Exploring non-linear gravitational waves in Horndeski gravity

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Compact objects in gravitational theories
Institut Pascal

Based on
J. BA, M. A. Gorji, H. Roussille [to appear '23]

Context and Motivations

Modified gravity:

- Parametrizing deviations w.r.t to GR
  $\rightarrow$ early and late cosmology (dark sector) / compact objects
- Scalar-tensor theories with $(g_{\mu\nu}, \phi)$: Horndeski $\rightarrow$ GLPV $\rightarrow$ DHOST theories
  [Horndeski '74] $\rightarrow$ [Langlois, Noui '15]

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Impact on gravitational wave production and propagation ?

- Effect 1: new scalar polarization and mixing between scalar/tensor sectors
  [Kobayashi, Yamaguchi, Yokoyama '11][Dalang, Fleury, Lombriser ' 19][Creminelli,Tambalo, Vernizzi, Yingcharoenrat '19][Kubota, Arai, Mukohyama '23]
  $\rightarrow$ Damped amplitude / change of the phase velocity on cosmological distance
  $\rightarrow$ Constraints from BBH mergers [Arai, Nishizawa '18] [Takeda, Morisaki, Nishizawa '22]
- Effect 2: Modified quasi-normal modes spectrum for black holes

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Studying the non-linear regime: two approaches

- 1) identify the symmetries at null-infinity in modified gravity and their flux-balance laws
  In GR, Bondi mass loss: $\dot{M}(u) = \mathcal{F} \rightarrow$ BMS flux-balance law
  Extension to Brans-Dicke [Tahura, Nichols, Shaffer, Stein, Yagi '20][Hou, Zhu '20][Seraj '21]

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- 2) construct exact non-perturbative radiative solutions :
  pp-waves, Kundt, Robinson-Trautman exact solutions in GR $\rightarrow$ useful laboratory

Outlines

- Disformal solution-generating method and Petrov classification
- Exact non-perturbative radiative solution in Horndeski
- Polarizations at the non-perturbative level: Penrose limit

Disformal solution-generating method and Petrov classification

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Disformal solution-generating map

- Disformal transformation (DT) :

$$(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu} = A g_{\mu\nu} + B \phi_\mu \phi_\nu, \phi) \tag{1}$$

with $A := A(\phi, X)$, $B := B(\phi, X)$ and $X = g^{\mu\nu}\phi_\mu\phi_\nu$.

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- Degenerate higher order scalar-tensor theories are organized into equivalent classes under DT [**BA**, Langlois, Noui '16]
- Provide a solution-generating map to explore the solution space [**BA**, Mukohyama, Liu '20]
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Petrov classification

- Classification from the 60's based on the algebraic properties (Petrov type) of the Weyl tensor
- Provide general theorems : Petrov type $\leftrightarrow$ behavior of light rays [Goldberg, Sachs '62]
- Provide a guiding map to derive new exact solutions in GR (among which Kerr)

$\to$ Can we keep under control how the Petrov type change under disformal transformation ?

Disformal solution-generating method and Petrov classification

Petrov classification in a nutshell

- Pick up a null tetrad E^μ_A

$$g_{\mu\nu} = E^A_\mu E^B_\nu \eta_{AB} \qquad \eta_{AB} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{2}$$

associated to four null vectors $\ell^\mu = E^\mu_0$, $n^\mu = E^\mu_1$, $m^\mu = E^\mu_2$, $\bar{m}^\mu = E^\mu_3$

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- Decompose the Weyl tensor into 5 complex scalars

$$\Psi_0 = C_{ABCD}\ell^A m^B \ell^C m^D \qquad \Psi_2 = C_{ABCD}\ell^A m^B \bar{m}^C n^D \qquad \Psi_4 = C_{ABCD} n^A \bar{m}^B n^C \bar{m}^D \tag{4}$$

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$$(\ell, n, m, \bar{m}) \to (\ell + b^* m + b\bar{m} + bb^* n, n, m + bn, \bar{m} + b^* n) \tag{5}$$

$$\Psi_0 \to \Psi_0 + 4b\Psi_1 + 6b^2\Psi_2 + 4b^3\Psi_3 + b^4\Psi_4 \tag{6}$$

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  Can set $\Psi_0 = 0$ in a suitable frame by solving for b.
- Multiplicity of the root b defines the Petrov type: four simple roots $\to$ Petrov type I

Petrov classification: Lorentz invariant formulation

- Consider the Lorentz invariant spectral index:

$$I = \Psi_0\Psi_4 - 4\Psi_1\Psi_3 + 3\Psi_2^2 \qquad J = \det\begin{pmatrix} \Psi_4 & \Psi_3 & \Psi_2 \\ \Psi_3 & \Psi_2 & \Psi_1 \\ \Psi_2 & \Psi_1 & \Psi_0 \end{pmatrix} \qquad \boxed{S = I^3 - 27J^2}$$

If $S \neq 0$, then the geometry is not algebraically special: Petrov type I

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- The remaining types can be deduced from the quantities

$$K = \Psi_4^2\Psi_1 - 3\Psi_4\Psi_3\Psi_2 + 2\Psi_3^3 \qquad L = \Psi_4\Psi_2 - \Psi_3^2 \qquad N = 12L^2 - \Psi_4^2 I \tag{7}$$

Type II: $S = 0$
Type III: $I = J = 0$
Type D: $S = K = N = 0$
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- How does the Petrov type change under a disformal transformation ?
- Can we get close formula to keep control on this change ?

Disformal solution-generating method and Petrov classification

Disformal transformation on the tetrad field

- Usually, DT are written at the level of the metric

$$(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu} = A g_{\mu\nu} + B \phi_\mu \phi_\nu, \phi) \tag{8}$$

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$$J^A{}_B = \sqrt{A}\left(\delta^A{}_B + \frac{\beta}{1-\beta X}\phi^A\phi_B\right) \qquad \beta = \frac{1}{X}\left[1 - \frac{\sqrt{A}}{\sqrt{A+BX}}\right] \tag{9}$$

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$$\tilde{\Theta} = \frac{1}{\sqrt{A}}\left[\Theta - \ell^\mu \nabla_\mu(\sqrt{A}/J) - \frac{\sqrt{A}}{J}\nabla_\mu\left(\frac{\beta J}{\sqrt{A}}\ell^\alpha\phi_\alpha\phi^\mu\right)\right] \tag{12}$$

Disformal solution-generating method and Petrov classification

Disformal transformation on the tetrad field

- Usually, DT are written at the level of the metric

$$(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu} = A g_{\mu\nu} + B\phi_\mu\phi_\nu, \phi) \qquad (8)$$

  with $A := A(\phi, X)$, $B := B(\phi, X)$ and $X = g^{\mu\nu}\phi_\mu\phi_\nu$.
- To understand the change in the Newman-Penrose quantities, implement DT on the tetrad
- Introduce the J-map

$$J^A{}_B = \sqrt{A}\left(\delta^A{}_B + \frac{\beta}{1-\beta X}\phi^A\phi_B\right) \qquad \beta = \frac{1}{X}\left[1 - \frac{\sqrt{A}}{\sqrt{A+BX}}\right] \qquad (9)$$

  with $\phi_A = E^\mu_A\phi_\mu$ and $X = \phi_A\phi^A = \phi_\mu\phi^\mu$
- Allows one to implement DT in a local rest frame:

$$\boxed{\tilde{E}^A_\mu = J^A{}_B E^B_\mu \qquad \tilde{g}_{\mu\nu} = (J^A{}_C E^C_\mu)(J^B{}_D E^D_\nu)\eta_{CD} = A g_{\mu\nu} + B\phi_\mu\phi_\nu} \qquad (10)$$

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- Formula for the change of Weyl scalars and thus of Petrov type :
  $\to$ guide for new exact solutions [**BA**, De Felice, Gorji, Mukohyama, Pookilath '22]

Applications

- Disformed static spherically symmetric spacetime : for example Schwarzschild

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + g(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 \qquad \phi(t, r) \tag{13}$$

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- Disformed Kerr black hole

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\mathrm{Kerr}} + B_0\left(\mathrm{d}t + \frac{\sqrt{2Mr(r^2+a^2)}}{\Delta}\mathrm{d}r\right)^2 \qquad \phi(t,r) \tag{14}$$

Seed is Petrov type D → Disformed spectral index is

$$S = I^3 - 27J^2 = B_0^2\chi + \mathcal{O}(B_0^3) \neq 0 \tag{15}$$

Disformed Kerr geometry becomes of Petrov type I
Loss of symmetry : Killing-Yano tensor only for type D and N
[**BA**, De Felice, Gorji, Mukohyama, Pookilath '22]

- Use disformal solution-generating method to construct exact radiative solutions

Constructing exact non-perturbative radiative solution in Horndeski

The seed

- We need to identify a seed solution: consider the Einstein-Scalar system

$$\mathcal{S} = \frac{1}{2} \int d^4x \sqrt{|g|} \left(\mathcal{R} - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$$

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- Exact radiative solution of this system [Tahamtan and Svitec '15 '16]

$$\mathrm{d}s^2 = -\frac{r\partial_u F + K(x,y)}{F(u)}\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + \frac{r^2F^2(u) - C_0^2}{F(u)P^2(x,y)}(\mathrm{d}x^2 + \mathrm{d}y^2)$$

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where

$$F(u) = \gamma e^{\omega^2u^2} \qquad \Delta K(x,y) = 4C_0^2\omega^2 = \alpha^2 \qquad K(x,y) = P^2\left(\partial_{xx}^2 + \partial_{yy}^2\right)\log P = \Delta\log P$$

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$$P_0 = \frac{1 + x^2 + y^2}{2} \qquad K_0 = 1 \tag{16}$$

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- What are its properties ?

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- Better described in another coordinate system: $(u, r, x, y) \to (w, \rho, x, y)$

$$\mathrm{d}s^2 = -K(x,y)\mathrm{d}w^2 - 2\mathrm{d}w\mathrm{d}\rho + \frac{\rho^2 - \chi^2(w)}{P^2(x,y)}(\mathrm{d}x^2 + \mathrm{d}y^2)$$

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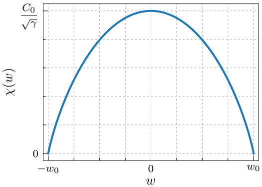

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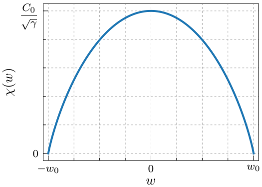

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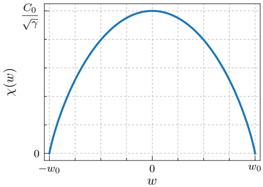

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$$X(0, \rho, x, y) = \frac{2\chi^2(0)K(x,y)}{(\rho^2 - \chi^2(0))^2} \neq 0 \tag{18}$$

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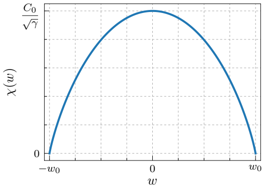

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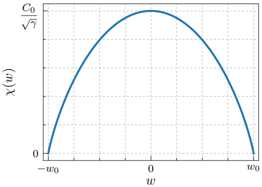

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- This solution describes a scalar pulse localized on the equator [**BA**, Gorji, Roussille '23]
- What type of gravitational wave propagate in this geometry ?

Scalar pulse and longitudinal mode

- Null directions and optical scalars:

$$\ell^\mu \partial_\mu = \partial_\rho \qquad n^\mu \partial_\mu = \partial_w - K(x,y)\partial_\rho \qquad m^\mu \partial_\mu = \frac{P(x,y)}{\sqrt{2(\rho^2 - \chi^2(w))}}(\partial_x + i\partial_y) \tag{19}$$

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$$\Theta(\rho, w) = -\frac{\rho}{\rho^2 - \chi^2(w)} \qquad \omega = 0 \qquad \sigma = 0 \tag{20}$$

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$\rightarrow$ compute the gaussian curvature $\mathcal{K}$ of the topological 2-sphere

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- Expected from a pure monopole scalar source ! $\phi(w,\rho) = \frac{1}{\sqrt{2}} \log\left[\frac{\rho - \chi(w)}{\rho + \chi(w)}\right]$

Scalar pulse in Horndeski theory

- What radiative gravitational field can be sourced by the same scalar pulse in Horndeski ?
- How does the mixing from higher order terms manifest at the fully non-linear level ?
- Accessible by a disformal transformation

Disformal map

- Consider the simplest disformal transformation of the Einstein-Scalar system

$$(g_{\mu\nu}, \phi) \to (\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_0 \phi_\mu \phi_\nu, \phi) \tag{22}$$

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  $\to$ different Petrov type
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- Disformal transformation of the seed solution reveals the effects of the higher order terms (controlled by B_0) in the presence of a monopole scalar source

The new exact solution

- The scalar profile remains unchanged: scalar monopole

$$\phi(w,\rho)=\frac{1}{\sqrt{2}}\log\left[\frac{\rho-\chi(w)}{\rho+\chi(w)}\right] \qquad \rightarrow \qquad \phi_\rho=\frac{2\chi(w)}{\rho^2-\chi^2} \qquad \phi_w=\frac{-2\rho\chi'(w)}{\rho^2-\chi^2} \tag{25}$$

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- What are the properties of this solution ? How does it deviate from the seed one ?

Scalar pulse in Horndeski theory

Asymptotic regimes $w \to \pm w_0$

- In the remote past and far future: non-spherically symmetric.

$$\lim_{w\to\pm w_0} \mathrm{d}s^2 = -\left[K(x,y) - \frac{Q}{\rho^2}\right]\mathrm{d}w^2 - 2\mathrm{d}w\mathrm{d}\rho + \frac{\rho^2}{P^2(x,y)}(\mathrm{d}x^2 + \mathrm{d}y^2) \tag{28}$$

$$\lim_{w\to\pm w_0} \phi = \frac{\sqrt{2}\chi(w)}{\rho} \to 0 \tag{29}$$

$$\lim_{w\to\pm w_0} X = 0 \tag{30}$$

- Qualitative difference with the GR solution: electric-like charge Q

$$Q^2 = \lim_{w\to\pm w_0} 4B_0(\chi')^2$$

Scalar pulse in Horndeski theory

Asymptotic regimes $w \to \pm w_0$

- In the remote past and far future: non-spherically symmetric.

$$\lim_{w\to\pm w_0} \mathrm{d}s^2 = -\left[K(x,y) - \frac{Q}{\rho^2}\right]\mathrm{d}w^2 - 2\mathrm{d}w\mathrm{d}\rho + \frac{\rho^2}{P^2(x,y)}(\mathrm{d}x^2 + \mathrm{d}y^2) \tag{28}$$

$$\lim_{w\to\pm w_0} \phi = \frac{\sqrt{2}\chi(w)}{\rho} \to 0 \tag{29}$$

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Maximum of the pusle: $w = 0$

- The metric and the scalar profile become

$$\lim_{w\to 0} \mathrm{d}s^2 = -K(x,y)\mathrm{d}w^2 - 2\mathrm{d}w\mathrm{d}\rho + \frac{4B_0C_0^2}{\gamma(\gamma\rho^2 - C_0^2)^2}\mathrm{d}\rho^2 + \frac{\rho^2}{P^2(x,y)}(\mathrm{d}x^2 + \mathrm{d}y^2) \tag{31}$$

$$\lim_{w\to 0} \phi = \frac{1}{\sqrt{2}}\log\left[\frac{\sqrt{\gamma}\rho - C_0}{\sqrt{\gamma}\rho + C_0}\right] \tag{32}$$

- The electric-like charge Q has disappeared but new contribution in $g_{\rho\rho}$
- Kinetic energy of the scalar:

$$X = \frac{4\gamma K(x,y)C_0^2}{(\rho^2 - C_0^2)^2 + 4B_0\gamma C_0^2 K(x,y)B_0} \neq 0 \tag{33}$$

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- The optical scalars reveal the non-linear superposition of breathing and shearing modes

$$\tilde{\Theta} = \Theta + B_0\Theta_1 + B_0^2\Theta_2$$
$$\tilde{\sigma} = B_0^2\sigma_2$$

with

$$\sigma_2 = \frac{(\rho^2 - \chi^2)}{96\chi^7}\left[30\rho^5\chi - 80\rho^3\chi^3 + 66\rho\chi^5 + 15\sqrt{2}(\rho^2 - \chi^2)^3\phi\right]\Psi_0 \qquad (34)$$

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- Does this effect survive a perturbative analysis ?

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The non-perturbative origin of the shear

- Most natural set-up to perform a perturbative approach, take the scalar profile and consider a small pulse $\chi := \epsilon(w)$

$$\phi(w, \rho) \simeq \frac{\epsilon(w)}{\rho} \tag{35}$$

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Further understanding the generation of the shear

- Newman-Penrose equations for optical scalars in GR:

$$D\Theta = \omega^2 - \Theta^2 - \sigma^2 - \Phi_{00} \qquad \text{Raychaudhuri} \tag{37}$$

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- How are they modified in Horndeski theory ? In modified gravity ? [**BA**, Roussille - in progress]

Identifying the polarizations at the non-perturbative level: Penrose limit

A tool to identify the polarizations: Penrose limit

- In the linearized theory

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \tag{40}$$

After gauge-fixing, polarization can be read off from $h_{\mu\nu}$. Not available in the fully non-linear regime.

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- H_+ and $H_\times$ encode the spin-2 shearing modes / $H_\circ$ is the trace mode

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$$(g_{\mu\nu}, \gamma) \qquad \rightarrow \qquad \text{pp-wave geometry} \tag{42}$$
[Penrose '76]

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- Applied to a radiative geometry, it allows one to read the polarizations of the propagating wave inducing these non-linear tidal effects from $(H_\circ, H_+, H_\times)$

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with the Weyl scalar Ψ_0 given by

$$\Psi_0 = B_0^2 \frac{\chi^4 P \left(P \partial_{\bar{z}\bar{z}} K + 2 \partial_{\bar{z}} K \partial_{\bar{z}} P \right)}{(\rho^2 - \chi^2)^5} + \mathcal{O}(B_0^3) \tag{44}$$

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- Allow to compute and compare the memory effects in the GR and Horndeski solutions [**BA**, Gorji, Roussille '23]

Conclusion and perspectives

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- Fluxes-balance laws by covariant phase space methods in modified gravity
  Exist for Brans-Dicke [Tahura, Nichols, Shaffer, Stein, Yagi '20][Hou, Zhu '20][Seraj '21]
  What about higher order modified gravity ?

Singularity and horizons

- Singularity in the GR solution: $\rho = \chi(w)$
- Additional singularity in the Horndeski solution:

$$\rho^4 - 2\left(\rho^2 + B_0 K(x,y)\right)\chi^2 + \chi^4 - 4B_0\rho\chi\chi' = 0 \tag{45}$$

At $w = 0$, singularity located at

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- Null directions

$$\ell^\mu \partial_\mu = \partial_\rho \qquad m^\mu \partial_\mu = \frac{P(x,y)}{\sqrt{2(\rho^2 - \chi^2(w))}}\left((M_x + iM_y)\partial_\rho + \partial_x + i\partial_y\right) \tag{47}$$

$$n^\mu \partial_\mu = \partial_w + \frac{P(x,y)^2}{\rho^2 - \chi(w)^2}(M_x\partial_x + M_y\partial_y) - \left[\frac{K(x,y)}{2} - \frac{P(x,y)^2}{2(\rho^2 - \chi(w)^2)}\left(M_x^2 + M_y^2\right)\right]\partial_\rho \tag{48}$$

which is orthogonal to the surface $\rho = M(x,y)$ and satisfies the standard orthogonality relations

$$\ell^\mu n_\mu = -1\,, \qquad m^\mu \bar{m}_\mu = 1 \tag{49}$$

while $l^\mu m_\mu = n^\mu m_\mu = 0$.

- Expansions

$$\Theta_\ell = \frac{\rho}{\rho^2 - \chi(w)^2}\,, \qquad \Theta_n = \frac{\Delta_S M - \rho k(x,y) - 2\chi(w)\chi'(w)}{2(\rho^2 - \chi(w)^2)} - \frac{\rho\|\nabla_S M\|^2}{2(\rho^2 - \chi(w)^2)^2} \tag{50}$$

- Equation for the dynamical apparent horizon $\Theta_n = 0$
- In general, very hard to solve .... even in GR

Penrose limit

- Concretely, pick up a null geodesic γ and construct null Fermi coordinates $X^A = (U, V, X^i)$ with $i \in (1, 2)$ adapted to the region around the geodesic

$$X^A = E^A_a x^a + E^A_\mu \bar{\Gamma}^\mu{}_{ab} x^a x^b + \mathcal{O}((x^a)^3) \tag{51}$$

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- In the region around the geodesic γ, the gravitational field can be described as

$$\begin{aligned} \mathrm{d}s^2 = {} & 2\mathrm{d}U\mathrm{d}V + \delta_{ij}\mathrm{d}X^i\mathrm{d}X^j - \bar{R}_{\lambda i \lambda j}(U) X^i X^j \mathrm{d}U^2 \\ & - \frac{4}{3}\bar{R}_{\lambda j i k}(U) X^j X^k \mathrm{d}U\mathrm{d}X^i - \frac{1}{3}\bar{R}_{ijk\ell}(U) X^k X^\ell \mathrm{d}X^i \mathrm{d}X^j \\ & + \mathcal{O}(X^3) \end{aligned} \tag{52}$$

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- Peeling behavior of the Weyl scalars
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$$\begin{aligned} \mathrm{d}s^2 &= 2\mathrm{d}U\mathrm{d}V + \delta_{ij}\mathrm{d}X^i\mathrm{d}X^j - \bar{R}_{\lambda i \lambda j}(U)X^iX^j\mathrm{d}U^2 \\ &- \frac{4}{3}\bar{R}_{\lambda j i k}(U)X^jX^k\mathrm{d}U\mathrm{d}X^i - \frac{1}{3}\bar{R}_{ijk\ell}(U)X^kX^\ell\mathrm{d}X^i\mathrm{d}X^j \\ &+ \mathcal{O}(X^3) \end{aligned} \tag{52}$$

- Organize this expansion in x^a using conformal transformation of the transverse space: $(U, V, X^i) \to (U, \lambda^2 V, \lambda X^i)$
- Peeling behavior of the Weyl scalars

$$\Psi_i = \mathcal{O}(\lambda^{4-i}) \qquad \text{for } i \in (0, \ldots, 4) \tag{53}$$

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  [Penrose '76][Blau '19]

- Concretely, pick up a null geodesic γ and construct null Fermi coordinates $X^A = (U, V, X^i)$ with $i \in (1,2)$ adapted to the region around the geodesic

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- Read the polarizations from the matrix $A_{ij}(U)$
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- Also powerful to compute the memory effects explicitly