Running of α_s with 3-loop accuracy

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in collaboration with

A. Bauer, R. Harlander, J. Salomon, M. Steinhauser

Running of couplings

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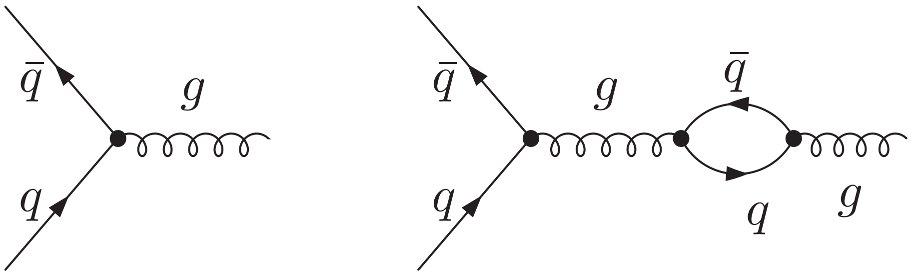

$\sim g_s$ $\qquad$ $\sim g_s^3$

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  - Vacuum can screen or anti-screen the gauge charges.
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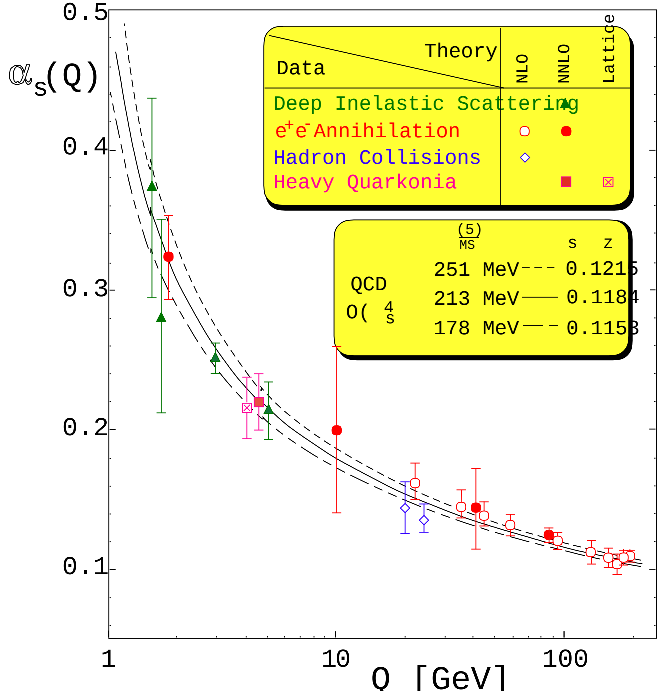

Grand Unification

- Gauge symmetry increases with energy [Georgi, Quinn, Weinberg '74]

	Low energy		$\Rightarrow$	High energy
$SU_c(3)\otimes$	$SU_L(2)\otimes$	$U_Y(1)$	$\Rightarrow$	G_{GUT}
gluons	W, Z	photon	$\Rightarrow$	gauge bosons
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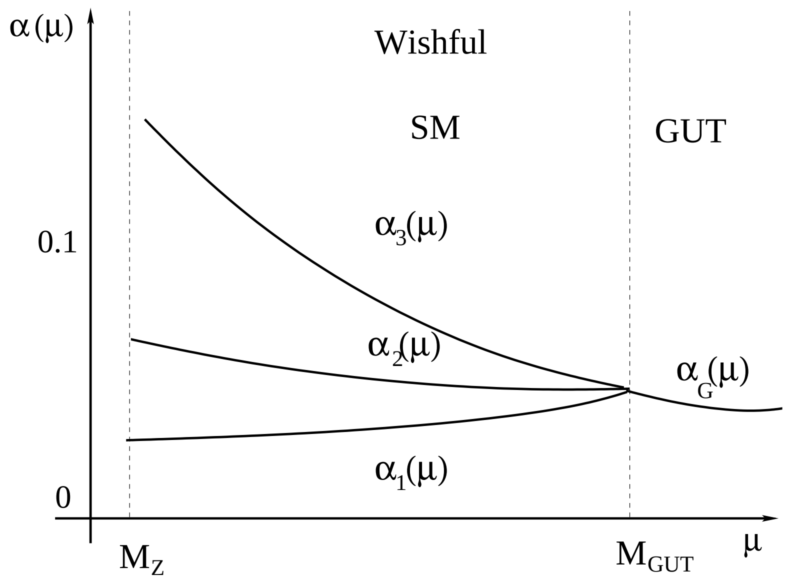
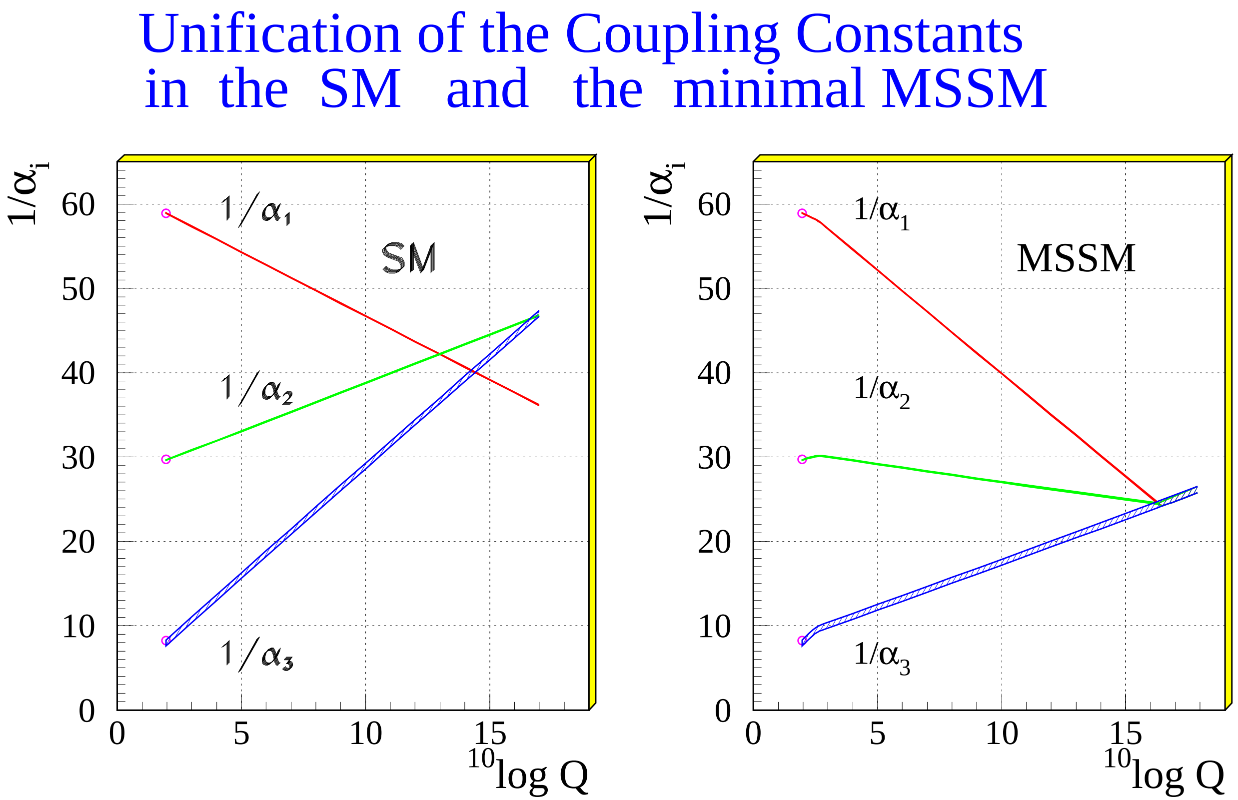
- Low energy interactions = branches of the unique interaction of a simple gauge group.

$$\alpha_i \equiv g_i^2/4\pi : \qquad \begin{aligned} \alpha_1(M_Z) &= 0.017\,, \\ \alpha_2(M_Z) &= 0.034\,, \\ \alpha_3(M_Z) &= 0.118\,, \\ M_Z &= 91.1876 \text{ GeV}\,. \end{aligned}$$



MSSM and LEP data



[Amaldi, Furstenau, de Boer]

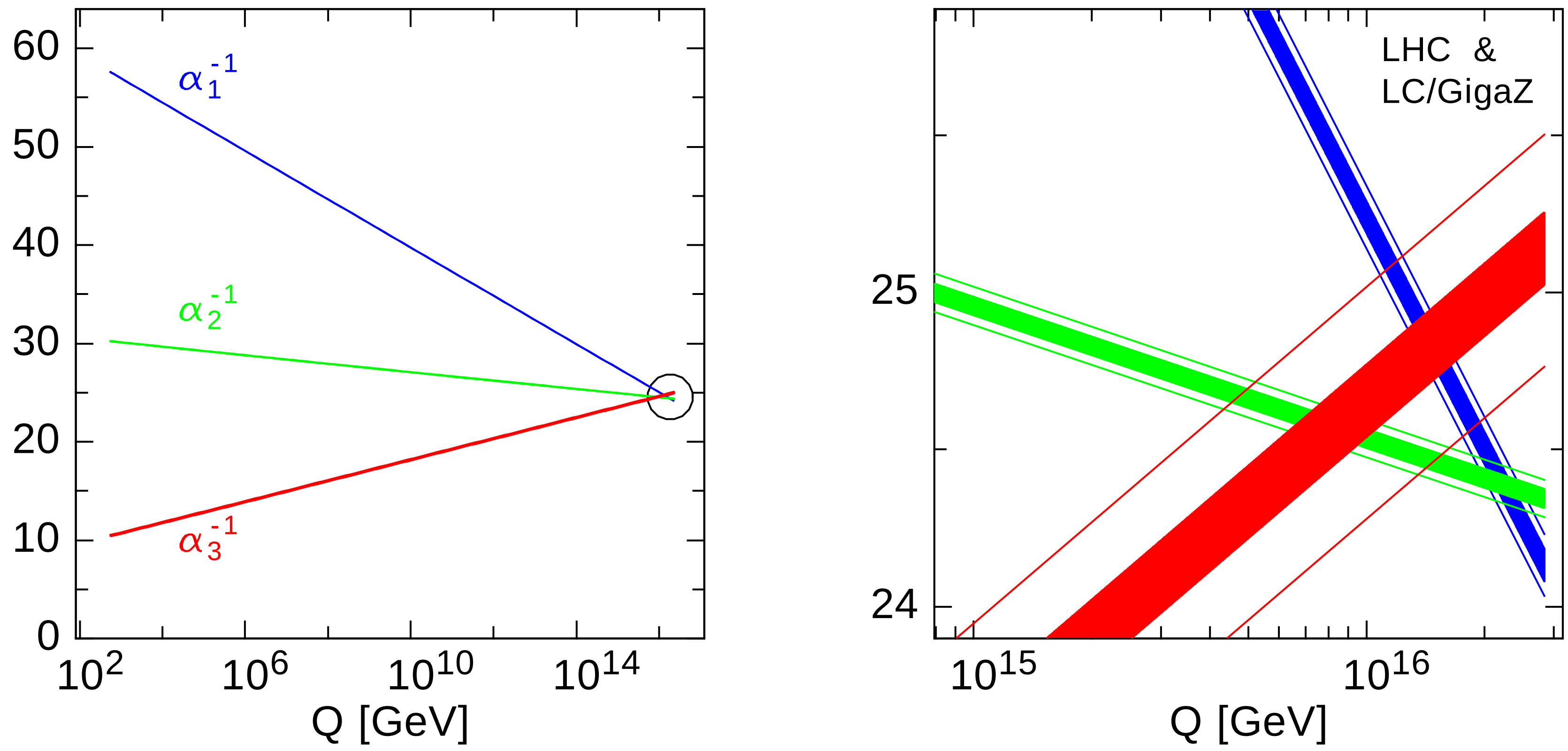
[Langacker, Luo]

[Ellis, Kelley, Nanopoulos]

- Gauge Coupling unification within SM excluded by about $12\,\sigma$.
- Gauge coupling Unification within SUSY GUTs works extremely well: it fits within 3σ the present low energy data.

High precision data

[Blair, Porod, Zerwas '04]



- Computation: common SUSY mass scale $\simeq$1 TeV
  2-loop Renormalization Group Running 1-loop threshold corrections at the weak scale (M_Z)
- Our aim: improve theoretical accuracy on $\alpha_s(M_{\mathrm{GUT}})$ calculated from $\alpha_s(M_{\mathrm{Z}})$

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  - Our aim: 3-loop RGEs for SUSY-QCD sector
    2-loop threshold corrections *R. Harlander, L. M., M. Steinhauser '05*
    $\Rightarrow$ $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ with 3-loop accuracy

Evolution of the strong coupling

- Precision calculations in MSSM require a manifestly SUSY and gauge invariant Regularization scheme $\Rightarrow$ DRED

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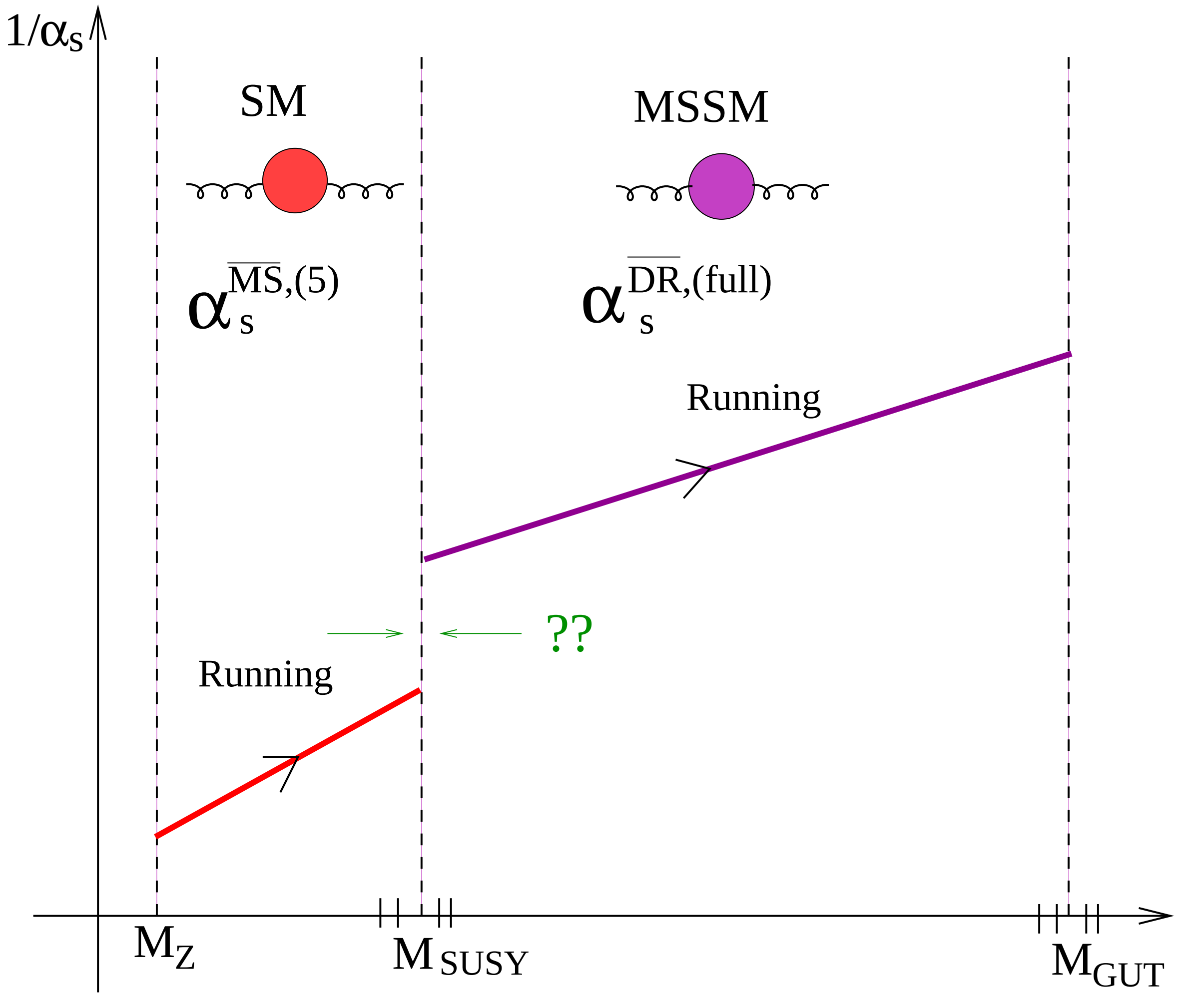
- Mass-independent Renormalization scheme
  Decoupling Theorem does not hold $\Rightarrow$ threshold effects should be added *by hand*
  - SUSY models with severely split mass spectrum
    Multi-Scale Approach: each particle decoupled at its own threshold
  - SUSY models with roughly degenerate mass spectrum
    Common Scale Approach: all SUSY particles decoupled at $\mu \simeq M_{\mathrm{SUSY}}$
    ! implemented in almost all currently available codes

Evolution of the strong coupling (2)

- Input parameter: $\alpha_s^{\overline{\mathrm{MS}},(5)}(\mathrm{M_Z})$ $\Rightarrow$ Output parameter: $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(\mathrm{M_{GUT}})$



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$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$$

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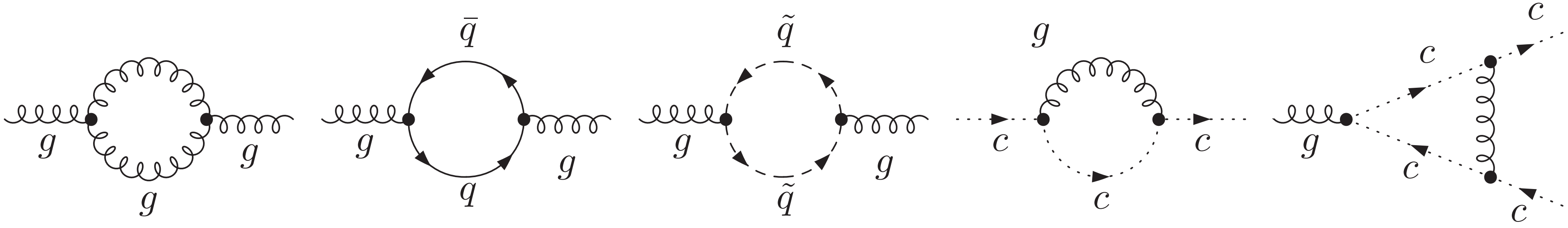

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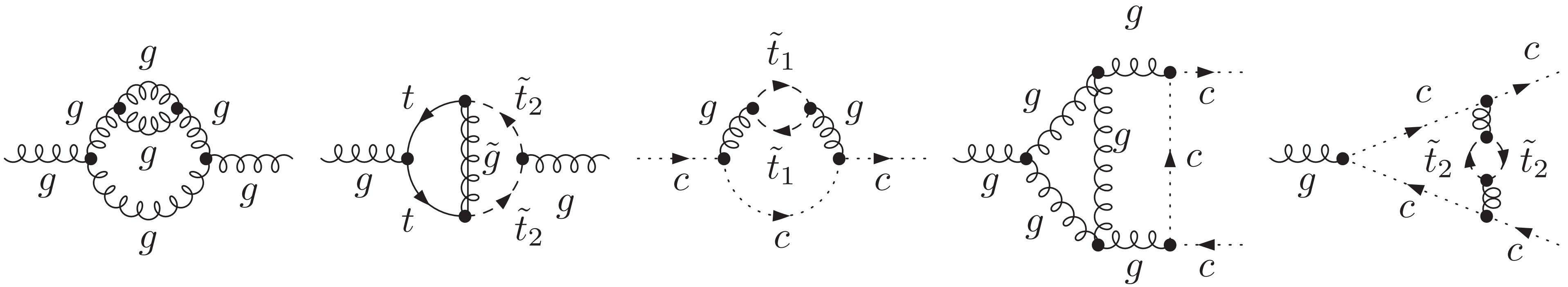

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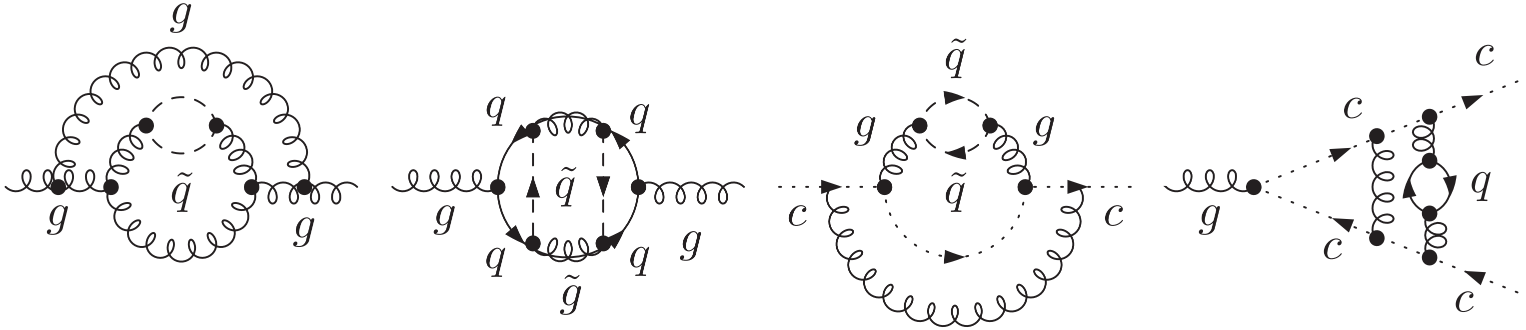

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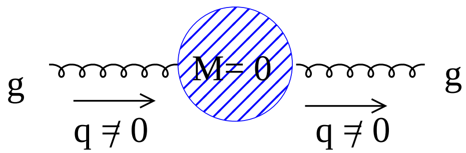
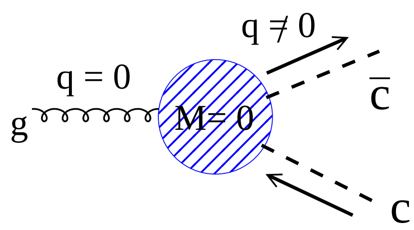
- Minimal Subtraction scheme & Dimensional Regularization (Reduction)
  - β_s: $1/\epsilon$-Pole of Z_s $\Rightarrow$ simplified kinematics





  $\Rightarrow$ Propagator-type integrals

  - Integration-by-Parts $\Rightarrow$ reduction to Master Integrals (MINCER)

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- 3-loop β_s in the MSSM
  - $\simeq$ 100.000 diagrams
  - Computer programs: QGRAF, FORM, EXP, MINCER, ...

    [Noguiera; Vermaseren; Larin, Tkachov; Steinhauser; Seidensticker, Harlander; ...]

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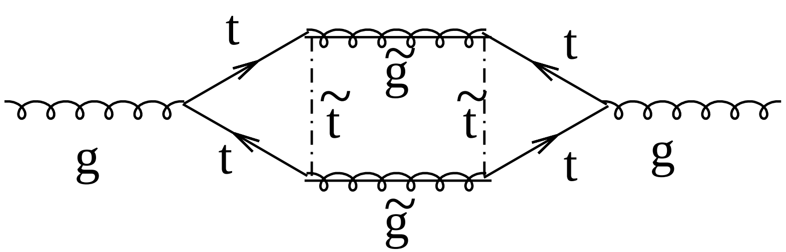
Majorana particles:



Renormalization & Regularization

Renormalization Scheme: Minimal Subtraction $\oplus$

- Dimensional Regularization (DREG) ['t Hooft and Veltmann'72] $\longrightarrow$ $\overline{\mathrm{MS}}$
  - higher order calculations in **Standard Model.**
  - DREG breaks **SUSY** : $N_{\mathrm{boson}} \neq N_{\mathrm{fermion}}$
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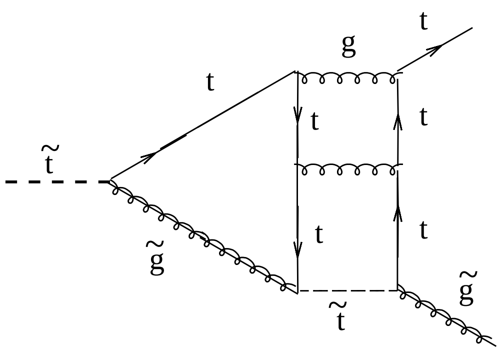
  - Removal of DRED inconsistencies *W. Siegel '80*, *D. Stöckinger '05*
  - Factorization Theorem within DRED proved *A. Signer and D. Stöckinger '05*
  - possible SUSY violation at HO *L. Avdeev, G. Chochia, A. Vladimirov '81*, *I. Jack and D. R. T. Jones '97*
  - SUSY preserved in all present 1-, 2- and 3-loop checks *W. Hollik and D. Stöckinger '05, R. Harlander, L.M., M. Steinhauser (in preparation)*

  γ_5-treatment:



DRED Framework

Quasi-4-dim. space (Q4S): $$4 = d \oplus 4 - d$$

- *Quasi*-4-dim metric tensor: $G_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}$
- Dirac matrices in Q4S: $\Gamma_\mu = \gamma_\mu + \tilde{\gamma}_\mu$
- space-time coordinates continued from 4 to $d \leq 4$ dim.
- the number of field components unchanged
  - 4-dim gluon field: $A^a_\mu = V^a_\mu + S^a_\mu\,,$

$$\begin{aligned} V^a_\mu &= g_{\mu\nu}\, A^a_\nu = && d\text{- dim. vector} \\ S^a_\mu &= \tilde{g}_{\mu\nu}\, A^a_\nu = && \varepsilon \text{ scalar} \end{aligned}$$

under gauge transformations

Conversion from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$

Input parameter: $\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z)$ $\quad\Rightarrow\quad$ Output parameter: $\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}(M_{\mathrm{GUT}})$

- 3-loop running analysis: 2-loop conversion relation $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$ for α_s

$$\alpha_s^{\overline{\mathrm{DR}},(5)} = \alpha_s^{\overline{\mathrm{MS}},(5)}\left[1+\frac{\alpha_s^{\overline{\mathrm{MS}},(5)}}{\pi}\frac{C_A}{12}+\left(\frac{\alpha_s^{\overline{\mathrm{MS}},(5)}}{\pi}\right)^2\frac{11}{72}C_A^2-\frac{\alpha_s^{\overline{\mathrm{MS}},(5)}}{\pi}\frac{\alpha_e^{(5)}}{\pi}\frac{1}{8}C_F T n_f\right]$$

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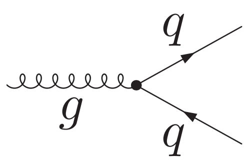
- What is $\alpha_e^{(5)}$?

DREG



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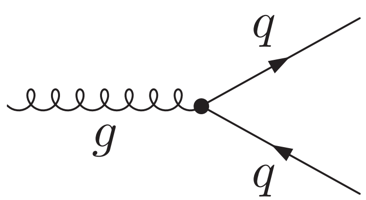
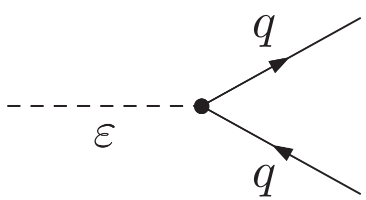
  - QCD & DRED: gluon and ε-scalar have different couplings





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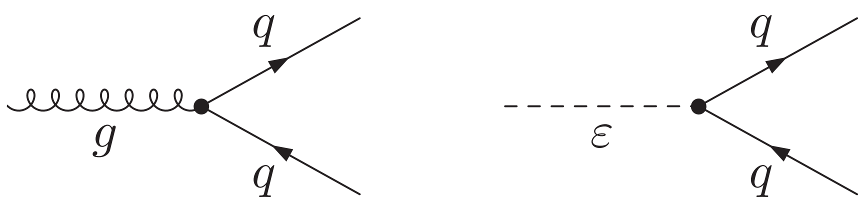
- What is $\alpha_e^{(5)}$?
  - SUSY-QCD & DRED: all couplings are equal



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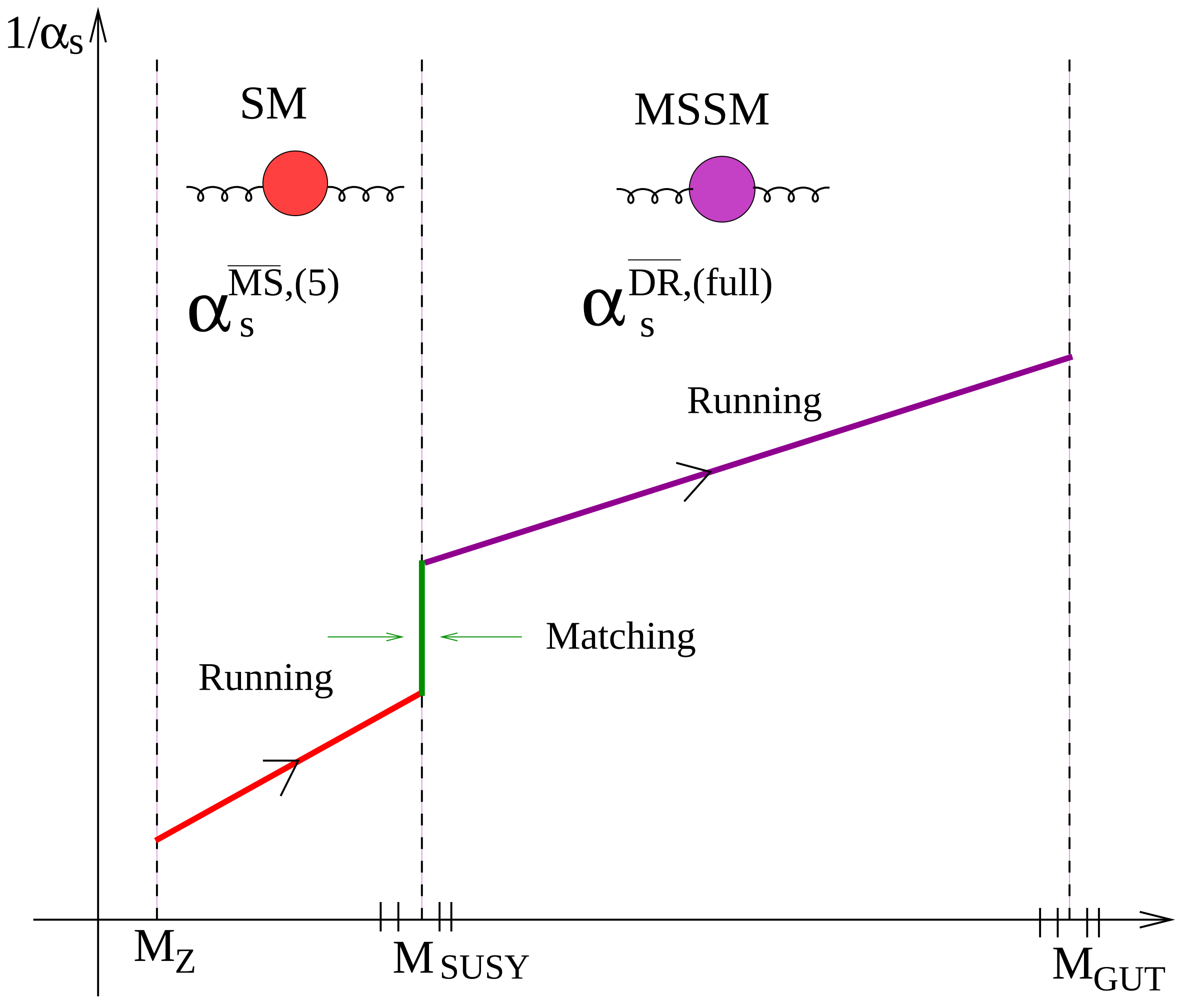
- What is $\alpha_e^{(5)}$?
  - QCD($n_f = 5$) as the low-energy effective theory of SUSY-QCD

$$\begin{aligned}\alpha_s^{\overline{\mathrm{DR}},(5)} &= \zeta_s^{(5)}\,\alpha_s^{\overline{\mathrm{DR}},(\mathrm{full})}\\ \alpha_e^{(5)} &= \zeta_e^{q}\,\alpha_e^{(\mathrm{full})}\end{aligned}$$

$\zeta_s^{(5)}$, ζ_e^{q}: needed at 2-loops

Matching

- Effective Field Theory:



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$$\mathcal{L}_{\mathrm{MSSM}}(\alpha_s^{(\mathrm{full})}, \ldots) \quad \rightarrow \quad \mathcal{L}(\alpha_s^{(5)}, \ldots) \quad \text{at energy } \mu$$

- "Matching" : low energy physics must be unchanged !!

$$\begin{aligned} \alpha_s^{(5)} &= \zeta_s \, \alpha_s^{(\mathrm{full})} \\ &\vdots \\ \zeta_s &= \zeta_s(\alpha_s, M_{\mathrm{SUSY}}, m_t, \mu) \\ \zeta_s &= \text{matching coefficient} \end{aligned}$$

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  - μ not predicted by theory
  - Physical quantities must be independent of μ
  - Quantum corrections improve stability

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Relate Green functions computed in the full and effective theory.

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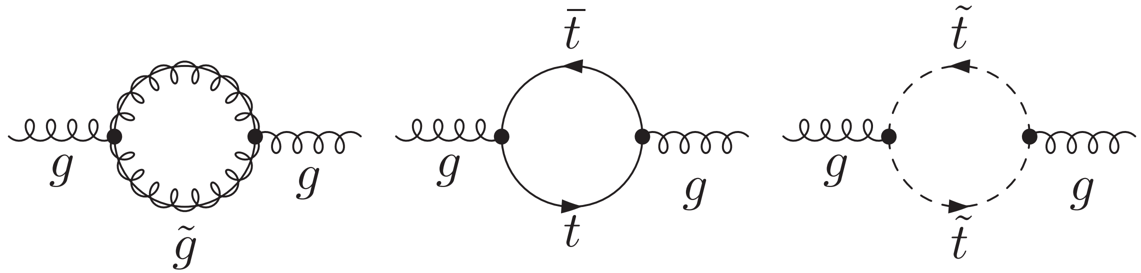

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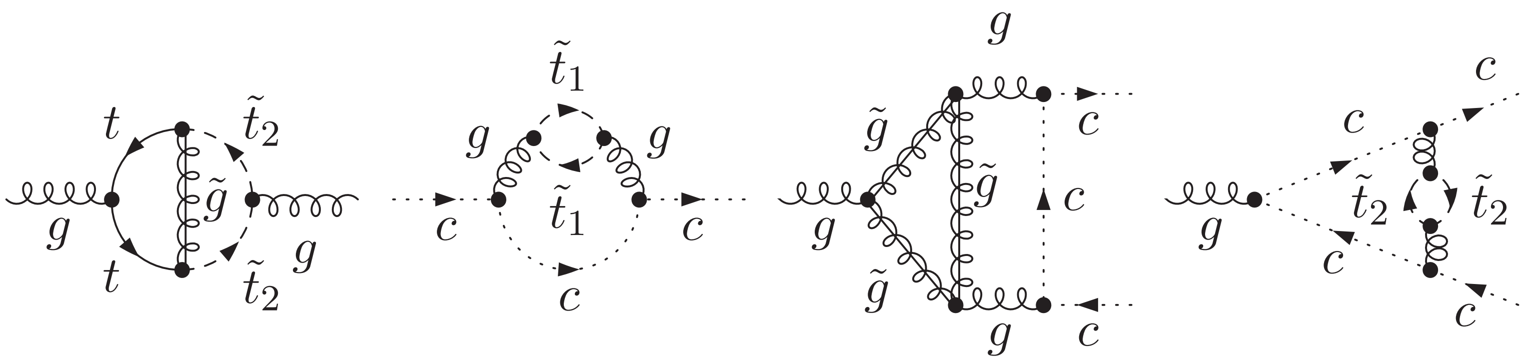
  - 2-loop ζ_s in MSSM [R. Harlander, L. M., M. Steinhauser '05, '07] [A. Bauer, L. M., J. Salomon '08]



Matching coefficient for α_s

Relate Green functions computed in the full and effective theory.

- Matching coefficient independent of external momenta $\Rightarrow p^2 = 0$
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$$\zeta_s^{(5)} = 1 + \frac{\alpha_s}{\pi}\zeta_{s1}^{(5)} + \left(\frac{\alpha_s}{\pi}\right)^2 \zeta_{s2}^{(5)} + \cdots$$

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$$
\begin{aligned}
\zeta_s^{(5)} &= 1 + \frac{\alpha_s}{\pi}\zeta_{s1}^{(5)} + \left(\frac{\alpha_s}{\pi}\right)^2 \zeta_{s2}^{(5)} + \cdots \\
\zeta_{s1}^{(5)} &= -\frac{1}{6}\ln\frac{\mu^2}{m_t^2} - \ln\frac{\mu^2}{\tilde{M}^2}\,, \qquad\qquad \tilde{M} = M_{\mathrm{SUSY}} \\
\zeta_{s2}^{(5)} &= -\frac{215}{96} - \frac{19}{24}\ln\frac{\mu^2}{m_t^2} - \frac{5}{2}\ln\frac{\mu^2}{\tilde{M}^2} + \left[\frac{1}{6}\ln\frac{\mu^2}{m_t^2} + \ln\frac{\mu^2}{\tilde{M}^2}\right]^2 \\
&\quad + \left(\frac{m_t}{\tilde{M}}\right)^2\left(\frac{5}{48} + \frac{3}{8}\ln\frac{m_t^2}{\tilde{M}^2}\right) - \frac{7\pi}{36}\left(\frac{m_t}{\tilde{M}}\right)^3 \\
&\quad + \left(\frac{m_t}{\tilde{M}}\right)^4\left(\frac{881}{7200} - \frac{1}{80}\ln\frac{m_t^2}{\tilde{M}^2}\right) + \frac{7\pi}{288}\left(\frac{m_t}{\tilde{M}}\right)^5
\end{aligned}
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Matching

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Matching

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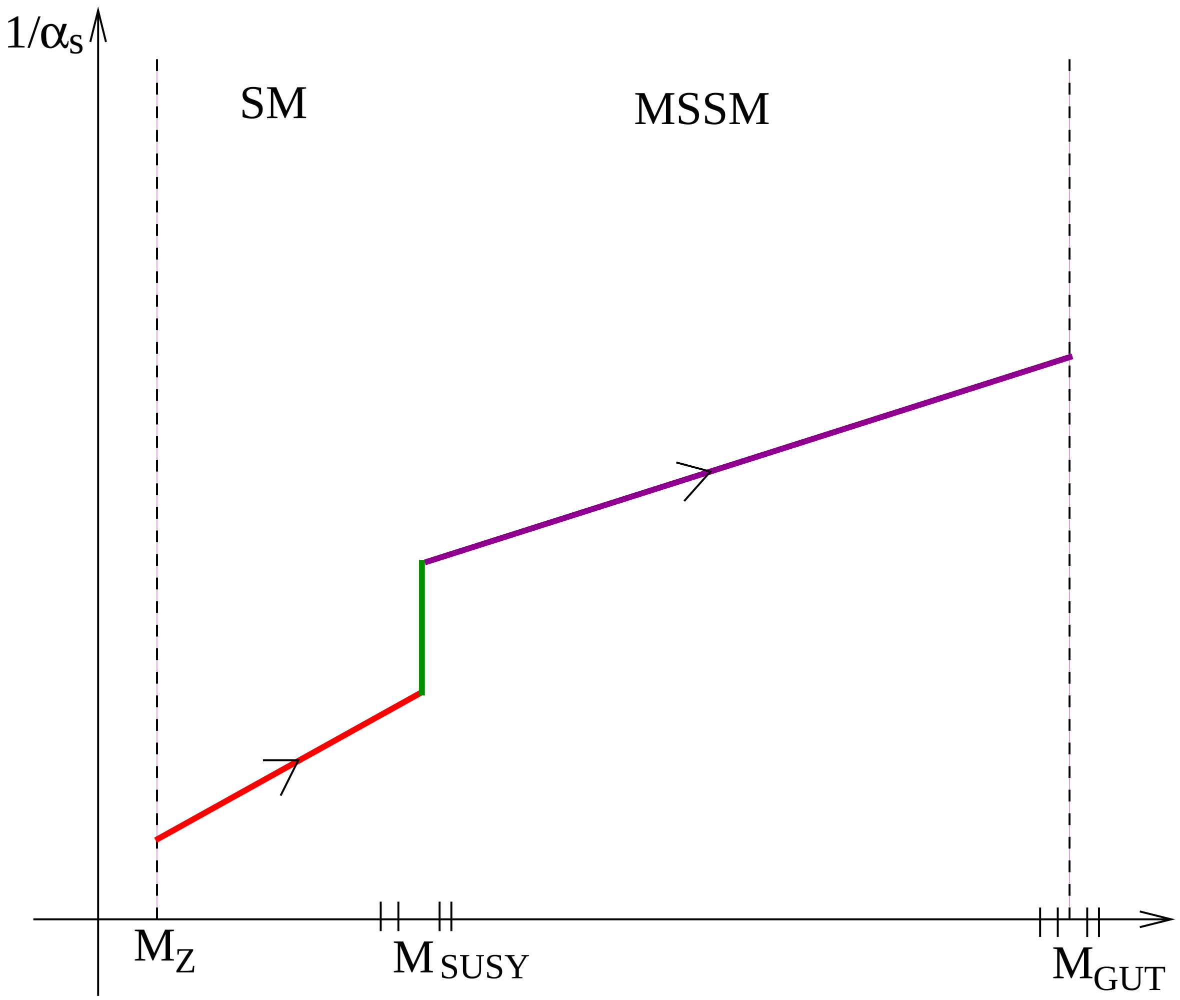

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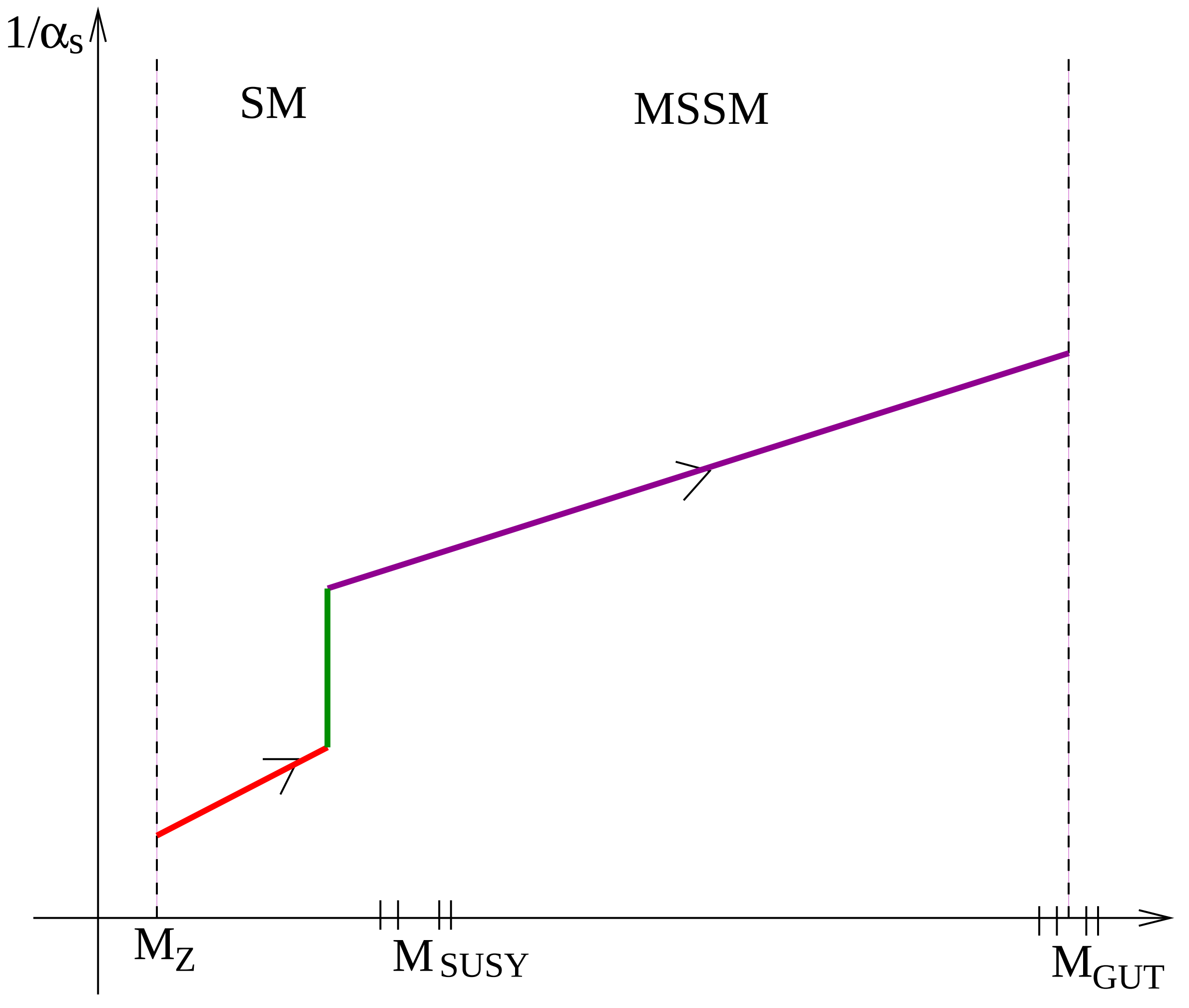

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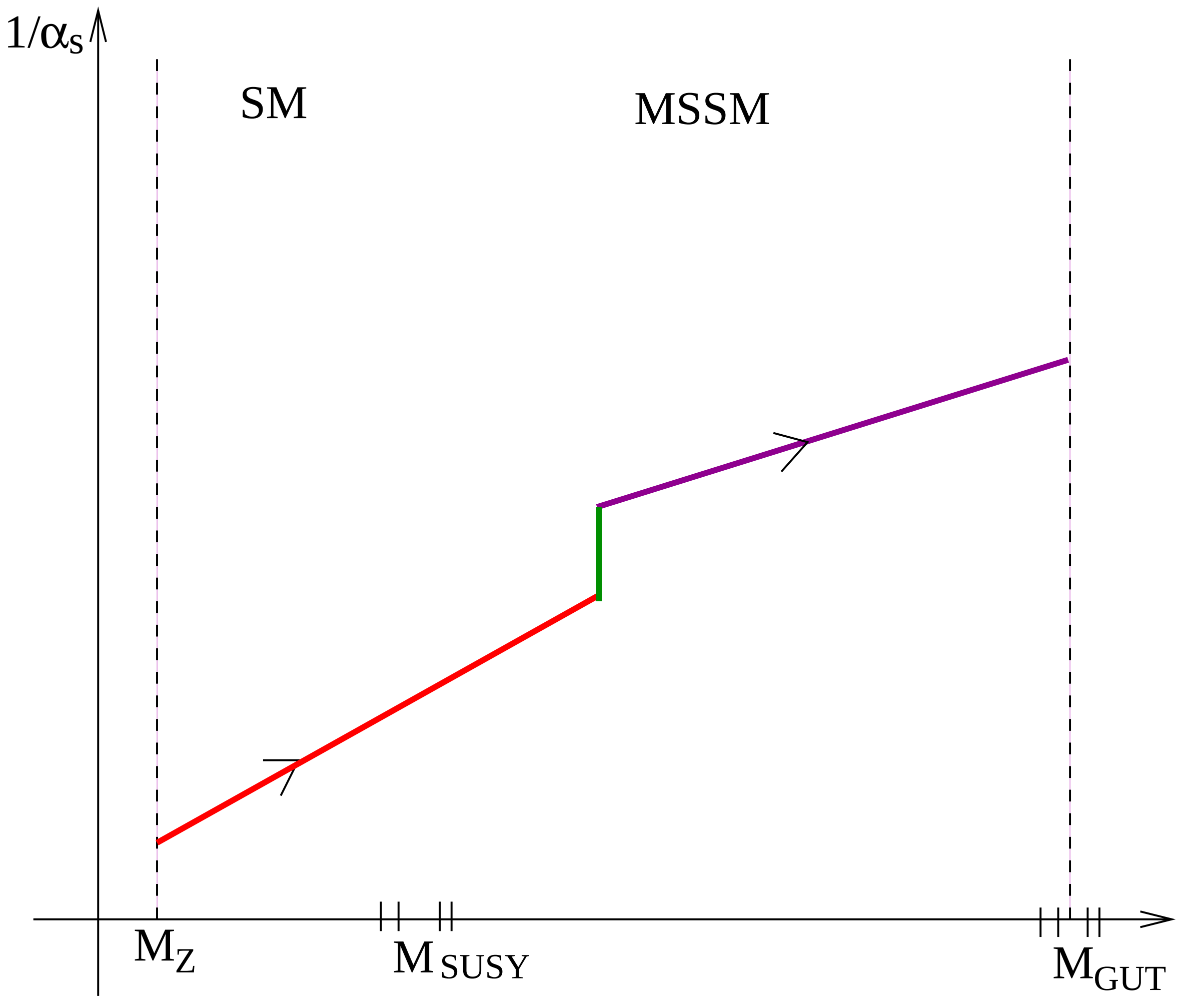
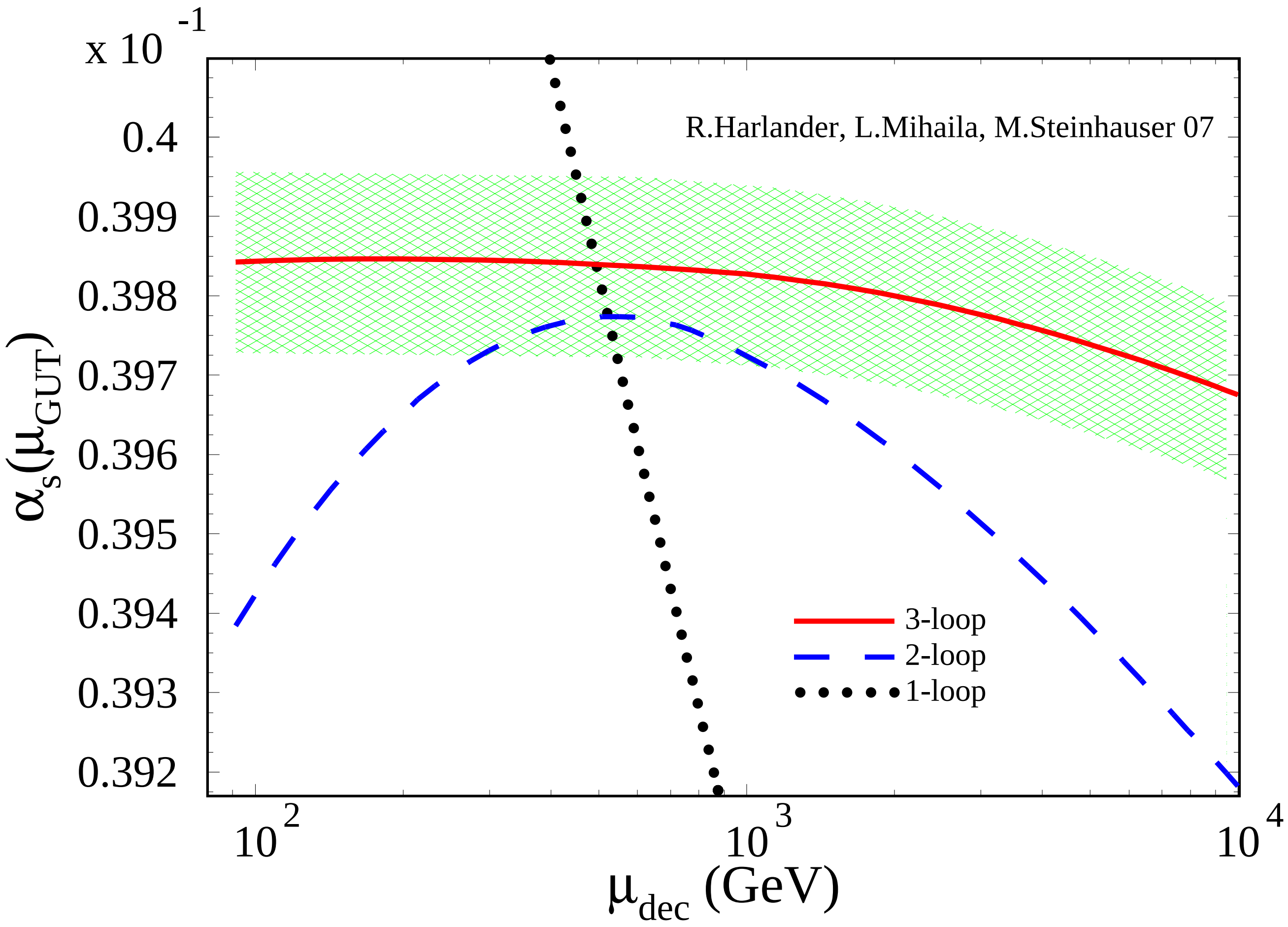


$\alpha_s(M_{\mathrm{GUT}})$

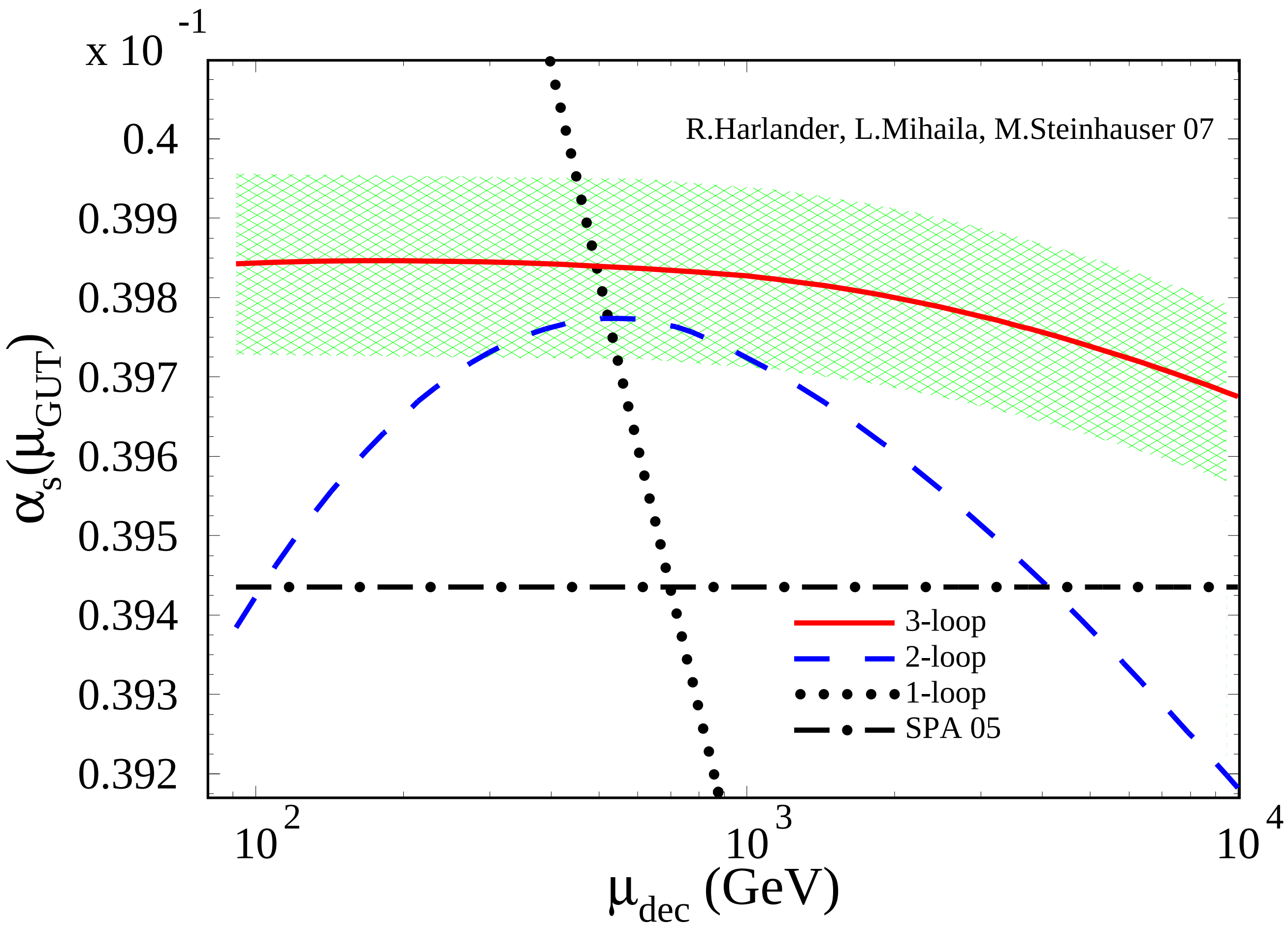
$\alpha_s^{\overline{\mathrm{MS}},(5)}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06], $M_Z = 91.1876$ GeV ,

and $\tilde{M} = m_{\tilde{q}} = m_{\tilde{g}} = 1000$ GeV SPS1a '05



$\alpha_s(M_{\mathrm{GUT}})$

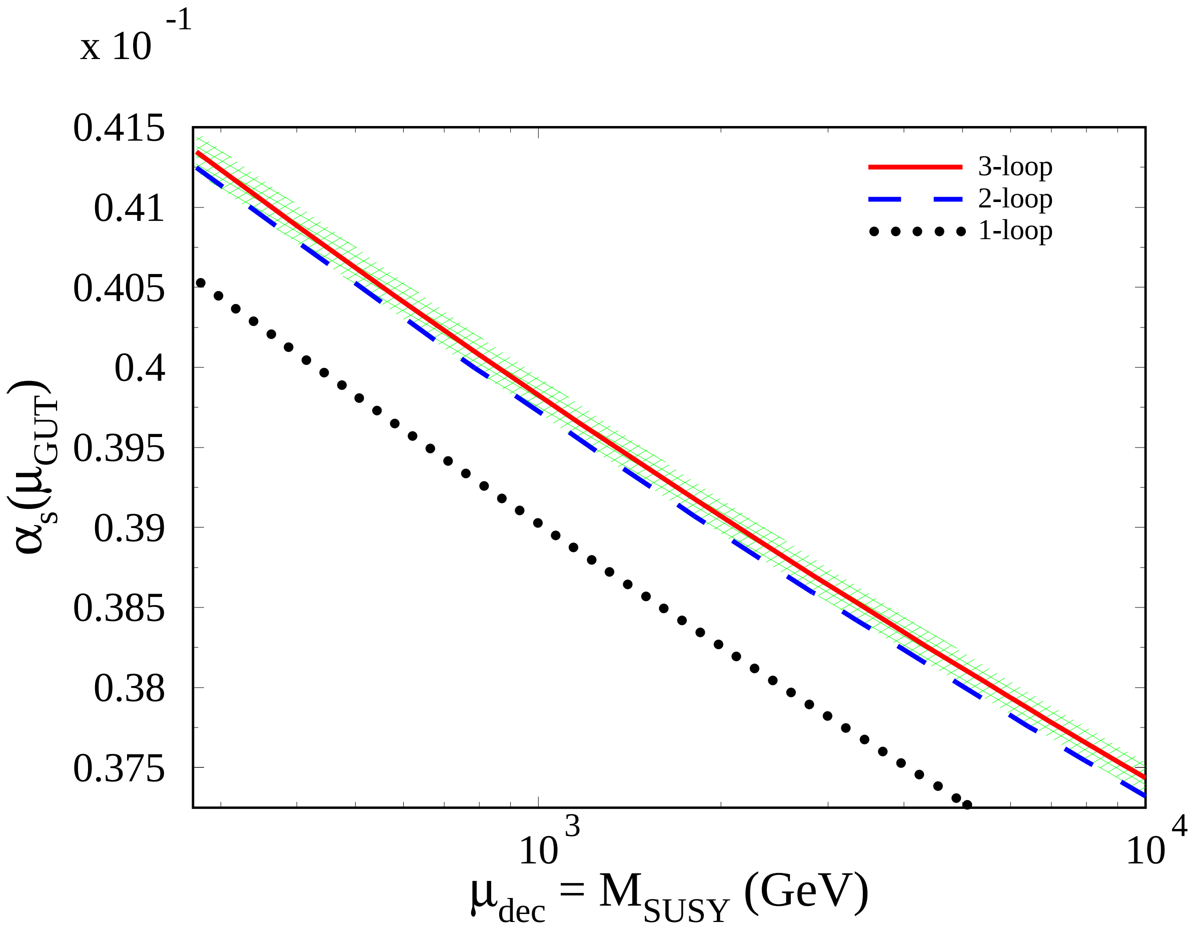
- Comparison with the Leading-Log Approximation SPA-Convention'05



SPA: state of the art at < 2007

M_{SUSY}

- Sensitivity of $\alpha_s(M_{\text{GUT}})$ to SUSY-mass scale:



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Input: $\alpha_s^{\overline{\text{MS}},(5)}(M_Z) = 0.1189 \pm 0.001$ [Bethke '06], $M_Z = 91.1876$ GeV ,

MSSM parameters: SPS1a' scenario

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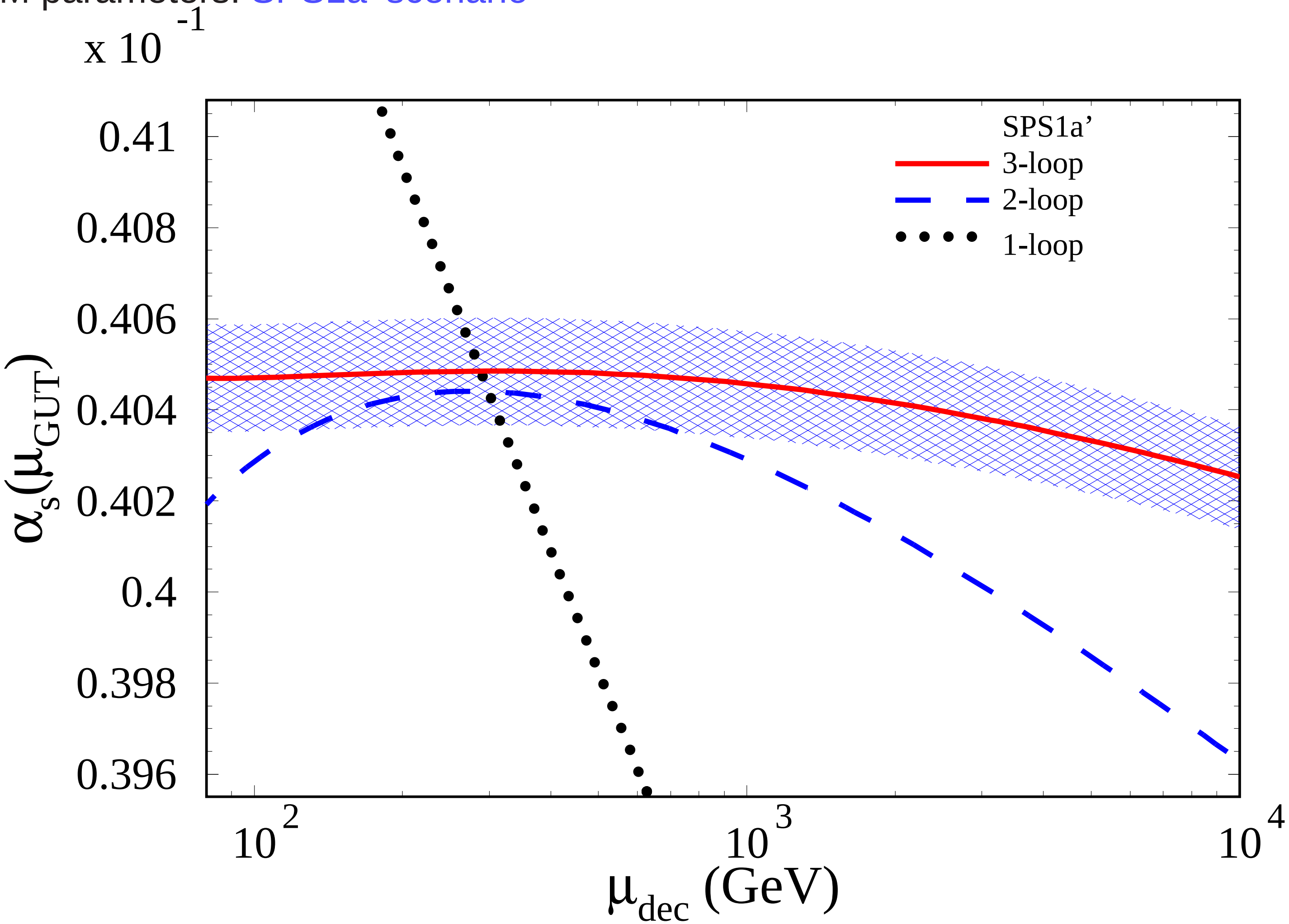

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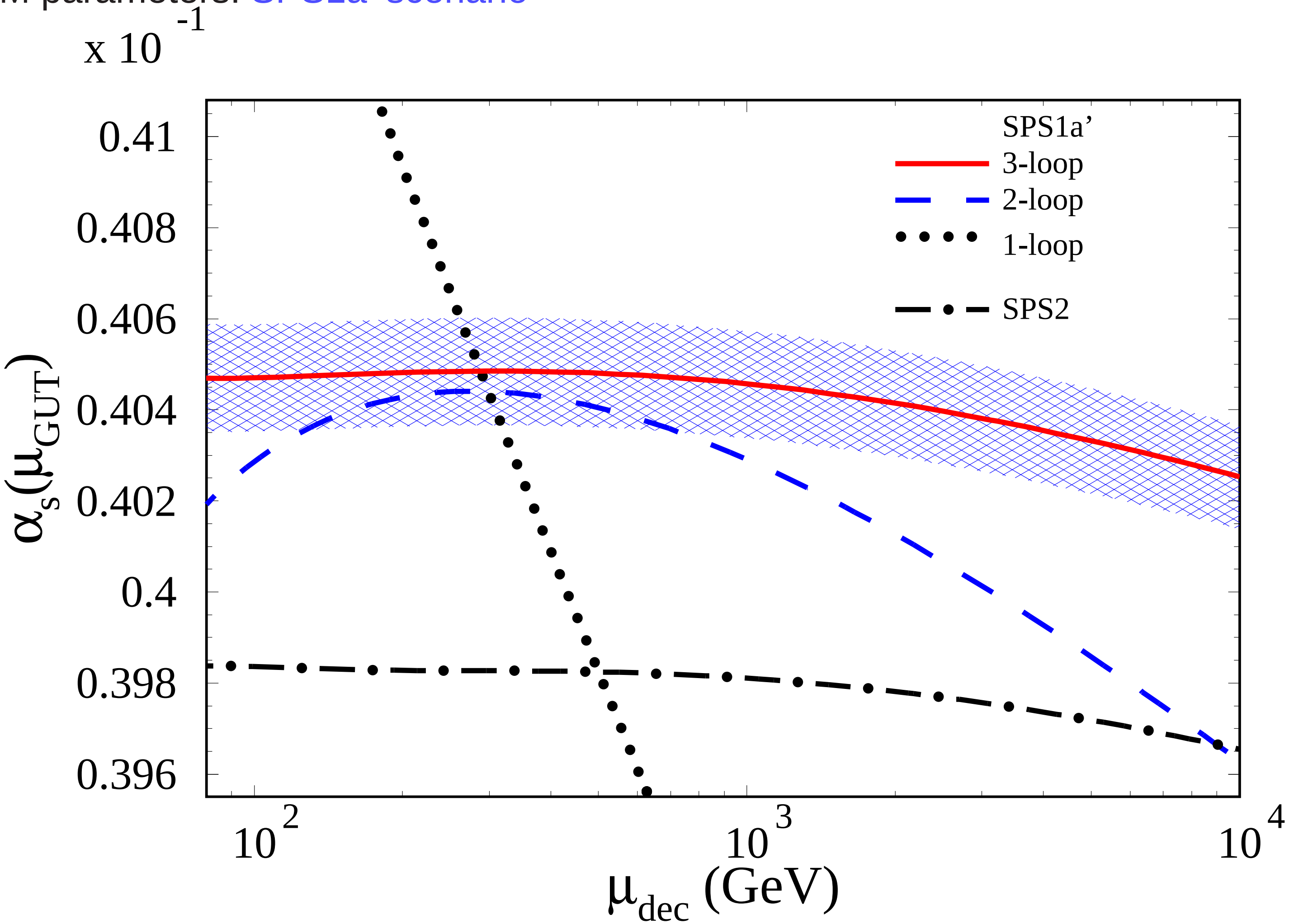

Conclusions

- A consistent approach to compute $\alpha_s^{\overline{\rm DR}}(M_{\rm GUT})$ with 3-loop accuracy
- The 3-loop effects comparable with the experimental accuracy for α_s
- $\alpha_{\rm s}(M_{\rm GUT})$ very sensitive to SUSY-mass scale

- ToDo:
  - combine running analysis for all 3 couplings
  - extend analysis to other parameters *e.g.* Yukawa couplings, Higgs sector, . . .