NCBJ

SIDIS and Inclusive DIS: Beyond Eikonal Order

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Dipole Approximation: SIDIS

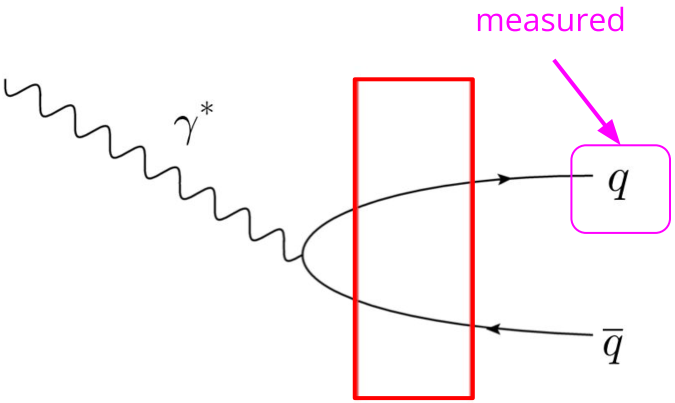
- In TMD factorization,

$$\frac{d\sigma}{d\mathcal{P}} \propto \int \frac{dz}{z_f} \frac{D(z)}{z_f^2} f(q_\perp, x) \times H(\xi, k_\perp)$$

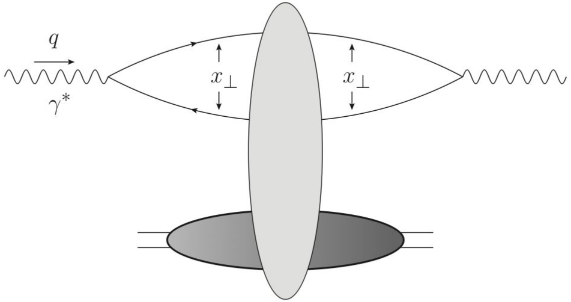
- In Dipole factorization sea quark TMD is recovered.
  (Marquet, Xiao, Yuan [arXiv:0906.1454])

$$f(q_\perp, x) \propto \mathcal{C} \otimes S(r_\perp, b_\perp)$$

- But in this model contribution coming due to valence quarks are not included.



Dipole Approximation: Inclusive DIS



From Dipole approximation, we can write total cross-section for DIS as

$$\sigma_{L,T}^{\gamma * p}(x, Q^2) = \sum_f \int d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} \left|\Psi_{\gamma_{L,T}^* \to q\bar{q}}\right|^2 \sigma_{q\bar{q}}(x, r)$$

- This model is well sufficient and explains contribution coming due to sea quarks at low-x.
- But when integration $z \longrightarrow 1$ or $z \longrightarrow 0$, dipole approximation is not justified.
- We have to include corrections to explain kinematics near limits 1 and 0.
- Also, in this model contribution coming due to valence quarks are not included.

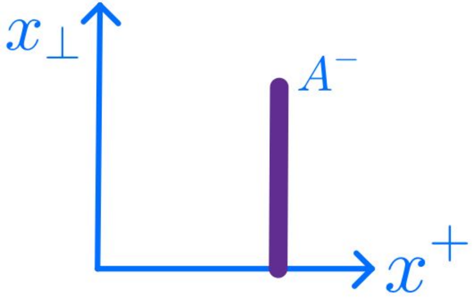
Generally in saturation physics in Color Glass Condensate (CGC) framework 2 approximations:

- **Semi-classical approximation:**

  Dense target given by **Strong semi-classical gluon field $A_\mu(x)\sim 1/g>>1$**

- **Eikonal approximation :**

  Limit of infinite boost of $A_\mu(x)$



- Taking into account **only leading power in** terms of high energy : (here, leading order component w.r.t. γ_t)
- Good enough approximation to describe physics at very high energy accelerators.

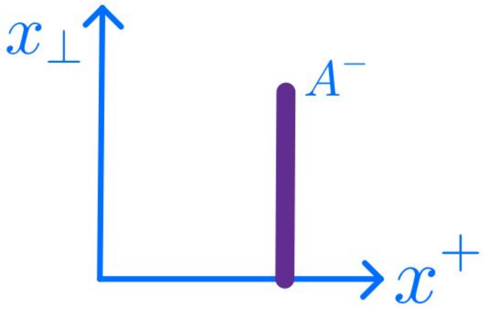
Eikonal Order: For $A_\mu(x)$

Eikonal Order

1. Shockwave approx.: target is localised in the longitudinal direction $x^+ = 0$ (**zero width**).
2. **Only leading - component of target considered**, subleading components are neglected (suppressed by γ_t)
3. Time dilation and static approximation: **x^- dependence of target neglected**

In light-cone coordinate, w.r.t. Lorentz boost factor of target (γ_t)

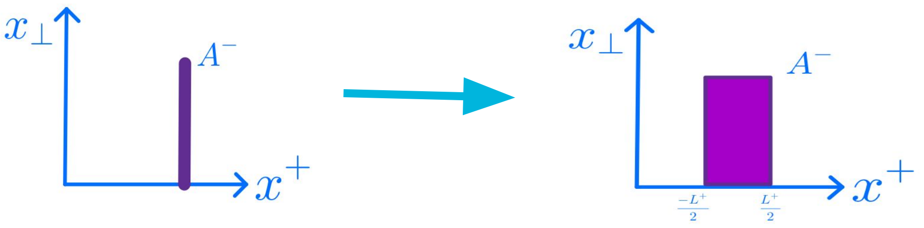
$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$



w.r.t. Lorentz boost factor of target (γ_t)

$$A^- = \mathcal{O}(\gamma_t) >> A^j = \mathcal{O}(1) >> A^+ = \mathcal{O}(1/\gamma_t)$$

Going Beyond Eikonal Order: For $A_\mu(x)$



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Next-to-eikonal Order

1. Instead of infinite thin shockwave as a target, we consider **finite width** of a target.
2. Include **transverse component** of background field(target).
3. Consider background field is x^- dependent: **dynamics of the target are considered**.

Pedro, Guillaume's Talk

Computations in : T. Altinoluk et.al. (2212.10484), P. Agostini et.al. (2403.04603) etc.

Going Beyond Eikonal Order: Quark Background Field

- Due to large boost of the target along x^-: its localized in longitudinal x^+ direction around small support (**Similar shockwave as Gluon**).
- If we consider projections on quark background field then,

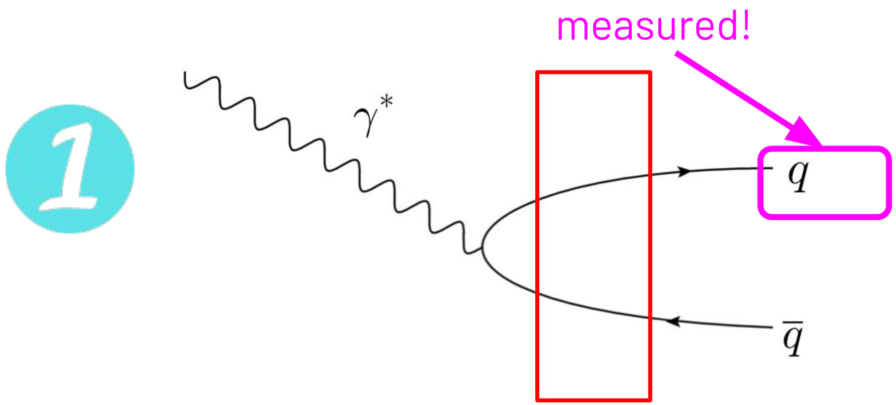
$$\Psi(z) = \frac{\gamma^+\gamma^-}{2}\Psi(z) + \frac{\gamma^-\gamma^+}{2}\Psi(z) = \Psi^-(z) + \Psi^+(z)$$

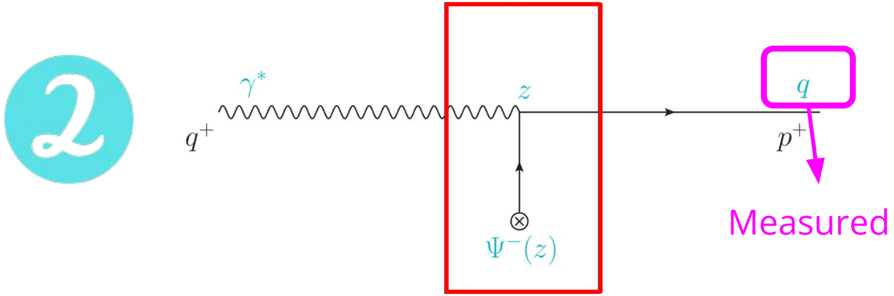
$$\Psi^-(z) \to \mathcal{O}(\sqrt{\gamma_t}) \qquad \Psi^+(z) \to \mathcal{O}(1/\sqrt{\gamma_t})$$

- For **Next-to-eikonal (NEik) corrections, only - component** considered and + component is neglected (contribute at NNEik only).
- Contribution at NEik order represent t-channel quark exchange.

Semi Inclusive Deep Inelastic Scattering (SIDIS):

- At low-x, for this process: two kinds of contributions!
- Each of them are expected to be dominant in different kinematic regions.





- Contribution (1) is studied by Marquet, Xiao, Yuan [arXiv:0906.1454]. There is contribution at eikonal order.
- In this talk contribution coming due to (2) is discussed. No contribution at eikonal order.

SIDIS: S-matrix computation

- S-matrix at NEik order calculated : only Ψ^-(z) of component considered

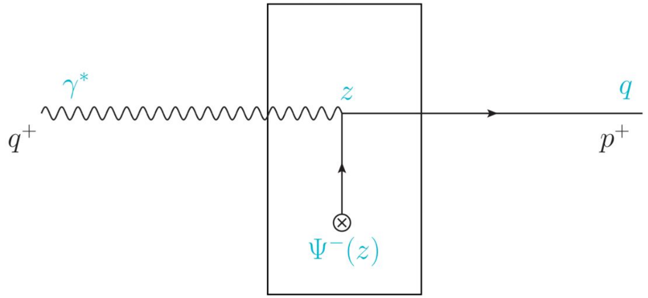
$$S_{\gamma*\to q} = \lim_{x^+\to\infty} \int d^2x_\perp \int dx^- \; e^{i\check{p}\cdot x} \int d^4z \; \epsilon_\mu^\lambda(q) \; e^{-iq\cdot z} \; \overline{u}(p,h) \; \gamma^+ \; S_F(x,z)|_{Eik}^{IA}(-iee_f\gamma^\mu)\Psi^-(z)$$

- Two polarizations of photons are considered:
  - Longitudinal Polarization:

    no contribution at NEik order

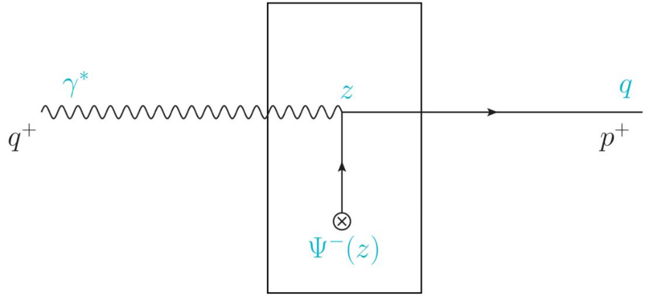
  - Transverse Polarization:

    Contribution at NEik order



Similar calculations in case of q-g dijets are done by Altinoluk, Armesto, & Beuf (arXiv:2303.12691)

SIDIS: S-matrix computation



Finally, S-matrix for SIDIS process:

$\mathcal{O}(1/\gamma_t)$

$$S_{\gamma_T^*\to q} = 2\pi\delta(q^+ - p^+) \int dz^+ \int d^2 z_\perp \; e^{i(q_\perp - p_\perp)z_\perp} \; \overline{u}(p,h) \times \epsilon_\lambda^j (iee_f) U_F(\infty, z^+, z_\perp) \left(\frac{\gamma^j \gamma^+ \gamma^-}{2}\right) \Psi(z)$$

$\mathcal{O}(\sqrt{\gamma_t})$

- Cross-section:

$$\frac{d^2\sigma^{\gamma_T^*\to q}}{d^2 p_\perp} = \frac{e^2 e_f^2}{(2\pi)^2} \frac{1}{2} \frac{1}{2q^+} \int d^2 z'_\perp \int d^2 z_\perp \; e^{i(q_\perp - p_\perp)(z_\perp - z'_\perp)} \int dz'^+ \int dz^+ \times \left\langle \overline{\Psi}(z')\gamma^- \; \mathcal{U}_F^\dagger(\infty, z'^+, z'_\perp) \; \mathcal{U}_F(\infty, z^+, z_\perp) \; \Psi(z) \right\rangle$$

Over all suppression of $\mathcal{O}(1/\gamma_t)$: NEik order

SIDIS: Relation at small-x between CGC and TMD calculations

- In Unpolarized target, the CGC-like target average $\langle \mathcal{O} \rangle$ is proportional to the quantum expectation value in the momentum state of target.

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

- Using this relation, we can relate obtained cross-section with unpolarized transverse momentum dependent (TMD) quark distribution.
- By comparing with quark TMD function, we get cross section:

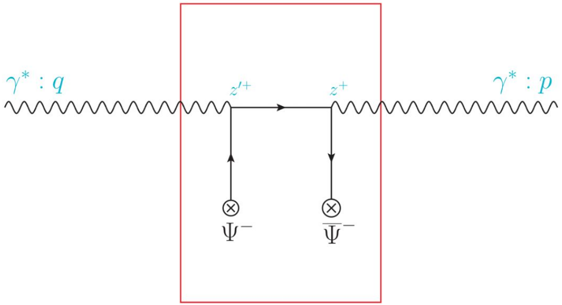
$$\frac{d^2\sigma^{\gamma_T^* \to q}}{d^2 p_\perp} = \frac{\pi e^2 e_f^2}{W^2} \, f_1^q(x = 0, p_\perp - q_\perp)$$

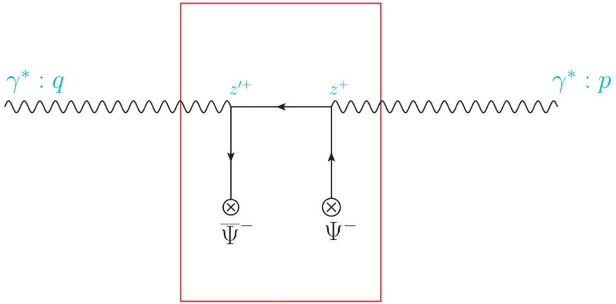
Suppression by centre of mass energy $1/W^2$ characterizes NEik contribution in terms of t channel quark exchange!

TMD-[arXiv:2303.12691]

Inclusive DIS:

- Two contributions added together:
  - From quark propagator
  - From antiquark propagator
- No contribution at Eikonal order due to quark background field.
- Two polarizations of photons are considered:
  - Longitudinal Polarization: no contribution at NEik order
  - Transverse Polarization: Contribution at NEik order



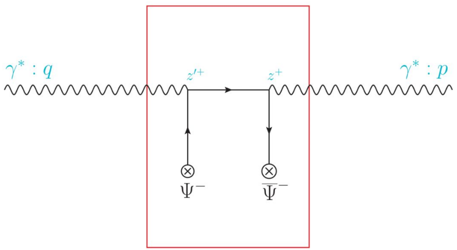


Inclusive DIS: Cross-section Computation

For contribution due to from inside to inside the medium quark propagator

- S-matrix:

$$S^{q}_{\gamma^*\to\gamma^*} = 2\pi\delta(p^+ - q^+)\ (e^2 e_f^2)\ \epsilon^{i\ *}_{\lambda_2}\ \epsilon^{j}_{\lambda_1} \int d^2 z_\perp \int dz^+ \int dz'^+\ \theta(z^+ - z'^+)\ e^{-i(p_\perp - q_\perp) z_\perp}$$
$$\times\ \overline{\Psi}_\beta(z^+, z_\perp)\ U_F(z^+, z'^+, z_\perp)_{\beta\alpha}\ \Big(\frac{\gamma^i\gamma^j\gamma^-}{2}\Big)\ \Psi_\alpha(z'^+, z_\perp)$$



- From Optical theorem:

$$\sigma^{\gamma*}_{\lambda} = 2\mathrm{Im}\mathcal{M}_{\gamma^*_\lambda\to\gamma^*_\lambda} = 2\mathrm{Re}(-i)\mathcal{M}_{\gamma^*_\lambda\to\gamma^*_\lambda}$$

- Cross-section:

$$\sigma^{\gamma^*}_{\mathrm{T}}|^q = \mathrm{Re}\Big\{\frac{(e^2 e_f^2)}{2q^+}\int d^2 z_\perp \int dz^+ \int dz'^+\ \theta(z^+ - z'^+)\ \overline{\Psi}(z^+, z_\perp)\ U_F(z^+, z'^+, z_\perp)\gamma^-\ \Psi(z'^+, z_\perp)\Big\}$$

Inclusive DIS: Relation at small-x between CGC and Integrated PDF calculations

$$\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} \,|\, \mathcal{O} \,|\, P_{tar} \rangle}{\langle P'_{tar} \,|\, P_{tar} \rangle}$$

Similar to SIDIS, we will use relation of operators

$$\sigma_{T,L} = \frac{(2\pi)^2 \alpha_{em}}{Q^2} F_{T,L} = \frac{\pi e^2}{Q^2} F_{T,L}$$

Structure function can be written in terms of cross section

$$F_L^q = 0 \quad F_2^q = F_T^q = \frac{Q^2}{W^2} \sum_f e_f^2 \, q_f(x)|_{x=0}$$

we recover quark contribution to structure function in terms of quark PDF

We can also obtain similar contribution in the case of antiquark

Summary

- Beyond eikonal order corrections (next-to-eikonal order) to cross-sections were computed by including quark background field effect comes from t-channel quark exchange.
- Cross-sections in CGC for SIDIS and Inclusive DIS were compared with TMD and PDF computations at small-x.
  - SIDIS Cross-section at small-x, in case of longitudinal incoming and outgoing momentum are equal, can be written in terms of TMD and its Next-to-eikonal order.
  - Quark contribution to PDF at small-x can be written in terms of CGC cross-section.
- In both SIDIS and Inclusive DIS corrections effects of valence quarks as long with small-x sea quarks were computed.

Thank you!