Power corrections to back-to-back DIS dijets: Next-to-Eikonal versus twist 3

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with Tolga Altinoluk, Alina Czajka and Cyrille Marquet, (*to appear*).

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TMD vs CGC approaches

For a process with a hard $\mathbf{P}$ and a not so hard $\mathbf{k}$ transverse momenta:

- TMD factorization: leading power (twist 2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$
- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$

Consistency of both approaches shown in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ at leading power (Dominguez, Marquet, Xiao, Yuan, 2011)

Power corrections in $|\mathbf{k}|/|\mathbf{P}|$ in the regime $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ studied from the CGC approach (Altinoluk, Boussarie, Kotko, 2019)

$\Rightarrow$ What about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

Eikonal approximation in the CGC

High-energy dense-dilute scattering in the CGC : Semiclassical and Eikonal approx.

Dense target represented by a **strong semiclassical gluon field** $\mathcal{A}^{\mu}(x) \propto 1/g$
$\Rightarrow$ Perturbative expansion in g needs improvement by all order resummation of $(g\,\mathcal{A}^{\mu}(x))^n$

Eikonal approx. : limit of **infinite boost** of $\mathcal{A}^{\mu}(x)$ along x^-:

- $\mathcal{A}^{\mu}(x)$ independent on x^- (static limit) due to Lorentz time dilation
  $\Rightarrow$ No p^+ transfer from the target
- Lorentz contraction of $\mathcal{A}^{\mu}(x)$ (shockwave limit)
  $\Rightarrow$ Partons from the projectile interact instantly in x^+ with the target, without transverse motion within the target
- Under a boost of parameter γ_t along the "$-$" direction, $\mathcal{A}^-$ is enhanced and $\mathcal{A}^+$ is suppressed: $\mathcal{A}^- = O(\gamma_t) \gg \mathcal{A}_\perp = O(1) \gg \mathcal{A}^+ = O(1/\gamma_t)$

Background field in the eikonal limit: $\mathcal{A}^{\mu}(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -}\mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

$\Rightarrow$ Only $\left(g\mathcal{A}^-(x^+, \mathbf{x})\right)^n$ needs all orders resummation $\Rightarrow$ Wilson line along x^+

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

$\rightarrow$ They arise from relaxing either of the 3 approximations:

1. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation
   $\rightarrow$ Treated as gradient expansion around a common x^- value:
   $\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)$
   $\Rightarrow$ Possibility of (small) p^+ exchange with the target
2. Target with finite width
   $\Rightarrow$ transverse motion of the projectile partons within the target
3. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$

Note: Background quark field of the target also relevant at NEik.

- Separate contribution not included in this talk (See Altinoluk, Armesto, GB, 2023).
- See talk from Swaleha Mulani for the cases of DIS and SIDIS

Full NEik quark propagator through a gluon background field

Propagator from y before the target to x after the target:

$$S_F(x,y) = \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+$$
$$\times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big)$$
$$+\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)}\Big[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\,\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{z})\Big]\Bigg\} \frac{(\check{\not{k}}+m)}{2k^+}$$
$$+\text{NNEik}$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

- Generalized Eikonal contribution: also includes the NEik non-static corrections: overall z^- dependence of the Wilson line.

$$\mathcal{U}_F(x^+,y^+;\mathbf{z},z^-) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\,\mathcal{P}_+\left[-ig\int_{y^+}^{x^+} dz^+\, t\cdot\mathcal{A}^-(z)\right]^N$$

- NEik contributions beyond the shockwave approx or due to $\mathcal{A}_\perp$.

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Propagator from y before the target to x after the target:

$$
\begin{aligned}
S_F(x,y) = & \int \frac{dq^+ d^2\mathbf{q}}{(2\pi)^3} \int \frac{dk^+ d^2\mathbf{k}}{(2\pi)^3}\, \theta(q^+)\,\theta(k^+)\, e^{-ix\cdot\check{q}}\, e^{iy\cdot\check{k}}\, \frac{(\check{\not{q}}+m)}{2q^+}\gamma^+ \\
& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \Bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big) \\
& + \frac{2\pi\delta(q^+-k^+)}{(q^++k^+)} \Bigg[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \Bigg] \Bigg\} \frac{(\check{\not{k}}+m)}{2k^+} \\
& + \text{NNEik}
\end{aligned}
$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

NEik correction due to the overall transverse drift of the quark during its interaction with the target:

$$
\begin{aligned}
\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) &= \int dz^+\, \mathcal{U}_F\Big(+\infty,z^+;\mathbf{z}\Big) \overleftrightarrow{\mathcal{D}_{\mathbf{z}^j}} \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big) \\
&= 2i \int dz^+\, z^+\, \mathcal{U}_F(+\infty,z^+;\mathbf{z})\, gt\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{z})\, \mathcal{U}_F(z^+,-\infty;\mathbf{z})
\end{aligned}
$$

Full NEik quark propagator through a gluon background field

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\begin{aligned}
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& \times \int d^2\mathbf{z}\, e^{-i\mathbf{z}\cdot(\mathbf{q}-\mathbf{k})} \bigg\{ \int dz^-\, e^{iz^-(q^+-k^+)}\, \mathcal{U}_F\Big(+\infty,-\infty;\mathbf{z},z^-\Big) \\
& + \frac{2\pi\delta(q^+-k^+)}{(q^+ + k^+)} \bigg[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) \bigg] \bigg\} \frac{(\check{\not{k}}+m)}{2k^+} \\
& + \text{NNEik}
\end{aligned}
$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

NEik correction due to the transverse Brownian motion of the quark during its interaction with the target:

$$
\begin{aligned}
\mathcal{U}^{(2)}_F(\mathbf{z}) &= \int dz^+\, \mathcal{U}_F\Big(+\infty, z^+;\mathbf{z}\Big) \overleftarrow{\mathcal{D}_{\mathbf{z}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{z}^j}}\, \mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big) \\
&= -\int dz^+ \int^{z^+} dz'^+\, (z^+ - z'^+)\, \mathcal{U}_F(+\infty, z^+, \mathbf{z})\, gt\cdot\mathcal{F}_j^{\;-}(z^+,\mathbf{z}) \\
&\quad \times\, \mathcal{U}_F(z^+, z'^+;\mathbf{z})\, gt\cdot\mathcal{F}_j^{\;-}(z'^+,\mathbf{z})\, \mathcal{U}_F(z'^+,-\infty;\mathbf{z})
\end{aligned}
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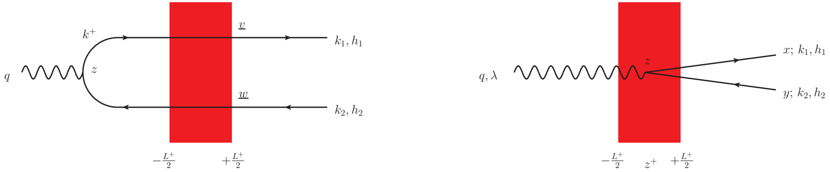
$$+\frac{2\pi\delta(q^+-k^+)}{(q^++k^+)}\bigg[-\frac{(\mathbf{q}^j+\mathbf{k}^j)}{2}\,\mathcal{U}^{(1)}_{F;j}(\mathbf{z}) - i\,\mathcal{U}^{(2)}_F(\mathbf{z}) + \frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{z})\bigg]\bigg\}\frac{(\check{\not{k}}+m)}{2k^+}$$
$$+\text{NNEik}$$

Altinoluk, G.B, Czajka, Tymowska (2021); Altinoluk, G.B (2022)

NEik coupling between the light-front helicity of the quark and the longitudinal chromomagnetic field of the target $\mathcal{F}_{ij}$:

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{z}) = \int dz^+\, \mathcal{U}_F\Big(+\infty,z^+;\mathbf{z}\Big)\, gt\cdot\mathcal{F}_{ij}(z^+,\mathbf{z})\,\mathcal{U}_F\Big(z^+,-\infty;\mathbf{z}\Big)$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*



DIS dijet cross calculated at NEik accuracy, at LO in α_s in the CGC.
(Altinoluk, G.B., Czajka, Tymowska, (2023))
See previous talk from Pedro Agostini for study in the dilute limit.

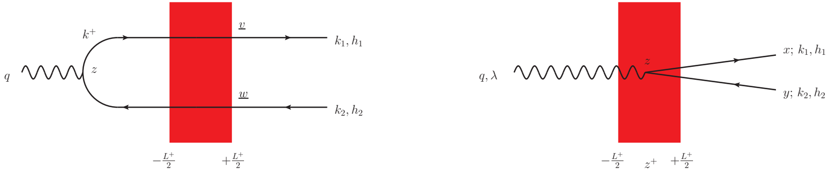
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case.

S-matrix at NEik accuracy: $S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$

$$S^{\text{Gen. Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = -2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+ v(2)\,\frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2}\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\ \mathrm{K}_0\left(\hat{Q}\,|\mathbf{v}-\mathbf{w}|\right)\int db^-\ e^{ib^-(k_1^+ + k_2^+ - q^+)}\left[\mathcal{U}_F\left(\mathbf{v}, b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) - 1\right]$$

$$\hat{Q}^2 = m^2 + \frac{(q^+ + k_1^+ - k_2^+)(q^+ - k_1^+ + k_2^+)}{4(q^+)^2}Q^2\,.$$

DIS dijet at NEik accuracy: S-matrix for γ_L^*



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$$S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\; iQ\, \frac{ee_f}{2\pi}\, \bar{u}(1)\gamma^+ v(2)\, \frac{(k_1^+ - k_2^+)}{(q^+)^2} \int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1} \int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$

$$\times \left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right) - \frac{\left(\bar{Q}^2 - m^2\right)}{2\bar{Q}}\, |\mathbf{v}-\mathbf{w}|\, \mathrm{K}_1\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right) \right] \left[\mathcal{U}_F\left(\mathbf{v}, b^-\right) \overleftrightarrow{\partial_{b^-}} \mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) \right]\Big|_{b^-=0}$$

$$\bar{Q}^2 = m^2 + \frac{k_1^+ k_2^+}{(q^+)^2}\, Q^2$$

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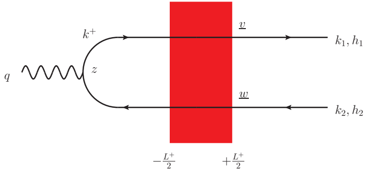

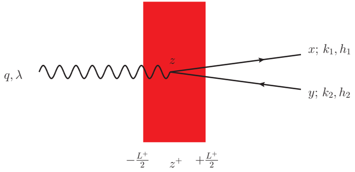

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$$S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\,\frac{ee_f}{2\pi}\,(-1)Q\,\frac{k_2^+}{(q^+)^2}\int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1}\int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\,\mathrm{K}_0\left(\bar{Q}\,|\mathbf{v}-\mathbf{w}|\right)$$
$$\times\ \bar{u}(1)\gamma^+\left[\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_F(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v})\left(\frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j}\right)\right]\mathcal{U}_F^\dagger(\mathbf{w})\,v(2)$$

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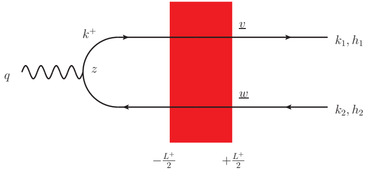

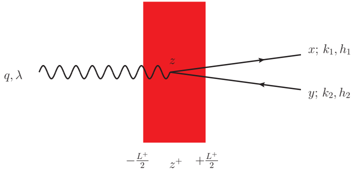

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$$\times\;\bar{u}(1)\gamma^+\left[\mathcal{U}_F(\mathbf{v})\left(\frac{[\gamma^i,\gamma^j]}{4}\,\mathcal{U}^{(3)\dagger}_{F;ij}(\mathbf{w}) - i\,\mathcal{U}^{(2)\dagger}_F(\mathbf{w}) + \left(\frac{i}{2}\overleftarrow{\partial_{\mathbf{v}^j}} - \frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2}\right)\mathcal{U}^{(1)\dagger}_{F;j}(\mathbf{w})\right)\right]v(2)$$

Change of variables and back-to-back limit

Back-to-back limit of dijets are conveniently expressed in terms of:

(dijet momentum imbalance) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

$$\boxed{\mathbf{k}_1 = \mathbf{P} + z_1\mathbf{k}} \qquad \boxed{\mathbf{k}_2 = -\mathbf{P} + z_2\mathbf{k}}$$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

such that

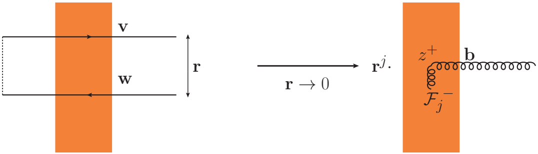
$$\boxed{\mathbf{v} = \mathbf{b} + z_2\,\mathbf{r}} \qquad \boxed{\mathbf{w} = \mathbf{b} - z_1\,\mathbf{r}}$$

back-to-back correlation limit: $|\mathbf{r}| \ll |\mathbf{b}|$

Small $\mathbf{r}$ expansion for the eikonal contribution (1)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b} + z_2\,\mathbf{r}, b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b} - z_1\,\mathbf{r}, b^-\Big) - 1\Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b}, b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b}, b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b}, b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b}, b^-)\right) + O(\mathbf{r}^2)\Big]$$
$$= \mathbf{r}^j\, t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}, b^-\Big)_{ab} (-ig)\mathcal{F}_j^{b\;-}(z^+, \mathbf{b}, b^-) + O(\mathbf{r}^2)$$



Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

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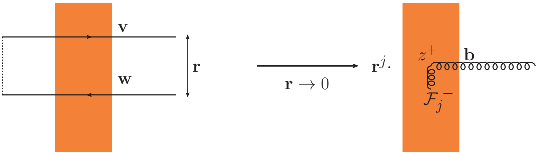
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[z_2\mathbf{r}^j\left(\partial_j\mathcal{U}_F(\mathbf{b},b^-)\right)\mathcal{U}_F^\dagger(\mathbf{b},b^-) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b},b^-)\left(\partial_j\mathcal{U}_F^\dagger(\mathbf{b},b^-)\right)+O(\mathbf{r}^2)\right]$$
$$= \mathbf{r}^j\,t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty,z^+;\mathbf{b},b^-\Big)_{ab}(-ig)\mathcal{F}_j^{b\ -}(z^+,\mathbf{b},b^-)+O(\mathbf{r}^2)$$



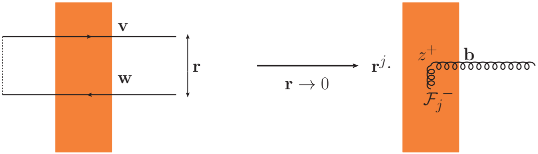
Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

However: the aim is to study the interplay between subleading power corrections
$\Rightarrow$ Terms of order $\mathbf{r}^2$ needed as well!

Small $\mathbf{r}$ expansion for the eikonal contribution (2)

Open dipole from the Generalized Eikonal term for $\mathbf{r} = \mathbf{v} - \mathbf{w} \to 0$:

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r},b^-\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r},b^-\Big)-1\Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Bigg[-i\Big(1+\frac{i(z_2-z_1)}{2}\mathbf{r}\cdot\mathbf{k}\Big)\mathbf{r}^j\,t^{a'}\int dv^+\,\mathcal{U}_A\left(+\infty,v^+;\mathbf{b},b^-\right)_{a'a}\,g\mathcal{F}_j{}^-{}_a(v^+,\mathbf{b},b^-)$$
$$-\frac{1}{2}\mathbf{r}^i\mathbf{r}^j\,t^{a'}t^{b'}\int dv^+\int dw^+\,\mathcal{U}_A\left(+\infty,v^+;\mathbf{b},b^-\right)_{a'a}\,g\mathcal{F}_i{}^-{}_a(v^+,\mathbf{b},b^-)$$
$$\times\mathcal{U}_A\left(+\infty,w^+;\mathbf{b},b^-\right)_{b'b}\,g\mathcal{F}_j{}^-{}_b(w^+,\mathbf{b},b^-)\;+O\left(|\mathbf{r}|^3\right)\Bigg]$$



$\Rightarrow$ Order $|\mathbf{r}|^2$ correction: contributions with either one or two field strength $\mathcal{F}_\perp^-$

Small $\mathbf{r}$ limit for the non-static NEik correction

For the open decorated dipole due to the dynamics of the target:

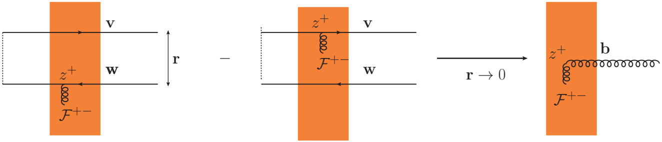
$$\left[\mathcal{U}_F\left(\mathbf{b}+z_2\,\mathbf{r},b^-\right)\overleftrightarrow{\partial_{b^-}}\mathcal{U}_F^\dagger\left(\mathbf{b}-z_1\,\mathbf{r},b^-\right)\right]\Big|_{b^-=0}$$

$$=\int_{z^+}\Big\{\mathcal{U}_F(\mathbf{b})\mathcal{U}_F^\dagger\left(z^+,-\infty;\mathbf{b}\right)igt\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F^\dagger\left(+\infty,z^+;\mathbf{b}\right)$$

$$-\mathcal{U}_F\left(+\infty,z^+;\mathbf{b}\right)(-ig)t\cdot\mathcal{F}^{+-}(z^+,\mathbf{b})\mathcal{U}_F\left(z^+,-\infty;\mathbf{b}\right)\mathcal{U}_F^\dagger(\mathbf{b})+O(|\mathbf{r}|)\Big\}$$

$$=2it^{a'}\int_{z^+}\mathcal{U}_A\left(+\infty,z^+;\mathbf{b}\right)_{a'a}g\mathcal{F}_a^{+-}(z^+,\mathbf{b})+O(|\mathbf{r}|)$$

Involves the longitudinal chromoelectric field $\mathcal{F}^{+-}$ instead of the transverse field $\mathcal{F}_j^{\ -}$

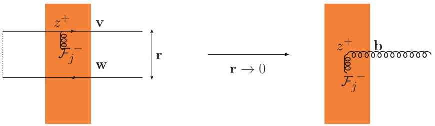


Note: Similar result for the NEik corrections with $\mathcal{U}^{(3)}_{F;ij}$, but with $\mathcal{F}_{ij}$ instead of $\mathcal{F}^{+-}$

Small $\mathbf{r}$ limit for the NEik corrections in $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_{F}$

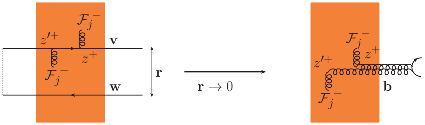
Terms with $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_{F}$ decorating the quark line (remembering that $|\mathbf{r}| \sim 1/|\mathbf{P}|$):

$$\int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big[- \Big(\mathbf{P}^j + \frac{(z_1-z_2)}{2}\mathbf{k}^j\Big)\, \mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\, \mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r})$$
$$+ \frac{i}{2}\, \mathcal{U}^{(1)}_{F;j}(\mathbf{b}+z_2\mathbf{r})\, \partial_j \mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r}) - i\, \mathcal{U}^{(2)}_{F}(\mathbf{b}+z_2\mathbf{r})\, \mathcal{U}^{\dagger}_{F}(\mathbf{b}-z_1\mathbf{r}) \Big]$$
$$= \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \Big\{ \Big[- \mathbf{P}^j + \frac{(z_2-z_1)}{2}\mathbf{k}^j \; - i z_2 \mathbf{P}^j (\mathbf{r}\cdot\mathbf{k}) \Big]\, \mathcal{U}^{(1)}_{F;j}(\mathbf{b})\, \mathcal{U}^{\dagger}_{F}(\mathbf{b})$$
$$+ \Big[\frac{i}{2}\, \delta^{ij} + \mathbf{P}^j \mathbf{r}^i \Big]\, \mathcal{U}^{(1)}_{F;j}(\mathbf{b})\, \partial_i \mathcal{U}^{\dagger}_{F}(\mathbf{b}) - i\, \mathcal{U}^{(2)}_{F}(\mathbf{b})\, \mathcal{U}^{\dagger}_{F}(\mathbf{b}) + O\left(|\mathbf{r}|\right) \Big\}$$



$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^{\dagger}_{F}\Big(\mathbf{b}\Big) \;=\; 2it^{a'} \int_{z^+} {\color{red} z^+}\, \mathcal{U}_A\Big(+\infty, z^+; \mathbf{b}\Big)_{a'a}\, g\mathcal{F}_j^{a\;-}(z^+,\mathbf{b})$$

Small $\mathbf{r}$ limit for the NEik corrections in $\mathcal{U}^{(1)}_{F;j}$ and $\mathcal{U}^{(2)}_F$



$$\mathcal{U}^{(2)}_F(\mathbf{b})\mathcal{U}^{\dagger}_F(\mathbf{b}) = -t^{a'}t^{b'}\int_{z^+,z'^+} (z^+-z'^+)\,\theta(z^+-z'^+)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{a'a}\, g\mathcal{F}^{a}_{j}{}^{-}(z^+,\mathbf{b}) \times\mathcal{U}_A\Big(+\infty,z'^+;\mathbf{b}\Big)_{b'b}\, g\mathcal{F}^{b}_{j}{}^{-}(z'^+,\mathbf{b})$$

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^{\dagger}_F(\mathbf{b}) = -\,2t^{a'}t^{b'}\int dz^+\int dz'^+\, z^+\,\mathcal{U}_A(+\infty,z^+;\mathbf{b})_{a'a}\, g\mathcal{F}^{a}_{j}{}^{-}(z^+,\mathbf{b}) \times\mathcal{U}_A(+\infty,z'^+;\mathbf{b})_{b'b}\, g\mathcal{F}^{b}_{i}{}^{-}(z'^+,\mathbf{b})$$

Like in the GEik term: contributions with either 1 or 2 $\mathcal{F}_{\perp}^{\;-}$,
but now with an extra factor z^+ or $(z^+-z'^+)$: NEik suppression with the target width.

Similar results for decorations on the antiquark line instead.

Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single $\mathcal{F}_\perp^-$ part of the Generalized Eikonal contribution in the back-to-back limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-}_{\text{Gen.Eik}} = g^2(ee_f)^2Q^2(q^++k_1^+-k_2^+)^2(q^+-k_1^++k_2^+)^2\frac{k_1^+k_2^+}{4(q^+)^6}\bigg[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4}-2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\hat{Q}^2]^4}$$
$$+16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\hat{Q}^2]^5}+O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\bigg](2q^+)\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}$$
$$\times\left\langle\mathcal{F}_i^{a\,-}\Big(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\Big[\mathcal{U}_A^\dagger\Big(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\Big)\mathcal{U}_A\Big(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\Big)\Big]_{ab}\mathcal{F}_j^{b\,-}\Big(z^+,\mathbf{b},\frac{\Delta b^-}{2}\Big)\right\rangle$$

Strict Eikonal result found by neglecting Δb^- in the fields:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-}_{\text{Strict.Eik}} = (2q^+)2\pi\delta(k_1^++k_2^+-q^+)(ee_f)^2g^24z_1^3z_2^3Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4}-2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4}+16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5}+O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle\mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

- **Correlator related to twist-2 gluon TMDs in the target**, with momentum fraction $\mathrm{x}=0$ and transverse momentum $\mathbf{k}$, with a *future staple* gauge link.
- **Kinematical twist 3 corrections**, suppressed by an extra $|\mathbf{k}|/|\mathbf{P}|$ in the back-to-back dijet limit $|\mathbf{k}|\ll|\mathbf{P}|$
- Not shown here: **Genuine twist 3 corrections**, involving a correlator of the type $\langle\mathcal{F}_\perp^-\mathcal{F}_\perp^-\mathcal{F}_\perp^-\rangle$

Back-to-back cross section: (Generalized) Eikonal piece

Squaring the single $\mathcal{F}_\perp^-$ part of the Generalized Eikonal contribution in the back-to-back limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Gen.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = g^2(ee_f)^2Q^2(q^+ + k_1^+ - k_2^+)^2(q^+ - k_1^+ + k_2^+)^2\frac{k_1^+k_2^+}{4(q^+)^6}\Bigg[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\hat{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\hat{Q}^2]^4}$$
$$+16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\hat{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\Bigg](2q^+)\int d(\Delta b^-)e^{i\Delta b^-(k_1^++k_2^+-q^+)}\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}$$
$$\times\left\langle \mathcal{F}_i^{a\,-}\left(z'^+,\mathbf{b}',-\frac{\Delta b^-}{2}\right)\left[\mathcal{U}_A^\dagger\left(+\infty,z'^+;\mathbf{b}',-\frac{\Delta b^-}{2}\right)\mathcal{U}_A\left(+\infty,z^+;\mathbf{b},\frac{\Delta b^-}{2}\right)\right]_{ab}\mathcal{F}_j^{b\,-}\left(z^+,\mathbf{b},\frac{\Delta b^-}{2}\right)\right\rangle$$

Strict Eikonal result found by neglecting Δb^- in the fields:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|_{\text{Strict.Eik}}^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2g^2 4z_1^3z_2^3Q^2$$
$$\times\Bigg[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\Bigg]$$
$$\times\int_{\mathbf{b},\mathbf{b}'}e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle\mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

- Difference between Gen. Eik and strict Eik. : involves correlator $\langle\mathcal{F}_\perp^-\mathcal{F}_\perp^-\mathcal{F}^{+-}\rangle$ or $\mathbf{k}\langle\mathcal{F}_\perp^-\mathcal{F}^{+-}\rangle$

$\Rightarrow$ NEik correction but twist 4: beyond our accuracy here!

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$, of order Eik or NEik, and twist 2 or twist 3:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+ z_1 z_2} + \text{NNEik}\right]$$
$$\times \left\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab} \mathcal{F}_j^{b\;-}(z^+,\mathbf{b})\right\rangle$$

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_\perp^{\;-} \mathcal{F}_\perp^{\;-} \rangle$, of order Eik or NEik, and twist 2 or twist 3:

$$\frac{d\sigma_{\gamma_L^* \to q_1 \bar{q}_2}}{d\text{P.S.}}\Bigg|^{\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times \left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2 - z_1)\frac{(\mathbf{P}^i\mathbf{k}^j + \mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} \left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+ z_1 z_2} + \text{NNEik}\right]$$
$$\times \left\langle \mathcal{F}_i^{a\;-}(z'^+, \mathbf{b}') \left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}')\mathcal{U}_A(+\infty, z^+; \mathbf{b})\right]_{ab} \mathcal{F}_j^{b\;-}(z^+, \mathbf{b})\right\rangle$$

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_\perp^{\;-} \mathcal{F}_\perp^{\;-} \rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Notation: TMD $\langle \mathcal{F}^{\mu\nu}\mathcal{F}^{\rho\sigma} \rangle$ correlator in unpolarized target:

$$\Phi^{\mu\nu;\rho\sigma}(\text{x}, \mathbf{k}) \equiv \frac{1}{\text{x}P_{tar}^-}\frac{1}{(2\pi)^3}\int d^2\mathbf{z}\, e^{-i\mathbf{k}\cdot\mathbf{z}} \int dz^+\, e^{i\text{x}P_{tar}^- z^+} \left\langle P_{tar}\middle| \mathcal{F}_a^{\mu\nu}(0)\left[\mathcal{U}_A^\dagger(+\infty, 0; 0)\mathcal{U}_A(+\infty, z^+; \mathbf{z})\right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+, \mathbf{z})\middle| P_{tar}\right\rangle$$
$$= \frac{1}{(2\pi)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+, z'^+} e^{i\text{x}P_{tar}^-(z^+ - z'^+)} \left\langle \mathcal{F}_a^{\mu\nu}(z'^+, \mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty, z'^+; \mathbf{b}')\mathcal{U}_A(+\infty, z^+; \mathbf{b})\right]_{ab} \mathcal{F}_b^{\rho\sigma}(z^+, \mathbf{b})\right\rangle,$$

Back-to-back cross section: twist 2 TMDs contribution

Including all contributions of the form $\langle \mathcal{F}_{\perp}^{\;-}\mathcal{F}_{\perp}^{\;-}\rangle$, of order Eik or NEik, and twist 2 or twist 3:

- NEik corrections and kinematic twist 3 corrections to the $\langle \mathcal{F}_{\perp}^{\;-}\mathcal{F}_{\perp}^{\;-}\rangle$ contribution factorize from each other!
- Result of the same form can be obtained for transverse photon case.

Notation: TMD $\langle \mathcal{F}^{\mu\nu}\mathcal{F}^{\rho\sigma}\rangle$ correlator in unpolarized target:

$$\Phi^{\mu\nu;\rho\sigma}(\mathrm{x},\mathbf{k}) \equiv \frac{1}{\mathrm{x}P_{tar}^-}\frac{1}{(2\pi)^3}\int d^2\mathbf{z}\, e^{-i\mathbf{k}\cdot\mathbf{z}}\int dz^+\, e^{i\mathrm{x}P_{tar}^- z^+}\Big\langle P_{tar}\Big|\mathcal{F}_a^{\mu\nu}(0)\Big[\mathcal{U}_A^{\dagger}(+\infty,0;0)\,\mathcal{U}_A(+\infty,z^+;\mathbf{z})\Big]_{ab}\mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{z})\Big|P_{tar}\Big\rangle$$
$$= \frac{1}{(2\pi)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+} e^{i\mathrm{x}P_{tar}^-(z^+-z'^+)}\Big\langle \mathcal{F}_a^{\mu\nu}(z'^+,\mathbf{b}')\Big[\mathcal{U}_A^{\dagger}(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\Big]_{ab}\mathcal{F}_b^{\rho\sigma}(z^+,\mathbf{b})\Big\rangle\,,$$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|^{\mathcal{F}_{\perp}^{\;-}\mathcal{F}_{\perp}^{\;-}} = \alpha_{\mathrm{em}}\,\alpha_s\, e_f^2\, \mathcal{C}_{T,L}^{ij}(z_1,\mathbf{P},\mathbf{k})\bigg\{\bigg[1+\frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1z_2W^2}\,\partial_{\mathrm{x}} + \mathrm{NNEik}\bigg]\Big[\mathrm{x}\,\Phi^{i-;j-}(\mathrm{x},\mathbf{k})\Big]\bigg\}\bigg|_{\mathrm{x}=0}$$
$$= \alpha_{\mathrm{em}}\,\alpha_s\, e_f^2\, \mathcal{C}_{T,L}^{ij}(z_1,\mathbf{P},\mathbf{k})\bigg\{\Big[\mathrm{x}\,\Phi^{i-;j-}(\mathrm{x},\mathbf{k})\Big]\bigg|_{\mathrm{x}=\frac{(\mathbf{P}^2+\bar{Q}^2)}{z_1z_2W^2}} + \mathrm{NNEik}\bigg\}$$

Non-zero value of momentum fraction x in the twist 2 gluon TMDs recovered from NEik corrections!

Reminder:

$$\Phi^{i-;j-}(\mathrm{x},\mathbf{k}) = \frac{\delta^{ij}}{2}\, f_1^g(\mathrm{x},\mathbf{k}) + \Big[\mathbf{k}^i\mathbf{k}^j - \frac{\mathbf{k}^2}{2}\,\delta^{ij}\Big]\frac{1}{2M^2}\, h_1^{\perp g}(\mathrm{x},\mathbf{k})$$

Back-to-back cross section: twist 3 TMDs from NEik

From the interference between the non-static NEik correction and the strict Eikonal amplitudes: terms in $\langle \mathcal{F}^{+-}\mathcal{F}_{\perp}^{-}\rangle$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|_{NEik}^{\mathcal{F}_{\perp}^{-}\mathcal{F}^{+-}} = \alpha_{\rm em}\,\alpha_s\, e_f^2\,\frac{1}{2q^+}\mathcal{C}^{j}_{T,L}(z_1,\mathbf{P})\Big[\mathrm{x}\,\Phi^{j-;+-}(\mathrm{x},\mathbf{k}) + \mathrm{x}\,\Phi^{+-;j-}(\mathrm{x},\mathbf{k})\Big]\Bigg|_{\mathrm{x}=0}$$

$\Rightarrow$ NEik. correction beyond the static approximation for the target involves a **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)) with momentum fraction $\mathrm{x}=0$.

From the interference between the NEik correction with $\mathcal{U}^{(3)}_{F;ij}$ and the strict Eikonal amplitude: terms in $\langle \mathcal{F}^{ij}\mathcal{F}_{\perp}^{-}\rangle$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|_{NEik}^{\mathcal{F}_{\perp}^{-}\mathcal{F}^{ij}} = \alpha_{\rm em}\,\alpha_s\, e_f^2\,\frac{1}{2q^+}\mathcal{C}^{ijl}_{T,L}(z_1,\mathbf{P})\Big[\mathrm{x}\,\Phi^{l-;ij}(\mathrm{x},\mathbf{k}) + \mathrm{x}\,\Phi^{ij;l-}(\mathrm{x},\mathbf{k})\Big]\Bigg|_{\mathrm{x}=0}$$

- Contribution from another type of **twist-3 gluon TMD**, (Mulders, Rodrigues (2001)), still with $\mathrm{x}=0$.
- Note: term absent in in the γ^*_L: $\mathcal{C}^{ijl}_{L}(z_1,\mathbf{P})=0$ due to Dirac algebra.

Parametrization (Lorcé, Pasquini (2013)):

$$\Phi^{j-;+-}(\mathrm{x},\mathbf{k}) + \Phi^{+-;j-}(\mathrm{x},\mathbf{k}) = \frac{2\mathbf{k}^j}{P^-_{tar}}\, f^{\perp g}(\mathrm{x},\mathbf{k}) \quad \text{and} \quad \Phi^{l-;ij}(\mathrm{x},\mathbf{k}) + \Phi^{ij;l-}(\mathrm{x},\mathbf{k}) = \epsilon^{ij}\,\epsilon^{ln}\,\frac{2\mathbf{k}^n}{P^-_{tar}}\,\bar{g}^{\perp g}(\mathrm{x},\mathbf{k})$$

$$\frac{d\sigma_{\gamma^*_{L,T}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\Bigg|_{NEik}^{\mathcal{F}_{\perp}^{-}\mathcal{F}^{+-}+\mathcal{F}_{\perp}^{-}\mathcal{F}^{ij}} = \alpha_{\rm em}\,\alpha_s\, e_f^2\left\{\frac{2\mathbf{k}^j}{W^2}\,\mathcal{C}^{j}_{T,L}(z_1,\mathbf{P})\,\mathrm{x}f^{\perp g}(\mathrm{x},\mathbf{k}) + \frac{2\mathbf{k}^n}{W^2}\,\epsilon^{ij}\,\epsilon^{ln}\,\mathcal{C}^{ijl}_{T,L}(z_1,\mathbf{P})\,\mathrm{x}\bar{g}^{\perp g}(\mathrm{x},\mathbf{k})\right\}\Bigg|_{\mathrm{x}=0}$$

Back-to-back cross section: 3-body correlators

Leftover contributions in $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$, of order Eik or NEik, starting at twist 3.

Various terms in the cross section, with a correlator of the form:

$$
\begin{aligned}
&2\mathrm{Re}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{v^+,w^+,v'^+}\left[d^{a'b'c'}\;\;\text{or}\;\; i\,f^{a'b'c'}\right]\left[1+i\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+z_1z_2}\,\phi(v^+-v'^+,w^+-v'^+)\right]\\
&\times\Big\langle \mathcal{U}_A(+\infty,v'^+;\mathbf{b}')_{c'c}\,g\mathcal{F}_l^{c\;-}(v'^+,\mathbf{b}')\,\mathcal{U}_A(+\infty,v^+;\mathbf{b})_{a'a}\,g\mathcal{F}_i^{a\;-}(v^+,\mathbf{b})\,\mathcal{U}_A(+\infty,w^+;\mathbf{b})_{b'b}\,g\mathcal{F}_j^{b\;-}(w^+,\mathbf{b})\Big\rangle
\end{aligned}
$$

with several possible functions ϕ in the NEik correction.

In the TMD formalism, such 3 body correlator should in principle depend on two momentum fractions x_1 and x_2 via phase factors.

Not entirely clear at the moment how to resum the NEik corrections unambiguously in order to recover this dependence on x_1 and x_2.

Summary

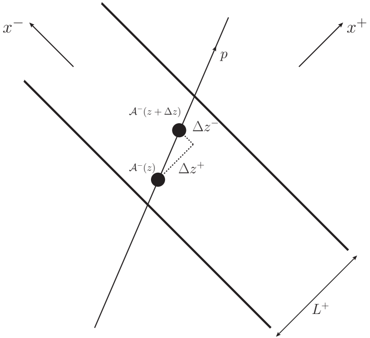
To further understand the interplay between CGC and TMD, we studied the NEik DIS dijet cross-section in the back-to-back jets limit, including twist 3 power corrections. Various types of contributions are obtained:

- $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \rangle$: twist 2 gluon TMDs
  - Factorization of kinematic twist 3 and of NEik corrections
  - NEik correction is the first order correction in the Taylor expansion of the TMDs around $\mathrm{x} = 0$
    $\Rightarrow$ x dependence of the TMDs recovered by resumming terms of all powers beyond the eikonal approximation
- 3-body twist 3 correlators $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \mathcal{F}_l^{\ -} \rangle$: beyond TMD partonic distributions
  - Already appear in Eikonal contributions
  - NEik corrections are related to the dependence on x_1, x_2
  - But unclear how to perform their resummation
- Twist 3 gluon TMDs, of the type $\langle \mathcal{F}_i^{\ -} \mathcal{F}^{+-} \rangle$ and (for γ_T^*) $\langle \mathcal{F}_l^{\ -} \mathcal{F}_{ij} \rangle$, found as further contributions to the cross section at NEik and twist 3 order.

More about NEik corrections beyond the static approx

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator:

$$\mathcal{A}^-(z^- + \Delta z^-) - \mathcal{A}^-(z^-) \simeq \Delta z^- \; \partial_- \mathcal{A}^-(z^-)$$



- Slow z^- dependence from time dilation:

$$\partial_- \mathcal{A}^- \propto \frac{1}{\gamma_t} \mathcal{A}^-$$

- Small Δz^- displacement of the trajectory within the target width L^+:

$$\Delta z^- \sim \frac{p^-}{p^+} \Delta z^+ \leq \frac{p^-}{p^+} L^+ = O\left(\frac{1}{\gamma_t}\right)$$

Double power suppression, beyond static approx and beyond shockwave approx:

$\Rightarrow$ NNEik effect within a single propagator!

More about NEik corrections beyond the static approx

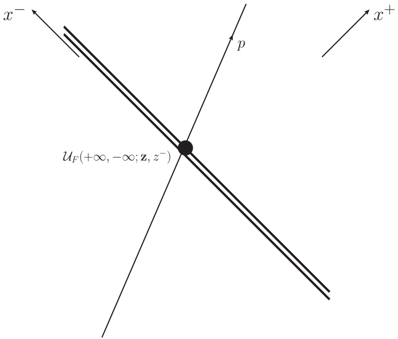
Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator is NNEik.

However, dependence on average z^- is suppressed only once.

$\Rightarrow$ Use Wilson lines with overall z^- dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma_t} \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

$\rightarrow$ Accounts for NEik effects beyond static approx



More about NEik corrections beyond the static approx

Effect of relative z^- dependence of $\mathcal{A}^-$ insertions along one propagator is NNEik.

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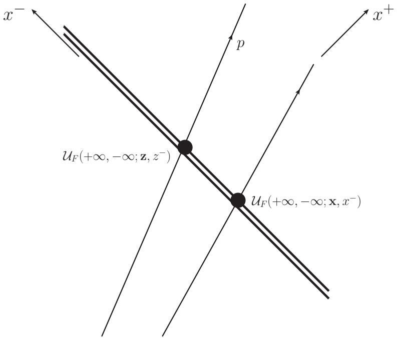
$\Rightarrow$ Use Wilson lines with overall z^- dependence

$$\partial_- \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-) \propto \frac{1}{\gamma t}\, \mathcal{U}_F(+\infty, -\infty; \mathbf{z}, z^-)$$

$\rightarrow$ Accounts for NEik effects beyond static approx

In particular: NEik corrections induced by the difference in z^- between different Wilson lines.



Recap: Corrections beyond the static approximation

- Relative z^- dependence along the same propagator : NNEik, higher twist ?
- Relative z^- dependence between Wilson lines in the amplitude: NEik, twist 3
- Relative z^- dependence between amplitude and cc. amplitude: NEik, twist 4
- Overall z^- dependence at cross section level: Gone when performing target average

All these non-static corrections can be written as insertions of the longitudinal chromoelectric field $\mathcal{F}^{+-}$ of the target.