Singularity-Free Spherical Black Holes with LQG corrections

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Outline

- Covariant deformations of GR
- The effective quantum Schwarzschild black hole
- Charged black holes in cosmological backgrounds
- Gravitational collapse: outlining effective LTB models

Covariant Deformations of General Relativity

We will demand the following:

- The derivative structure is the same as in GR
- The constraints form an anomaly-free algebra
- The theory is embeddable in a 4D manifold
- The GR Hamiltonian stands as a particular limit
- The model admits matter
- There is an explicit vacuum limit

```
brackets[expr1_, expr2_] :=
 Module[{table1variables, table1moments, table2variables,
   table2moments},
  table1variables =
   Table[
    Sum[(-1)^n D[D[expr1, D[variables[[i]], {x, n}]], {x, n}],
     {n, 0, derorder}], {i, 1, variablesnumber}];
  table1moments =
   Table[Sum[(-1)^n D[D[expr1, D[moments[[i]], {x, n}]],
      {x, n}], {n, 0, derorder}], {i, 1, variablesnumber}];
  table2variables =
   Table[
    Sum[(-1)^n D[D[expr2, D[variables[[i]], {x, n}]], {x, n}],
     {n, 0, derorder}], {i, 1, variablesnumber}];
  table2moments =
   Table[
    Sum[(-1)^n D[D[expr2, D[moments[[i]], {x, n}]], {x, n}],
     {n, 0, derorder}], {i, 1, variablesnumber}];
  -(Dot[table1variables, table2moments] -
     Dot[table1moments, table2variables])]

var[expr_, f_] :=
 Module[
  {order =
    Max[Join[{0}, Map[(# // Head // Head)[[1]] &,
       Cases[expr, Derivative[_][f][x], Infinity]]]]},
  Sum[(-1)^n D[D[expr, D[f[x], {x, n}]],
    {x, n}], {n, 0, order}]]

onshell = {diff :> (0 &), ham :> (0 &)};
```

Covariant Deformations of General Relativity

Requirements:

- Derivatives as in GR
- Anomaly-free algebra
- Embeddable in 4D
- GR limit
- Admits matter
- Vacuum limit

$$\{K_x(x_1), E^x(x_2)\} = \{\mathcal{K}_\varphi(x_1), \mathcal{E}^\varphi(x_2)\} = \delta(x_1, x_2)$$

$$\mathcal{D} = -K_x E^{x\prime} + \mathcal{K}_\varphi' \mathcal{E}^\varphi,$$

$$\mathcal{H} = -\frac{\mathcal{E}^\varphi}{2\sqrt{1+\lambda^2}\sqrt{E^x}}\left(1 + \frac{\sin^2(\lambda\mathcal{K}_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x \frac{\sin(2\lambda\mathcal{K}_\varphi)}{\sqrt{1+\lambda^2}\lambda}\left(1 + \left(\frac{\lambda E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2\right)$$
$$+ \left(\frac{(E^{x\prime})^2}{8\sqrt{E^x}\mathcal{E}^\varphi} - \frac{\sqrt{E^x}}{2\mathcal{E}^{\varphi 2}}E^{x\prime}\mathcal{E}^{\varphi\prime} + \frac{\sqrt{E^x}}{2\mathcal{E}^\varphi}E^{x\prime\prime}\right)\frac{\cos^2(\lambda\mathcal{K}_\varphi)}{\sqrt{1+\lambda^2}} + \frac{\sqrt{E^x}\mathcal{E}^\varphi}{2\sqrt{1+\lambda^2}}\left(\Lambda + \frac{Q^2}{(E^x)^2}\right)\Bigg)$$

Which are the effects on spacetime?

Covariant Deformations of General Relativity

The effective Hamiltonian satisfies the hypersurface deformation algebra (by construction, questions are welcome!)

$$\{D[s_1], D[s_2]\} = D[s_1 s_2' - s_1' s_2],$$
$$\{D[s_1], H[s_2]\} = H[s_1 s_2'],$$
$$\{H[s_1], H[s_2]\} = D[F(s_1 s_2' - s_1' s_2)].$$

with $$F = \frac{\cos^2(\lambda \mathcal{K}_\varphi)}{1+\lambda^2}\left(1+\left(\frac{\lambda E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2\right)\frac{E^x}{\mathcal{E}^{\varphi 2}}$$

note that it is nowhere negative!

roots of the cosine are new roots of F

GR: $\lambda \to 0$

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GR: $\lambda \to 0$

Three assumptions:

(I) The lapse and the shift are defined in the same way as in GR

(II) Gauge transformations describe coordinate changes

(III) The area of the spheres is not affected by the corrections

$$ds^2 = -N(t,x)^2 dt^2 + \frac{1}{F}\left(dx^2 + N^x(t,x)dt^2\right) + r(t,x)^2 d\Omega^2$$

Covariant Deformations of General Relativity

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$$
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$$

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Condition (ii) is highly non-trivial, but again, by construction, the structure function F satisfies this requirement!

given a generic vector

$$\xi^A \partial_A = \xi^t \partial_t + \xi^x \partial_x$$

$$\xi^t \partial_t \left(\frac{1}{F}\right) + \xi^x \partial_x \left(\frac{1}{F}\right) + \frac{2}{F}\left(N^x \partial_x \xi^t + \partial_x \xi^x\right) = \left\{\frac{1}{F}, H[\xi^t N] + D[\xi^t N^x + \xi^x]\right\}$$

coordinate transformations (Lie derivative) gauge transformations (Poisson bracket)

Covariant Deformations of General Relativity

the effective Hamiltonian

$$\mathcal{D} = -K_x E^{x\prime} + \mathcal{K}_\varphi' \mathcal{E}^\varphi,$$

$$\mathcal{H} = -\frac{\mathcal{E}^\varphi}{2\sqrt{1+\lambda^2}\sqrt{E^x}}\left(1+\frac{\sin^2(\lambda\mathcal{K}_\varphi)}{\lambda^2}\right) - \sqrt{E^x}K_x\frac{\sin(2\lambda\mathcal{K}_\varphi)}{\sqrt{1+\lambda^2}\lambda}\left(1+\left(\frac{\lambda E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2\right)$$
$$+\left(\frac{(E^{x\prime})^2}{8\sqrt{E^x}\mathcal{E}^\varphi} - \frac{\sqrt{E^x}}{2\mathcal{E}^{\varphi 2}}E^{x\prime}\mathcal{E}^{\varphi\prime} + \frac{\sqrt{E^x}}{2\mathcal{E}^\varphi}E^{x\prime\prime}\right)\frac{\cos^2(\lambda\mathcal{K}_\varphi)}{\sqrt{1+\lambda^2}} + \frac{\sqrt{E^x}\mathcal{E}^\varphi}{2\sqrt{1+\lambda^2}}\left(\Lambda+\frac{Q^2}{(E^x)^2}\right)\Bigg)$$

$$\{K_x(x_1), E^x(x_2)\} = \{\mathcal{K}_\varphi(x_1), \mathcal{E}^\varphi(x_2)\} = \delta(x_1, x_2)$$

Leading-order corrections go as λ^2

GR: $\lambda \to 0$

The “angular component of the curvature” shows a reflection symmetry at $\pi/(2\lambda)$

$$ds^2 = -N(t,x)^2 dt^2 + \frac{1}{F}\left(dx^2 + N^x(t,x)dt^2\right) + r(t,x)^2 d\Omega^2$$

the metric

$$F = \frac{\cos^2(\lambda\mathcal{K}_\varphi)}{1+\lambda^2}\left(1+\left(\frac{\lambda E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2\right)\frac{E^x}{\mathcal{E}^{\varphi 2}}$$

Roots of F

Covariant Deformations of General Relativity

There is a simpler way of writing the metric.

We just need to use the mass function (the quantity that is a constant in vacuum):

$$m := \frac{\sqrt{E^x}}{2}\left(1 + \frac{\sin^2(\lambda \mathcal{K}_\varphi)}{\lambda^2} - \left(\frac{E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2 \cos^2(\lambda \mathcal{K}_\varphi)\right)$$

$$F = \frac{\cos^2(\lambda \mathcal{K}_\varphi)}{1+\lambda^2}\left(1 + \left(\frac{\lambda E^{x\prime}}{2\mathcal{E}^\varphi}\right)^2\right)\frac{E^x}{\mathcal{E}^{\varphi 2}} = \left(1 - \frac{2\lambda\!\!\!^{-} m}{\sqrt{E^x}}\right)\frac{E^x}{\mathcal{E}^{\varphi 2}}$$

with

$$\lambda\!\!\!^{-} := \frac{\lambda^2}{1+\lambda^2}$$

this is the physically meaningful constant of the model
it takes values in $(0,1)$
the limit to 0 corresponds to GR

$$ds^2 = -N^2 dt^2 + \left(1 - \frac{2\lambda\!\!\!^{-} m}{\sqrt{E^x}}\right)^{-1}\frac{\mathcal{E}^{\varphi 2}}{E^x}\left(dx + N^x dt\right)^2 + E^x d\Omega^2$$

Effective Quantum Schwarzschild Black Hole

Static chart

$$ds^2 = -\left(1 - \frac{2M}{\tilde{r}}\right) d\tilde{t}^2 + \left(1 - \frac{2\lambda M}{\tilde{r}}\right)^{-1} \left(1 - \frac{2M}{\tilde{r}}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2.$$

Schwarzschild-like coordinates

$$ds^2 = -\left(1 - \frac{2\lambda M}{T}\right)^{-1} \left(\frac{2M}{T} - 1\right)^{-1} dT^2 + \left(\frac{2M}{T} - 1\right) dX^2 + T^2 d\Omega^2$$

Homogeneous chart

The covering chart

where $r(z)$ is completely determined by $$\frac{dr(z)}{dz} = \mathrm{sgn}(z)\sqrt{1 - \frac{2\lambda M}{r(z)}},$$

$$ds^2 = -\left(1 - \frac{2M}{r(z)}\right) d\tau^2 + 2\sqrt{\frac{2M}{r(z)}}\, d\tau dz + dz^2 + r(z)^2\, d\Omega^2.$$

$$r_0 := 2\lambda M$$

The positive minimum of the area-radius function!

Effective Quantum Schwarzschild Black Hole

- Curvature scalars attain their maximum at the transition surface
- Every definition of mass provides the ***same*** exact value in every region of the spacetime
- Although the transition surface appears always inside the horizon, the modifications affect the whole spacetime
- Quantum effects are measurable! There are already have some bounds…

Ricci scalar

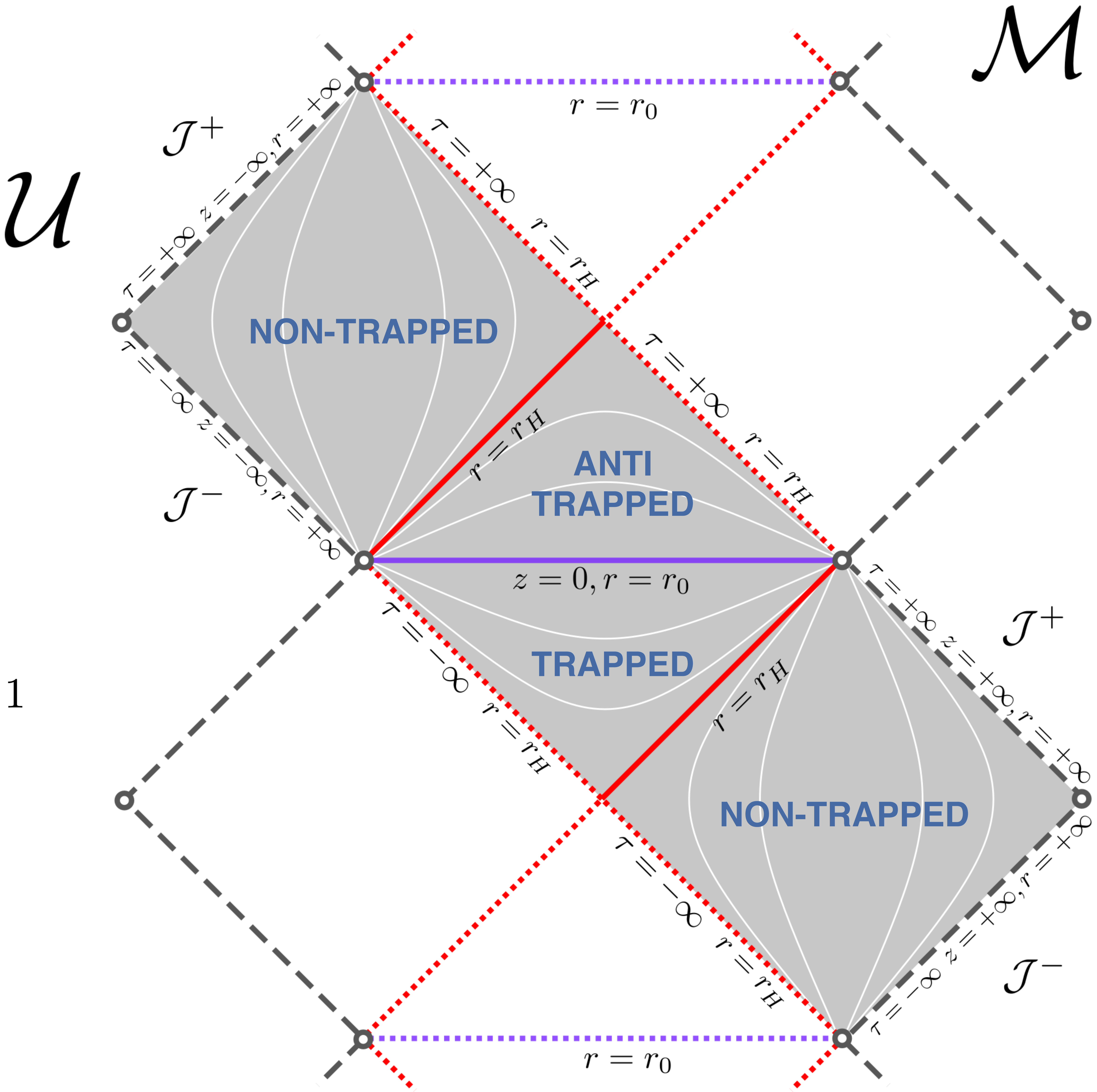
$$\mathcal{R} = \frac{6\lambda M^2}{r^4}$$

Characterization of the new constant

$$\lambda = \lim_{r\to\infty} \frac{M_H}{M_K} - 1$$

mean curvature vector: $H^\mu = (2/r)\,\nabla^\mu r$

$$H^\mu \partial_\mu = \mathrm{sgn}(z)\frac{2}{r}\sqrt{1-\frac{2\lambda M}{r}}\left(\sqrt{\frac{2M}{r}}\partial_\tau + \left(1-\frac{2M}{r}\right)\partial_z\right)$$



Charged BHs in Cosmological Backgrounds

$$ds^2 = -\left(1 - \frac{2M}{r(z)}\right) d\tau^2 + 2\sqrt{\frac{2M}{r(z)}}\, d\tau dz + dz^2 + r(z)^2\, d\Omega^2$$

Change of coordinates to static/homogeneous charts:

$$d\tau = dT + \left(1 - \frac{2m\big(r(z)\big)}{r(z)}\right)^{-1} \sqrt{\frac{2m\big(r(z)\big)}{r(z)}} dz$$

$$ds^2 = -\left(1 - \frac{2m\big(r(z)\big)}{r(z)}\right) d\tau^2 + 2\sqrt{\frac{2m\big(r(z)\big)}{r(z)}}\, d\tau dz + dz^2 + r(z)^2 d\Omega^2$$

[This is a solution to the Hamiltonian equations, and not just an extension of the vacuum metric]

mass function

$$m(r) := M - \frac{Q^2}{2r} + \frac{\Lambda}{6} r^3$$

$$\left(\frac{dr(z)}{dz}\right)^2 = 1 - \frac{2\bar{\lambda} m\big(r(z)\big)}{r(z)}$$

implicit definition of $r(z)$

Are these spacetimes free of singularities?

Charged BHs in Cosmological Backgrounds

- Ricci scalar

$$\mathcal{R} = 4\Lambda\left(1+\frac{\lambda}{2}\right) + 2\lambda\left(\frac{3M^2}{r^4} + \frac{Q^2}{r^4}\left(1-\frac{4M}{r}+\frac{Q^2}{r^2}\right) - \Lambda\left(\frac{4M}{r}+\Lambda r^2\right) + \frac{4\Lambda Q^2}{3r^2}\right)$$

The singularity is resolved by the appearance of a positive lower bound for r

- Kretschmann scalar

$$\mathcal{R}_{abcd}\mathcal{R}^{abcd} = \frac{8\Lambda^2}{3}(1+\lambda) + \frac{48M^2}{r^6} - \frac{96MQ^2}{r^7} + \frac{56Q^4}{r^8} - \lambda\left(\frac{8}{3}\Lambda^3 r^2 - \frac{152Q^6}{r^{10}} - \frac{240M^3}{r^7} + \frac{P_8(r)}{r^9}\right)$$

$$+\lambda^2\left(\frac{20}{27}\Lambda^4 r^4 - \frac{40}{27}\Lambda^3 r^2 + \frac{16}{3}M\Lambda^3 r + \frac{108Q^8}{r^{12}} + \frac{324M^4}{r^8} + \frac{P_{10}(r)}{r^{11}}\right)$$

Charged BHs in Cosmological Backgrounds

Domain of r restricted by the existence of a solution for

$$\left(\frac{dr(z)}{dz}\right)^2 = 1 - \frac{2\lambda m(r(z))}{r(z)}$$

which is equivalent to a 1D particle with zero total energy

The singularity is resolved by the appearance of a positive lower bound for r

$$\frac{1}{2}(r')^2 + V(r) = 0 \qquad \text{where} \qquad V(r) = -\frac{1}{2}\left(1 - \frac{2\lambda m(r)}{r}\right)$$

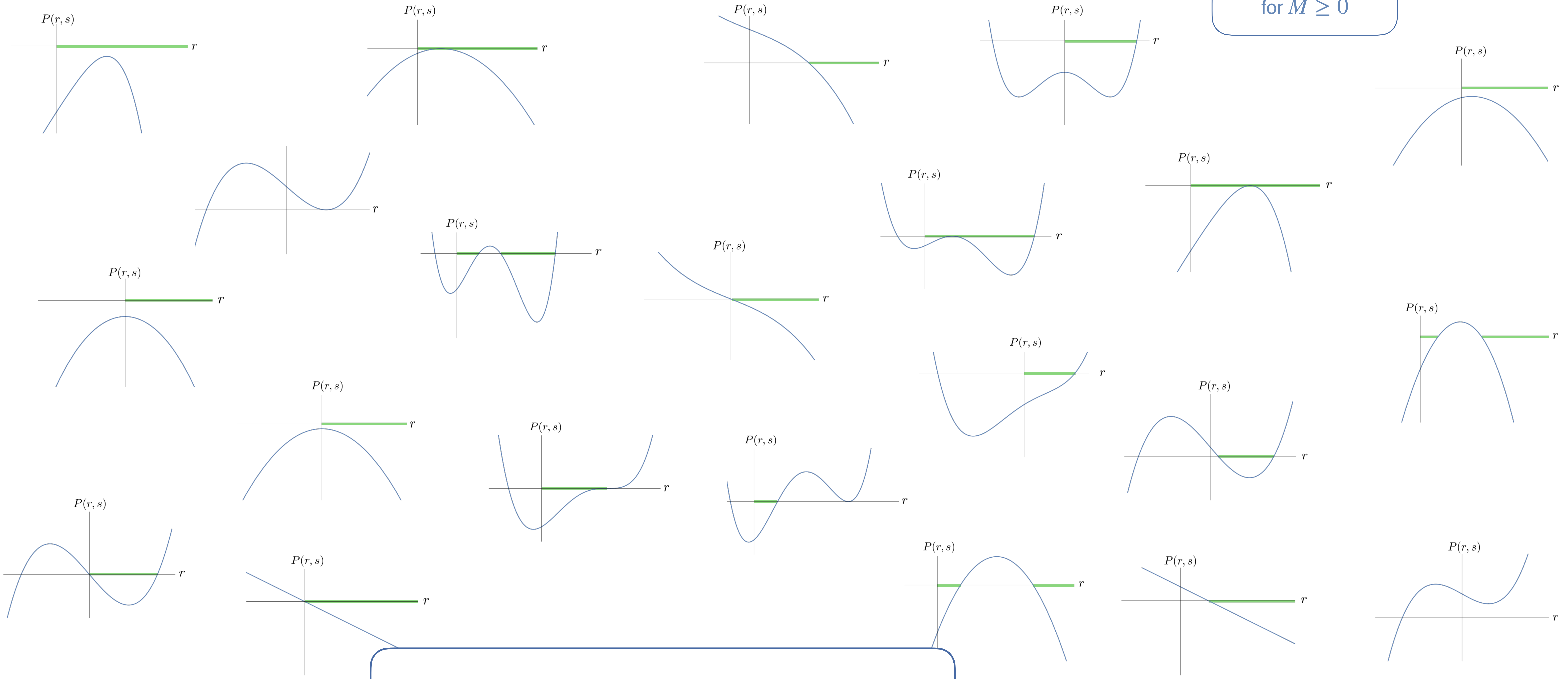
Allowed regions: $r > 0$ and $2r^\ell V(r) \leq 0$

Study of singularity resolution = study of the roots of $V(r)$

- Simple roots of $V(r)$: Turning points of $r(z)$
- Multiple roots of $V(r)$: Asymptotic values of z

Charged BHs in Cosmological Backgrounds

for $M \geq 0$

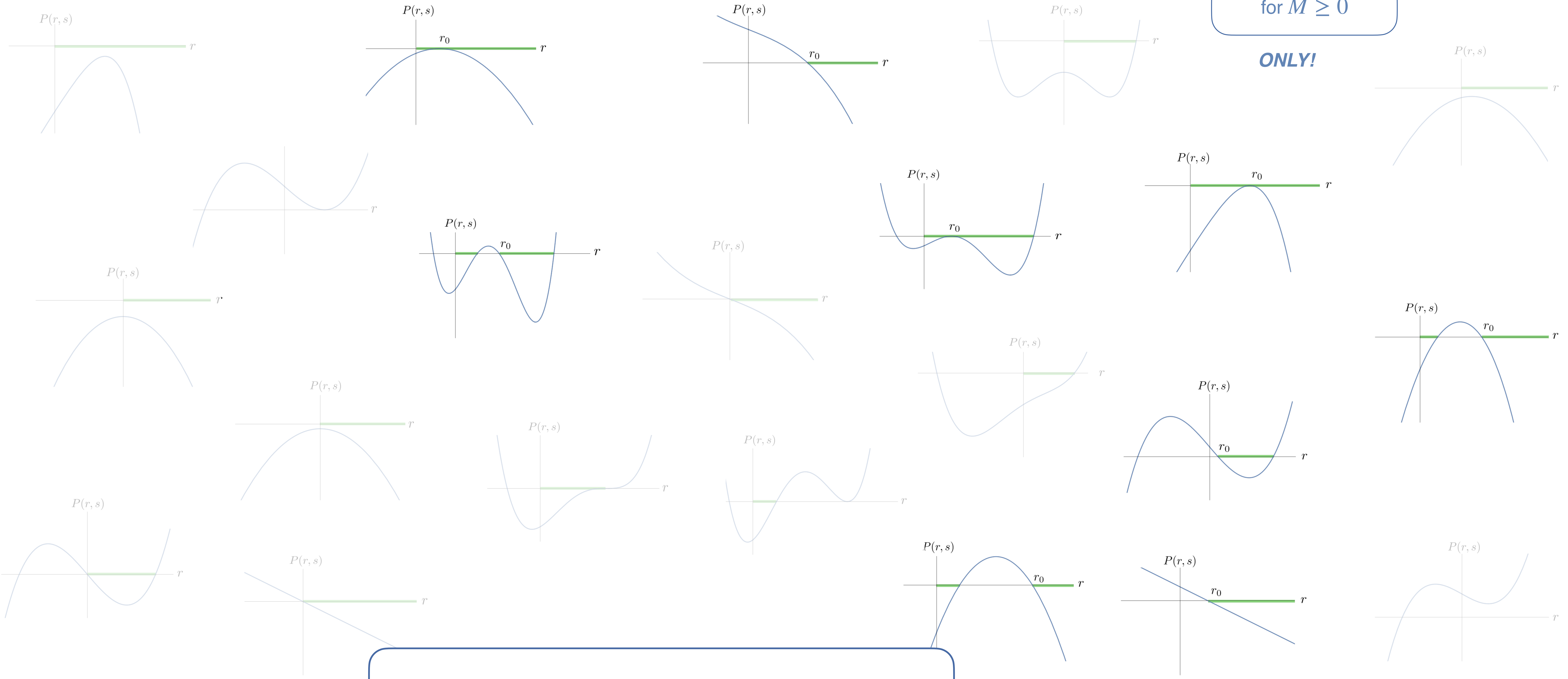


Allowed regions: $r > 0$ and $2r^{\ell}V(r) \leq 0$

Charged BHs in Cosmological Backgrounds

for $M \geq 0$

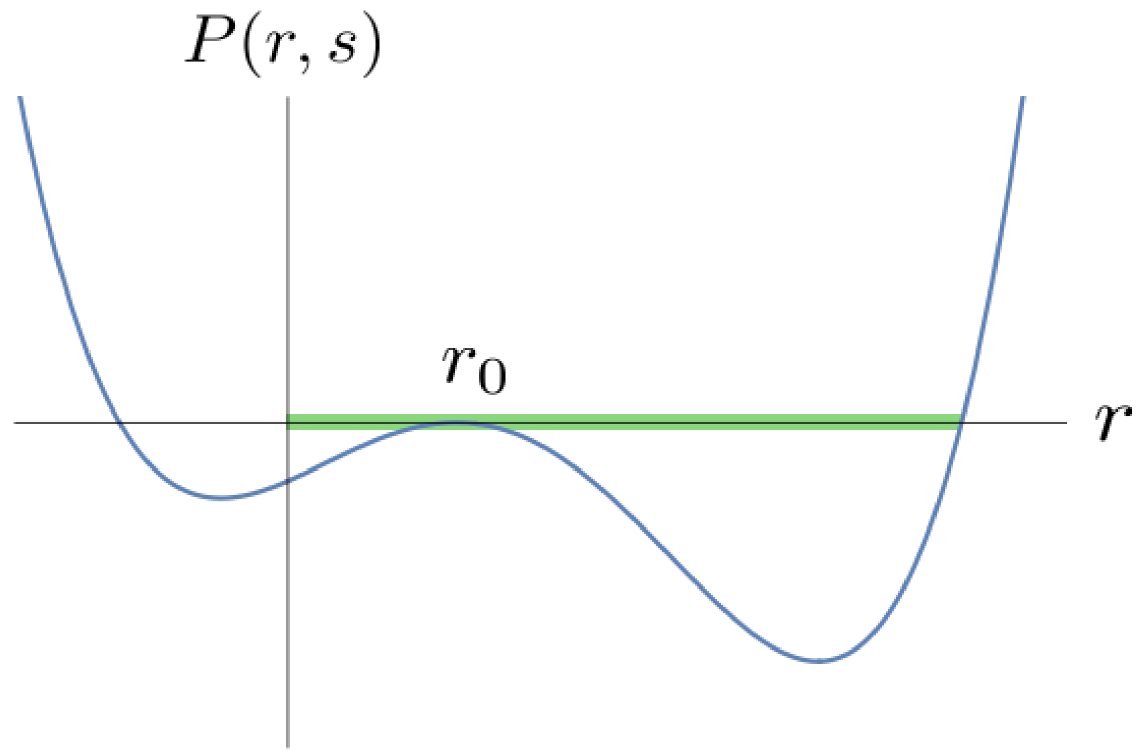
ONLY!

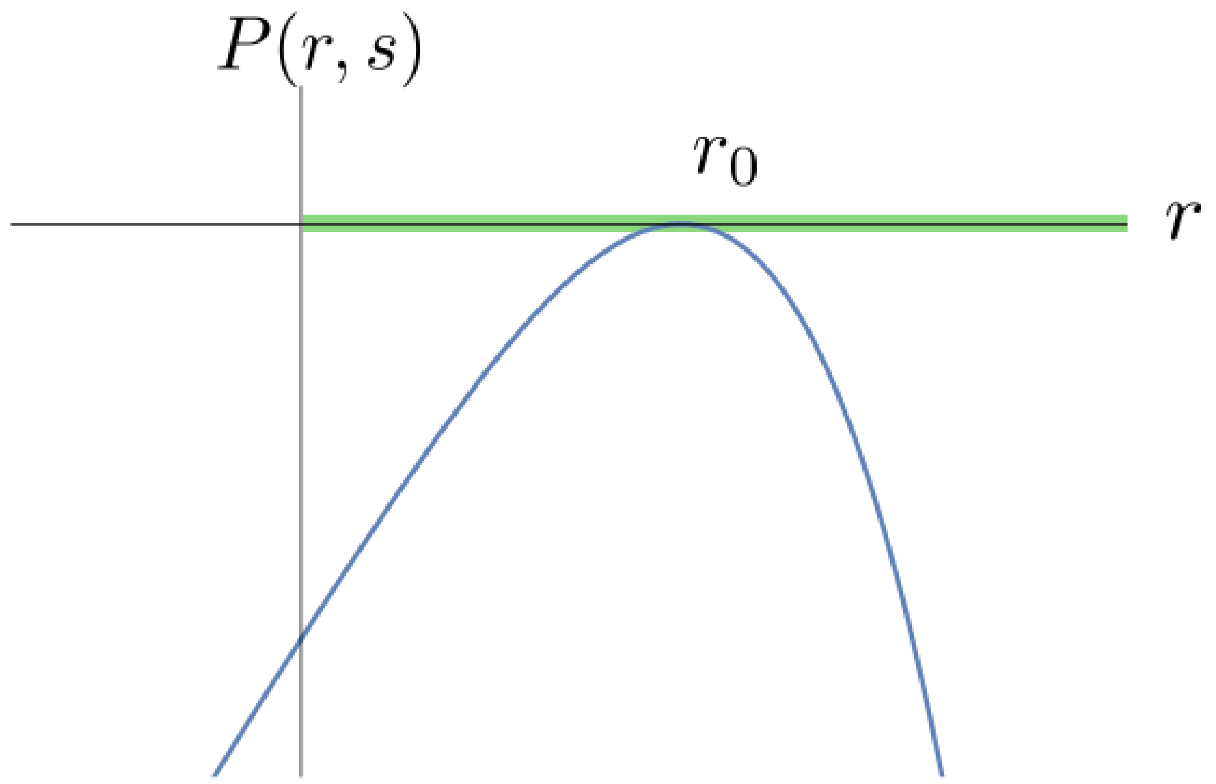


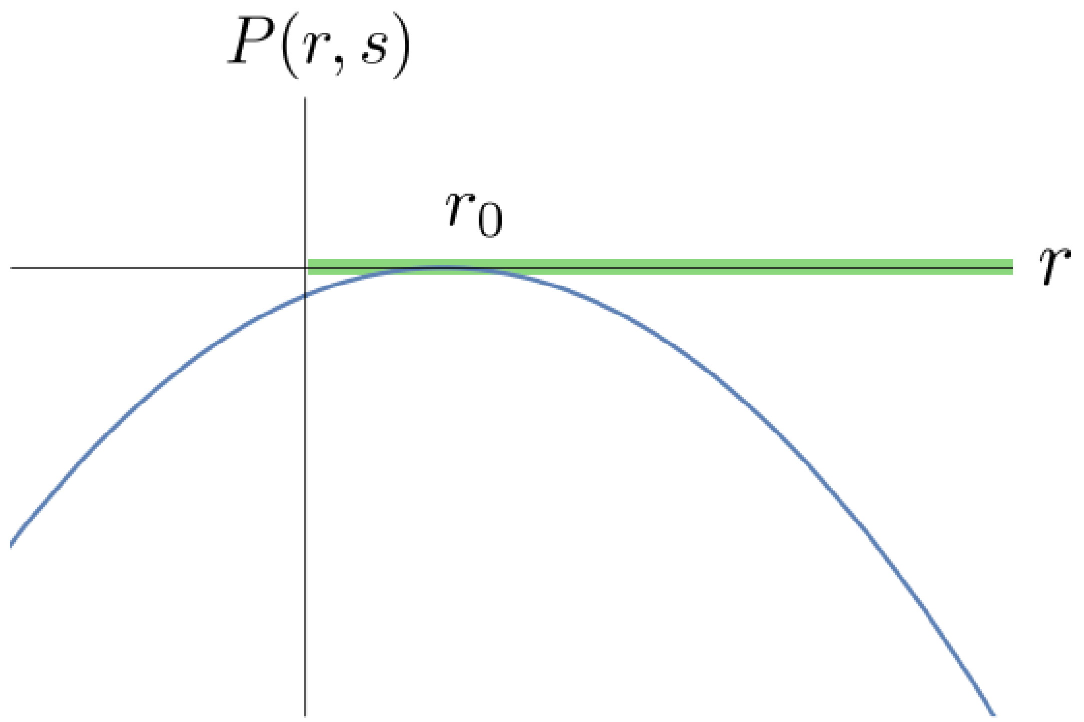
Allowed regions: $r > 0$ and $2r^\ell V(r) \leq 0$

Charged BHs in Cosmological Backgrounds

The infimum is a double root







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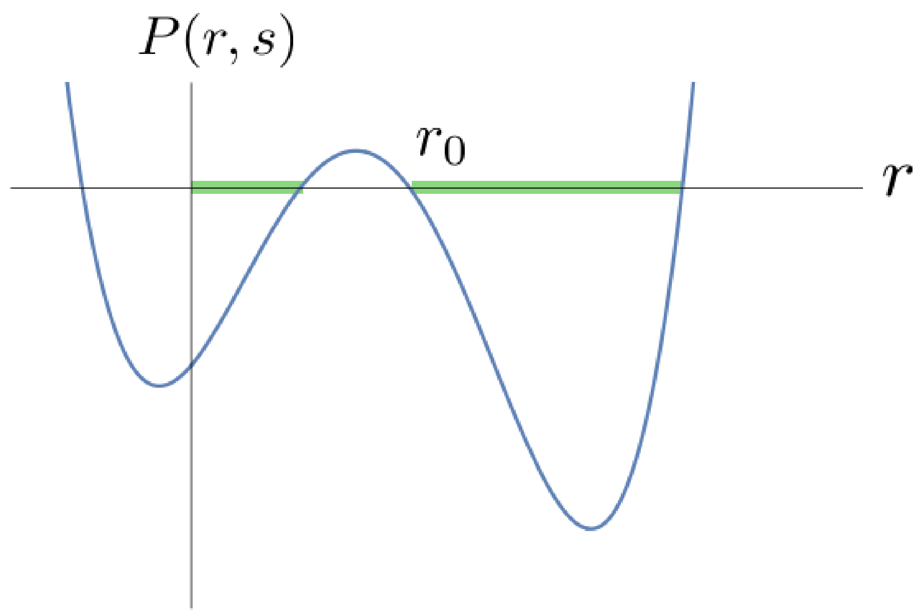

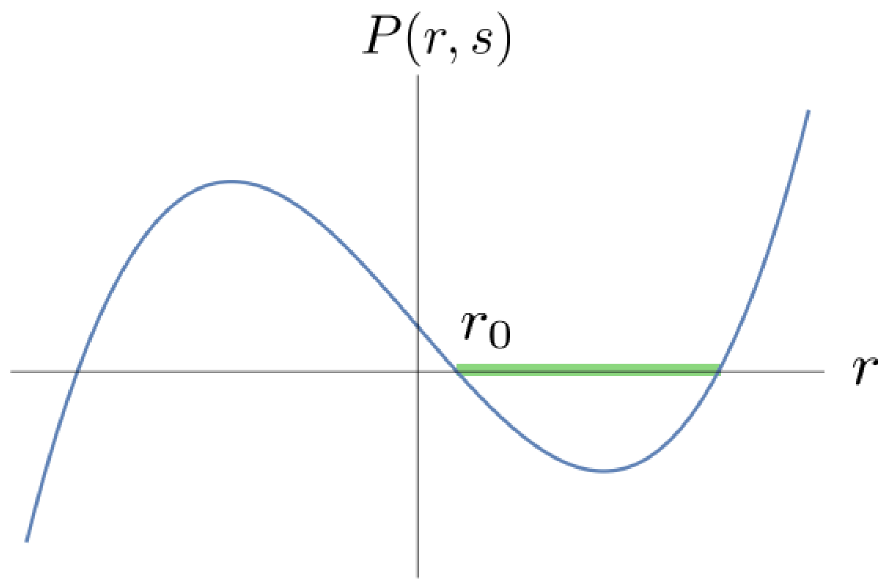

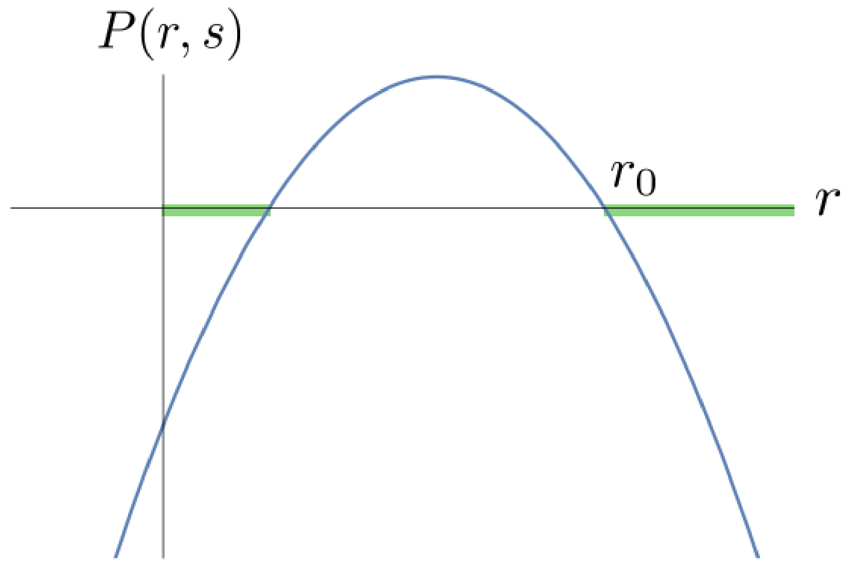

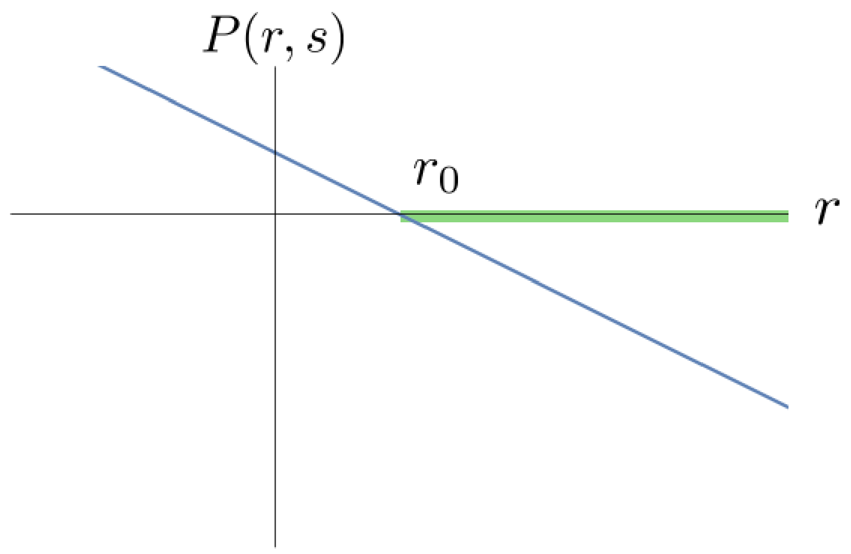

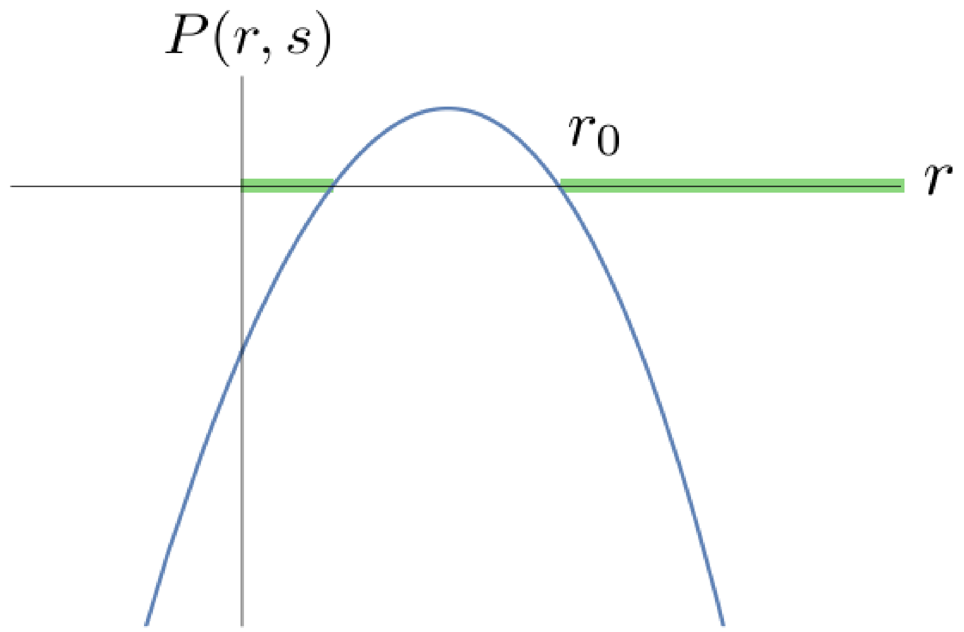

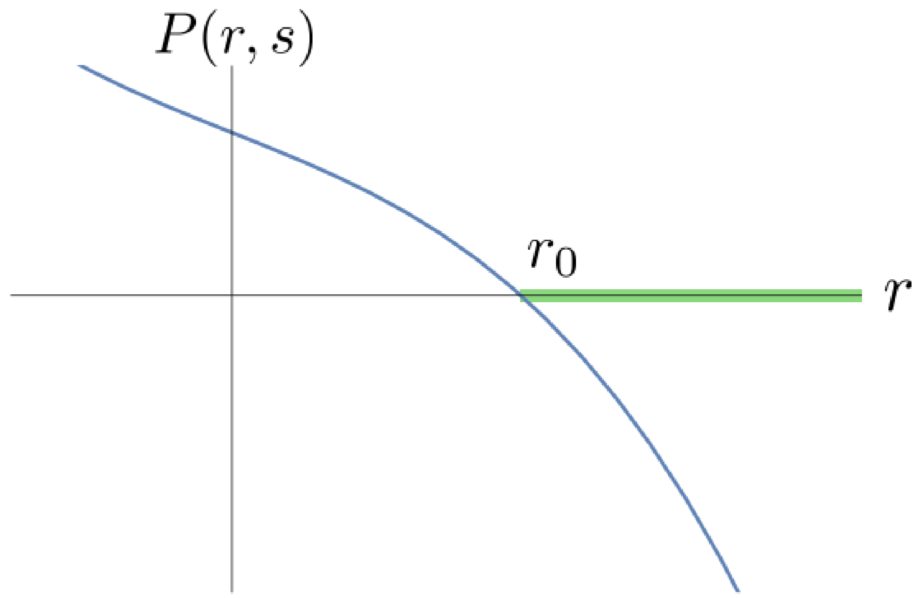

Charged BHs in Cosmological Backgrounds

- Ricci scalar

$$\mathcal{R} = 4\Lambda\left(1+\frac{\lambda\!\!\!^-}{2}\right) + 2\lambda\!\!\!^-\left(\frac{3M^2}{r^4} + \frac{Q^2}{r^4}\left(1-\frac{4M}{r}+\frac{Q^2}{r^2}\right) - \Lambda\left(\frac{4M}{r}+\Lambda r^2\right) + \frac{4\Lambda Q^2}{3r^2}\right)$$

The singularity is resolved by the appearance of a positive lower bound for r

- Kretschmann scalar

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But we also need a finite upper bound for r (when $\Lambda \neq 0$)

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r_0 ***"Replaces zero"***

r_∞ ***"Replaces infinity"***

But we also need a finite upper bound for r (when $\Lambda \neq 0$)

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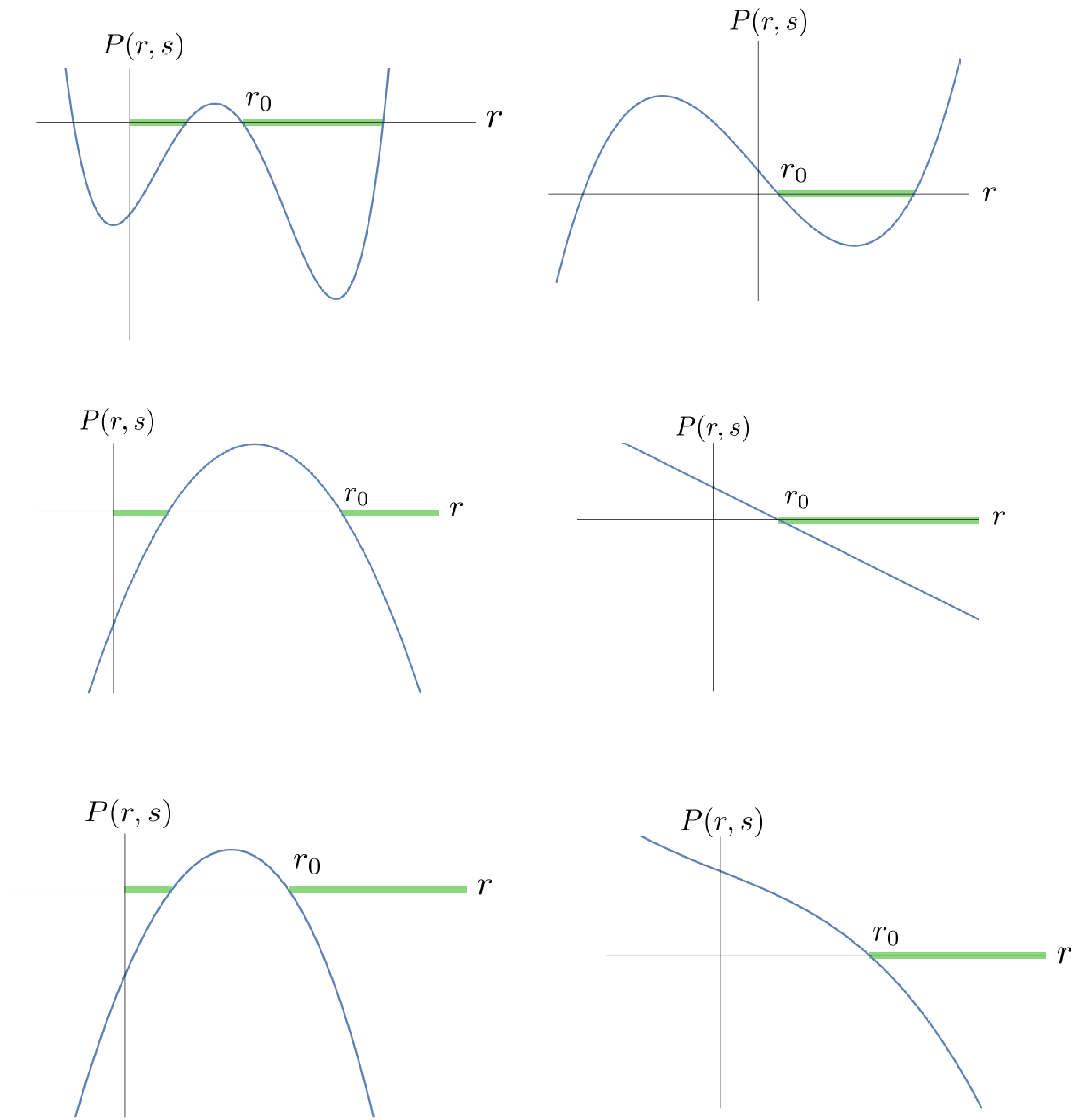
Charged BHs in Cosmological Backgrounds

The infimum is a single root



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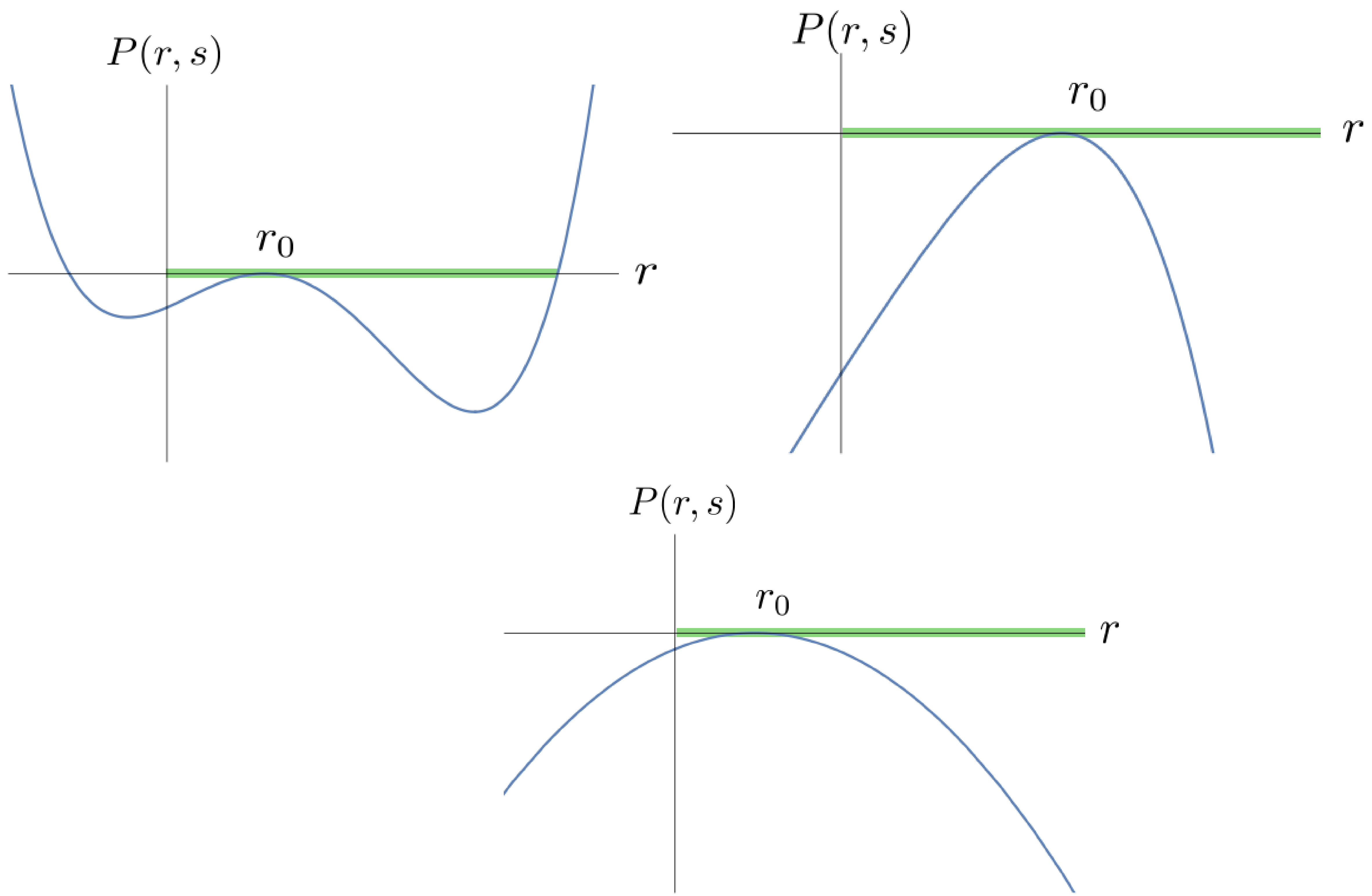

Charged BHs in Cosmological Backgrounds

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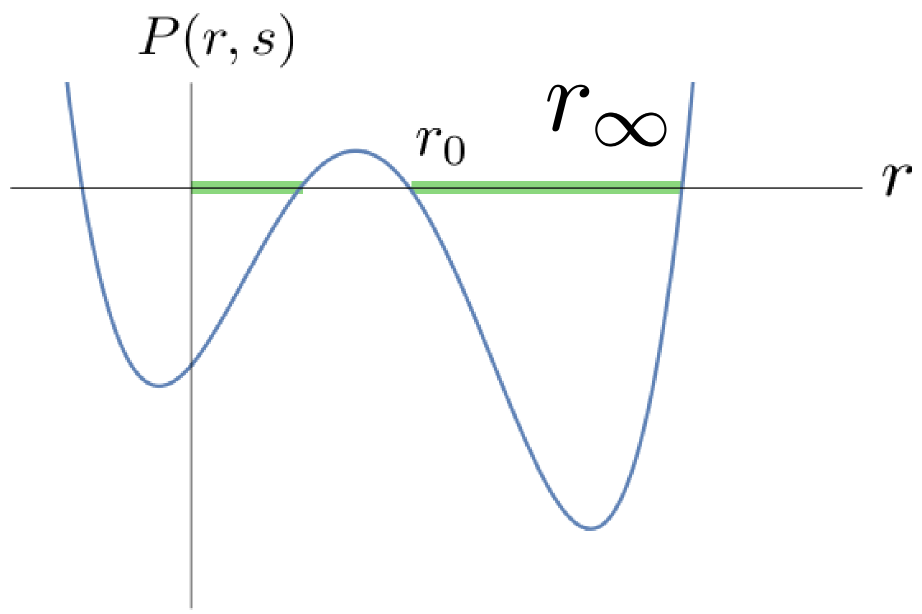
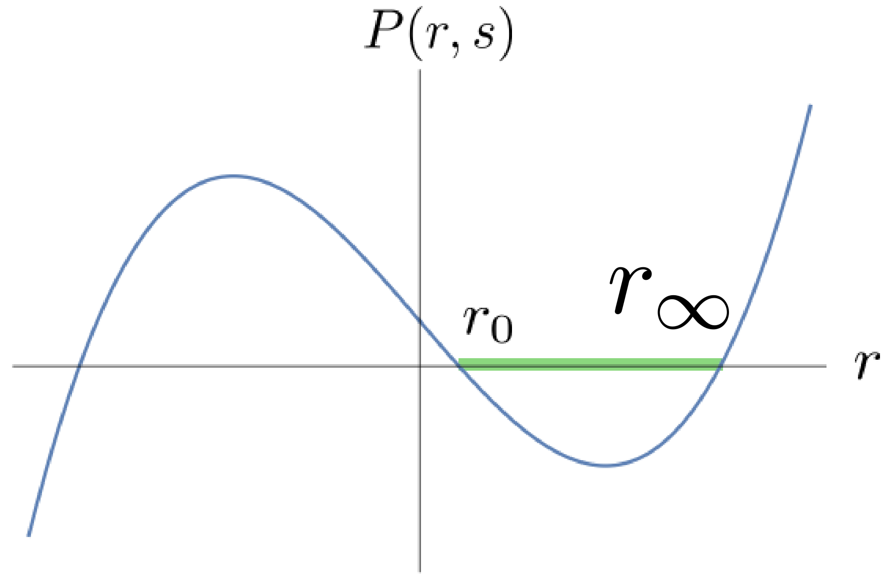
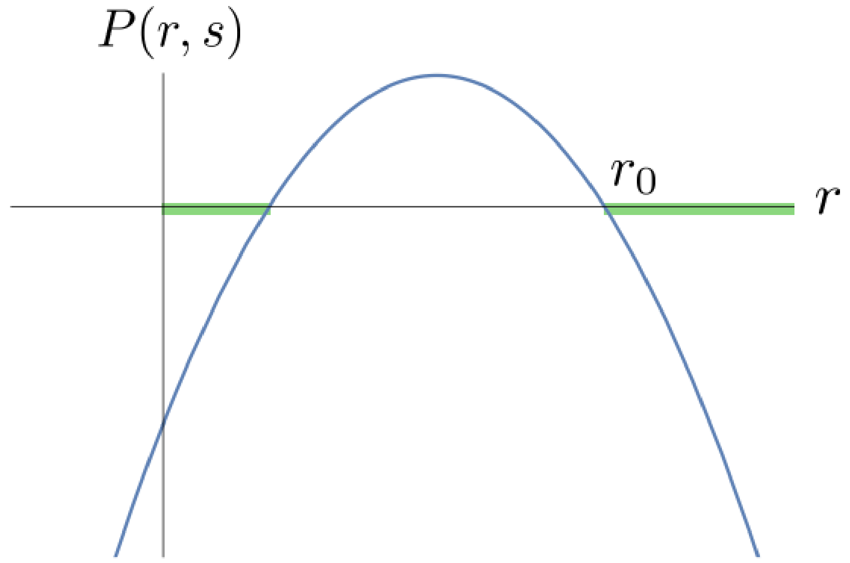
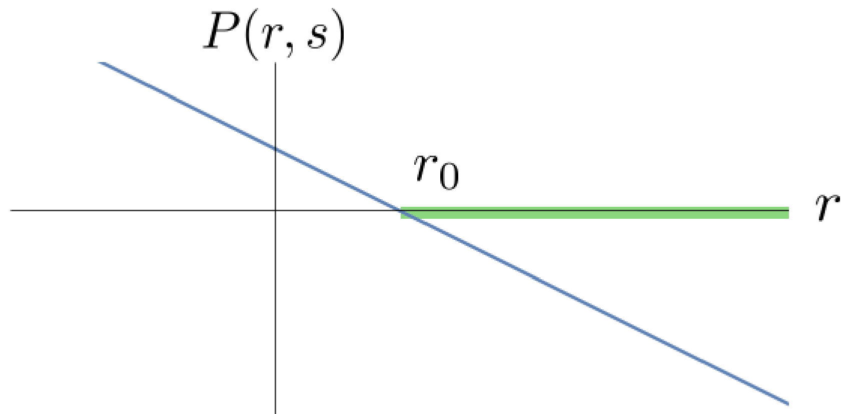
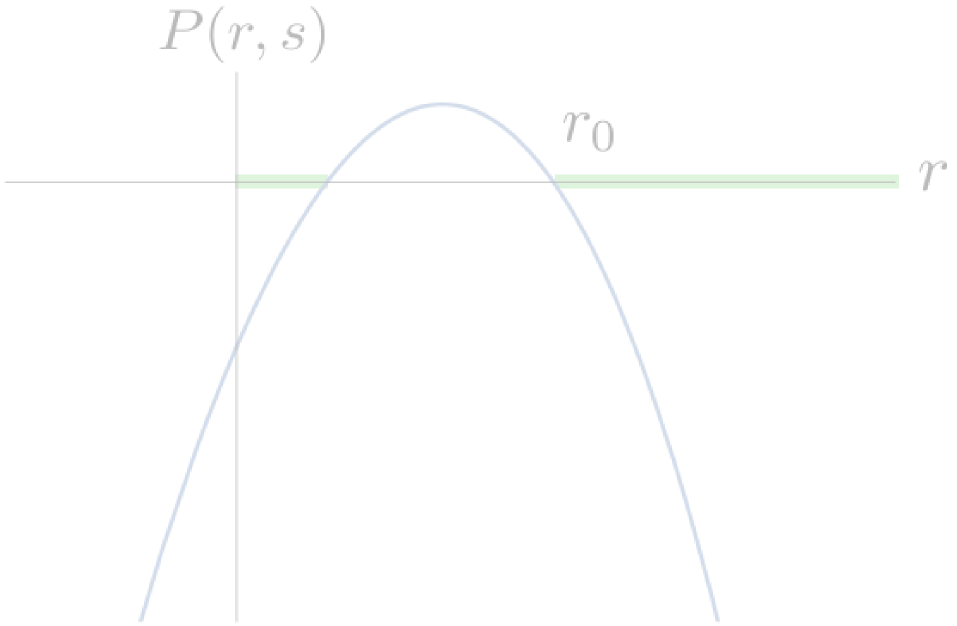
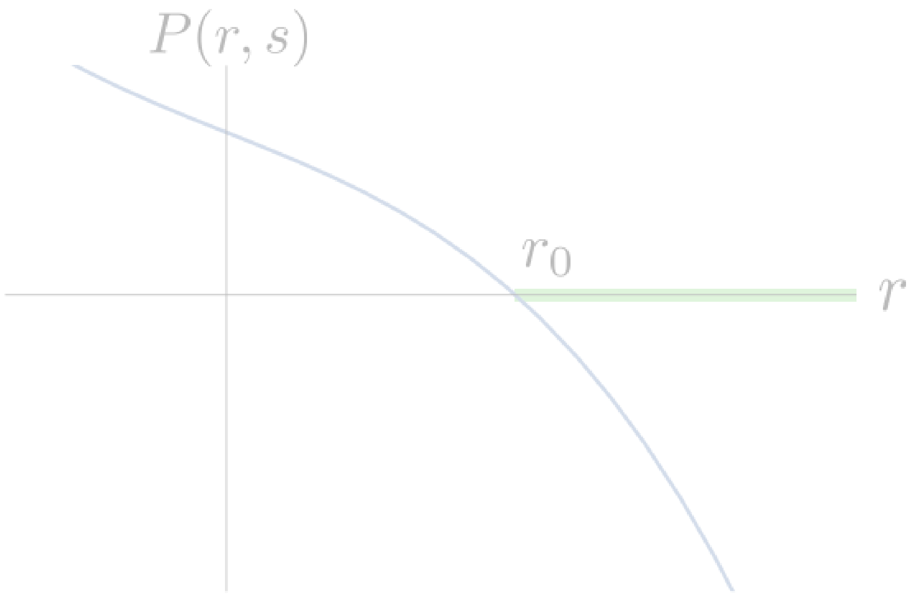




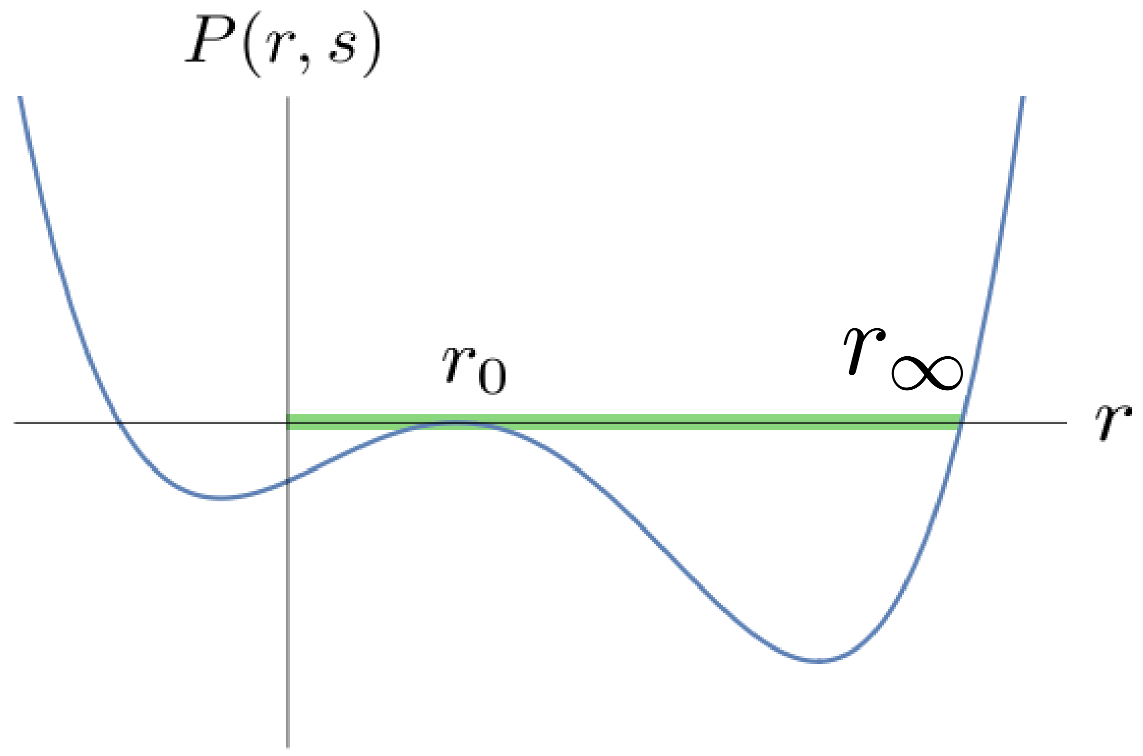


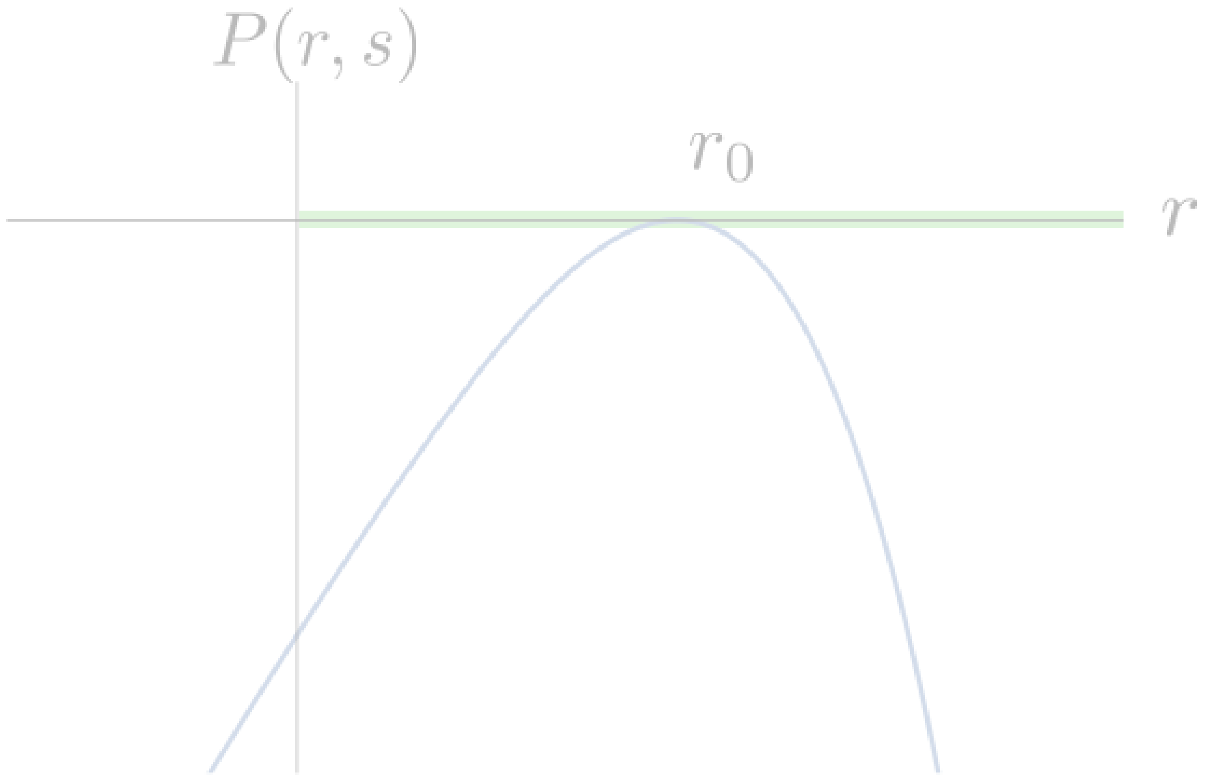


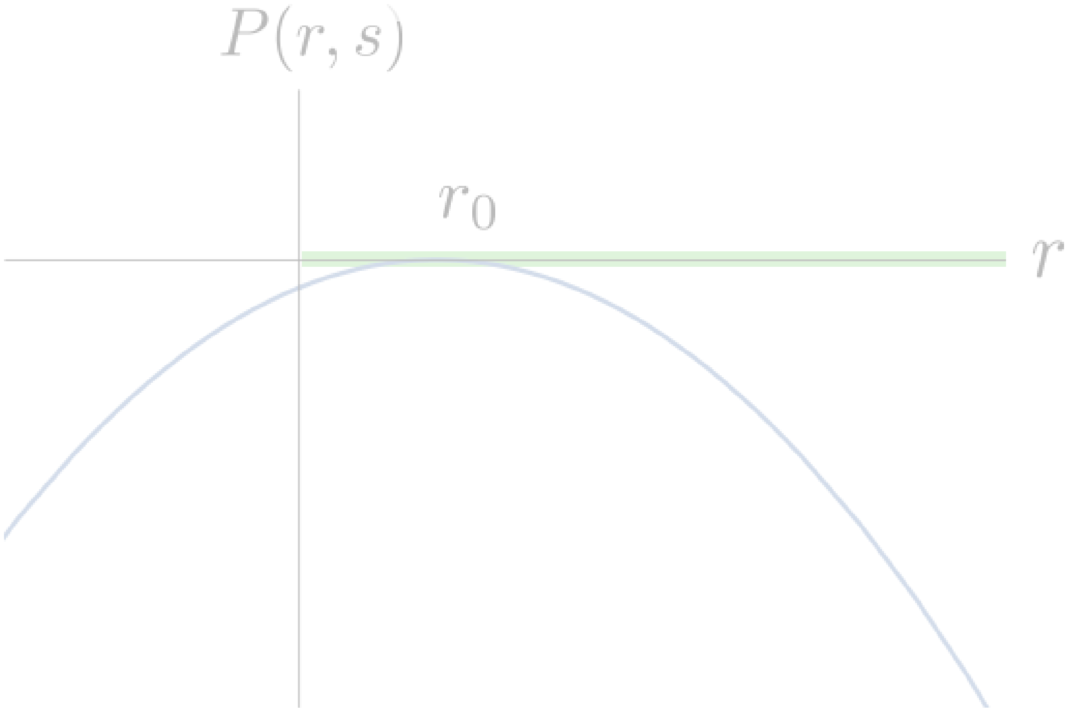




The infimum is a double root

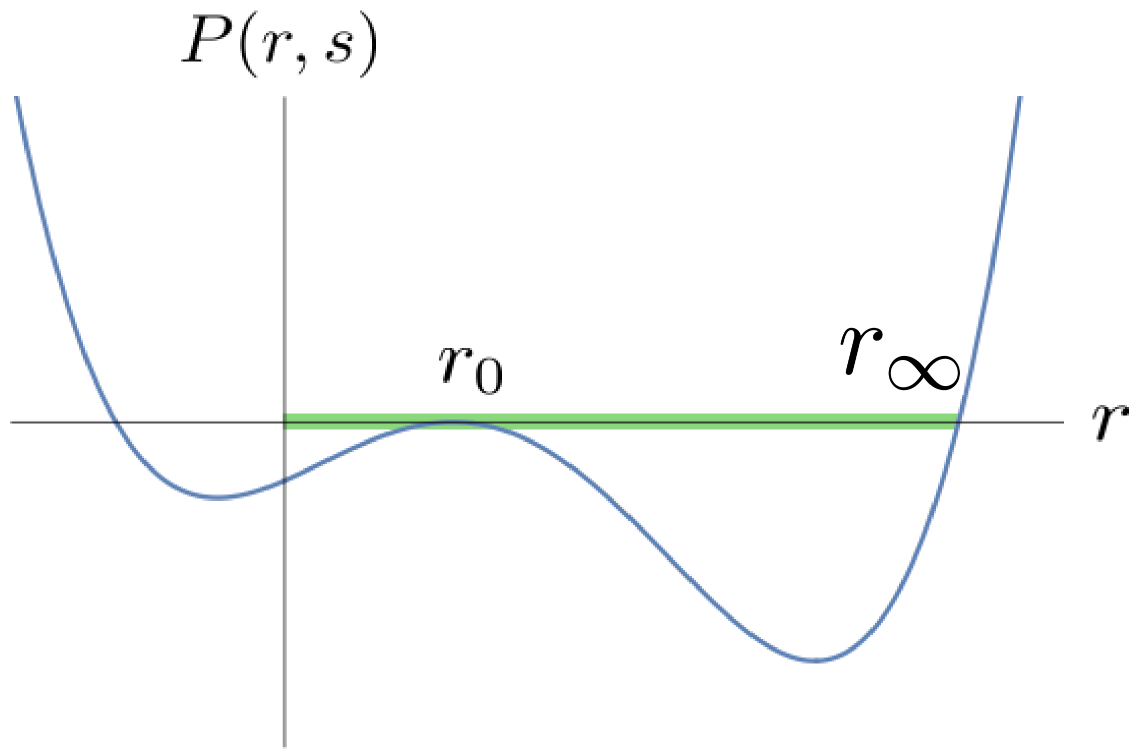


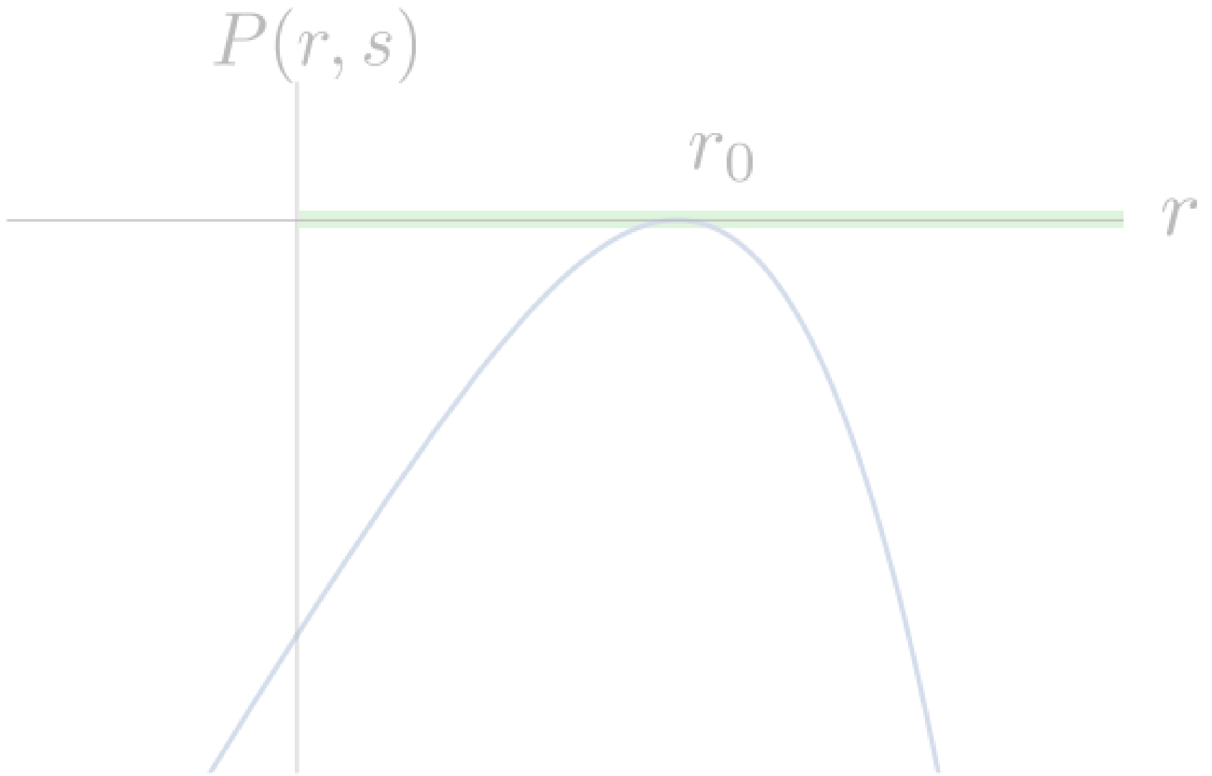




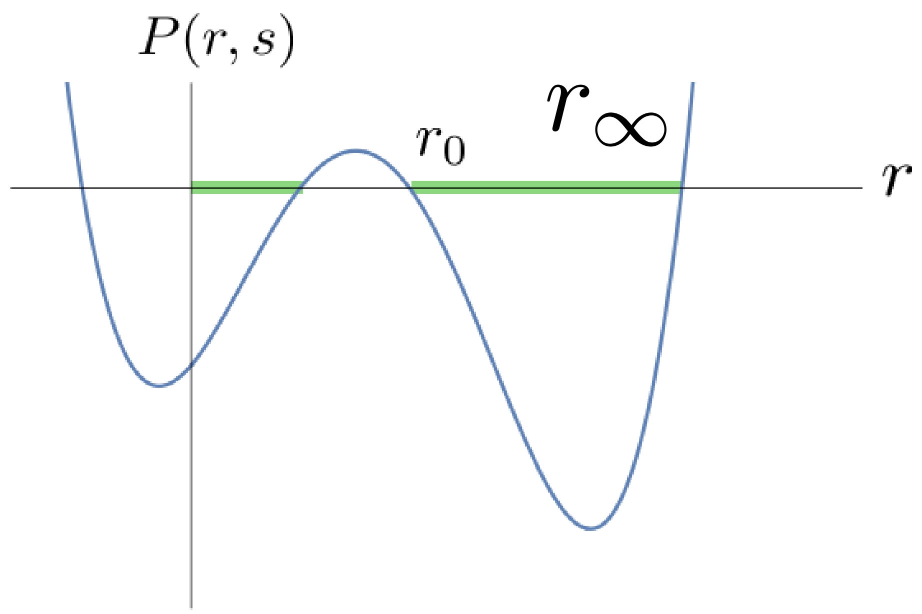
Charged BHs in Cosmological Backgrounds

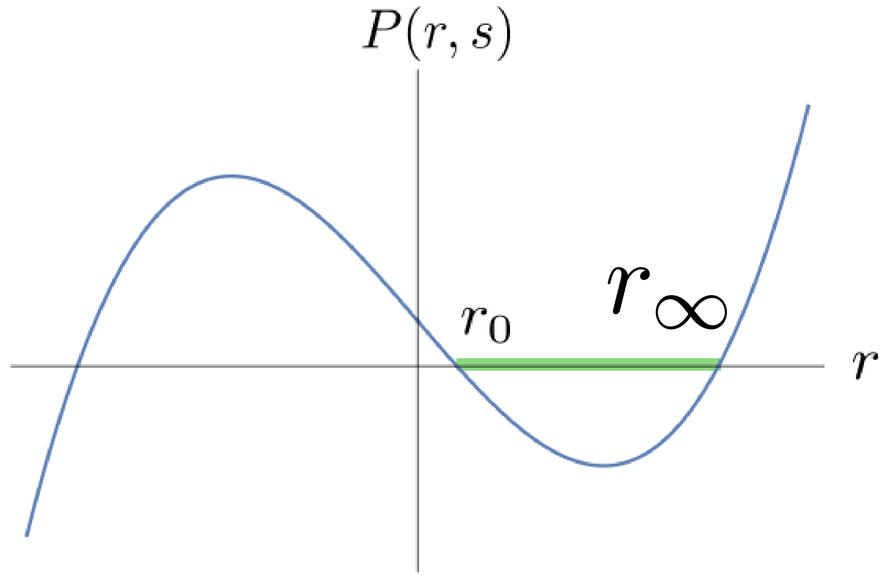
The infimum is a double root

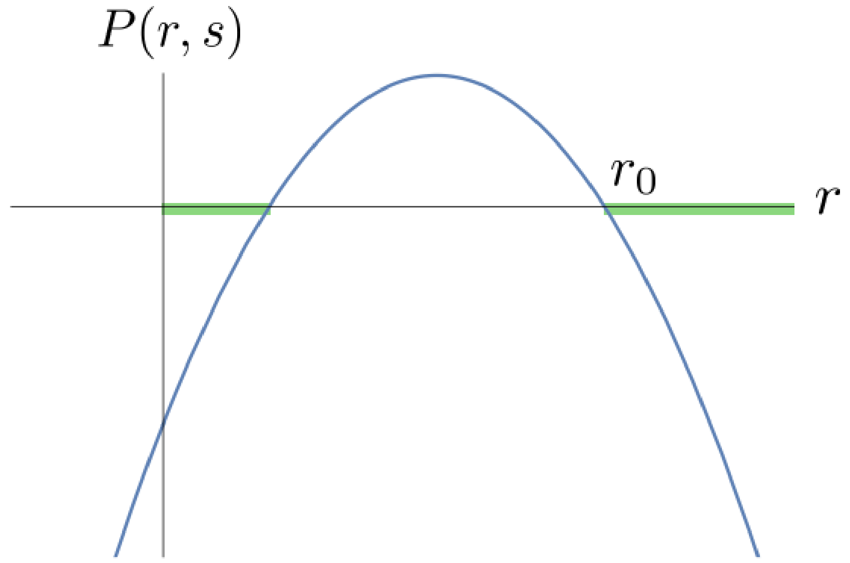


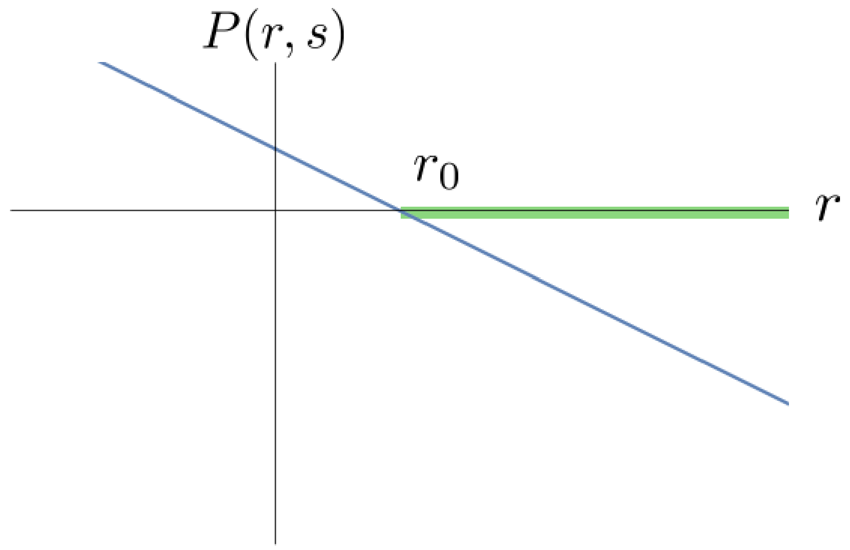


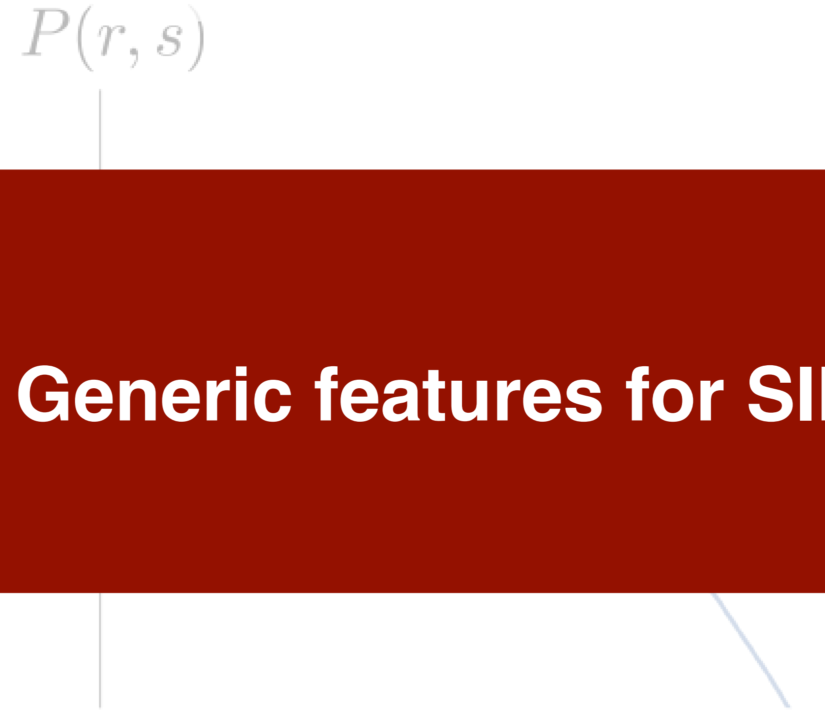
The infimum is a single root

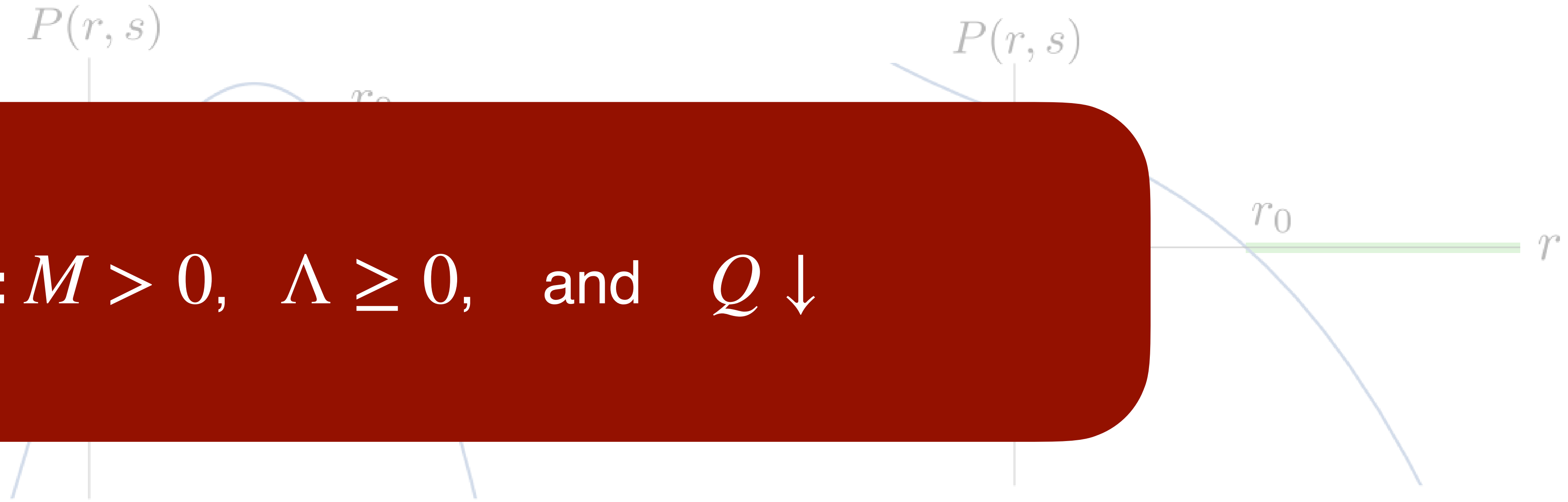












Generic features for SINGULARITY RESOLUTION: $M > 0, \quad \Lambda \geq 0, \quad$ **and** $\quad Q \downarrow$

Charged BHs in Cosmological Backgrounds

We study the cases that are free of curvature divergences

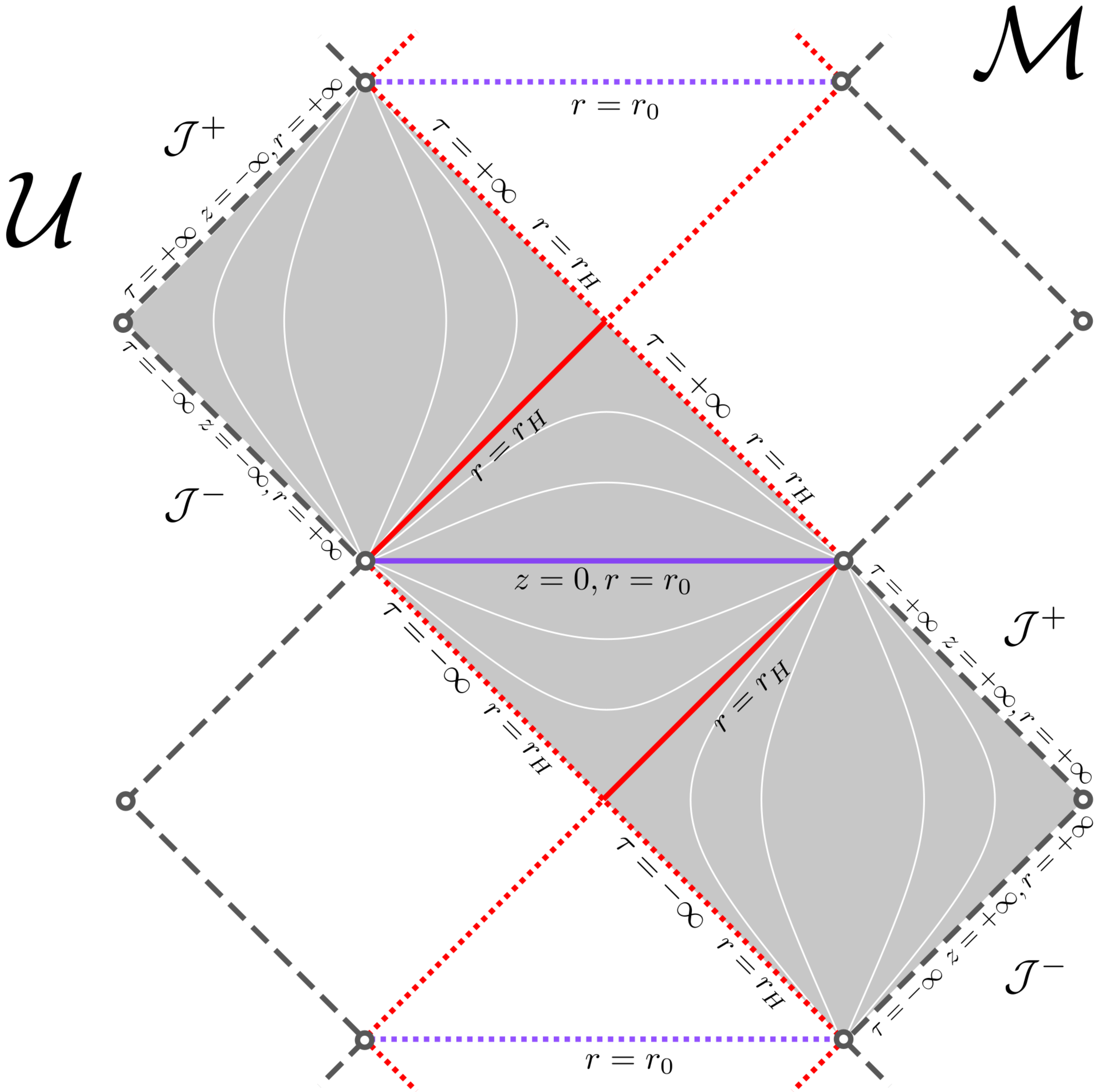
$$\lambda G(r) + 2V(r) = \lambda - 1$$

Norm of the Killing vector: $$G := -\tau^\mu \tau_\mu = 1 - \frac{2m(r)}{r}$$

The roots of $G(r)$ and $V(r)$ cannot coincide

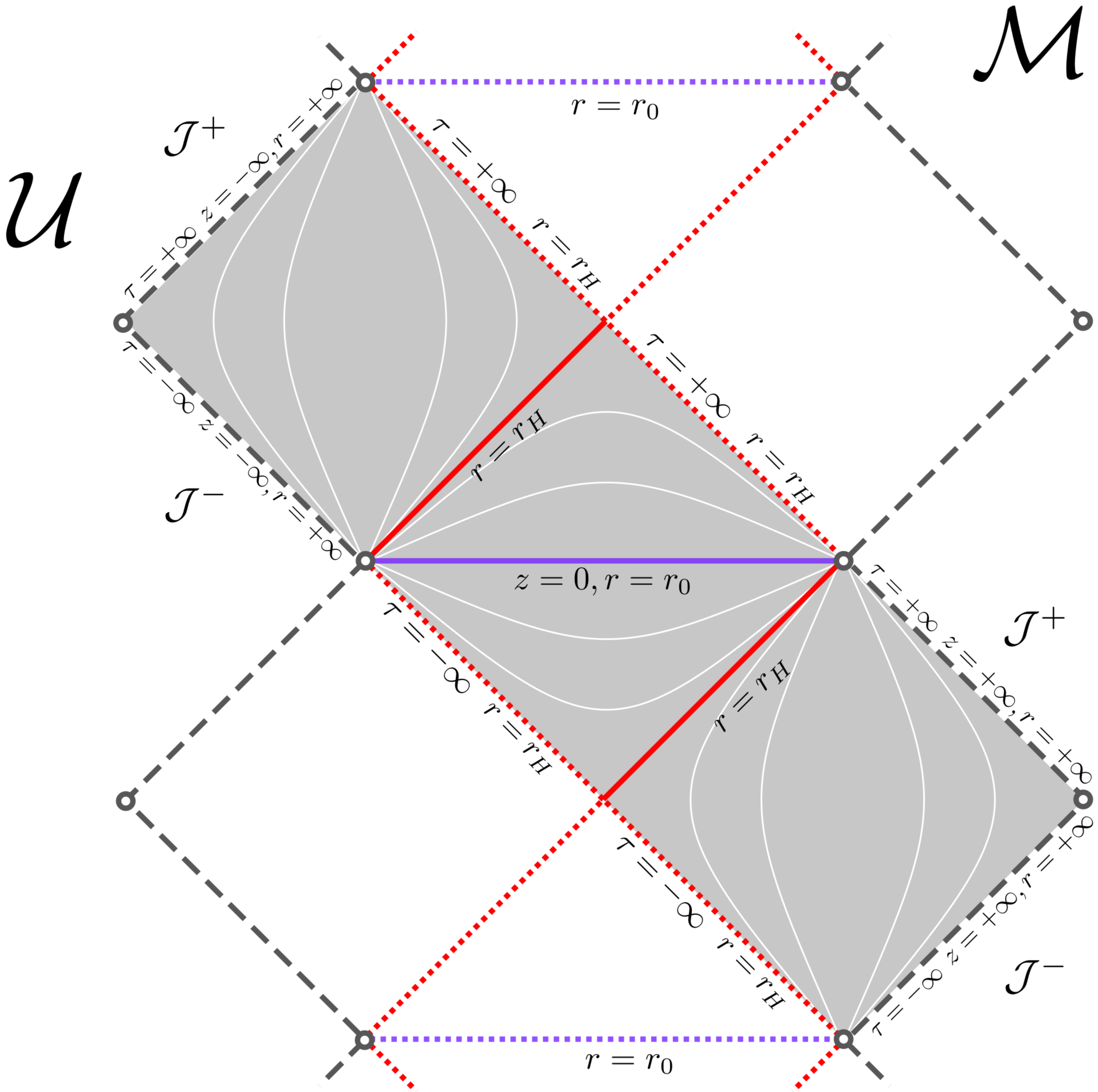
- All roots of $G(r)$ at $r = 2m(r)$ are null hypersurfaces (horizons)
- Simple roots of $V(r)$ at $r = r_0$ or $r = r_\infty$ are minimal spacelike hypersurfaces
- Double roots of $V(r)$ at $r = r_0$ are null (past or future) boundaries at infinity

Charged BHs in Cosmological Backgrounds



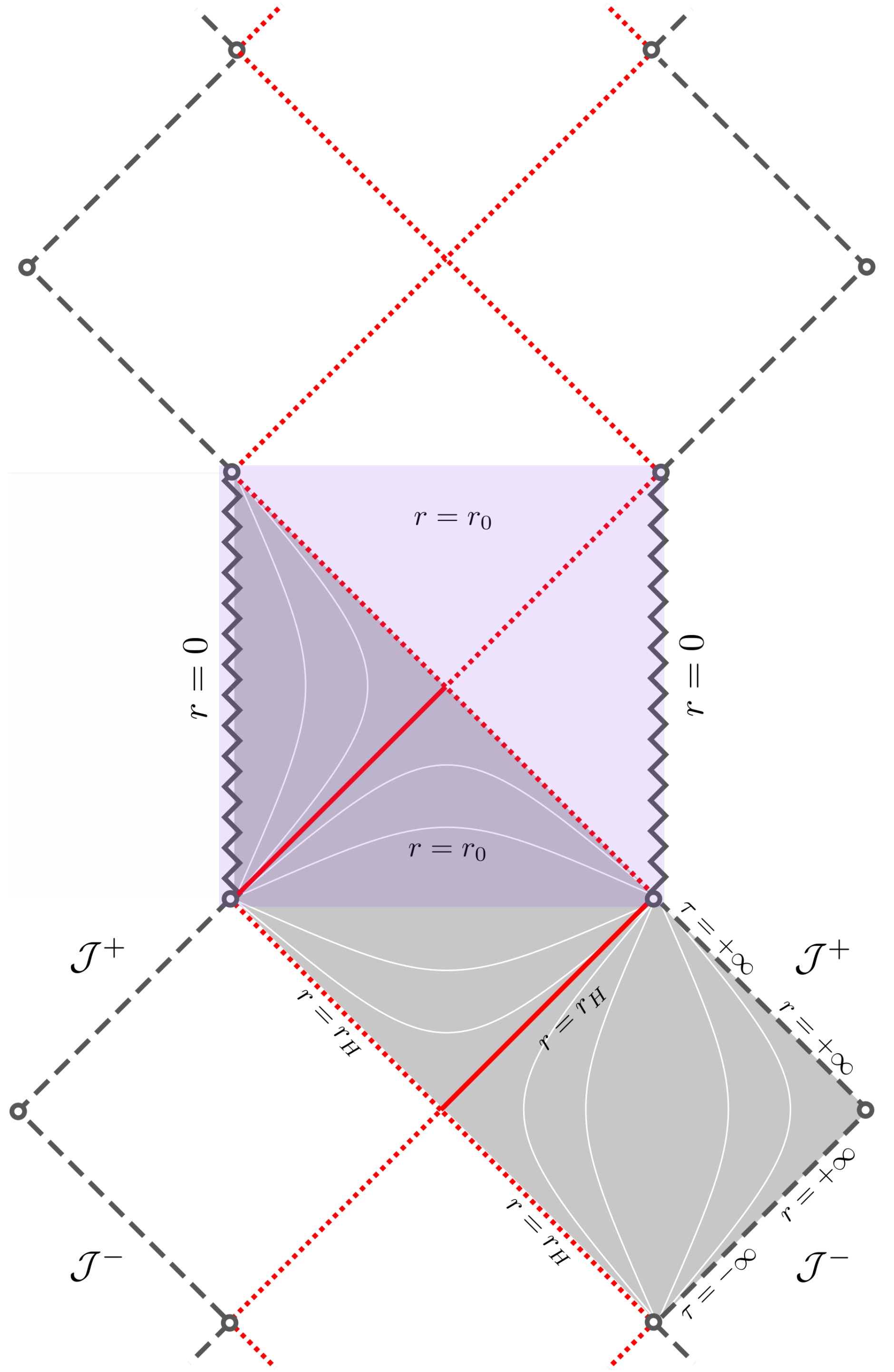
- Maximal extension $\mathcal{M}$
- Included in $\mathcal{U}$
- Domain $\mathcal{U}$ covered by (τ, z)
- Null infinities
- Timelike/Spacelike infinities
- Constant r hypersurfaces
- Critical hypersurfaces [roots of $V(r)$]
- Horizons [roots of $G(r)$]
- Asymptotic ends [$r \to \infty$]

Charged BHs in Cosmological Backgrounds

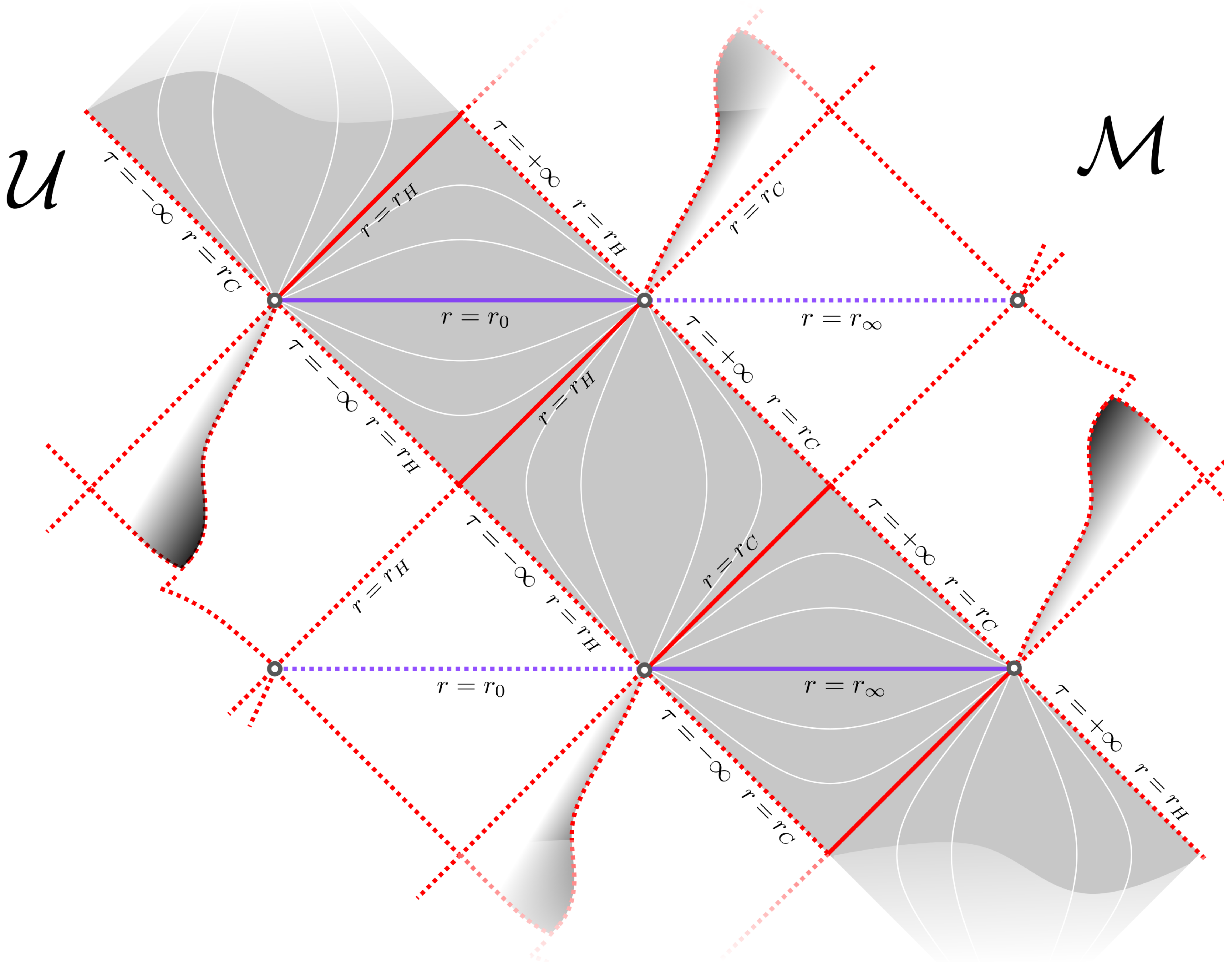


Reissner-Nordström and Schwarzschild

- It is the same diagram as in vacuum
- Small amounts of charge do not alter the causal structure of the spacetime
- The singularity (and all structure beyond the inner horizon in RN) is replaced by a minimal transition surface

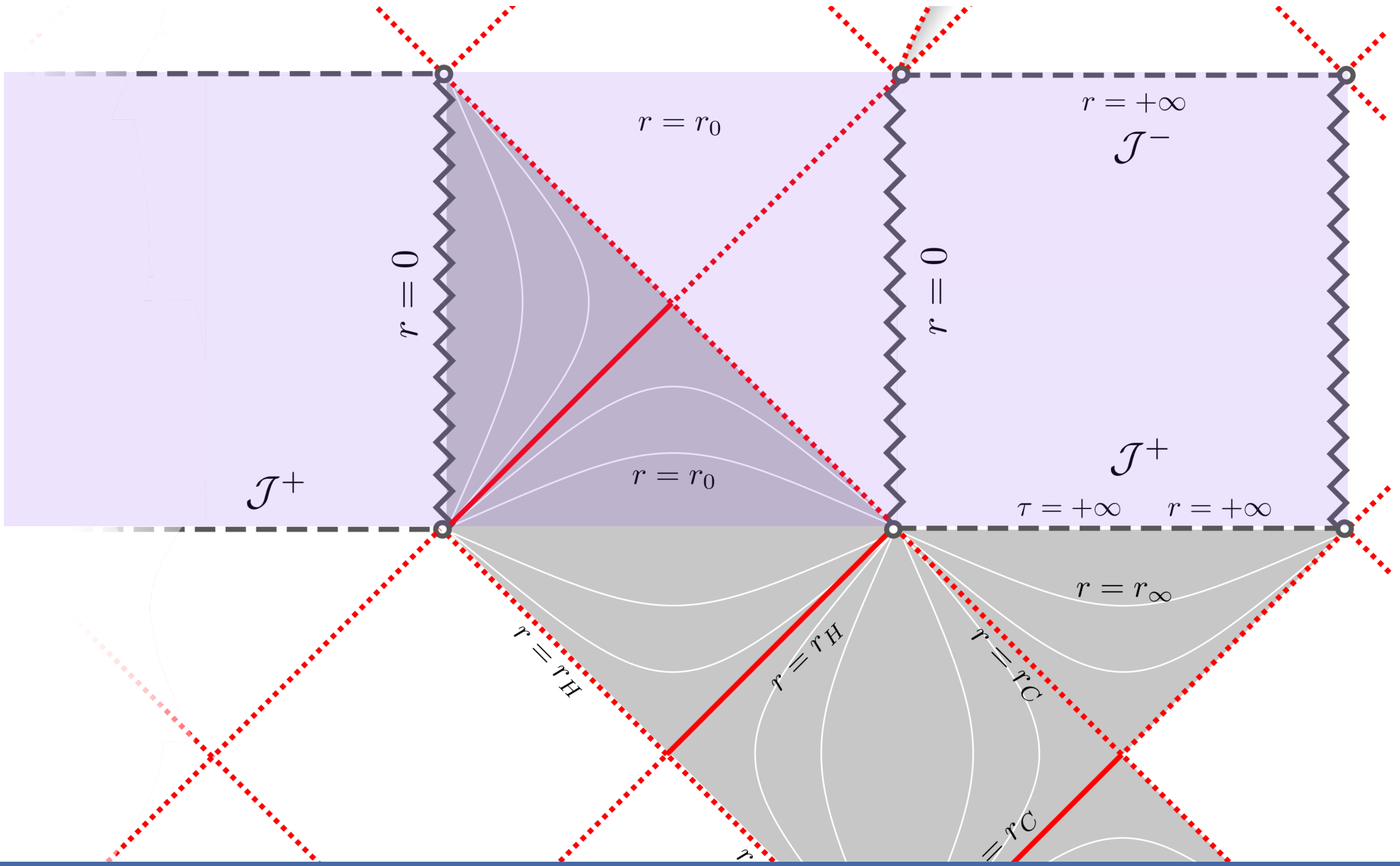


Charged BHs in Cosmological Backgrounds

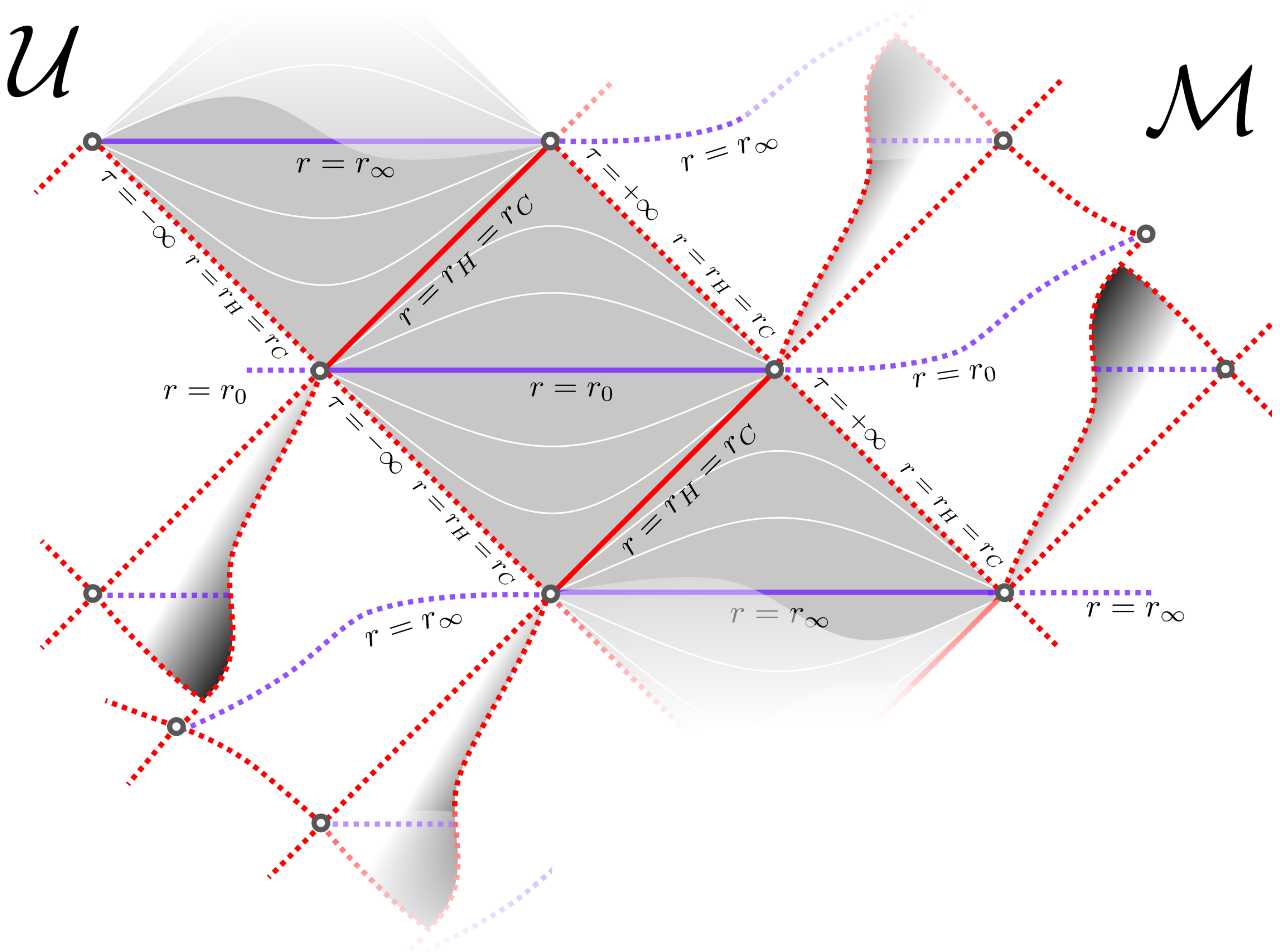


Reissner-Nordström-de Sitter and Schwarzschild-de Sitter

- Minimal spacelike hypersurfaces foliated by spheres of area $4\pi r_0^2$ replace the singularity
- Minimal spacelike hypersurfaces foliated by spheres of area $4\pi r_\infty^2$ replace asymptotic infinities
- Infinite copies of $\mathcal{U}$ along all directions, layering up around each ring in a helical (clockwise) manner



Charged BHs in Cosmological Backgrounds

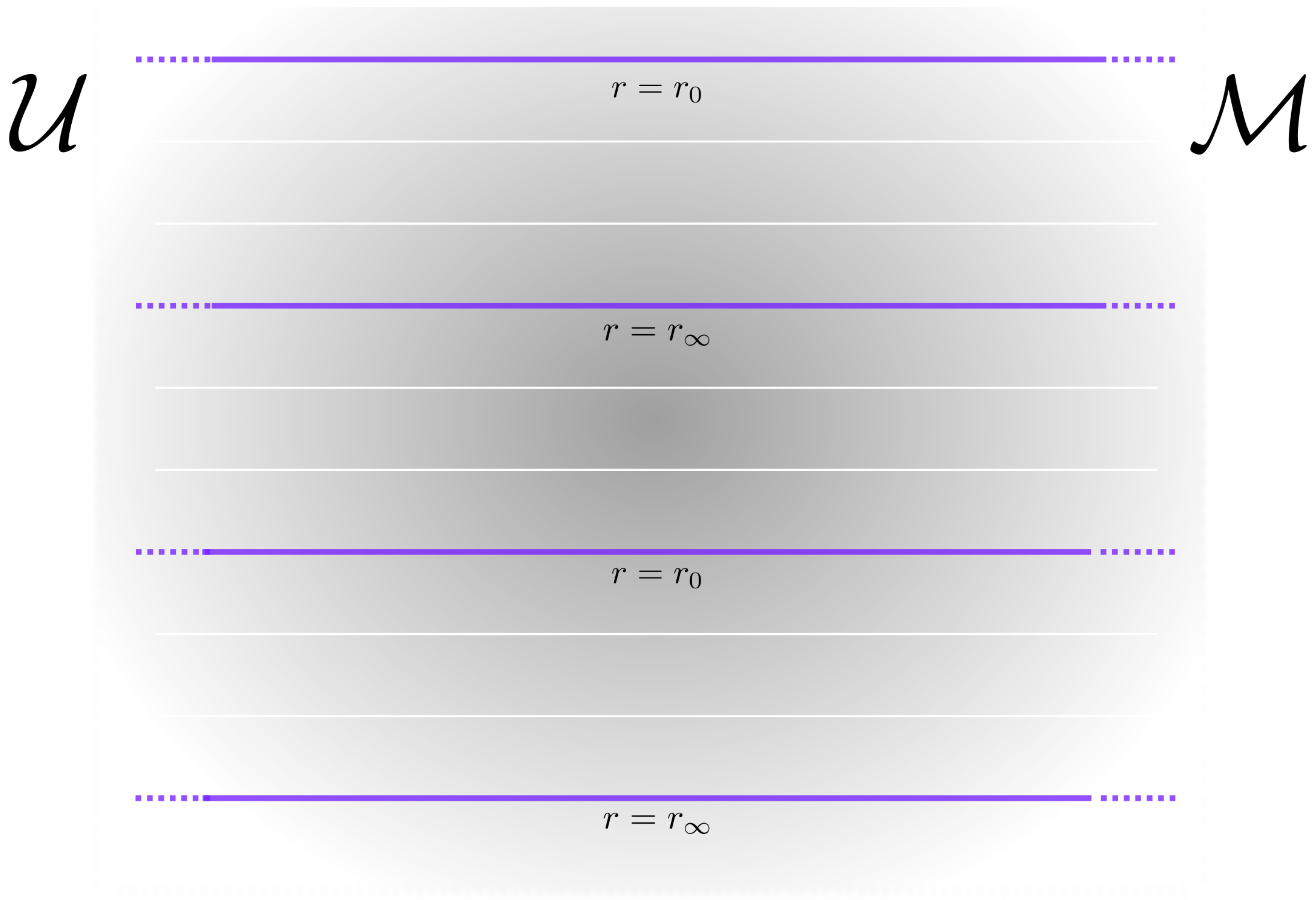


Reissner-Nordström-de Sitter

(one degenerate horizon)

- Same features as before BUT the two horizons degenerate into a single one
- There are no static regions in $\mathcal{M}$: This solution represents a periodic bouncing cosmology
- The existence of horizons allows accelerating observers to decouple from cosmic time and end at i^+

Charged BHs in Cosmological Backgrounds



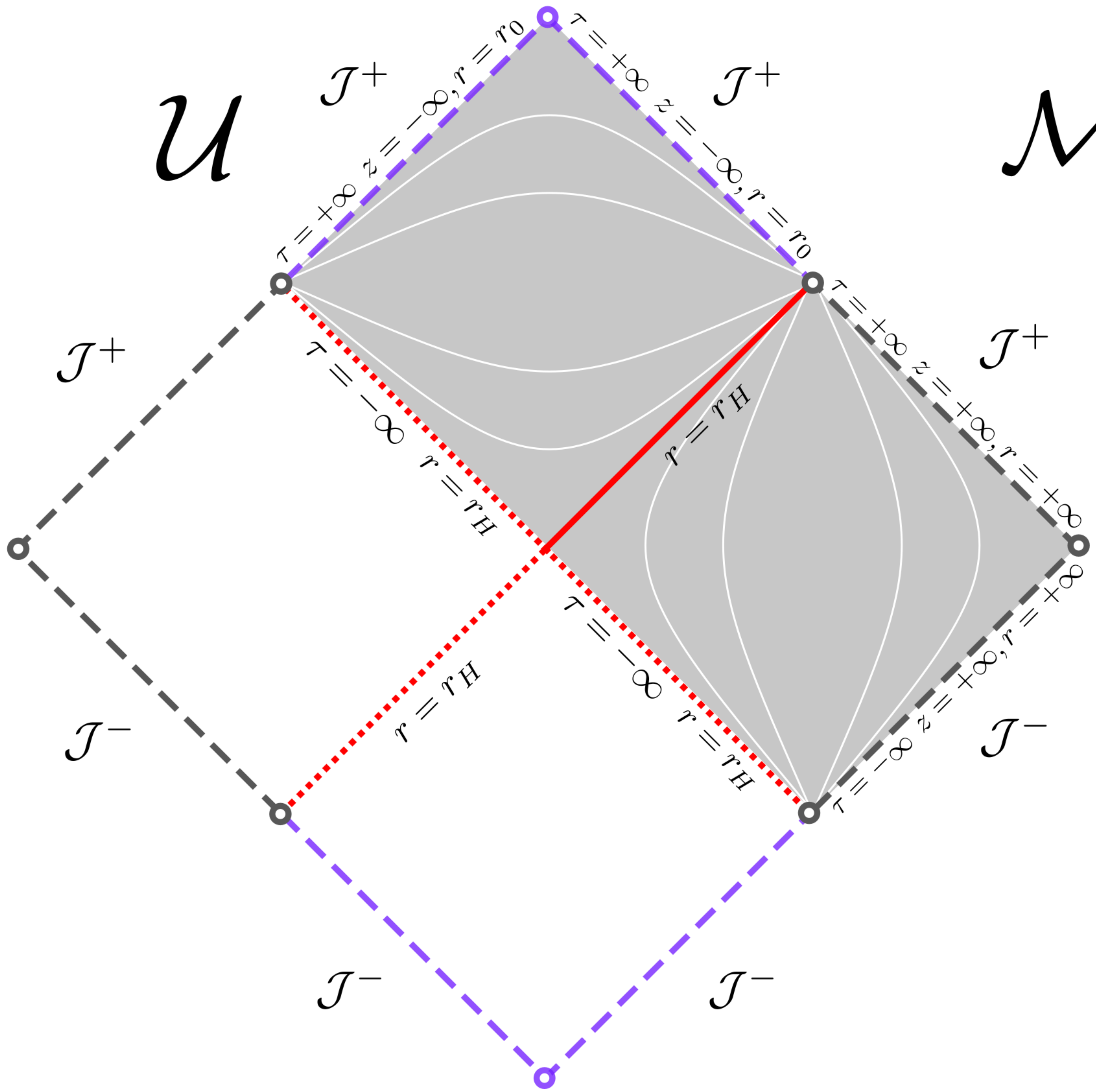
Reissner-Nordström-de Sitter

(no horizons)

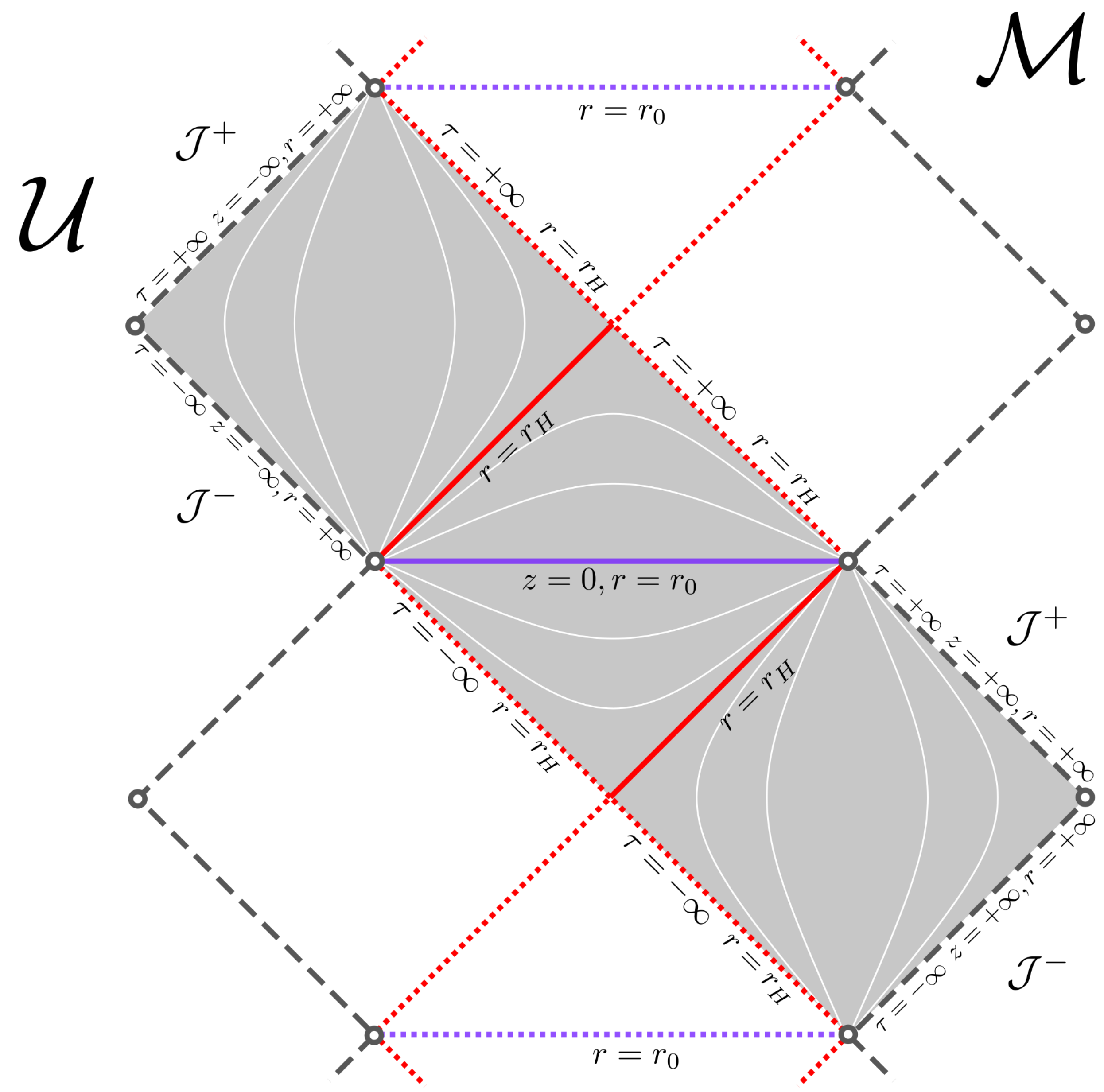
- All hypersurfaces of constant r are spacelike
- This is a cyclic cosmology, oscillating between hypersurfaces foliated by spheres of area $4\pi r_0^2$ and $4\pi r_\infty^2$
- The diagram is not compactified. Any observers cross infinitely many hypersurfaces of equal r

Charged BHs in Cosmological Backgrounds

"Extremal" cases

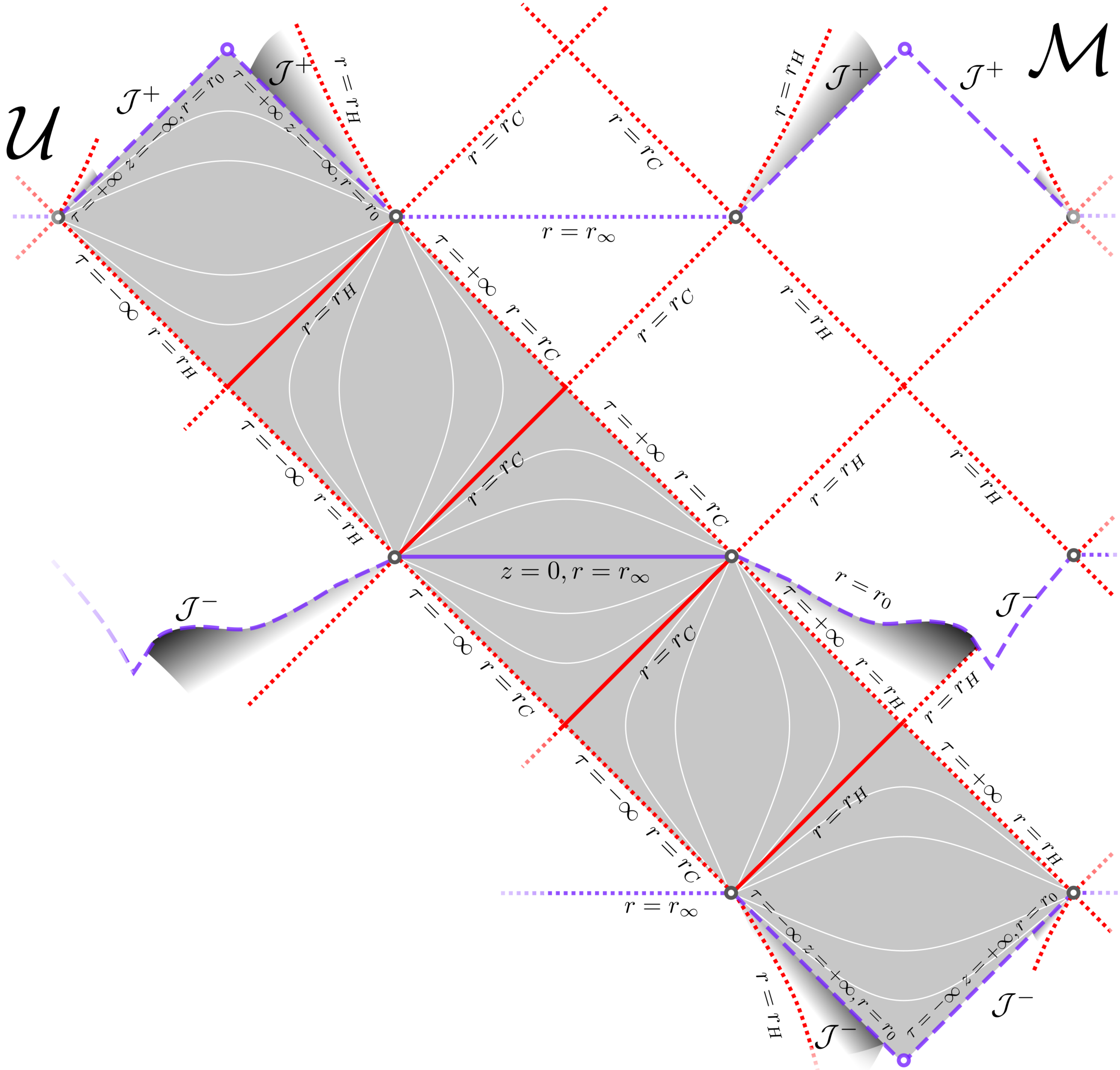


Reissner-Nordström (MAX. charge)



- The charge, however, has an upper limit that makes $r = r_0$ unreachable in finite proper time
- Although there is no singularity, observers crossing the horizon can never leave
- Radial travellers move forever towards either r_0 or ∞

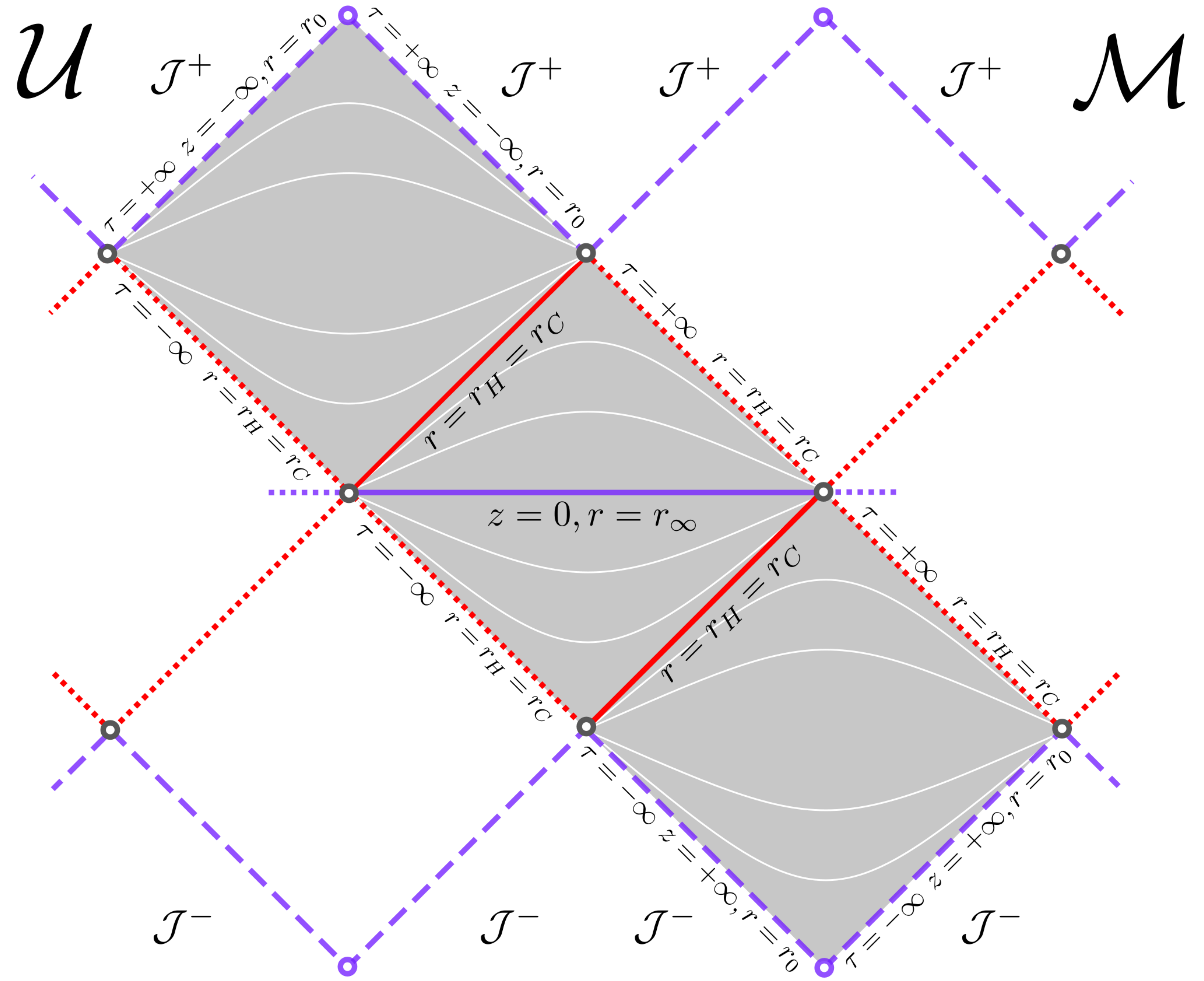
Charged BHs in Cosmological Backgrounds



Reissner-Nordström-de Sitter
(MAX. charge)

- The diagram for $\mathcal{U}$ is finite. Still, $\mathcal{M}$ unfolds at each ring
- Hypersurfaces $r = r_0$ become null infinities and cannot be crossed
- The maximum r_∞ of the area radius is a reflection-symmetry point

Charged BHs in Cosmological Backgrounds



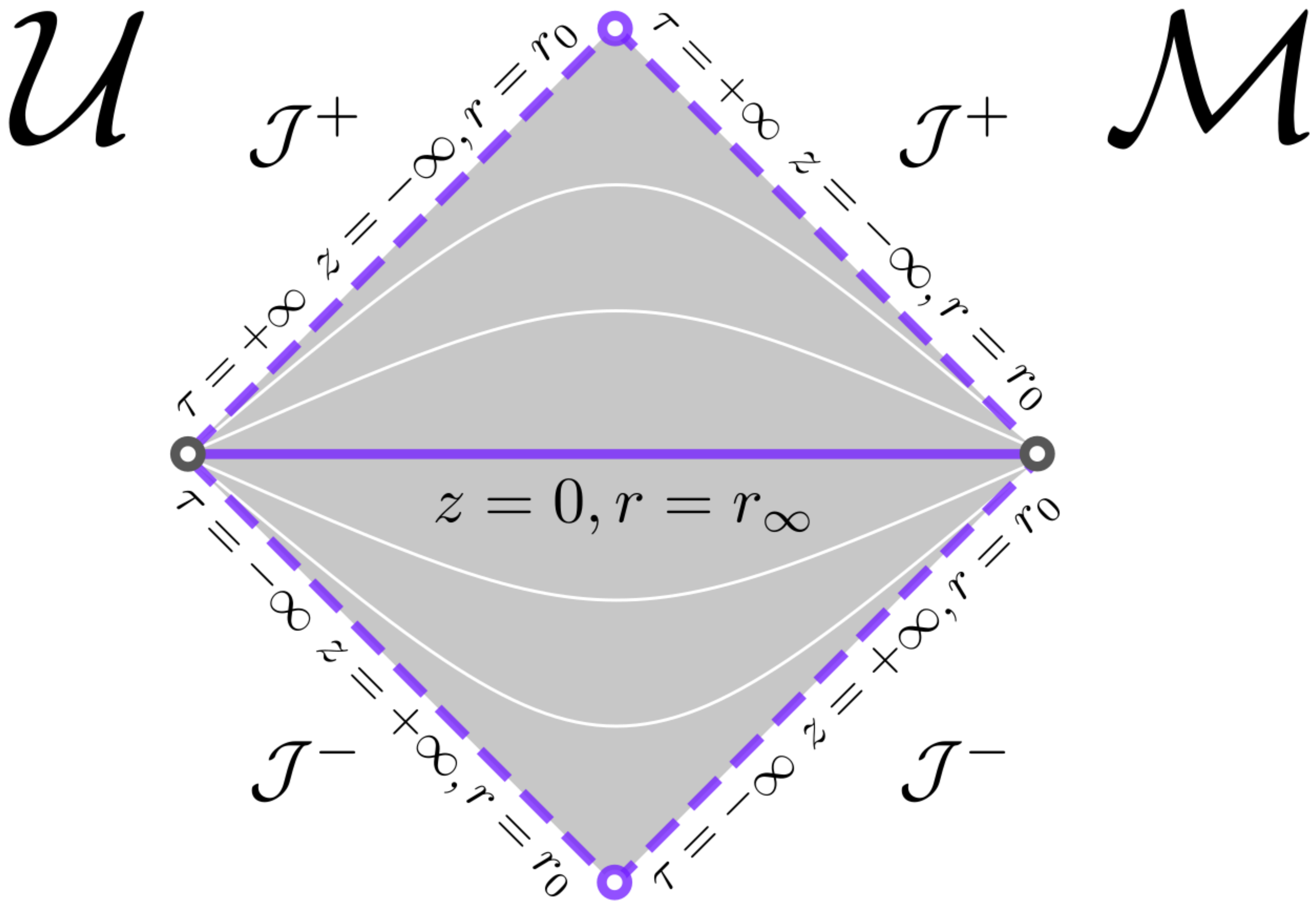
Reissner-Nordström-de Sitter

(MAX. charge — one degenerate horizon)

- This corresponds to the extremal case of the previous one, when both horizons coincide
- Homogeneous regions are bounded either by horizons or by null infinities foliated by spheres of area $4\pi r_0^2$
- Only accelerating observers may stay in regions where $r > r_H$

Charged BHs in Cosmological Backgrounds

"Extremal" cases



Reissner-Nordström-de Sitter

(MAX. charge — no horizons)

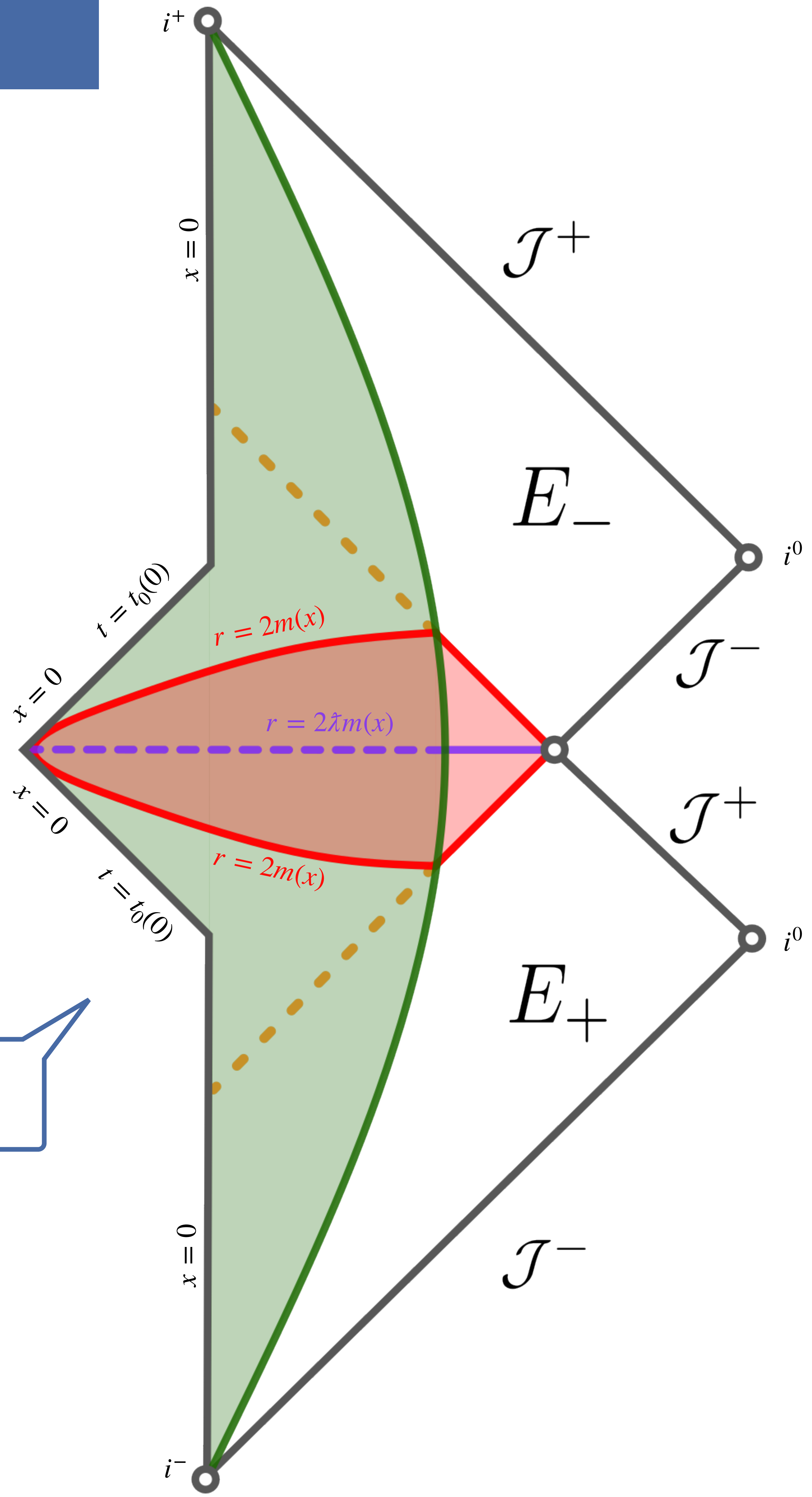
- The universe asymptotically expands from and contracts to a null hypersurface of spheres with minimum area
- All observers cross the unique $r = r_\infty$ hypersurface, which is a reflection-symmetry point
- This is a closed cosmology that solves both the Big Bang and the Big Crunch, with no need of any Big Bounces

Regular Gravitational Collapse

- Regular and analytical bounce on phase space
- MAIN PROBLEM: the geometry is singular!
- However, it is possible to overcome this situation:
  we are able to provide a completely regular spacetime
  for reasonable energy distributions ($m(0) = 0$, $m'(x) > 0$)
- The shell-crossing (soft) singularity is still present
- The surface $x = 0$ and $r = 0$ might be problematic…
  (we are working on it)
- Each layer of dust has a positive infimum (bigger as we
  move outwards in the star)

Work in progress

LTB model (outline)



Concluding Remarks

- **Effective theory with quantum corrections** λ
- **Vacuum solution:** always free of singularities
- **Charge and Cosmological constant:** $M > 0$, $\Lambda \geq 0$, **and** $Q \downarrow$
- **Minkowski is always a solution for any** λ
- **We recover general relativity in the limit** $\lambda \to 0$
- **Dynamical collapse (dust) free of curvature divergences**

The effective theory provides an entirely regular description for any spherical astrophysical black hole.

First positive results in the literature.
Covariance and matter

The effective corrections modify the whole spacetime, and also at large radii!

Aix*Marseille université Socialement engagée

CPT Centre de Physique Théorique

cnrs