QUASI-NORMAL MODES OF LQG BLACK HOLES

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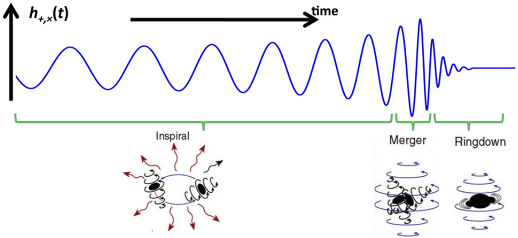
Gravitational perturbations

- Since the first detection of gravitaional waves in 2015, black holes perturbations have regain a lot of interest.



- Once the two black holes have merged, one is left with a very perturbed black hole which keeps emitting gravitational waves until it reaches an equilibrium state. This phase is called **ringdown** and is characterized by frequencies called **Quasi Normal Modes** (QNM).
- As we have a dissipative system, the ringdown signal is damped and the QNM are complex: $\omega = \omega_R + i\omega_I$.

spin s field perturbations

- Gravitational perturbations are studied by linearising Einstein equations.
- It is also possible to study the perturbations of a black hole due to a field of spin s, by looking at the propagation of a spin s field on a black hole.
- For a Schwarzschild black hole, we can summurize the perturbations of all spins in one equation:

$$\partial_x^2 \psi + (\omega^2 - V(r))\psi = 0,$$

with

$$V(r) := \frac{r - r_s}{r} \left[\frac{\ell(\ell+1)}{r^2} + \frac{r_s(1-s^2)}{r^3} \right].$$

Boundary conditions for QNM

- The effective potential $V(r)$ typically have the shape of a barrier, decaying at the horizon and at infinity:

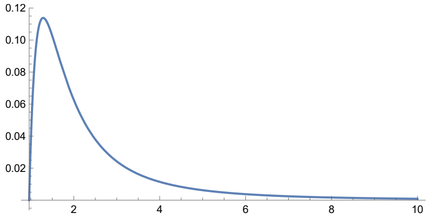


Figure: Typical shape of the effective potential V(r)

- It follows that the main equation reduces to a plane-wave equation both at the horizon and at infinity. Imposing only **ingoing** waves at the horizon and **outgoing** waves at infinity, we get the following asymptotic behaviour for Schwarzshild perturbations:

$$\psi \underset{x \to +\infty}{\sim} e^{i\omega x} \sim e^{i\omega r} r^{i\omega r_s},$$

$$\psi \underset{x \to -\infty}{\sim} e^{-i\omega x} \sim (r - r_s)^{-i\omega r_s}.$$

- Using this asymptotic behaviour we can construct an **ansatz** in the form of a power series satisfying the QNM border conditions:

$$\psi(r) = \left(\frac{r-r_s}{r^2}\right)^{-i\omega r_s} e^{i\omega(r-r_s)} \sum_{n=0}^{\infty} a_n \left(\frac{r-r_s}{r}\right)^n .$$

  We see that QNM are frequencies such that $\sum_{n=0}^{\infty} a_n$ converges.

- Inserting it in the main equation, we obtain a recurrence relation, which is of order two for Schwarzshild:

$$c0(1,\omega)a_1 + c_1(1,\omega)a_0 = 0,$$

$$c_0(n,\omega)a_n + c_1(n,\omega)a_{n-1} + c_2(n,\omega)a_{n-2} = 0 \quad \text{for } n < 1.$$

Continued fraction method

- Defining the ratio $R_n = -\frac{a_n}{a_{n-1}}$ and inserting it in the recurrence relation, we obtain a continued fraction:

$$R_n = \frac{c_2(n+1,\omega)}{c_1(n+1,\omega)-c_0(n+1,\omega)R_{n+1}} = \frac{c_2(n+1,\omega)}{c_1(n+1,\omega)-c_0(n+1,\omega)\frac{c_2(n+2,\omega)}{c_1(n+2,\omega)-c_0(n+2,\omega)\frac{c_2(n+3,\omega)}{c_1(n+3,\omega)-\ldots}}}.$$

- On one side we have $R_1 = \frac{c_1(1,\omega)}{c_0(1,\omega)}$ and on the other

$$R_1 = \frac{c_2(2,\omega)}{c_1(2,\omega)-c_0(2,\omega)\frac{c_2(3,\omega)}{c_1(3,\omega)-c_0(3,\omega)\frac{c_2(4,\omega)}{c_1(4,\omega)-\ldots}}}.$$

Equalizing the two gives:

$$c_1(1,\omega) - c_0(1,\omega)\frac{c_2(2,\omega)}{c_1(2,\omega)-c_0(2,\omega)\frac{c_2(3,\omega)}{c_1(3,\omega)-c_0(3,\omega)\frac{c_2(4,\omega)}{c_1(4,\omega)-\ldots}}} = 0.$$

Then the **QNM** correspond to the **roots** of this equation and can be computed numerically.

Schwarzschild QNM

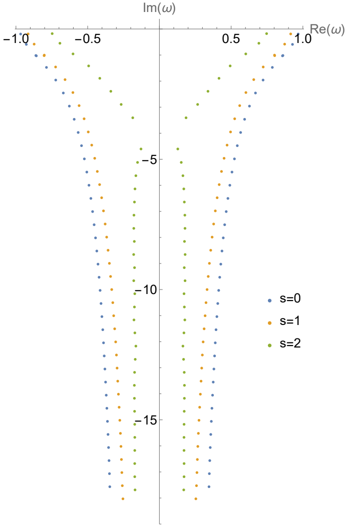


Figure: scalar, electromagnetic and gravitational (s=0,1,2 and l=2) QNM for a Schwarzshild black hole.

- We will focus on **scalar perturbations** for the rest of the talk and consider static black holes having spherical symmetry:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{g(r)} + h(r)\mathrm{d}\Omega^2,$$

- It has been shown that it is always possible to obtain a wave equation for the radial scalar perturbations of a static black hole having spherical symmetry:

$$\partial_x^2\psi + (\omega^2 - V(r))\psi = 0.$$

- x is called the turtoise coordinate and is defined as:

$$\frac{\mathrm{d}x}{\mathrm{d}r} = \frac{1}{\sqrt{f(r)g(r)}},$$

- The potential can be directly written in terms of the metric functions:

$$V(r) = l(l+1)\,\frac{f(r)}{h(r)} + \frac{1}{2}\sqrt{\frac{f(r)g(r)}{h(r)}}\left(\frac{f(r)g(r)}{h(r)}h'(r)\right)'$$

Modesto metric

- One of the first loop quantum black hole metric, developed by Modesto in 2008.

- The metric functions can be written in the Reisser-Nordström form:

$$f(r) = \frac{(r-r_+)(r-r_-)}{r^4+a_0^2}(r+r_0)^2,$$

$$g(r) = \frac{(r-r_+)(r-r_-)}{r^4+a_0^2}\frac{r^4}{(r+r_0)^2},$$

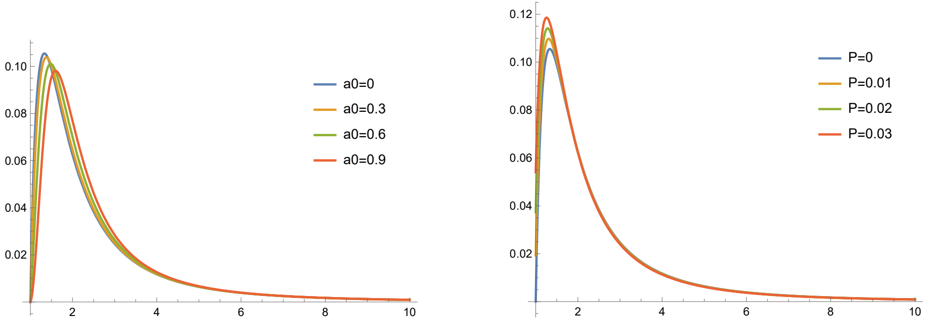
$$h(r) = r^2 + \frac{a_0^2}{r^2}.$$

$r_+ = \frac{2M}{(1+P)^2}$ is the outer horizon radius, $r_- = \frac{2MP^2}{(1+P)^2}$ and $r_0 = \frac{2MP}{(1+P)^2}$.

- a_0 is related to the minimum area gap of LQG and P is called the polymeric function.
- P is a free parameter but it has been constrained by astrophysical data.

Modesto metric - Potential behaviour

$$V(r) = \frac{(r - r_s)\left(2l(l+1)r^2 + r(r_0 + 2r_s) - 3r_0 r_s\right)}{2r^5}$$



(a) $P = 0$ (b) $a_0 = 0$

Figure: Effective potential versus the radial coordinate for $2M = 1$, $l = 0$ and several values of a_0 and P. The blue curve corresponds to the Schwarzshild potential.

Brizuela metric

- A recently obtained loop quantum metric black hole, obtained by Brizuela in 2022.
- The metric functions stands as:

$$f(r) = \frac{r - r_s}{r},$$
$$g(r) = \frac{r - r_0}{r} f(r),$$
$$h(r) = r^2.$$

where $r_0 < 2M$.

- The horizon is located at $2M$, similarly to what we have for Schwarzshild black hole.
- M is related to the ADM mass by $M_{ADM} = M + \frac{r_0}{2}$. We will scale the QNM with $2M$.

Brizuela metric - potential behaviour

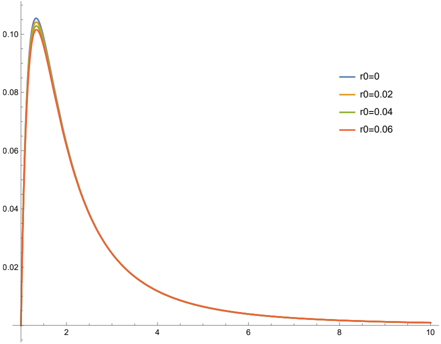


Figure: Effective potential versus the radial coordinate for $2M = 1$, $l = 0$ and several values of r_0. The blue curve corresponds to the Schwarzshild potential.

Results for the Brizuela metric

- The ansatz with correct asymptotic behaviour is:

$$\psi(r) = e^{ir\omega}(r - r_0)^{\frac{ir_0\omega}{2} + ir_s\omega - 1}\left(\frac{r - r_s}{r - r_0}\right)^{-\frac{ir_s\omega}{\sqrt{1-\frac{r_0}{r_s}}}}\sum_{n=0}^{\infty} a[n]\left(\frac{r - r_s}{r - r_0}\right)^n$$

- We obtain a four terms recurrence relation,

$$\begin{cases} \alpha_0 a_1 + \beta_0 a_0 = 0, \\ \alpha_1 a_2 + \beta_1 a_1 + \gamma_1 a_0 = 0, \\ \alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} + \delta_n a_{n-2} = 0 \qquad \text{for } n \geq 2. \end{cases}$$

which can be reduced to a three terms one using Gauss reduction:

$$\begin{cases} \tilde{\alpha}_0 a_1 + \tilde{\beta}_0 a_0 = 0, \\ \tilde{\alpha}_n a_{n+1} + \tilde{\beta}_n a_n + \tilde{\gamma}_n a_{n-1} = 0 \qquad \text{for } n \geq 1, \end{cases}$$

where:

$$\begin{cases} \tilde{\alpha}_0 = \alpha_0,\ \tilde{\beta}_0 = \beta_0,\ \tilde{\gamma}_0 = \gamma_0, \\ \tilde{\alpha}_1 = \alpha_1,\ \tilde{\beta}_1 = \beta_1,\ \tilde{\gamma}_1 = \gamma_1; \\ \tilde{\alpha}_n = \alpha_n, \\ \tilde{\beta}_n = \beta_n - \frac{\tilde{\alpha}_{n-1}}{\tilde{\gamma}_{n-1}}\delta_n, \\ \tilde{\gamma}_n = \gamma_n - \frac{\tilde{\beta}_{n-1}}{\tilde{\gamma}_{n-1}}\delta_n \qquad \text{for } n \geq 2. \end{cases}$$

Results for the Brizuela metric

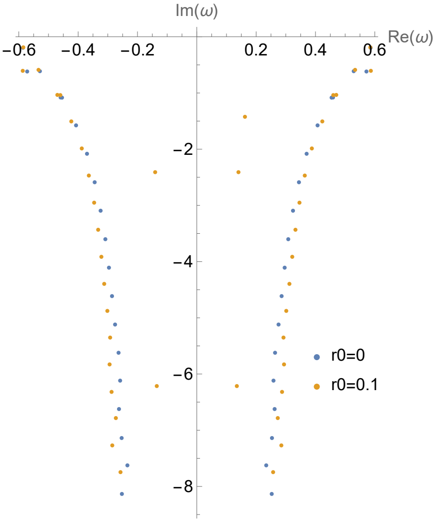


Figure: QNM frequencies for $s = 0, l = 1$.

Results for the Brizuela metric

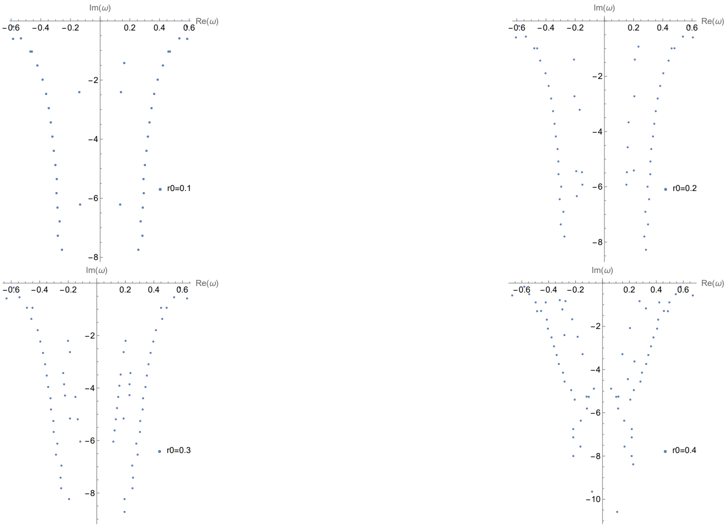


Figure: QNM frequencies for $s = 0, l = 1$ and $r_0 = 0.1, 0.2, 0.3, 0.4$.

Results for the Brizuela metric

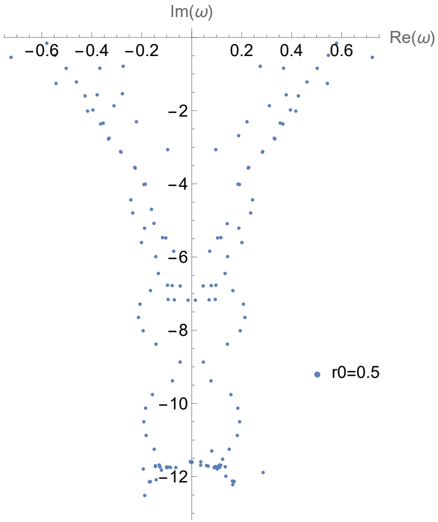


Figure: QNM frequencies for $s = 0, l = 1, r0 = 0.5$.

Results for the Modesto metric

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- The ansatz with correct asymptotic behaviour is:

$$\Psi(r) = e^{i\omega(r-r_+)}(r-r_-)^{i\omega(r_-+r_+)}\left(\frac{r-r_+}{r-r_-}\right)^{i\omega\frac{a_0^2+r_+^4}{(r_--r_+)r_+^2}}\sum_{n=0}^{\infty}a_n\left(\frac{r-r_+}{r-r_-}\right)^n.$$

- We obtain a fifteen terms recurrence equation, which I won't detail here...
- We use Gauss reduction to reduce it to three terms.

Results for the Modesto metric

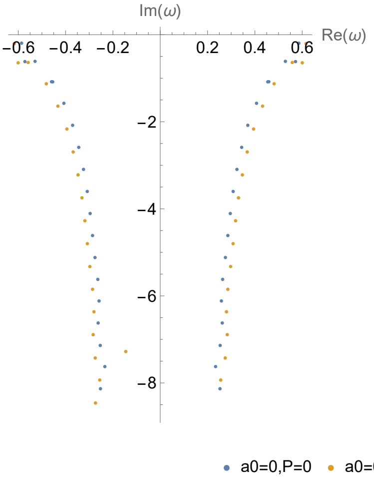

Figure: QNM frequencies for $a0 = 0$ and $P = 0.02$ ($s = 0, l = 1$)

Results for the Modesto metric

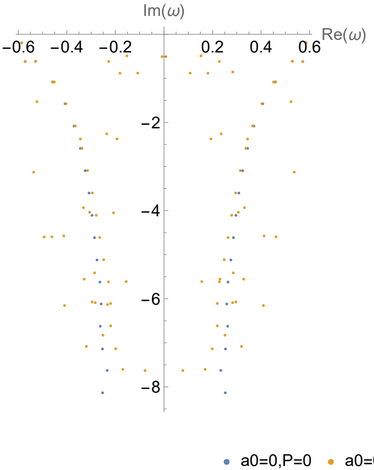

Figure: QNM frequencies for $a0 = 0.05$ and $P = 0$ ($s = 0, l = 1$)

Results for the Modesto metric

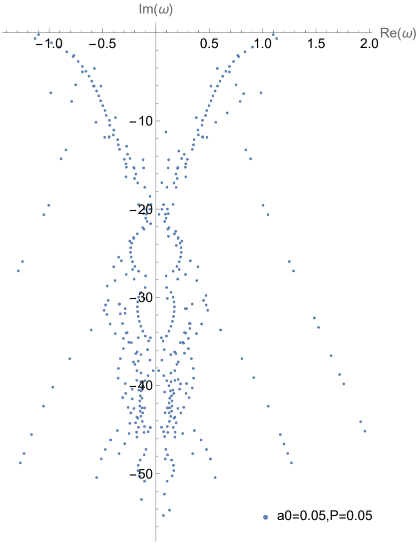

Figure: QNM frequencies for $a0 = 0.05$ and $P = 0.05$ ($s = 0, l = 2$)