salvatore ribisi @ Grenoble, LQG meeting
CPT - Marseille 13/12/2023

# LIGHT-CONE THERMODYNAMICS

based on arXiv:2307.12031 Alejandro Perez, SR
and De Lorenzo, Perez 2018

# LIGHT-CONE THERMODYNAMICS

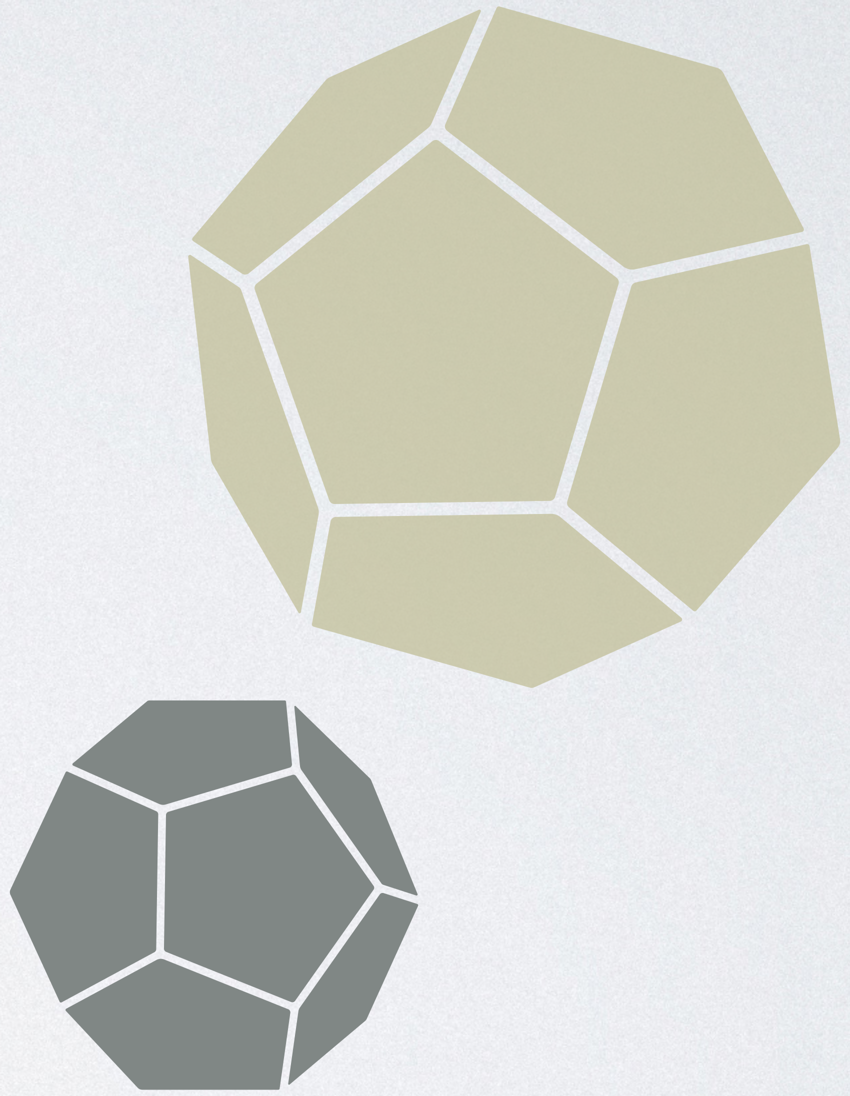
**Light-cones in 4D Minkowski spacetime are conformal Killing horizons**

The conformal group is isomorphic to SO(5,1).

Any generator $\xi$ defines a Conformal Killing Field such that

$$\mathscr{L}_\xi \eta_{\mu\nu} = \frac{\psi}{2}\eta_{\mu\nu},$$

$$\psi = \nabla_\mu \xi^\mu$$



T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

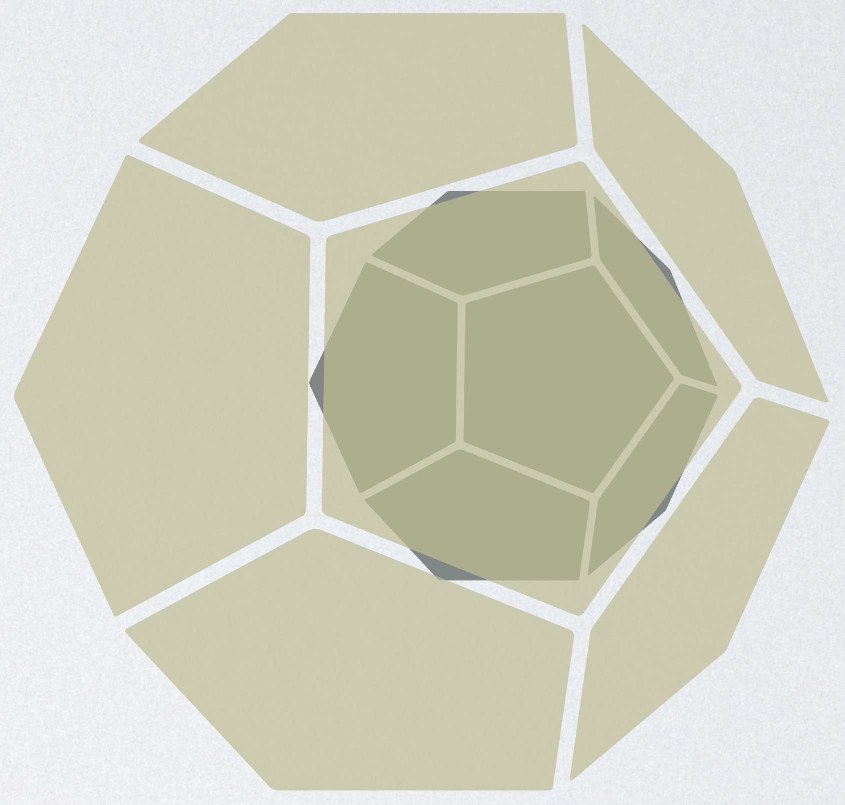
# LIGHT-CONE THERMODYNAMICS

The only generators that don't contain angular components are

$$D = r\partial_r + t\partial_t$$

$$P_0 = \partial_t$$

$$K_0 = -2tD - (r^2 - t^2)P_0$$



The most general radial MCKF is

$$\xi = -aK_0 + bD + cP_0$$
$$= (av^2 + bv + c)\partial_v + (au^2 + bu + c)\partial_u$$

$$u = t - r, \quad v = t + r$$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

# LIGHT-CONE THERMODYNAMICS

The norm of $\xi = (av^2 + bv + c)\partial_v + (au^2 + bu + c)\partial_u$

is given by $\xi \cdot \xi = -(av^2 + bv + c)(au^2 + bu + c)$

It's null along the light cones given by

$$u = u_\pm = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$v = v_\pm = u_\pm$$

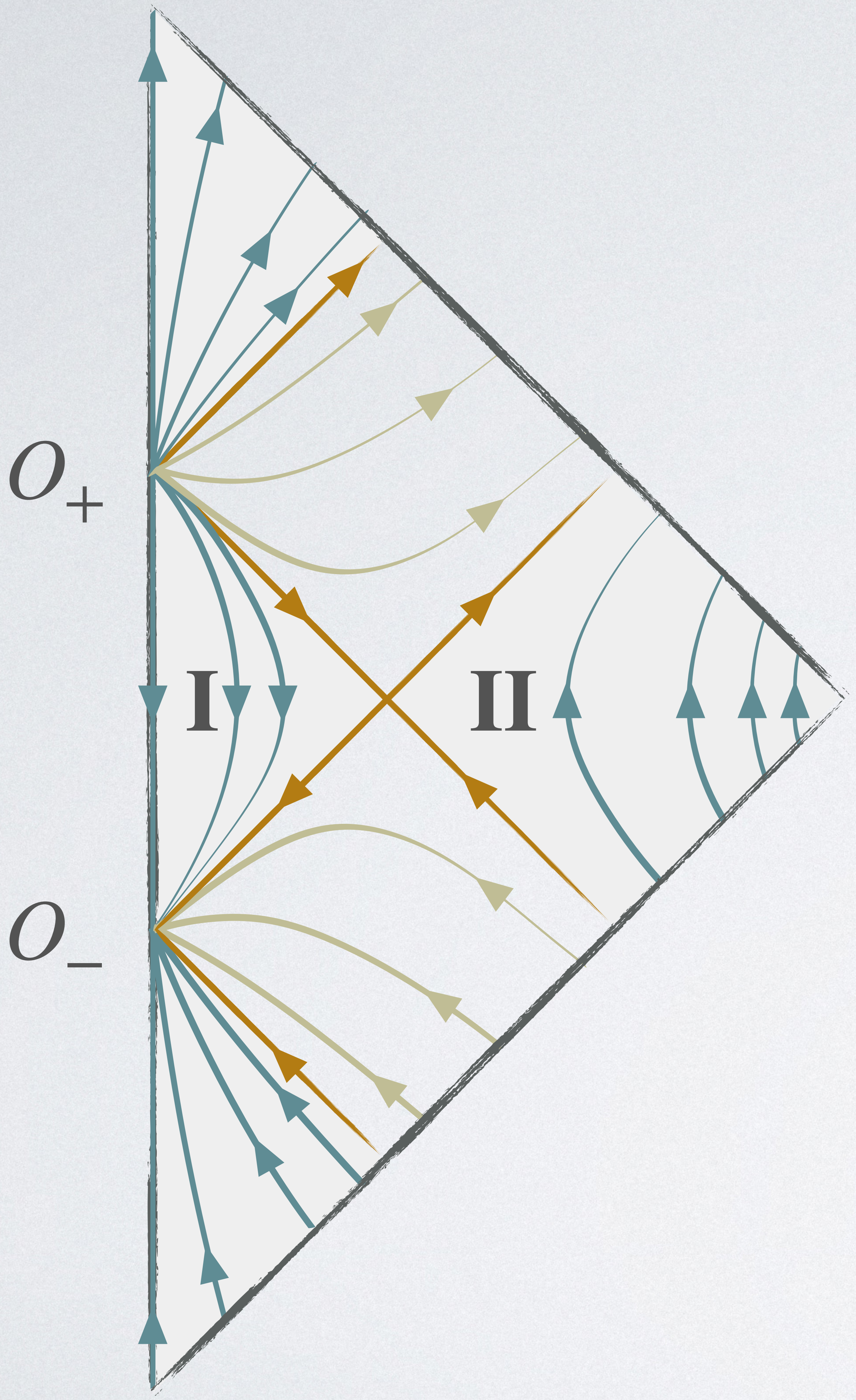
$\xi$ vanishes at the intersection of $u = u_-$, $v = v_+$

$$t_H := -\frac{b}{2a} \qquad r_H := \frac{v_+ - u_-}{2} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

# LIGHT-CONE THERMODYNAMICS

Martinetti, Rovelli (2003)
Kay, Wald (1991)
Hislop, Longo (1982)
Jacobson, Visser (2022)

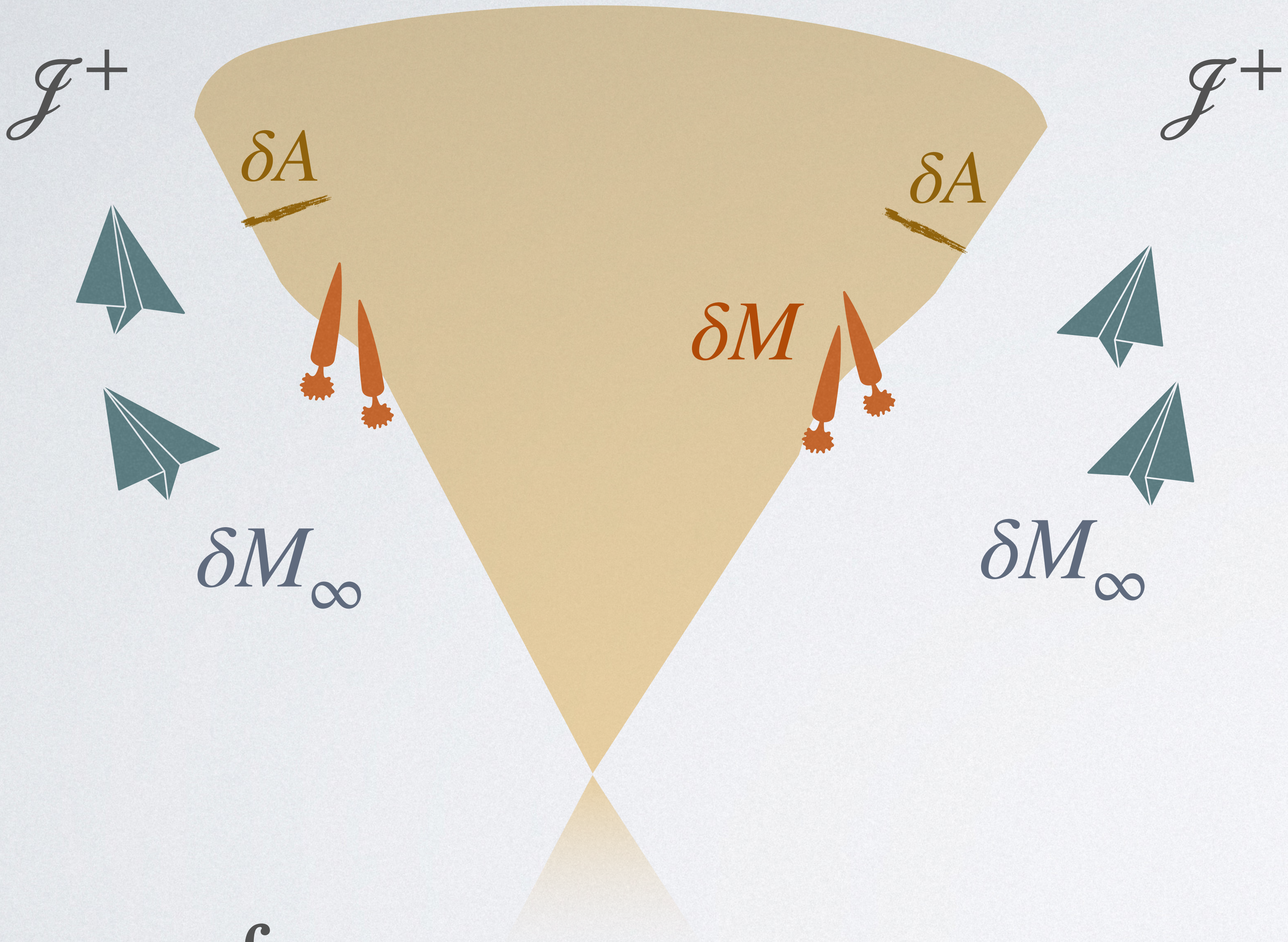



- $\xi$ defines two Conformal Killing Horizons at the past and future light cones of $O_\pm = (t = v_\pm, r = 0)$
- Each horizon has constant (conformally invariant) surface gravity defined via $\nabla_\mu (\xi \cdot \xi) \hat{=} -2\kappa \eta_{\mu\nu} \xi^\nu$
- Events in spacetime are separated as in a spherical charged black hole.

T De Lorenzo and A Perez. Light Cone Thermodynamics. Phys. Rev. D, 97(4):044052, 2018

# LAWS OF LIGHT-CONE THERMODYNAMICS

from T De Lorenzo, A Perez (2018)




$$M := \int_\Sigma T_{\mu\nu}\xi^\mu d\Sigma^\nu$$

$$\delta A := \Delta A - \Delta A_{\mathrm{vac}}$$

0. constant surface gravity $\kappa$ on the conformal Killing horizon
1. under conformally-invariant matter perturbations $\delta M = \frac{\kappa}{2\pi}\frac{\delta A}{4} + \delta M_\infty$
2. $\delta A \geq 0$
3. extremal radial **MCKFs** have vanishing "temperature" and vanishing "entropy"

# LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

A Perez, SR 2023
Unruh 1976

**Goal: writing the Minkowski vacuum $|0\rangle_M$ as a superposition of particle states associated to $\xi$**

how to characterize positive frequency solutions of the KG equation with respect to inertial time on a light cone?

$$\Box\Phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)\Phi = 0$$

$$= \left(-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)\Phi(x)$$

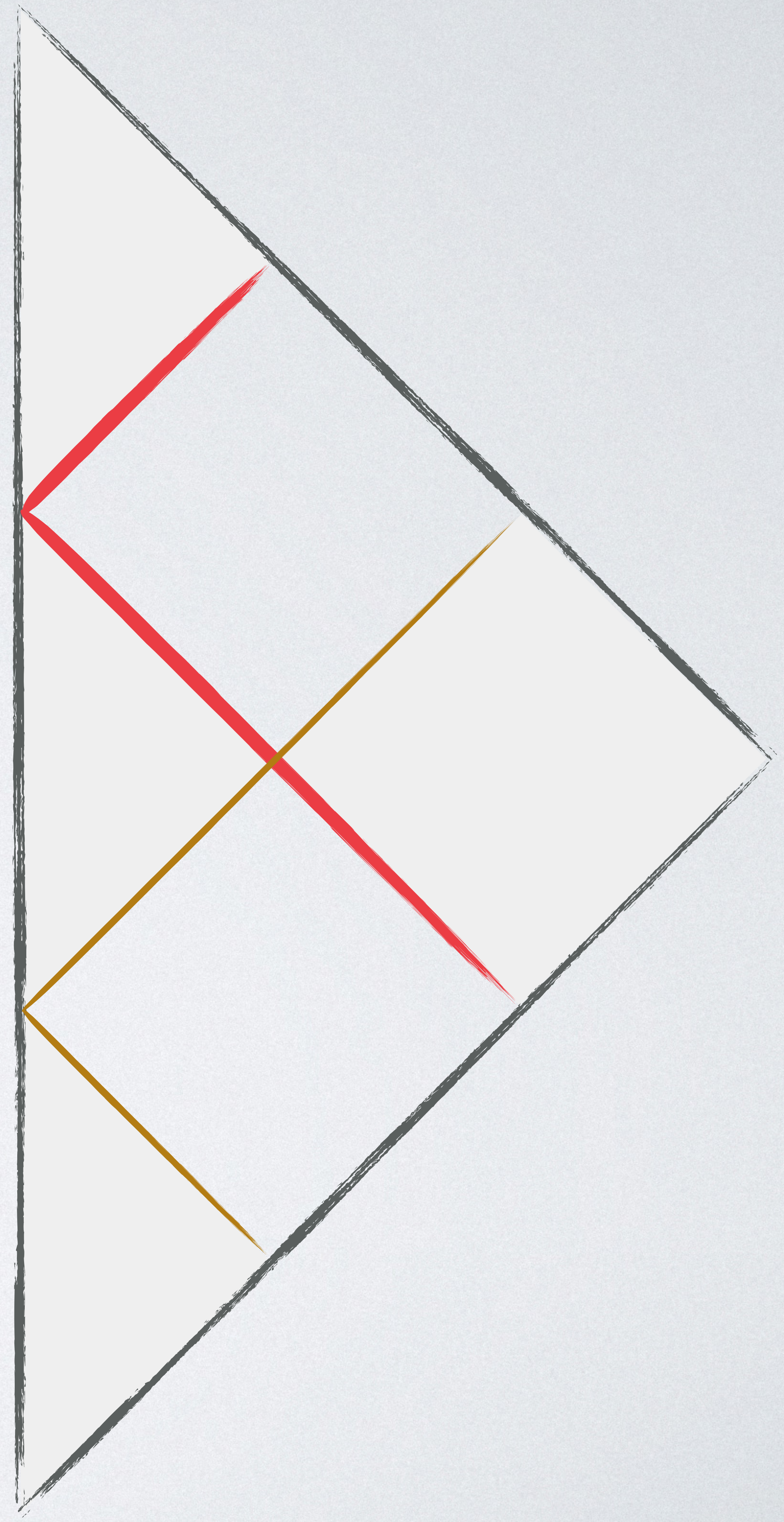
# LIGHT-CONE THERMODYNAMICS

A Perez, SR 2023

decomposition of the Minkowski vacuum

$$\Phi(x) = e^{-i\omega t} Y_{\ell m}(\theta, \varphi) R_\ell(r)$$

KG equation: $$\left( \omega^2 + \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{r} - \frac{\ell(\ell+1)}{r^2} \right) R_\ell = 0$$



solved by the spherical Bessel functions

$$R_\ell(r) = j_\ell(\omega r)$$

Solutions are completely characterized in the union of the past and future light cone.

# LIGHT-CONE THERMODYNAMICS

A Perez, SR 2023

## decomposition of the Minkowski vacuum

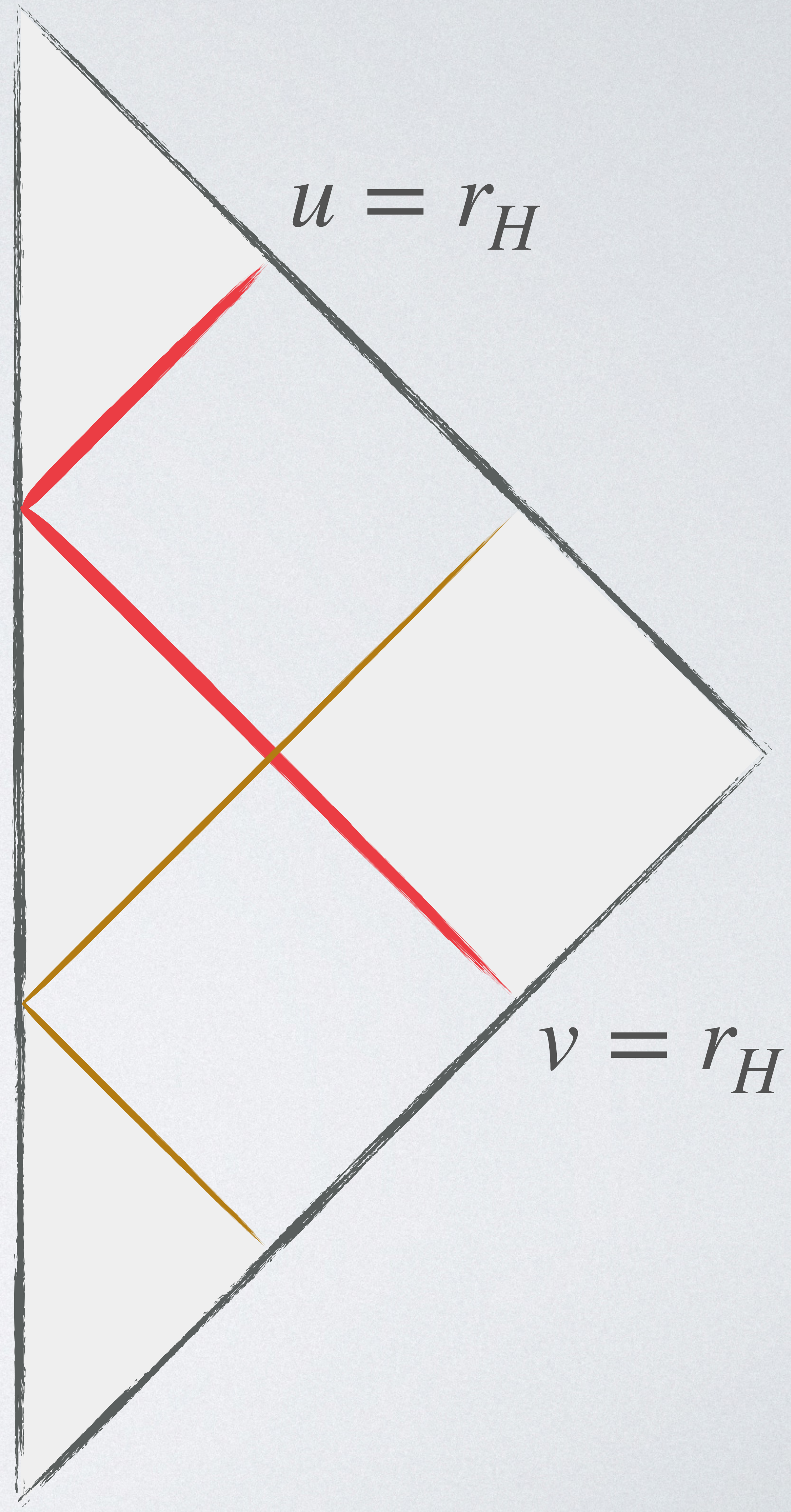
$$\Phi(x) = e^{-i\omega t} Y_{\ell m}(\theta, \varphi) j_\ell(\omega r) \qquad t = \frac{v+u}{2}, \quad r = \frac{v-u}{2}$$

By means of a coordinate transformation we can set $b = 0$, $v_\pm := \pm r_H$

at $u = r_H$, $$\Phi = e^{-i\omega\left(\frac{v+r_H}{2}\right)} Y_{\ell m} j_\ell \left(\omega \frac{v - r_H}{2}\right), \qquad v > r_H$$

at $v = r_H$, $$\Phi = e^{-i\omega\left(\frac{u+r_H}{2}\right)} Y_{\ell m} j_\ell \left(\omega \frac{r_H - u}{2}\right), \qquad u < r_H$$

which can be written in terms of a single variable which covers the whole real line.



# LIGHT-CONE THERMODYNAMICS

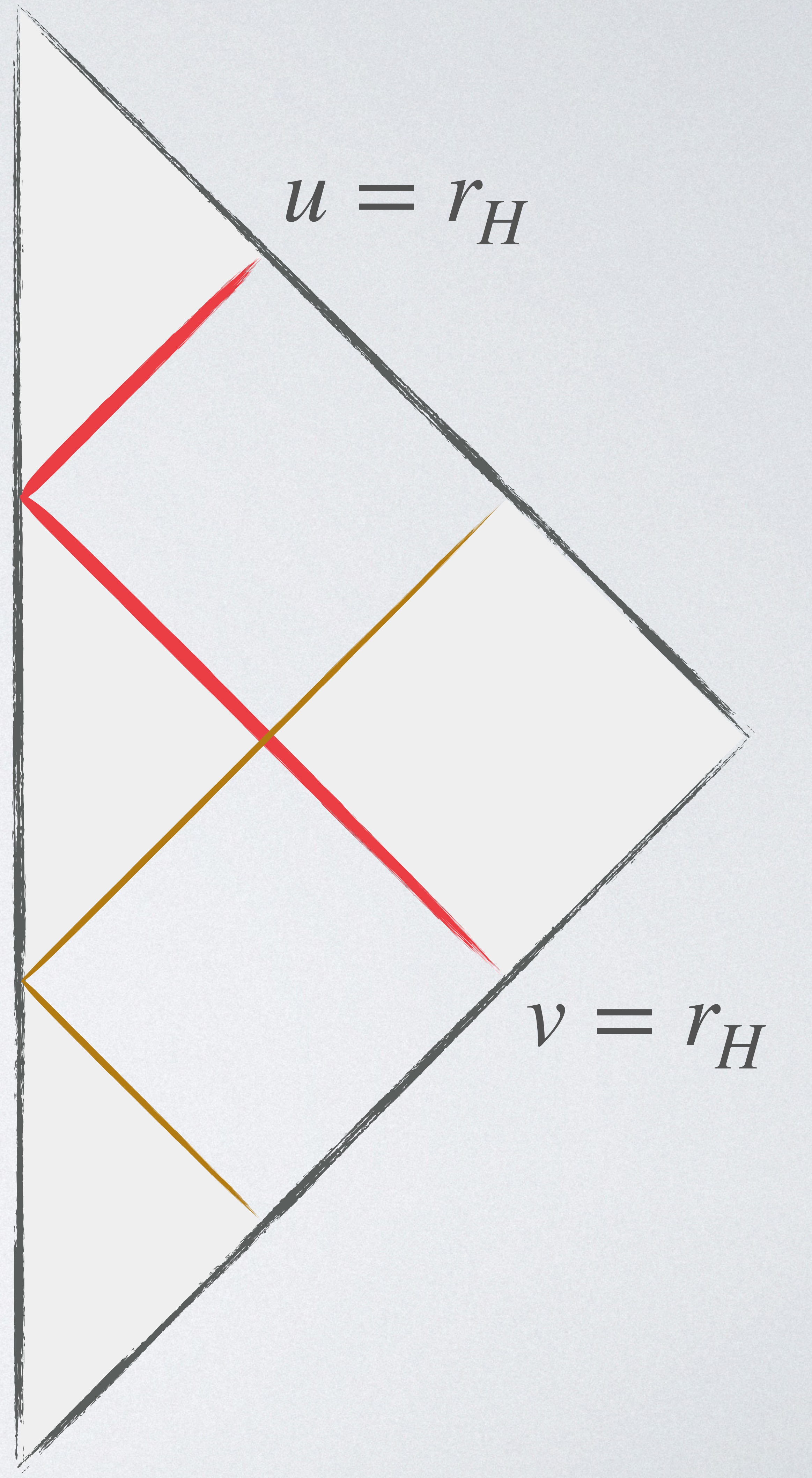
decomposition of the Minkowski vacuum

A Perez, SR 2023

In terms of a single null variable $z$ spanning from $-\infty$ to $+\infty$, solutions of the KG equation take the form

$$\Phi = e^{-i\omega\left(\frac{z+v_+}{2}\right)} Y_{\ell m} j_\ell\left(\omega \frac{z-v_+}{2}\right),$$

$\Phi$ is analytic. We want to characterize $\omega > 0$ solutions.
Let us look where they are bounded. For large $|z|$ :



# LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

A Perez, SR 2023

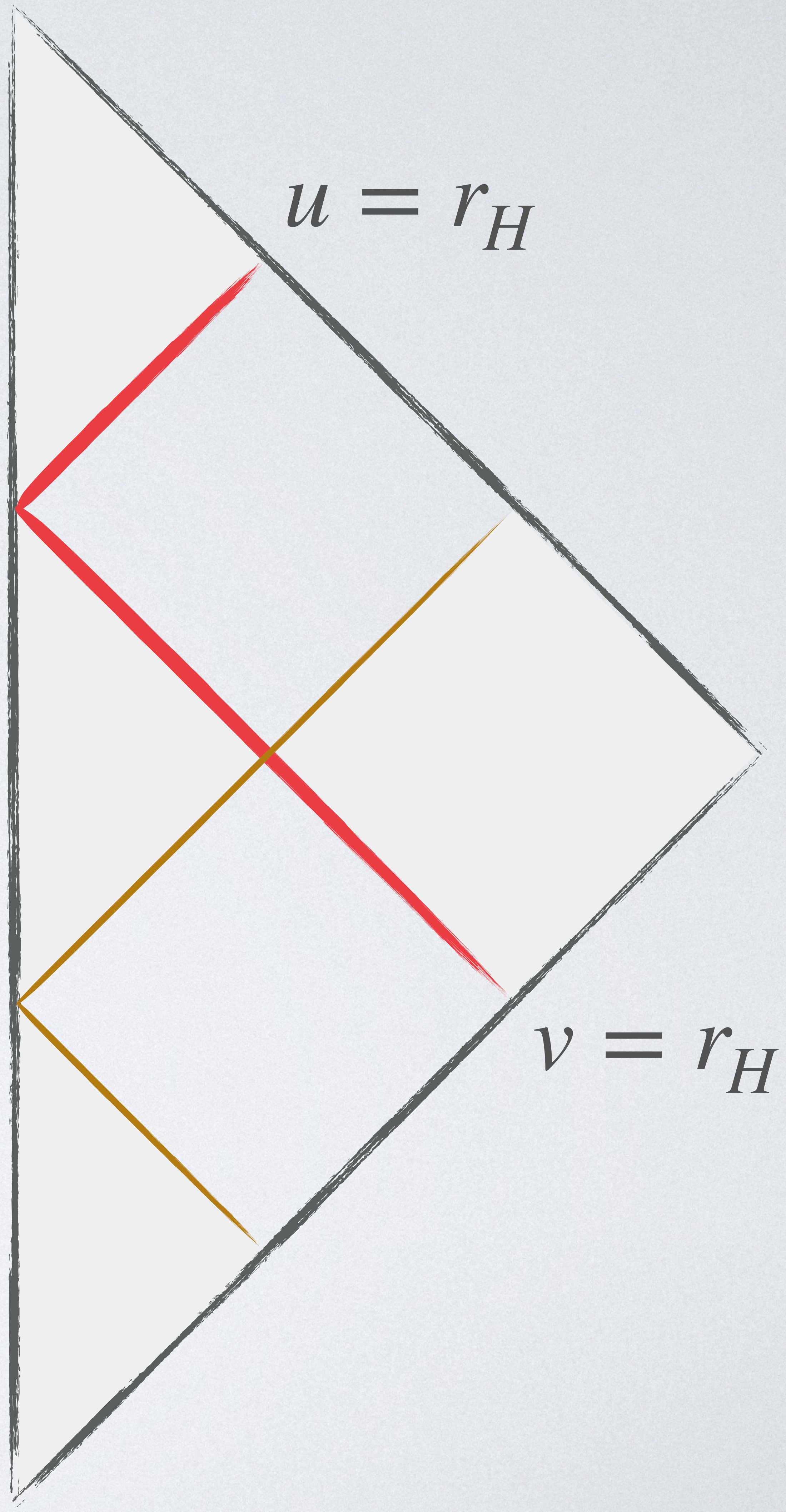
$\Phi$ is analytic. We want to characterize $\omega > 0$ solutions.
Let us look where they are bounded. For large $|z|$ :

$$\Phi \approx e^{-i\omega\left(\frac{z+r_H}{2}\right)} Y_{\ell m} \sin\left(\omega\frac{z-r_H}{2} - \frac{\ell\pi}{2}\right) / \left(\omega\left(z-r_H\right)\right),$$

$$= Y_{\ell m}\frac{A}{z-r_H}e^{-i\omega z}$$

bounded for $\mathbf{Im}(z) < 0$

$\omega > 0$ solutions are **analytic** functions of $z$ **bounded** in the lower-half complex plane $\mathbf{Im}(z)$



# LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

A Perez, SR 2023

What are the positive-frequency solutions associated to $\xi$ ?

Consider this coordinate transformation

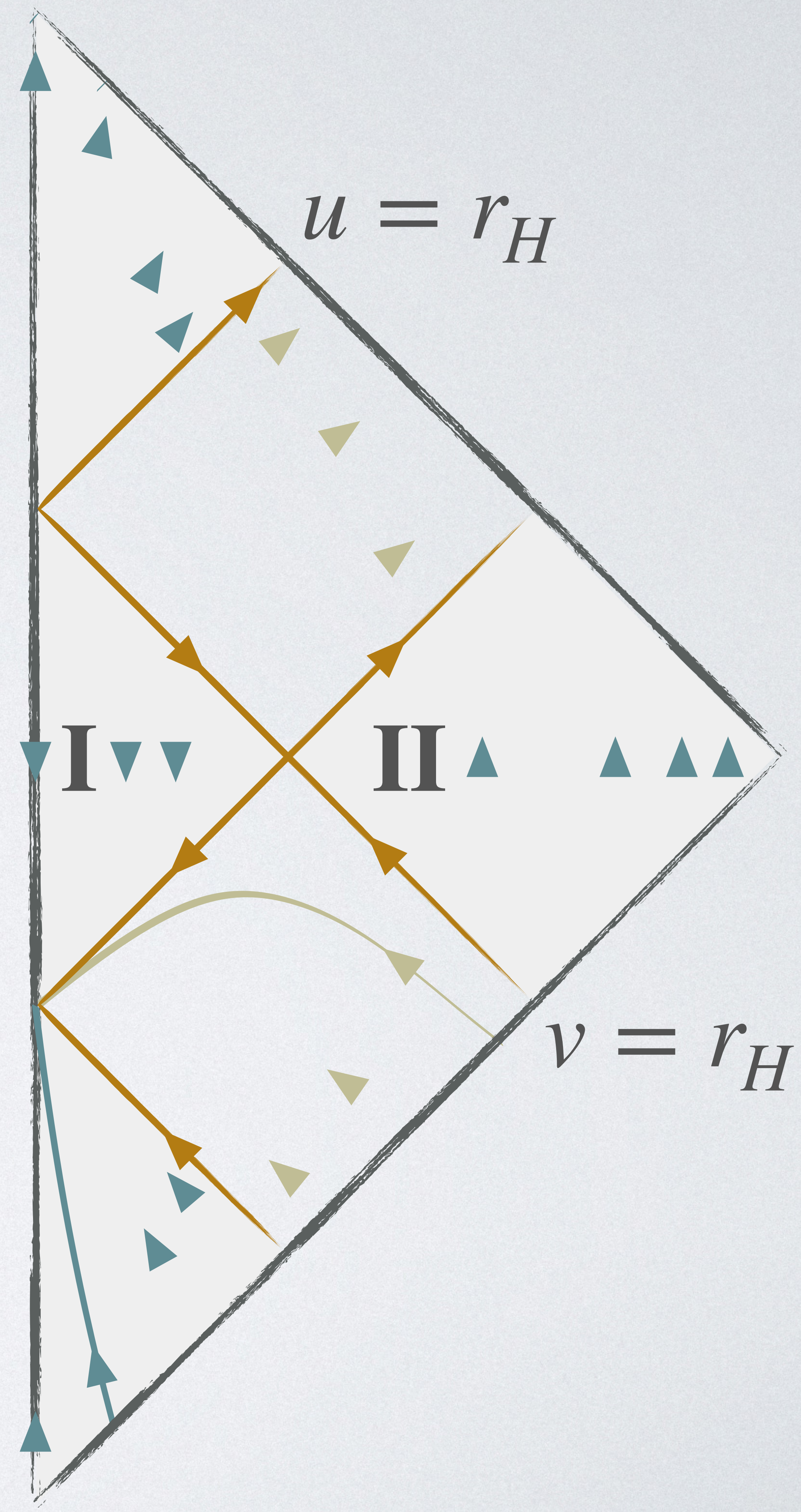
$$t = \frac{r_H \sinh(\kappa\tau)}{\cosh(\kappa\rho) - \cosh(\kappa\tau)}$$

$$r = -\frac{r_H \sinh(\kappa\rho)}{\cosh(\kappa\rho) - \cosh(\kappa\tau)}$$

$$u = -r_H \coth\left(\frac{\kappa\tilde{u}}{2}\right)$$

$$v = -r_H \coth\left(\frac{\kappa\tilde{v}}{2}\right)$$

$$ds_M^2 = \Omega_{\mathbf{II}}(\tau,\rho)\left(-d\tau^2 + d\rho^2 + \kappa^{-2}\sinh^2(\kappa\rho)dS^2\right)$$



# LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

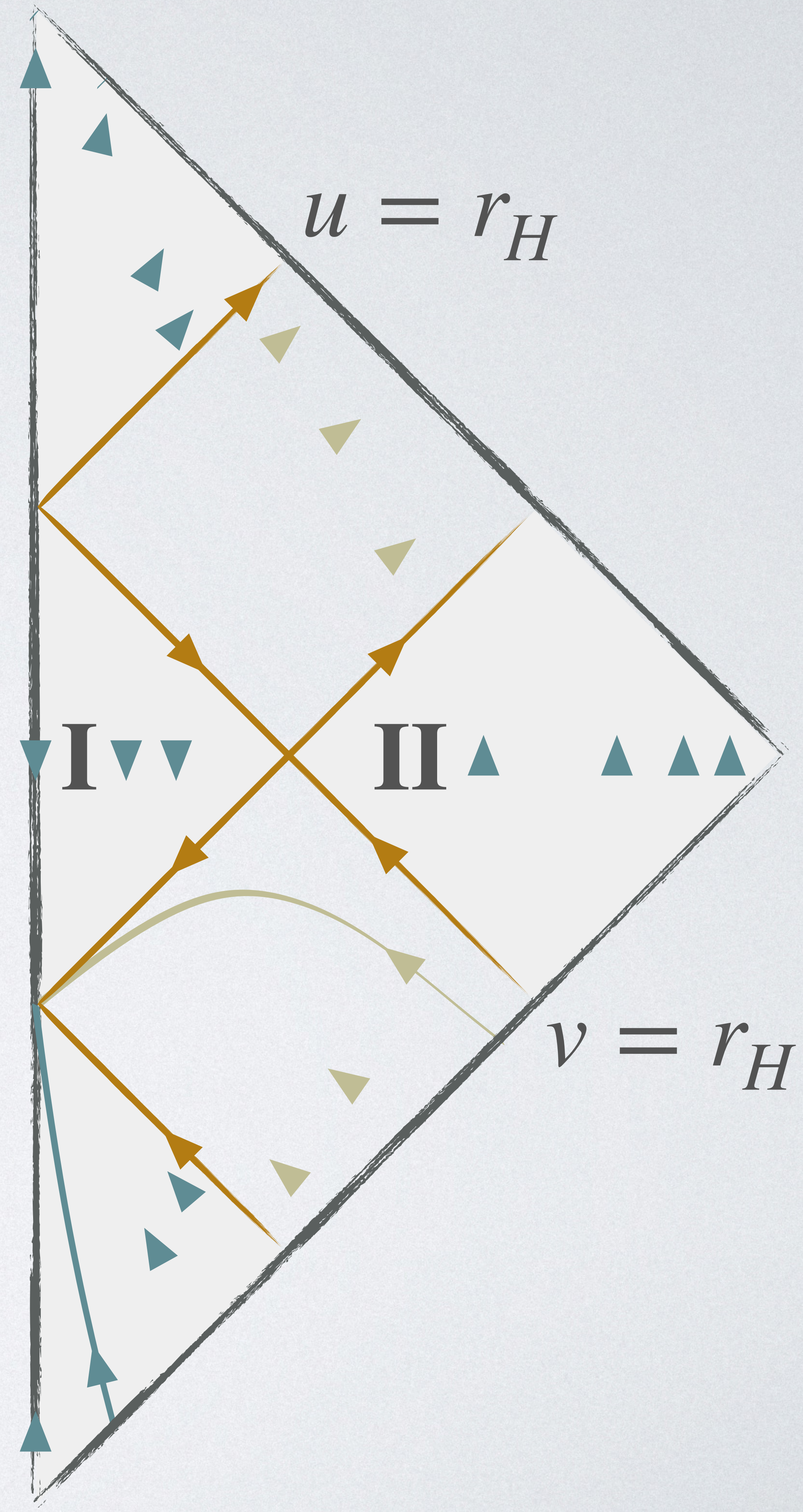
A Perez, SR 2023

$$ds_M^2 = \Omega_{\mathbf{II}}(\tau,\rho)\left(-d\tau^2 + d\rho^2 + \kappa^{-2}\sinh^2(\kappa\rho)dS^2\right)$$

Under a conformal transformation $g_{\mu\nu} \to g'_{\mu\nu} = C^2 g_{\mu\nu}$ ,

solutions of $\left(\Box - \frac{1}{6}R\right)U = 0$ are mapped via $\Phi \to C^{-1}\Phi$.

Solutions take the form: $$U_{\omega\ell m} = e^{-i\omega\tau}Y_{\ell m}\frac{R_{\omega\ell\pm}(\rho)}{\sinh(\kappa\rho)}$$



# LIGHT-CONE THERMODYNAMICS

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decomposition of the Minkowski vacuum

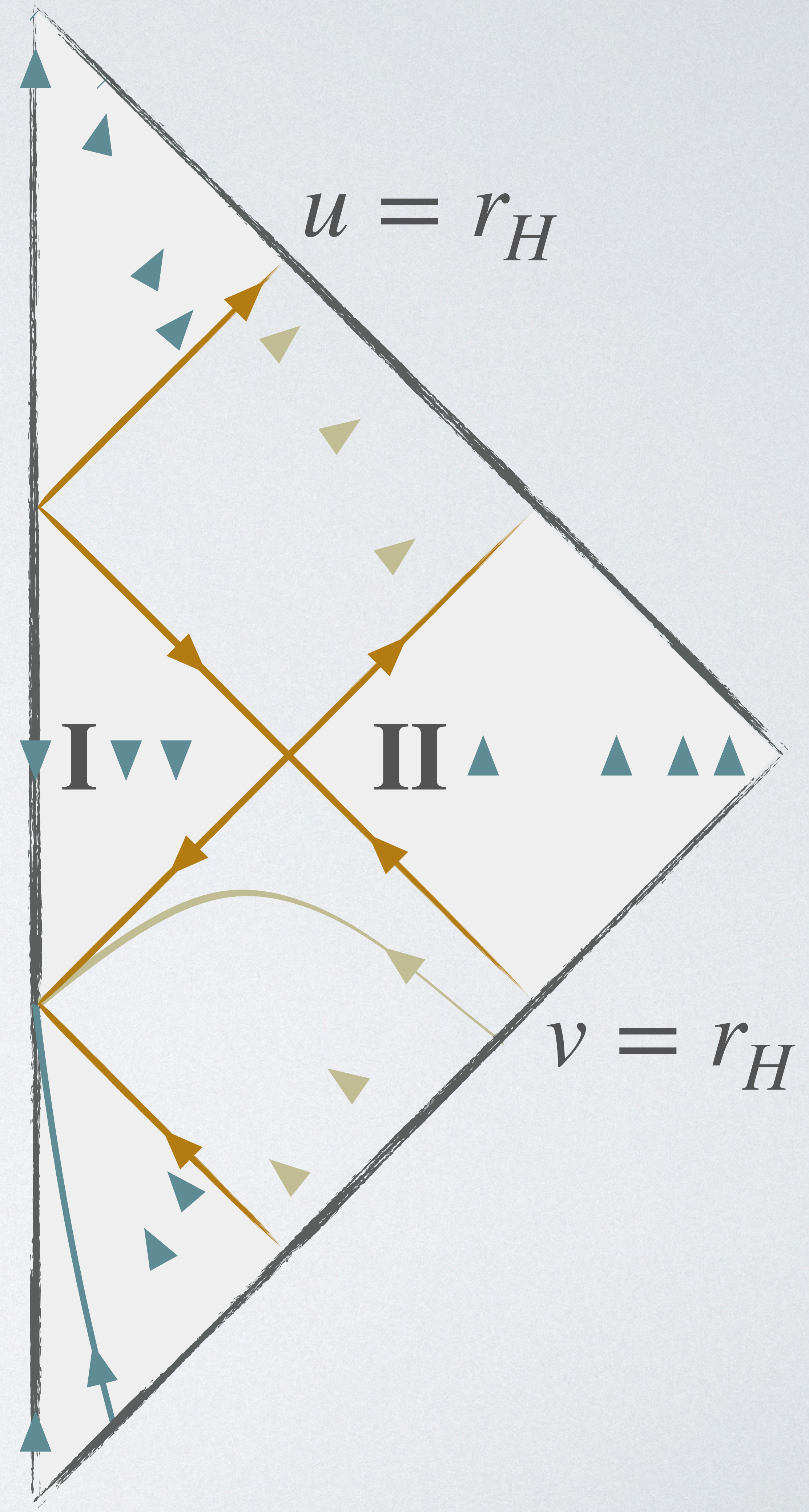
$$U_{\omega\ell m} = e^{-i\omega\tau} Y_{\ell m} \frac{R_{\omega\ell\pm}(\rho)}{\sinh(\kappa\rho)}$$

KG equation:

$$\left( \frac{\partial^2}{\partial\rho^2} + \omega^2 - \frac{\ell(\ell+1)\kappa^2}{\sinh^2(\kappa\rho)} \right) R_{\omega\ell\pm}(\rho) = 0$$

Near the past boundary of region II ($v = r_H$) $\rho \to +\infty$ and the effective potential vanishes. Thus

$$U_{\omega\ell m} = e^{-i\omega(\tau \pm \rho)} \frac{Y_{\ell m}}{\sinh(\kappa\rho)}$$



# LIGHT-CONE THERMODYNAMICS

A Perez, SR 2023

## decomposition of the Minkowski vacuum

studying solutions near the light cone in region **II** (and **III**) gives

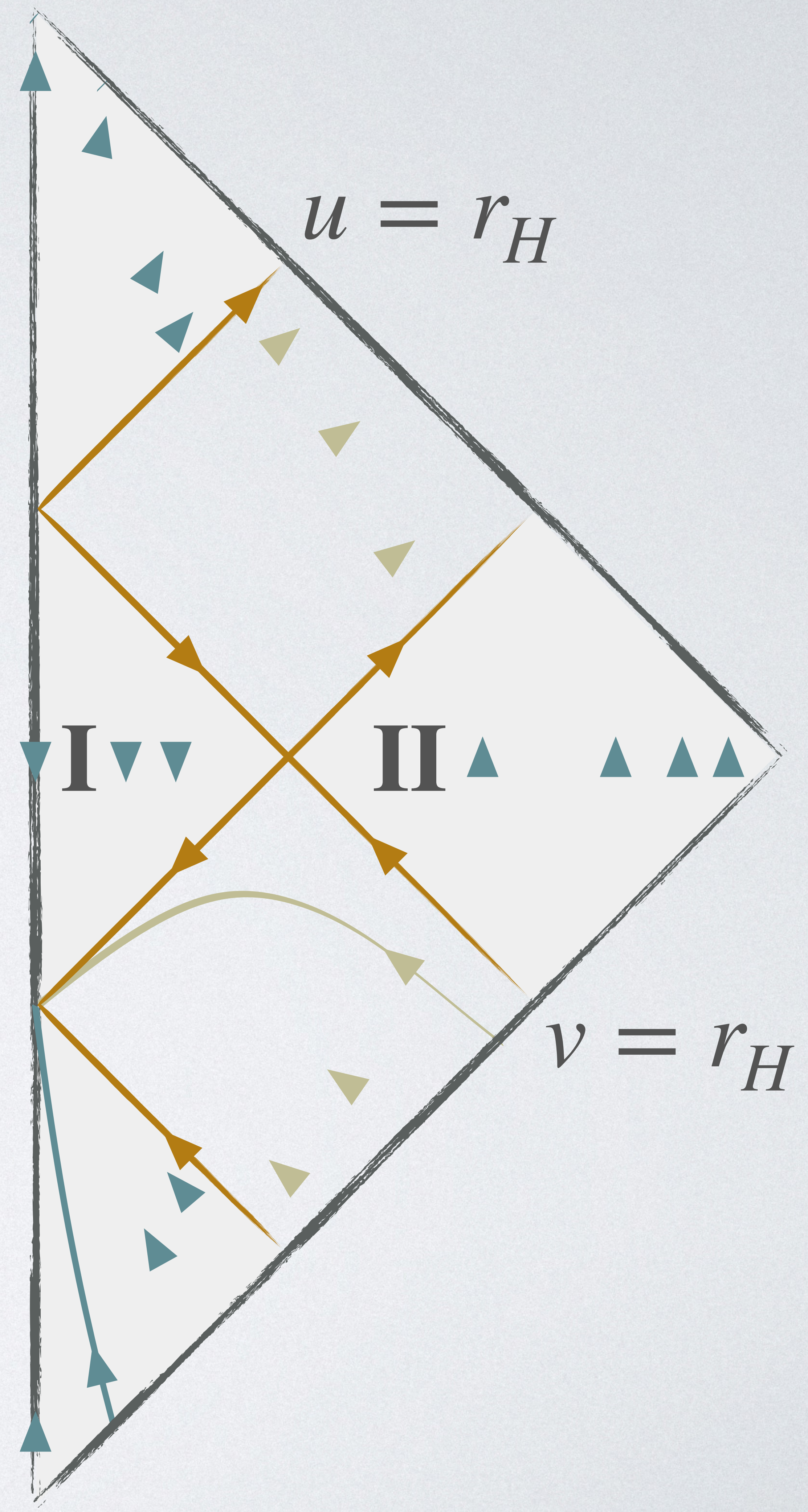
$$\Phi_{\mathbf{II}}^{\omega\ell m} = \Omega_{\mathbf{II}}^{-1} U_{\omega\ell m} = \frac{1}{r} Y_{\ell m} e^{-i\omega(\tau - \rho)}$$

$$= \frac{1}{r_H - u} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{u - r_H}{u + r_H}\right)}$$

Similarly

$$\Phi_{\mathbf{I}}^{\omega\ell m} = \frac{1}{r_H - u} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{r_H + u}{r_H - u}\right)}$$

at $v = r_H$



# LIGHT-CONE THERMODYNAMICS

decomposition of the Minkowski vacuum

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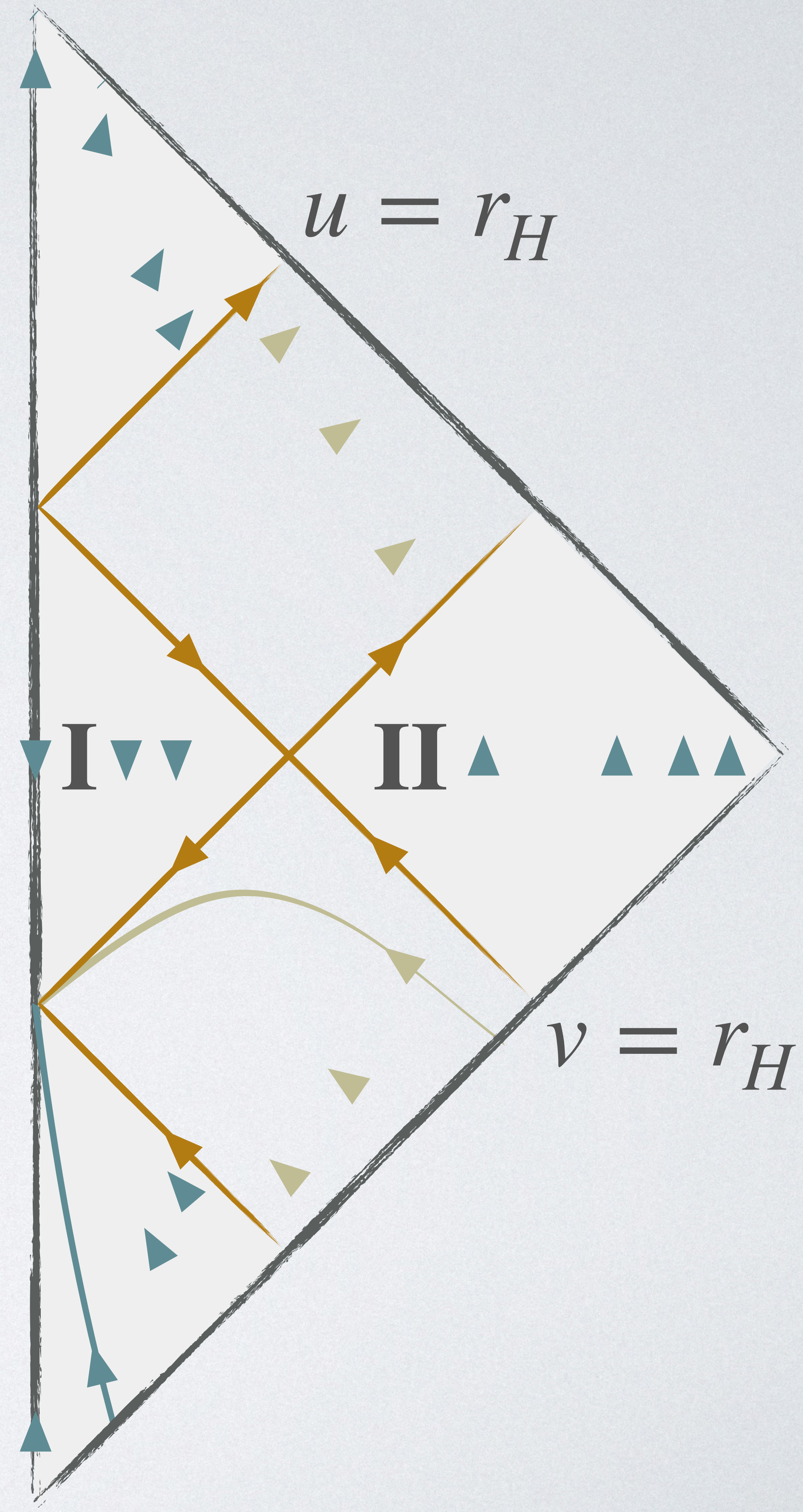
$$\Phi_{\mathbf{II}}^{\omega\ell m} = \frac{1}{r_H - z} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{z - r_H}{z + r_H}\right)} \qquad z < -r_H,\ z > r_H$$

$$\Phi_{\mathbf{I}}^{\omega\ell m} = \frac{1}{r_H - z} Y_{\ell m} e^{-i\frac{\omega}{\kappa}\log\left(\frac{r_H + z}{r_H - z}\right)} \qquad -r_H < z < r_H$$

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evaluate the complex function $F_\omega = \frac{1}{r_H - z} e^{-i\frac{\omega}{\kappa}\log\left(\frac{z - r_H}{z + r_H}\right)}$

at $z = x - i\epsilon,\ \epsilon > 0$

# LIGHT-CONE THERMODYNAMICS

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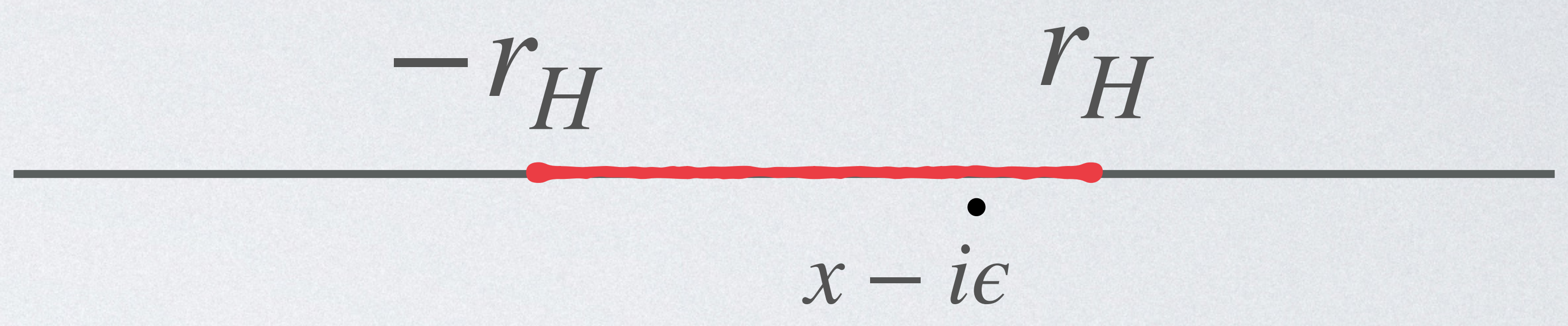
## decomposition of the Minkowski vacuum

$$F_\omega = \frac{1}{r_H - x + i\epsilon} e^{-i\frac{\omega}{\kappa}\log\left(\frac{x - r_H - i\epsilon}{x + r_H + i\epsilon}\right)}$$




for $-r_H < x < r_H$ we get

$$\log\left(\frac{x - r_H - i\epsilon}{x + r_H - i\epsilon}\right) = \log\left(\frac{r_H - x - i\epsilon}{x + r_H - i\epsilon} e^{-i(\pi - \mathcal{O}(\epsilon))}\right) = \log\left(\frac{r_H - x - i\epsilon}{x + r_H - i\epsilon}\right) - i\pi$$

$$F_\omega \approx \frac{e^{-\frac{\pi\omega}{\kappa}}}{r_H - x} e^{-i\frac{\omega}{\kappa}\log\left(\frac{x - r_H}{x + r_H}\right)} = e^{-\frac{\pi\omega}{\kappa}} \overline{\Phi_{\mathbf{I}}^{\omega}}$$

# LIGHT-CONE THERMODYNAMICS

## decomposition of the Minkowski vacuum

A Perez, SR 2023

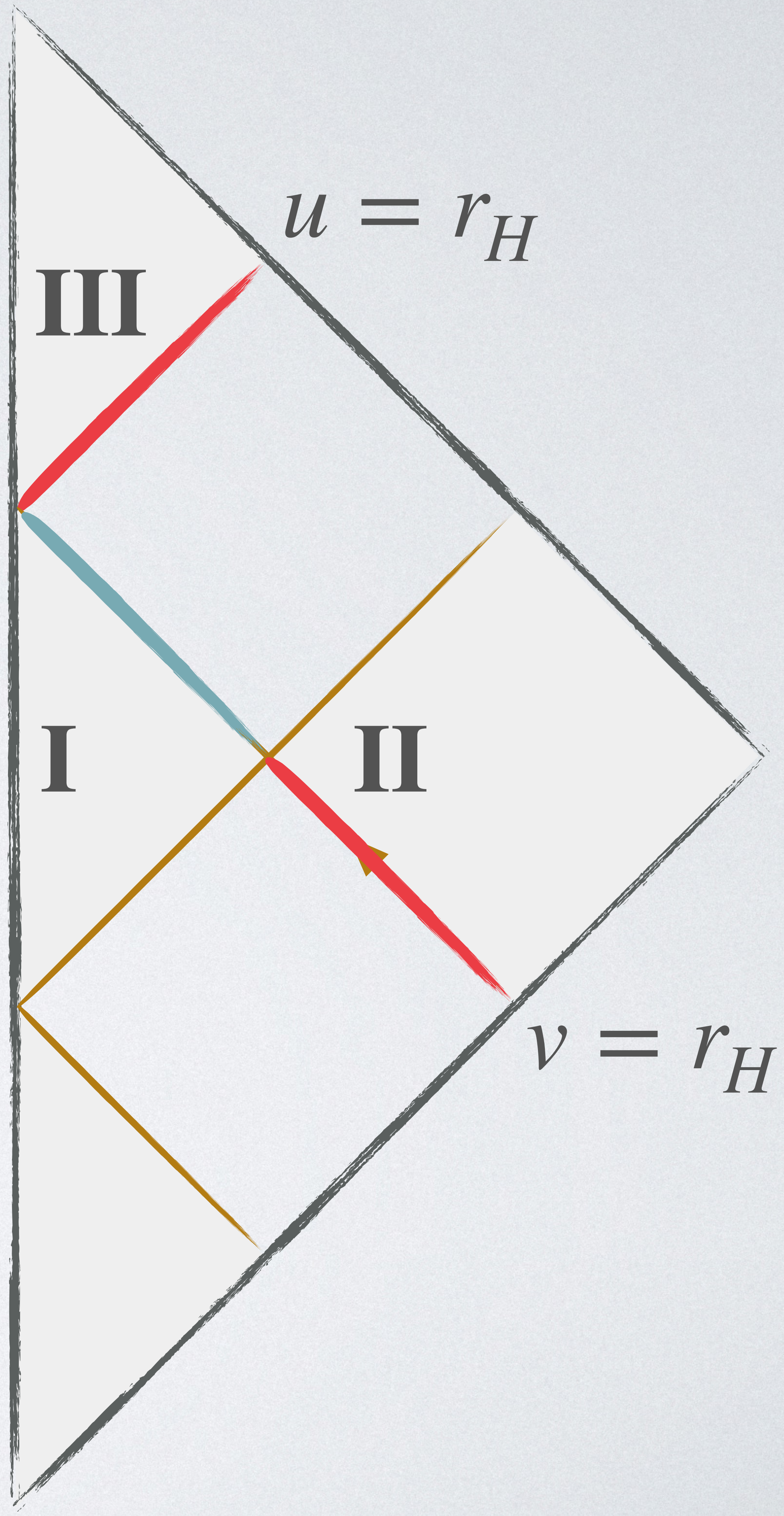
Fulling 1973, Davies 1975, Unruh 1976

similarly

$$F_\omega = \Phi^\omega_{\mathbf{II}} + e^{-\frac{\pi\omega}{\kappa}} \overline{\Phi}^\omega_{\mathbf{I}}$$

$$F'_\omega = \Phi^\omega_{\mathbf{I}} + e^{-\frac{\pi\omega}{\kappa}} \overline{\Phi}^\omega_{\mathbf{II}}$$

are analytic and bounded in the lower-half plane in terms of the Minkowski coordinate, thus **positive frequency solutions**.




$$\left(a_{\mathbf{II}\omega} + e^{-\frac{\pi\omega}{\kappa}} a^\dagger_{\mathbf{I}\omega}\right) \left|0\right\rangle_M = 0, \qquad \left(a_{\mathbf{I}\omega} + e^{-\frac{\pi\omega}{\kappa}} a^\dagger_{\mathbf{II}\omega}\right) \left|0\right\rangle_M = 0$$

$$U\left|0\right\rangle_M = \prod_i \left( \sum_n e^{-\frac{n\pi\omega_i}{\kappa}} \left|n,\omega_i\right\rangle_{\mathbf{I}} \otimes \left|n,\omega_i\right\rangle_{\mathbf{II}} \right)$$

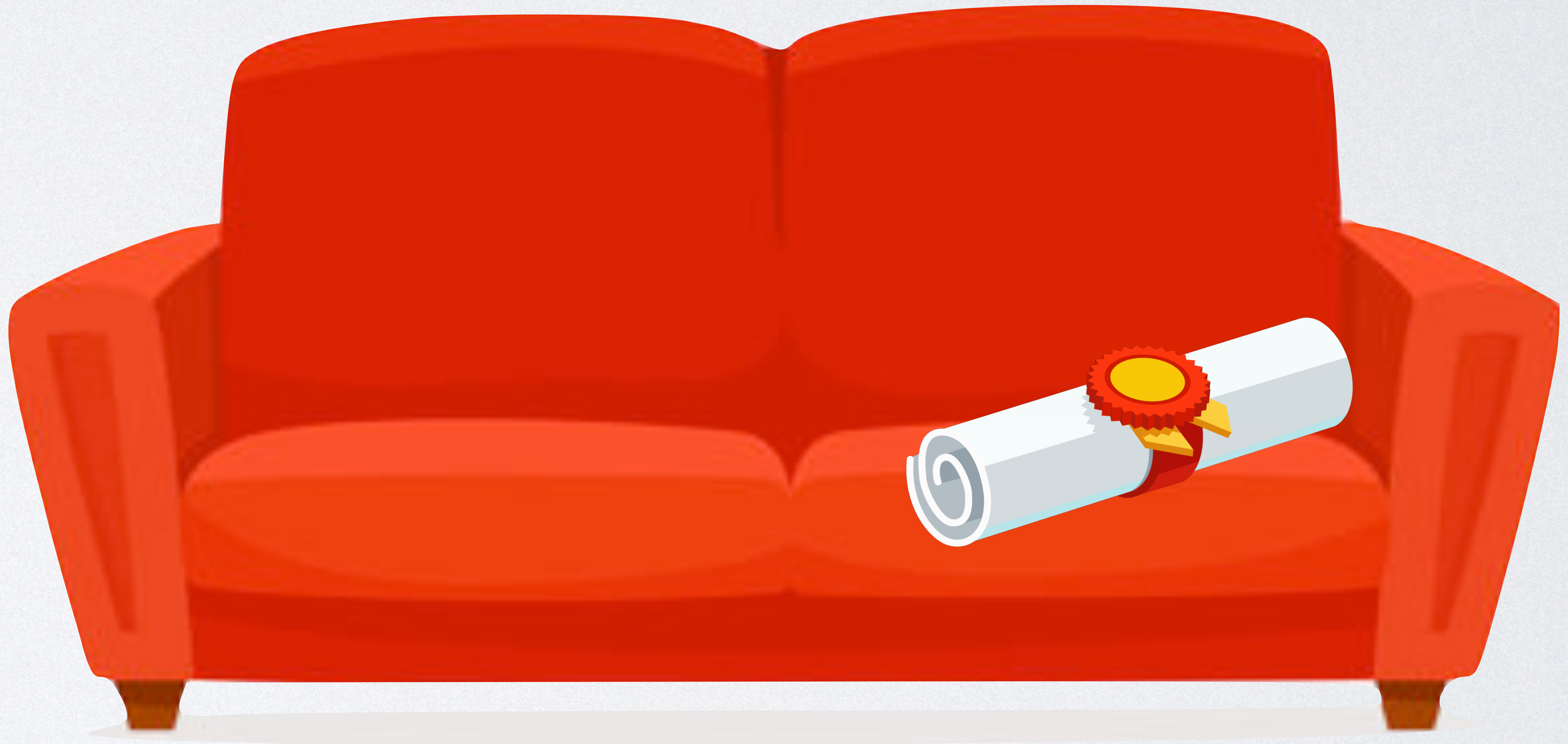
# PERSPECTIVES
## reading the signs

# RIBISI SALVATORE'S APPLICATIONS

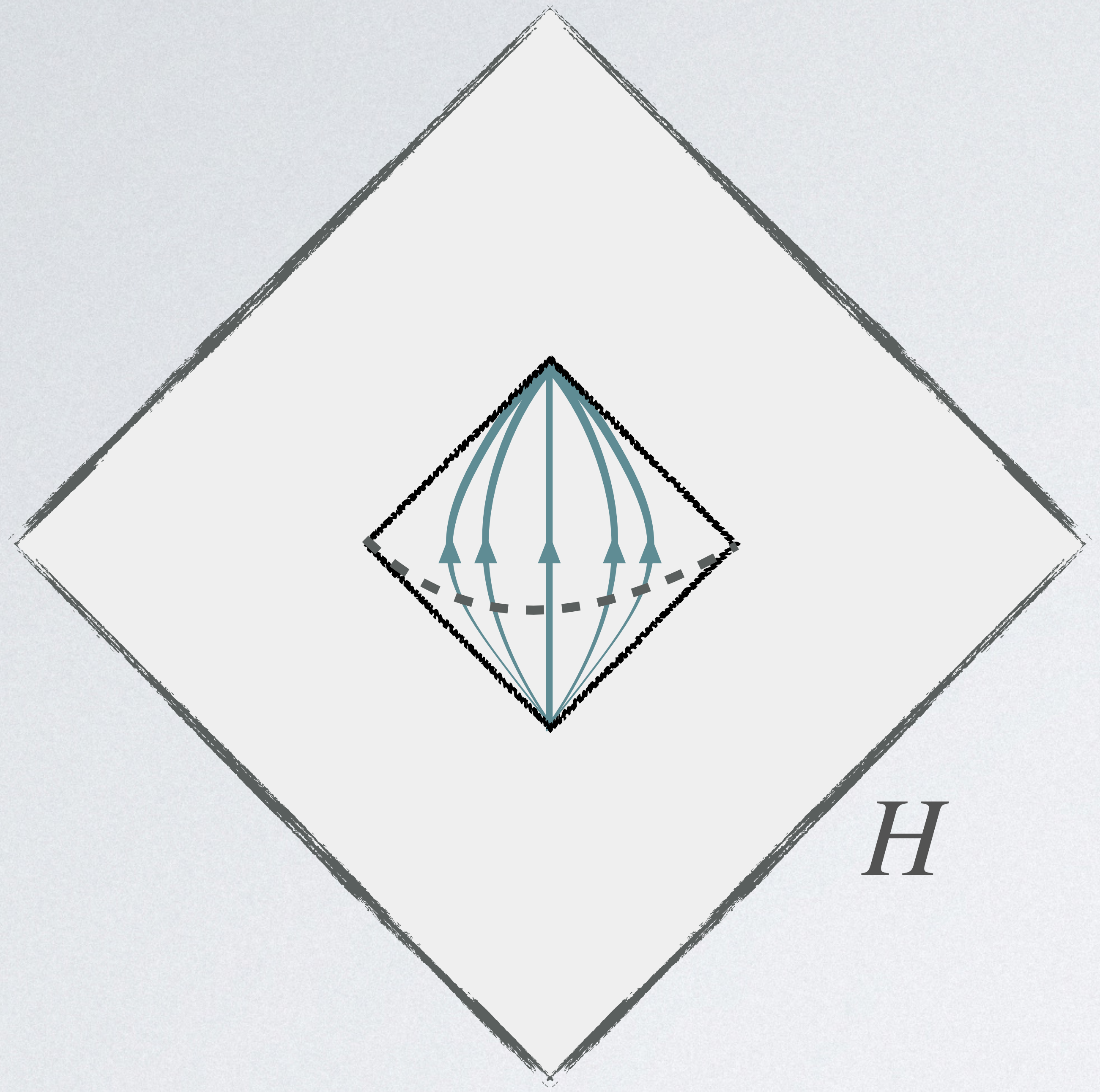

"Humility is a virtue, but not on a resume"

# PERSPECTIVES
## reading the signs

**R**IBISI
**S**ALVATORE'S
**A**PPLICATIONS



# PERSPECTIVES

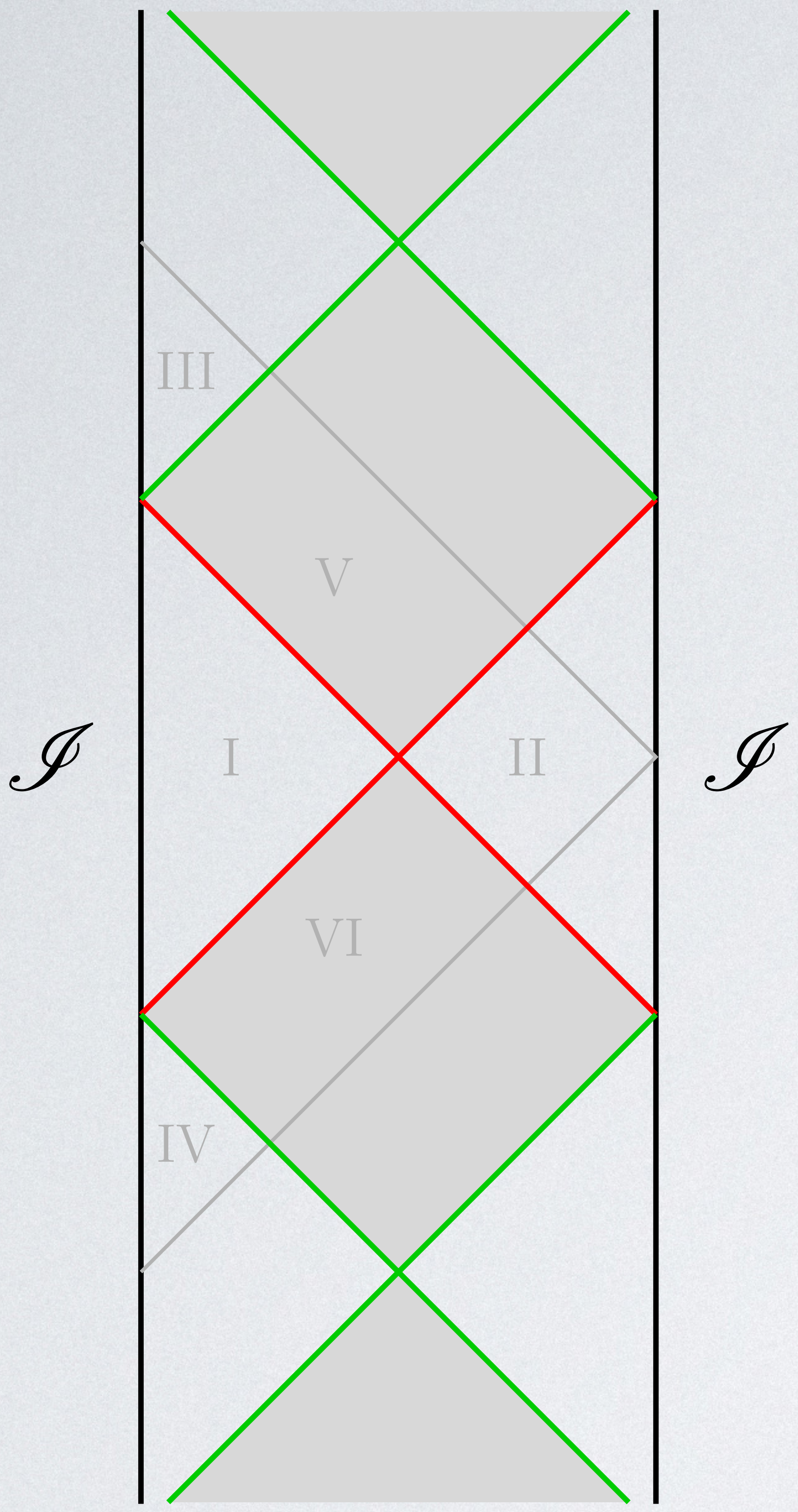



causal diamond in the static patch of deSitter spacetime. The boundary $H$ is the cosmological horizon. In blue the trajectories of the CKF. (Jacobson, Visser 2019)

- The same result can be obtained in (A)dS spacetime, relating (A)dS vacua to modes living in causally-disconnected regions of spacetime.

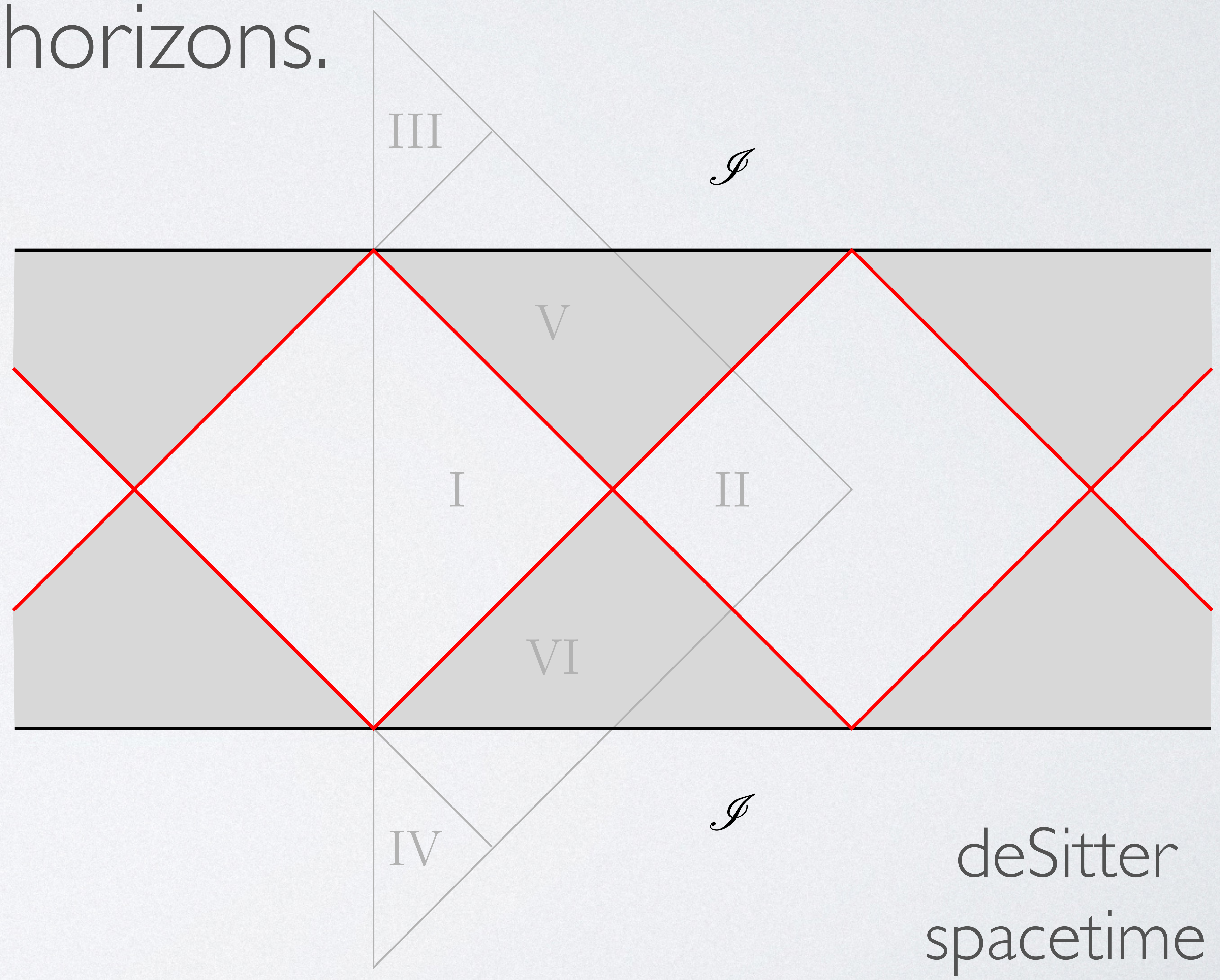
- We know the solutions of the Klein-Gordon equation in spherical coordinates, and the spherical trajectories leading to a thermal decomposition of the vacuum. We can use this knowledge to build **4D spherically symmetric moving mirror models.** (You're welcome)

# PERSPECTIVES




near-horizon
near-extremal RN

- Under a family of conformal transformations, $\xi$ is mapped into an actual Killing field. Our result straightforwardly translates to this class of spacetimes, among which we have deSitter and near-horizon near-extremal RN. The light cones are mapped into Killing horizons.




deSitter
spacetime

courtesy of T De Lorenzo and A Perez