Generalized holonomy corrections in effective LQC : Consistency and phenomenological consequences

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Gauge theories and constraint systems

Gauge theories = Theories in which the dynamical variables are specified w.r.t a reference frame whose choice is arbitrary at every instant of time

- Every gauge theory is a constraint system in its canonical form !

Singular Lagrangian

$$\det\left(\frac{\partial^2 L}{\partial\dot{q}^m\partial\dot{q}^n}\right) = 0 \quad \Rightarrow \quad \phi_m(q,p) = 0$$

Constraints

Dirac's conjecture : Constraints in a canonical formalism of a gauge theory might generate transformations which map a state to its equivalent state, gauge transformation

Generally covariant systems and GR

- GR as a canonical theory : (Einstein-Hilbert action)

$$S_{EH} = \int dt \int d\mathbf{x} \Big[(\kappa\gamma)^{-1} E^a_i \dot{A}^i_a - \underbrace{\left[\Lambda^i \mathcal{G}_i + N^a \mathcal{D}_a + N\mathcal{H} \right]}_{\text{Constraints}} \Big]$$

- **Gauss constraints** : internal rotations

$$\mathbb{G}[\Lambda^i] = (\kappa\gamma)^{-1} \int d\mathbf{x}\, \Lambda^i \mathcal{G}_i = (\kappa\gamma)^{-1} \int d\mathbf{x}\, \Lambda^i \left[\partial_a E^a_i + \epsilon^{\ l}_{ik} A^k_a E^a_l \right]$$

- **Diffeomorphism constraints** : spatial coordinate transformations

$$\mathbb{D}_{\mathfrak{g}}[N^a] = (\kappa\gamma)^{-1} \int d\mathbf{x}\, N^a \mathcal{D}_a = (\kappa\gamma)^{-1} \int d\mathbf{x} \left[\left(\partial_a A^j_b - \partial_b A^j_a \right) E^b_j + A^j_a \partial_b E^b_j \right]$$

- **Hamiltonian (scalar) constraint** : "time evolution"

$$\mathbb{H}_{\mathfrak{g}}[N] = (2\kappa)^{-1} \int d\mathbf{x}\, N\mathcal{H} = (2\kappa)^{-1} \int d\mathbf{x}\, N \frac{E^c_j E^d_k}{\sqrt{|\det E|}} \left[\epsilon^{jk}_i F^i_{cd} - 2\left(1+\gamma^2\right) K^j_{[c} K^k_{d]} \right]$$

Algebra of constraints

$$\{\mathcal{C}_i, \mathcal{C}_j\} = f^k_{ij}(E, A)\mathcal{C}_k$$

- Constraints on suitable initial values
- Generate space-time transformations on phase space functions
- Encodes the space-time structure

Effective LQC

- LQG describes space-time as,

$$(g, \pi) \longrightarrow (E, A) \longrightarrow (h_e, F_s)$$

  - **Inverse-volume (-triads) corrections** : cut-off functions of divergences of factors containing inverse components of the densitized triad

$$\frac{1}{\sqrt{|\det E|}} \longrightarrow \frac{\alpha(E)}{\sqrt{|\det E|}}$$

  - **Holonomy corrections** : higher powers of spatial curvature components

$$\mathfrak{c} \longrightarrow \frac{\sin \delta \mathfrak{c}}{\delta}$$

"Usual" holonomy correction

$$\mathfrak{c} \longrightarrow g(\mathfrak{c}, \mathfrak{p})$$

Generalized holonomy correction

- Quantum corrections (either HC or IV) introduce anomalies at the gauge algebra level, i.e.

$$\{\mathcal{C}_i, \mathcal{C}_j\} = f_{ij}^k(E, A)\mathcal{C}_k + \mathcal{A}_{ij}$$

Toward a consistent theory

- One has to get rid of those anomalies, mostly two explored ways with GHC

  - **Counter-terms method** : add counter-terms at the action level to ensure the closure of the algebra of constraints

$$\mathbb{H}_{\mathfrak{g}}^{\mathrm{HC}} + \mathrm{IV} \Rightarrow \mathcal{A}_{ij} \neq 0 \longrightarrow \mathbb{H}_{\mathfrak{g}}^{\mathrm{HC}} + \mathrm{IV} + \mathbb{H}_{\mathfrak{g}}^{\mathrm{CT}} \Rightarrow \mathcal{A}_{ij} = 0$$

[Bojowald, Hossain, Kagan, Cailleteau, Barrau, Mielczarek...]

  - **Restriction on the GHC** : restriction of the form of the GHC to ensure the closure of the algebra of constraints

$$\mathfrak{G} = \left\{ g(\mathfrak{c}, \mathfrak{p}) \mid g(\mathfrak{c}, \mathfrak{p}) \longrightarrow \mathfrak{c} \text{ at the classical limit} \right\}$$

    - Is there a subset of all the GHC $\mathfrak{G}_{\mathcal{A}} \subset \mathfrak{G} \Rightarrow \mathcal{A}_{ij} = 0$?

[Li, Wu 2023 & WiP]

Cosmological background dynamics and GHC

[Renevey, Martineau, Barrau 2023]

- Considering only the GHC : the closure of the algebra (via some counter-terms) is possible if,

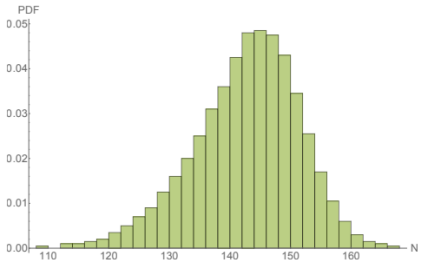
$$g(\mathfrak{c},\mathfrak{p}) - \mathfrak{c}\,\partial_{\mathfrak{c}} g(\mathfrak{c},\mathfrak{p}) - 2\mathfrak{p}\,\partial_{\mathfrak{p}} g(\mathfrak{c},\mathfrak{p}) = 0 \quad \Rightarrow g(\mathfrak{c},\mathfrak{p}) = \frac{f(\sqrt{\mathfrak{p}}\mathfrak{c})}{\sqrt{\mathfrak{p}}}$$

- **Background study** : focus on the duration of inflation

$\Rightarrow$ Geometrical explanation of inflation seems possible modulo a fine-tuning ! **What about the dilution of energy densities though ?**

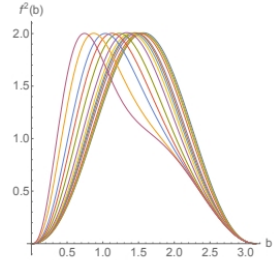
$\Rightarrow$ Effects on the duration of inflation due to the shape of the GHC are quite weak. **Still, one stays close to the standard case.**

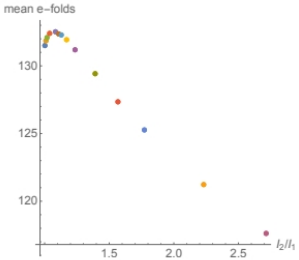
"Usual" LQC conclusions are robust in respect to GHC !



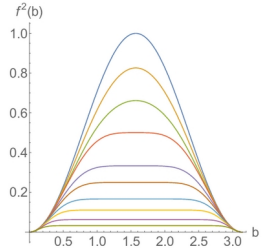
PDF of the efolds of Inflation

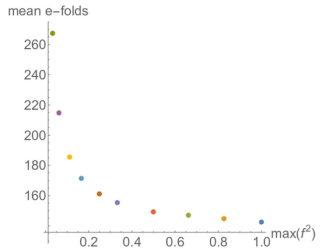
Asymmetry of the GHC





Effect of the maximum of the GHC



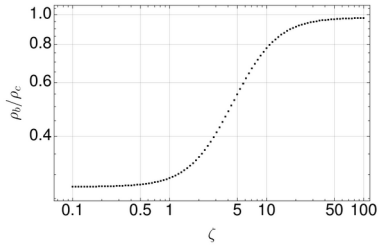


Cosmological perturbations theory and GHC (Deformed algebra approach)

- The closure of the algebra (via some counter-terms) allows the study of the gauge-invariant cosmological perturbations

[MDS, Martineau, Renevey, Barrau 2023]

$$f_\zeta(x) = \sqrt{\frac{\zeta^2+\pi^2}{\zeta^2+4(x-\pi/2)^2}}\sin x$$

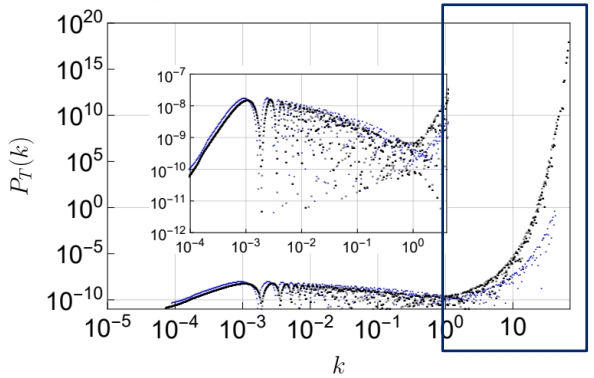


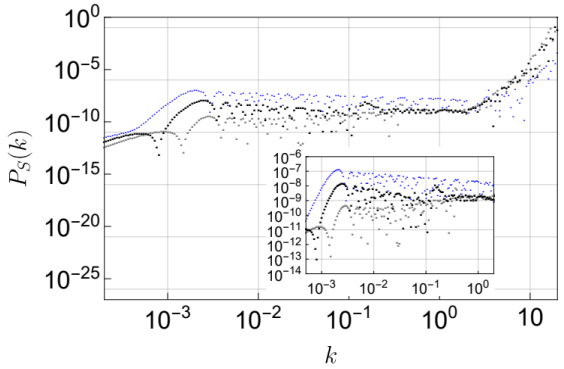
$$v''_{s/t} - \underbrace{\mathcal{G}^{(2)}(\mathfrak{c},\mathfrak{p})}_{=\frac{1}{2}\partial^2_{\mathfrak{c}} g^2(\mathfrak{c},\mathfrak{p})}\nabla^2 v_{s/t} + \underbrace{\frac{z''_{s/t}}{z_{s/t}}}_{\text{Background-related quantity}} v_{s/t} = 0$$

$$H^2 = \frac{\kappa}{3}\rho\left[f'\left(f^{-1}\left(\sqrt{\frac{\rho}{\rho_c}}\right)\right)\right]^2$$

Signature change

$$\left\{\mathbb{H}_{\rm tot}, \mathbb{H}_{\rm tot}\right\} = \mathcal{G}^{(2)}(\mathfrak{c},\mathfrak{p})\mathbb{D}_{\rm tot}$$





⟹ Weak effects on the cosmological observables due to the GHC !

⟹ The main drawback of the approach (the exp. Divergence) is maintained w/ GHC. **This is because of the signature change which is still there w/ GHC.**

"Usual" LQC conclusions are robust in respect to GHC !

Toward the most generic case : IV + GHC, closure of the algebra and restrictions ?

- Can one escapes the change of signature due to (G)HC without exotic assumptions ?

[MDS, Martineau, Barrau - Work in progress]

  - The answer seems to be yes : at least for the usual HC, by considering the full model IV + HC. **Still true with GHC ?**

Very tedious computation w/ ~60 anomalies to derive : **generic code to derive the full algebra being written currently and will be made public in a near future.**

  - A lot of questions remains and will (hopefully) get an answer in the near future :

⇒ Will we impose more restrictions on the correction in this most general case ?

⇒ How to link those restrictions to previous work on ambiguities ? (such as generic representation for example)

⇒ What is the fate of the signature change ?

⇒ Phenomenological consequences ? Link to CMB data ?

Thank you !