The particle is only color rotated through the eikonal **Wilson line**:

$$U_{[x^+,y^+]}(\mathbf{x}) = \mathcal{P}^+ \exp\left\{ -ig\int_{y^+}^{x^+} dz^+ A^-(z^+,\mathbf{z})\right\}$$



- CGC observables depend on $\langle A^- \, A^- \rangle$ for a Gaussian distribution

GOING BEYOND THE EIKONAL APPROXIMATION

- The eikonal approximation can be relaxed by expanding around the classical straight trajectory
- Type of corrections to the background field:
  - **Finite width** (L^+) of the target (allow for insertions in z^+)
  - Corrections to the classical transverse trajectory ($A_\perp^j$ **and** $\partial^j A^-$ **insertions**)

    Altinoluk, Armesto, Beuf, Martínez, Salgado: 1404.2219
    Altinoluk, Armesto, Beuf, Moscoso: 1505.01400
    Balitsky, Tarasov: 1505.02151
    Chirilli: 1807.11435

  - Corrections to the classical z^- trajectory (A^+ **and** $\partial^+ A^-$ **insertions**) **Not included in this work!**

    Jalilian-Marian: 1708.07533 *Altinoluk, Beuf: 2109.01620* *PA: 2307.13573*

- Apart from the $\langle A^- \, A^- \rangle$ correlator, now we also need other $\langle A^\mu \, A^\nu \rangle$ **correlators**.

For other type of corrections see Swaleha and Gabriel's talk!

THE NON-EIKONAL FIELD CORRELATOR

- We assume that:
  - The target is composed by $A \gg 1$ **nucleons**
  - Each nucleon is in a singlet state of N_c **quarks**
  - All the particles are **independent**, uncorrelated and have the **same momenta** $(P_q^-, 0_\perp)$
  - **Homogeneous** nucleus
- The non-linear terms of the Yang-Mills equations are subleading

Cougoulic, Kovchegov: 2005.14688

$$\langle A_a^\mu(\vec{x})\, A_b^\nu(\vec{y})\rangle = \delta^{ab}\delta(x^+ - y^+)\frac{\tilde{\mu}^2}{L^+}G^{\mu\nu}(\mathbf{x}-\mathbf{y})$$

$$G^{--}(\mathbf{r}) = \int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{1}{\mathbf{P}^4},$$

→ **MV MODEL**

$$G^{i-}(\mathbf{r}) = \frac{1}{2P_q^-}\int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{\mathbf{P}^i}{\mathbf{P}^4},$$

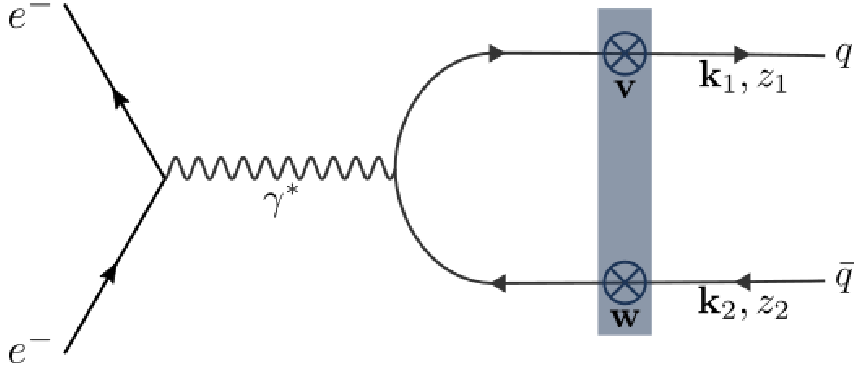
$$G^{ij}(\mathbf{r}) = \frac{1}{(2P_q^-)^2}\int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{\mathbf{P}^i\mathbf{P}^j + \epsilon^{im}\epsilon^{jn}\mathbf{P}^m\mathbf{P}^n}{\mathbf{P}^4}.$$

DIJET PRODUCTION IN DIS AT THE EIKONAL LEVEL

- The virtual photon splits into a quark-antiquark pair:
  - Depends on the polarization of the photon (L or T)
  - Can be computed using **perturbative QED**
- The pair interacts with the dense target:
  - Eikonal scattering depends only on **Wilson lines**

$$d(\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$

$$Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$

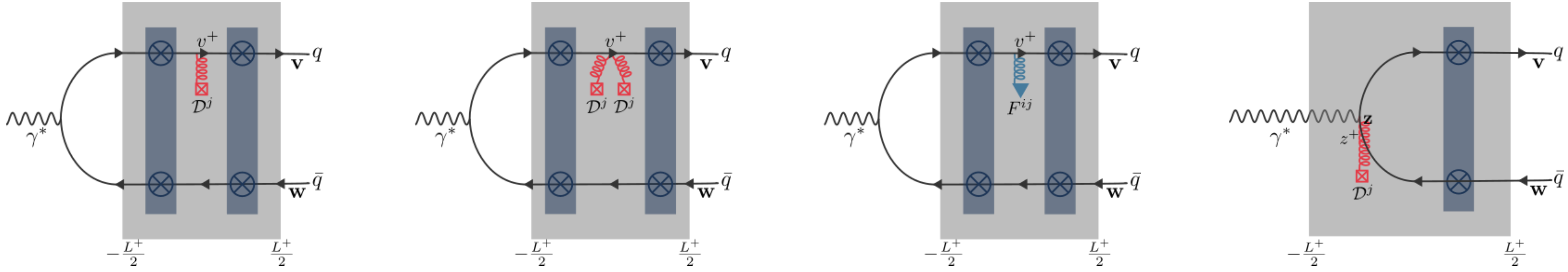


Dominguez, Marquet, Xiao, Yuan: 1101.0715

$$\left.\frac{d\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d^2\mathbf{k}_2 d\eta_1 d\eta_2}\right|_{\mathrm{Eik.}} = \int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})+i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\mathcal{C}_\lambda(\mathbf{w}'-\mathbf{v}',\mathbf{w}-\mathbf{v})$$
$$\times\left[Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) - d(\mathbf{w}',\mathbf{v}') - d(\mathbf{v},\mathbf{w}) + 1\right]$$

DIJET PRODUCTION BEYOND THE EIKONAL APPROXIMATION

- Next-to-eikonal diagrams to the dijet production amplitude (**only transverse corrections**):



Altinoluk, Beuf, Czajka, Tymowska: 2212.10484

- Depend on objects like:

$$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle$$

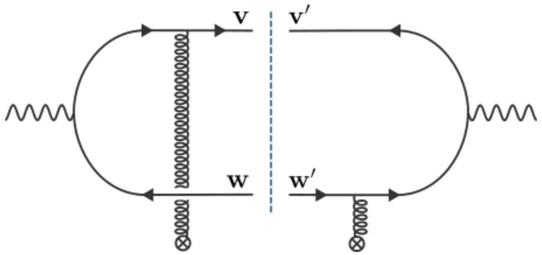
$$\mathcal{U}_j^{(1)}(\mathbf{z}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_{[\frac{L^+}{2}, v^+]}(\mathbf{z}) \overleftrightarrow{D_{\mathbf{z}^j}}(v^+) \mathcal{U}_{[v^+, -\frac{L^+}{2}]}(\mathbf{z})$$

$$D_{\mathbf{z}^j}(v^+) = \partial_{\mathbf{z}^j} - igA^j(v^+, \mathbf{z})$$

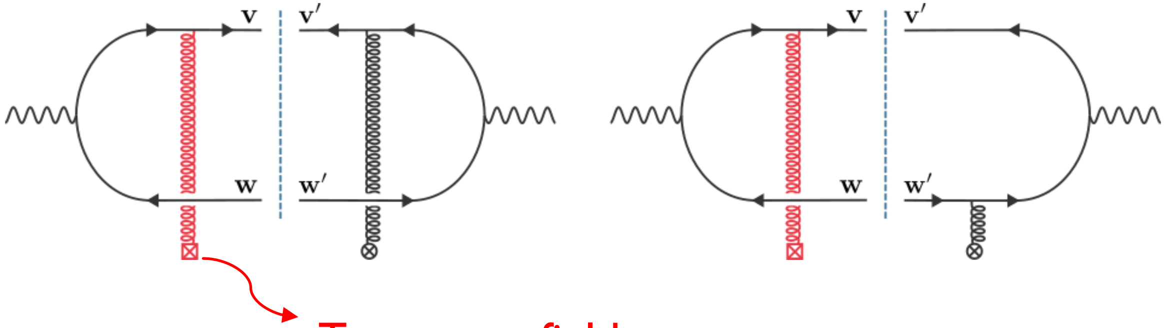
For the correlation limit see Guillaume's talk!

DIJET PRODUCTION IN THE DILUTE LIMIT

- For a dense target we have 2 challenges:
  - Resum the non-eikonal field correlators
  - Include non-linear terms in the YM equations
- We study the case where $k_{1\perp}, k_{2\perp} \gg \; Q_s$ (**dilute limit**):
  - $A^{\mu} \sim \; O(g^0)$: **perturbative expansion**
  - Only 2-gluon exchange



Eikonal term



Transverse field

RESULT

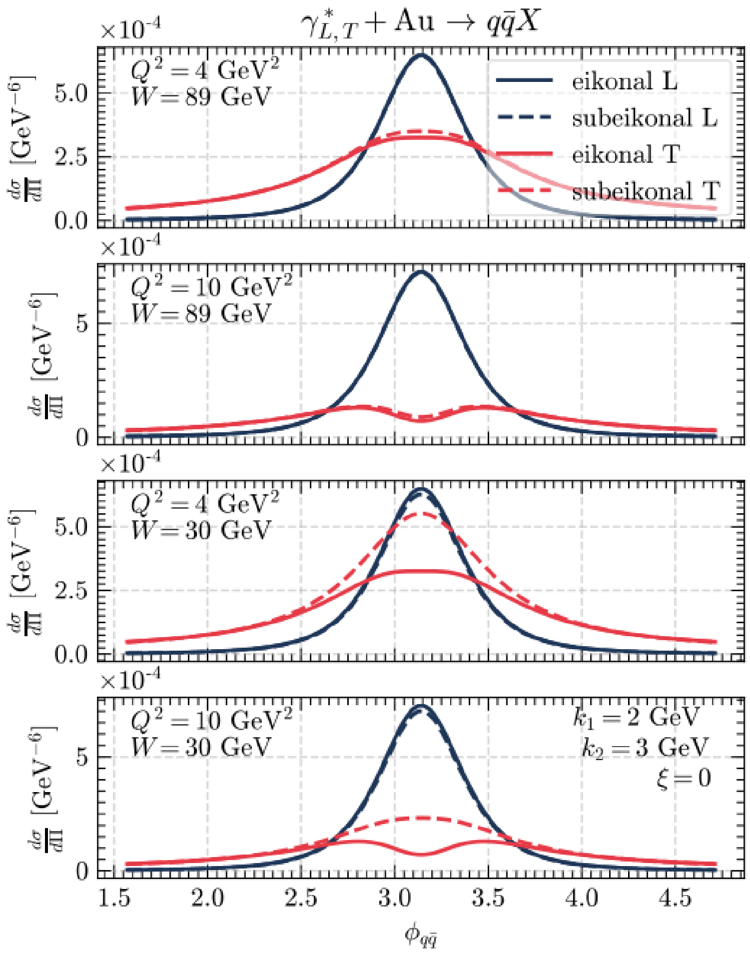
- Analytical solution for the differential cross section:

$$\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \frac{8N_c\alpha_{\rm em}e_f^2Q_s^2S_\perp}{(2\pi)^3}\delta_z z_1^3 z_2^3$$

$$\frac{Q^2(\mathbf{k}_1^2-\mathbf{k}_2^2)^2}{(\mathbf{k}_1+\mathbf{k}_2)^4(\mathbf{k}_1^2+\epsilon_f^2)^2(\mathbf{k}_1^2+\epsilon_f^2)^2}\left[1+\frac{N_c}{W^2}\left(\frac{\mathbf{k}_1^2+\epsilon_f^2}{z_1}-\frac{\mathbf{k}_2^2+\epsilon_f^2}{z_2}\right)\right]$$

subeikonal correction

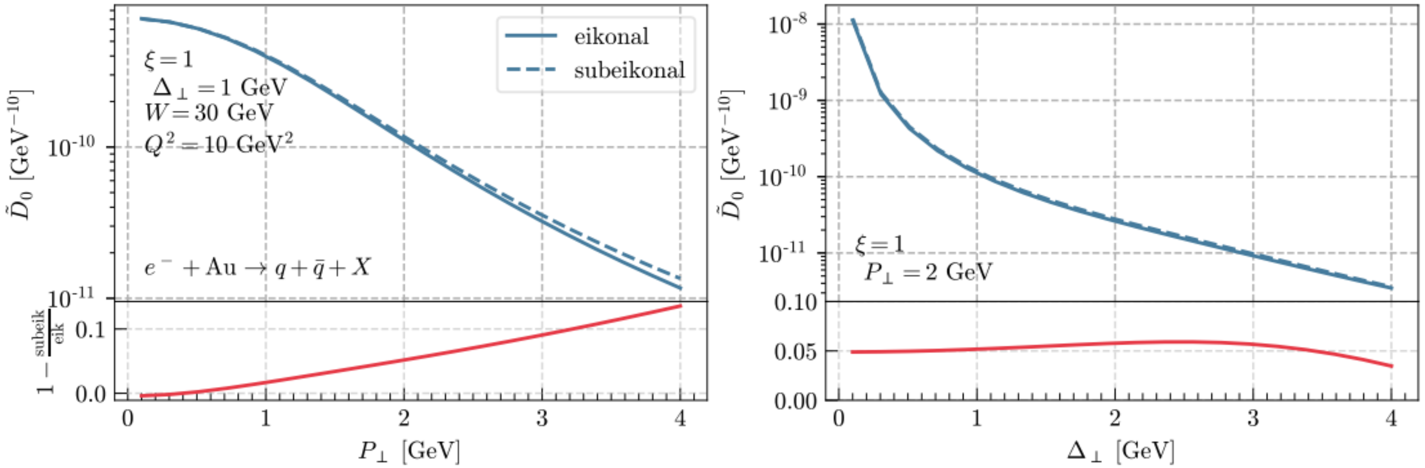
  - Similar expression for a transversely polarized photon
- W: Center of Mass energy of the $\gamma^* A$ system:
  - $W \to \infty$ in the eikonal approximation
- Longitudinal momentum fraction asymmetry: $\xi = \ln\frac{1-z_1}{z_1}$



NUMERICAL RESULTS

- Momentum Imbalance: $\mathbf{\Delta} = \mathbf{k}_1 + \mathbf{k}_2$
- Relative momentum: $\mathbf{P} = z_2\mathbf{k}_1 - z_1\mathbf{k}_2$
- Harmonic expansion w.r.t. the angle ϕ between P and Δ:

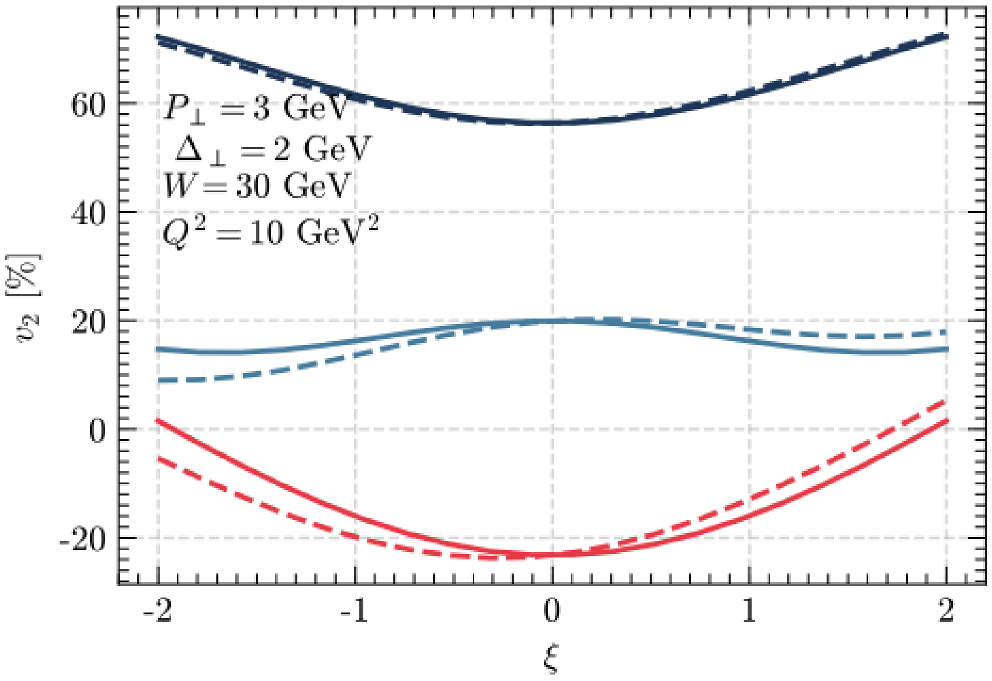
$$\frac{d\sigma_{\lambda}^{\gamma^* A\to q\bar{q}X}}{d\Pi} = D_{0,\lambda}(P_\perp,\Delta_\perp)\left[1+2\sum_{n=1}^{\infty} v_{n,\lambda}(P_\perp,\Delta_\perp)\cos\phi\right]$$

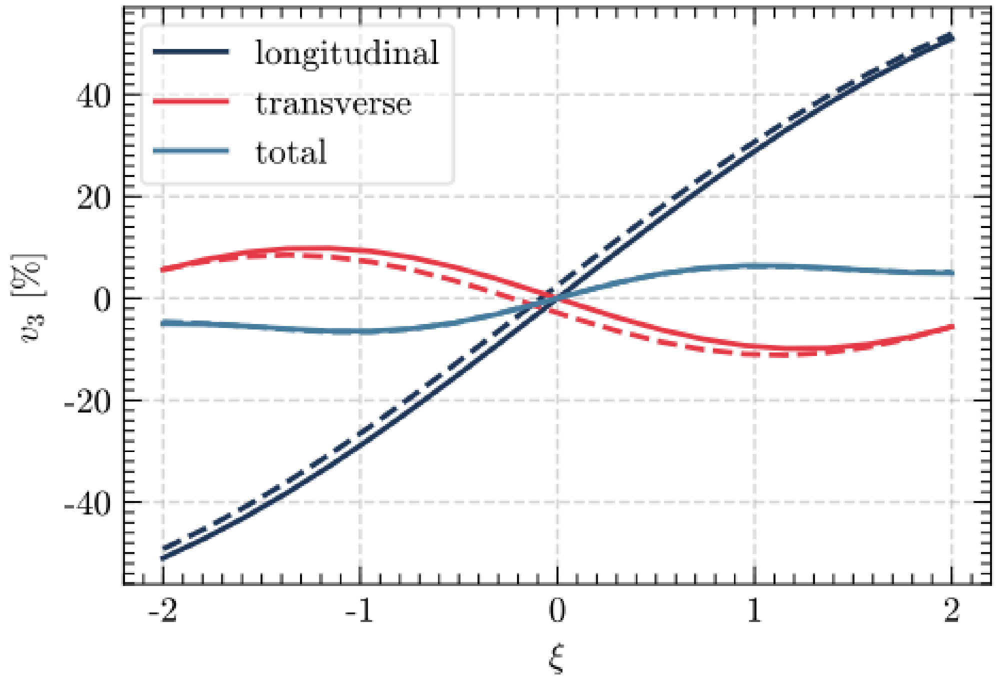


- EIC energy ($\sqrt{s} = 90$ GeV)
- 10% correction at relatively large $P_\perp$
- Weak dependence with $\Delta_\perp$

NUMERICAL RESULTS

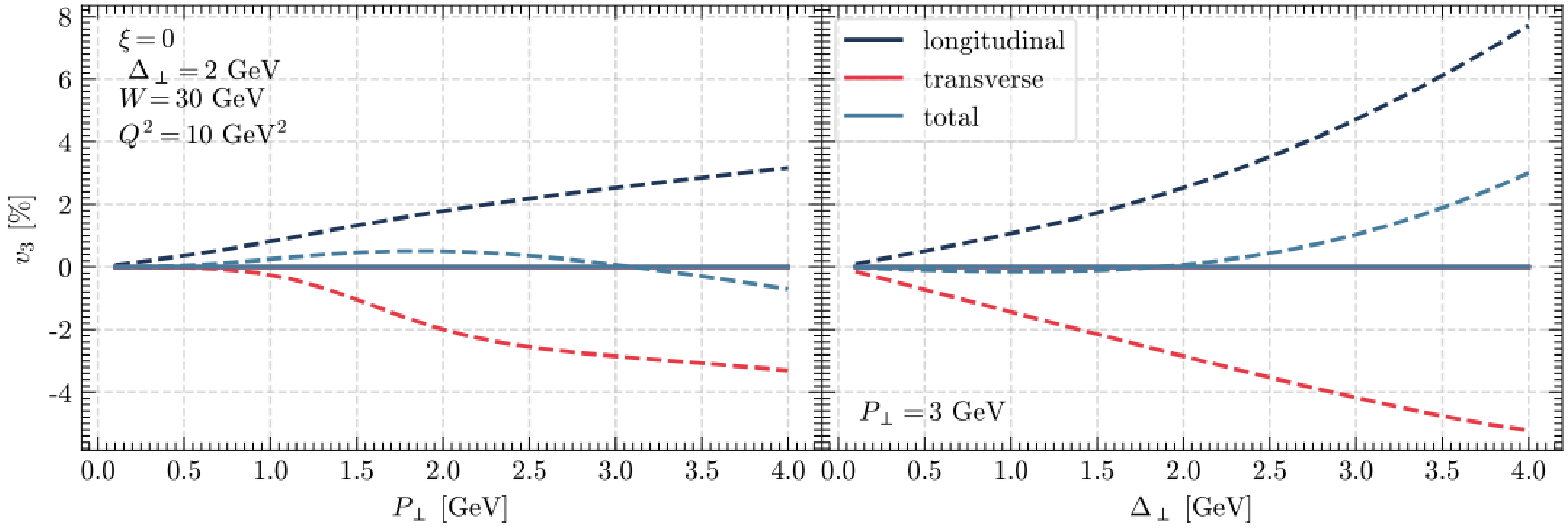
- Non-eikonal corrections break the (odd) even harmonics (anti)symmetry w.r.t. to ξ:





NUMERICAL RESULTS

- Finite odd harmonics when the particles have same longitudinal momenta ($\xi = 0$)



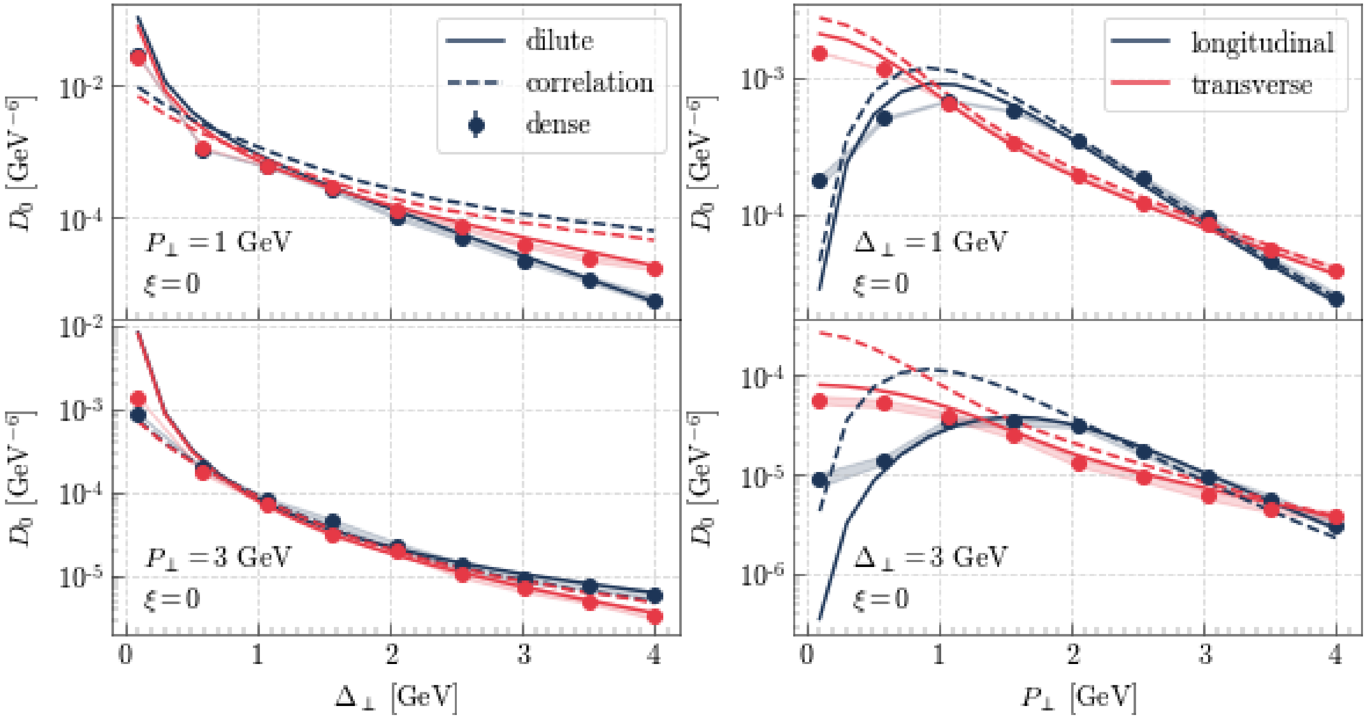
Analogous to double particle production in pA collisions

COMPARISON WITH THE DENSE AND CORRELATION LIMIT AT EIKONAL ACCURACY

- Our analysis is valid when $\Delta_\perp, P_\perp > Q_s$
- It matches the correlation limit when

$$P_\perp \gg \Delta_\perp > Q_s$$

- Very good agreement with the dense limit despite the simplicity of the model



CONCLUSIONS AND OUTLOOK

- We have computed the **field correlators** including all components
- We have analyzed the dijet differential cross section in the **dilute limit** and **beyond the eikonal approximation:**
  - **Analytical solution**
  - **O(10%) corrections** at relatively high momenta
  - **Non-zero odd harmonics**
- **Non-eikonal corrections are relevant at the EIC energies!**
- There is still room for improvement:
  - Studying the dense limit
  - Including other non-eikonal sources (z^- dependence, A^+, classical quarks)
  - More involved model for the target (include correlation, finite A effects, inhomogeneous nucleus, …)