Localized asymptotic energy via Wald-Zoupas

Antoine Rignon-Bret

Aix✳Marseille université

Rignon-Bret, Speziale, *Localized energy of asymptotic gravitational waves*, to appear soon

December 12, 2023

Motivations

Asymptotic symmetries in General Relativity Bondi,van der Burg,Metzner, Sachs

BMS symmetries, preserve the normal and the unphysical metric up to a conformal factor. We obtain the Lorentz transformations (rotations + boosts) and all the supertranslations $\Rightarrow$ each observer has her own clock.

The Noether charges are observable Ashtekar, Dray, Geroch, Streubel, Wald, Zoupas

The canonical charges associated to the BMS symmetries are known from the 80'. These are Noether charges. What do they measure ?
Energy loss formula, memory effect $\Rightarrow$ hard terms, soft terms.
Are there more observables that we can measure ?

Divergent S-matrix Strominger, Pasterski, Donnay, Ruzziconi

Asymptotic symmetries $\Leftrightarrow$ soft theorems $\Leftrightarrow$ memory effects. In the S-matrix, the aplitude associated to the soft terms diverge but not the ones associated to the hard terms.

Action of supertranslations

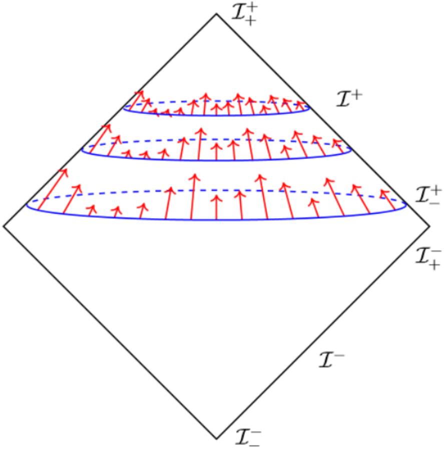


Figure 1: Action of an arbitrary supertranslation on future null infinity. The supertranslations can tend to a Dirac function centered on a point of the celestial sphere.

Energy of the gravitational waves

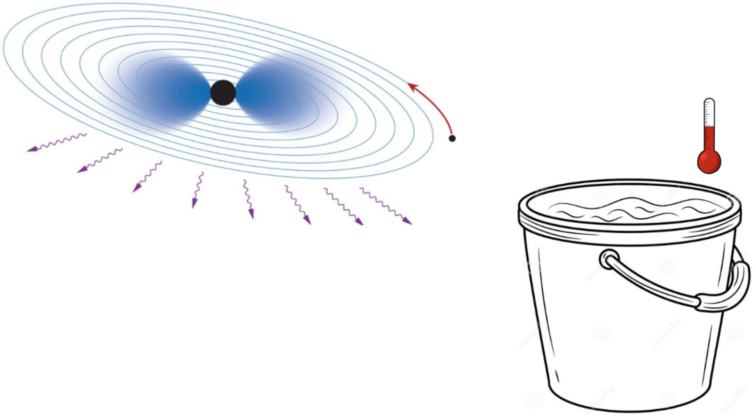


Figure 2: After the passage of a gravitational wave, the water in the bucket should boil. The temperature increases by which amount ? Energy of the gravitational wave is transferred to the water. What is the Noether flux ? What is the Noether charge ?

Symmetry and Noether charge

Variational principle

Variational principle $\delta L = E(g)\delta g + d\theta(g, \delta g, \chi)$.
The Lagrangian and the symplectic potential depend on the dynamical field and background fields $L(g, \chi)$ and $\theta(g, \delta g, \chi)$.

Noether current

We have $j_\xi = I_\xi \theta - i_\xi L = C_\xi + dq_\xi \hat{=} dq_\xi$

Symplectic form

Presymplectic form density $\omega = \delta\theta$. Presymplectic form $\Omega = \int_\Sigma \omega$

Ambiguities in the symplectic potential

⚠ L and θ and ambiguous up to exact terms in field space and spacetime
Indeed, $L \to L + dl$; $\theta \to \theta + \delta l - d\vartheta$;

What are good asymptotic charges ? Wald,Zoupas,99'

We want Wald,Zoupas,99'; Grant,Prabhu,Shezad,21'; Odak, ARB, Speziale,22'

- A symplectic potential $\bar{\theta} = \theta_*^{EH} + \delta l - d\vartheta$
- Covariance, i.e $(\delta_\xi - \mathcal{L}_\xi)\bar{\theta} = 0$
- Stationarity requirement $\Theta(g, \delta g) = 0$ in non-radiative spacetime

Charges Harlow,Wu,20'and Freidel,Geiller,Pranzetti,20'

- à la Noether $\bar{q}_\xi = \kappa_*^{EH} + i_\xi l - I_\xi \vartheta$
- à la Wald-Zoupas $-I_\xi \bar{\omega} + di_\xi \bar{\theta} = d\delta\bar{q}_\xi$ + one reference solution

Covariance $\Rightarrow d\bar{q}_\xi = I_\xi \bar{\theta}$

Ashtekar and Streubel flux and Dray-Streubel charges

We have $\bar{\theta}^{AS} = -\frac{1}{2}\hat{N}_{AB}\delta C^{AB}\varepsilon_l$, covariant and stationary
And the charges $\bar{Q}_\xi^{DS} = \int_S (4\tau\hat{M} + 2Y^A P_A)\varepsilon_S$

Introduction of the edge mode Donnelly,Freidel,16',Rovelli,13'

Supertranslation field Compère,Fiorucci,Ruzziconi,18'

This supertranslation field $u_0(x^A)$ plays the role of a reference frame for an asymptotic observer at $\mathscr{I}$. We start from the requirement $(\delta_\xi - \mathscr{L}_\xi)(u - u_0) = -\dot{\tau}(u - u_0)$ implying
$\delta_{\tau,Y} u_0 = Y^A \partial_A u_0 - T - \dot{\tau} u_0$

Covariant shear Donnay,Cheng,Ruzziconi,21'

The shear transforms inhomogenously. Consider
$\hat{C}_{AB} = C_{AB} - (u - u_0)\rho_{<AB>} - 2D_{<A}\partial_{B>} u_0$.
We can check that $(\delta_\xi - \mathscr{L}_\xi)\hat{C}_{AB} = -\dot{\tau}\hat{C}_{AB}$

Boundary condition

We will set the boundary condition that in the far future $\lim_{u \to +\infty} \hat{C}_{AB} = 0$

Define u_0 as $(C_{AB} - u\rho_{<AB>}) \underset{u \to +\infty}{=} 2D_{<A}\partial_{B>} u_0 - u_0 \rho_{<AB>}$

New symplectic potential and new charges

Corner ambiguity

We use the corner ambiguity to add the corner term
$\vartheta^{new} = -\frac{1}{2}\hat{C}_{AB}\delta(2D^A\partial^B u_0 + u_0\rho^{AB})\varepsilon_S$

New symplectic potential

$\bar{\theta}^{new} = \bar{\theta}_*^{AS} + d\vartheta^{new} = -\frac{1}{2}\hat{N}_{AB}\delta\hat{C}^{AB}$ It is covariant, conformally invariant and stationary.

New charges

$Q_S^{new} = \int_S 4\tau[\hat{M} - \frac{1}{8}\tau(u_0)(2D^A D^B + \rho^{AB})\hat{C}_{AB}]_S$
$+2Y^A\left[P_A + \frac{1}{4}\partial_A u_0(2D^B D^C + \rho^{BC})\hat{C}_{BC}\right]\varepsilon_S$ so $\rightarrow$ DS + f($\hat{C}_{AB}$)

Flux of supertranslation

Local energy flux $\Delta Q_T^{new} = -\frac{1}{2}T\hat{N}_{AB}\hat{N}^{AB}\varepsilon_{\mathscr{I}} \leq 0$

Symplectic form and bracket

No local symplectic form density

The corner term ϑ^{new} is defined at $\mathscr{I}$ only. The symplectic form is obtained by integrating on a Cauchy surface intersection $\mathscr{I}$
$\Omega_\Sigma = \int_\Sigma \omega^{EH} + d\vartheta^{new}$

Barnich-Troessart bracket

$\{Q_\xi, Q_\chi\}^{BT}|_S = \delta_\chi Q_\xi - i_\xi I_\chi \bar{\theta}|_S$ generates symmetry if $\bar{\theta} = 0$, i.e $\hat{N}_{AB} = 0$

No central extension Barnich,Troessart,11'

- $(\delta_\xi - \mathscr{L}_\xi)\bar{\theta} = 0 \Rightarrow \{Q_\xi, Q_\chi\}^{BT}|_{S_2} - \{Q_\xi, Q_\chi\}^{BT}|_{S_1} = Q_{[\xi,\chi]]}|_{S_2} - Q_{[\xi,\chi]]}|_{S_1} \Rightarrow$ central extension independent on S.
- As $\lim_{u\to+\infty} \hat{C}_{AB} = 0$, we have that $\{Q_\xi^{new}, Q_\chi^{new}\}^{BT}|_{S_{+\infty}} = \{Q_\xi^{BT}, Q_\chi^{new}\}^{BT}|_{S_{+\infty}} = \delta_\chi Q_\xi^{new/BT}|_{S_{+\infty}}$
- Therefore, on any cross section S : $\{Q_\xi^{new}, Q_\chi^{new}\}|_S = Q_{[\xi,\chi]]}^{new}|_S$

Why new charge is a good notion of local energy

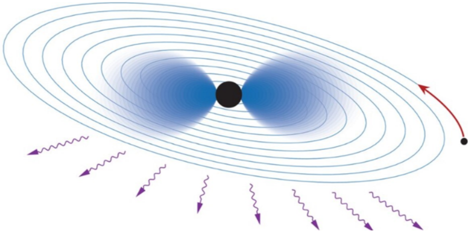
Properties of the new charge

- Is conserved in absence of radiation, and so is hamiltonian
- Vanishes in Minkowski spacetime
- Is the Noether charge associated to local time translations (positive supertranslations)
- Is positive and decreases if energy is carried away by gravitational waves. In particular, it decreases to an amount proportional to the temperature rise $\Delta T = \frac{\alpha_w}{32\pi G \rho_w c_w} \int_{u_0}^{u_1} \hat{N}_{AB}(\theta, \phi) \hat{N}^{AB}(\theta, \phi) du'$
- Is extensive, i.e the total energy is the sum of the local energies in the causally disconnected asymptotic regions
- In absence of radiataion, the charge generates the local time translations via the usual bracket

What about angular momentum ?

Does the water of in the bucket turn after the gravitational wave ?

What about angular momentum ?



THANK YOU !