Graviton corrections to Newton's potential as a test for non perturbative black hole models

Sami Viollet
Supervisors : Alejandro Perez and Simone Speziale
CPT Marseille

Motivations

-How a quantum perturbative treatment of GR gives predictions at low energy ?
[Donoghue ‘94, Donoghue Ivanov & Shkerin ‘17, Burgess ‘03]

-How this treatment allows to compute corrections to Newton’s potential ?
[Bjerrum-Bohr Donoghue & Holstein ‘02, Muzinich & Vokos ‘95]

-Can we test non-perturbative black hole models by asking them to reproduce the graviton corrections to Newton’s potential ?

Non renormalizability of GR

-Pure gravity at 1 loop -> **Finite** ['T Hooft & Veltman '74]

Divergences can be absorbed in a field redefinition.

Non renormalizability of GR

-Pure gravity at 1 loop -> **Finite** ['T Hooft & Veltman '74]

Divergences can be absorbed in a field redefinition.

-Gravity + massless scalar field at 1 loop -> **Infinite** ['T Hooft & Veltman '74]

Need to add a counter term in the Lagrangian : $\Delta\mathcal{L} = \frac{\sqrt{-g}}{\epsilon}\frac{1}{16\pi}\left[\frac{1}{240}R^2 + \frac{1}{120}R_{\mu\nu}R^{\mu\nu}\right]$

Non renormalizability of GR

-Pure gravity at 1 loop -> **Finite** ['T Hooft & Veltman '74]

Divergences can be absorbed in a field redefinition.

-Gravity + massless scalar field at 1 loop -> **Infinite** ['T Hooft & Veltman '74]

Need to add a counter term in the Lagrangian : $\Delta\mathcal{L} = \frac{\sqrt{-g}}{\epsilon}\frac{1}{16\pi}\left[\frac{1}{240}R^2 + \frac{1}{120}R_{\mu\nu}R^{\mu\nu}\right]$

-Pure Gravity at 2 loops -> **Infinite** [Goroff & Sagnotti '85]

Need to add a counter term in the Lagrangian : $\Delta\mathcal{L} = \frac{\sqrt{-g}}{\epsilon}\frac{209}{2880}\frac{G}{8\pi^3}R^{\mu\nu\alpha\beta}R_{\alpha\beta\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}$

Effective treatment of GR

We focus on the **low energy** regime.

Most general action : $S = \int d^4\sqrt{-g}\left[\frac{1}{16\pi G}R + \underbrace{c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + ...}_{\text{Higer derivatives}}\right]$

Renormalization affects the higher derivative terms, which are not important at low energies.

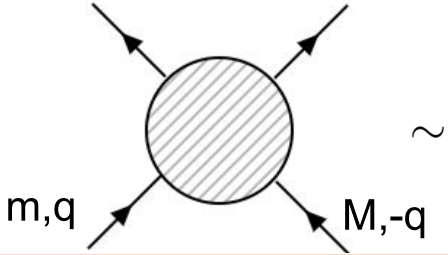
Effective treatment of GR

We focus on the **low energy** regime.

Most general action : $S = \int d^4 \sqrt{-g} \left[\frac{1}{16\pi G} R + \underbrace{c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + ...}_{\text{Higer derivatives}} \right]$

Renormalization affects the higher derivative terms, which are not important at low energies.

More explicitly, we have at one loop :



$$\sim \frac{GMm}{q^2} \left[1 + aG\sqrt{-m^2 q^2} + bGq^2 \log(-q^2) + f(c_1, c_2) Gq^2 + \mathcal{O}(q^2) \right]$$

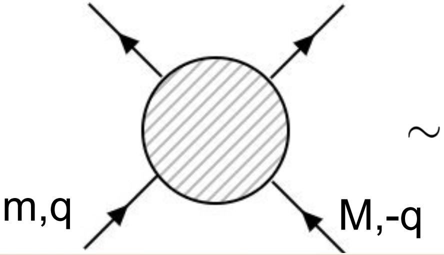
Effective treatment of GR

We focus on the **low energy** regime.

Most general action : $$S = \int d^4 \sqrt{-g} \left[\frac{1}{16\pi G} R + \underbrace{c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + ...}_{\text{Higer derivatives}} \right]$$

Renormalization affects the higher derivative terms, which are not important at low energies.

More explicitly, we have at one loop :

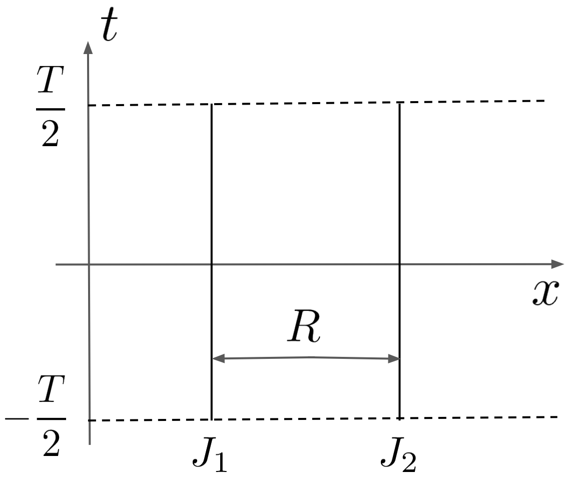


Local term in real space

$$\sim \frac{GMm}{q^2} \left[1 + aG\sqrt{-m^2 q^2} + bGq^2 \log(-q^2) + f(c_1, c_2) Gq^2 + \mathcal{O}(q^2) \right]$$

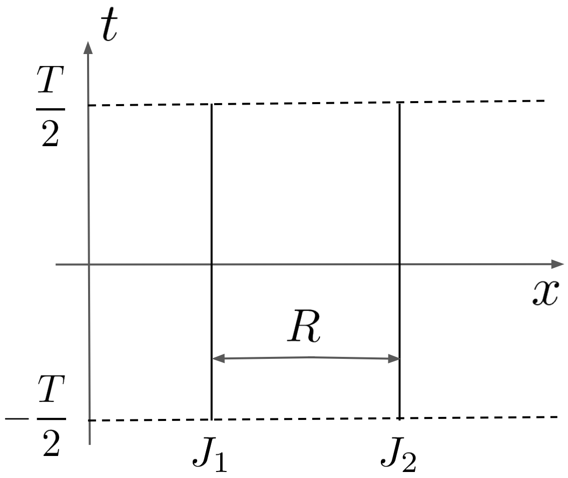
Static potential in QFT

We consider **static** sources i.e. $J^\mu(x) = (\rho\delta(\vec{x})\Pi(t), 0, 0, 0)$



Static potential in QFT

We consider **static** sources i.e. $J^{\mu}(x) = (\rho\delta(\vec{x})\Pi(t), 0, 0, 0)$



$$E(J) = \lim_{T\to+\infty} -\frac{1}{iT}\log\frac{Z(J)}{Z(0)}$$

-In the case of QED we have [Zee ‘10] : $E = -\dfrac{e^2}{4\pi R}$

-In the case of QCD we have [Bander ‘81] : $E \sim A + \dfrac{B}{R} + \sigma R$

Corrections to Newton's potential

[Muzinich & Vokos '95]

Static source in gravity : $T^{\mu\nu}(x) = (M\delta^{(3)}(\vec{x}) + m\delta^{(3)}(\vec{x} - \vec{R}))\delta^{\mu}_{0}\delta^{\nu}_{0}\Pi(t) \quad \text{with} \quad m \ll M$

Total action : $$S = \underbrace{-\frac{2}{\kappa^2}\int d^4x\sqrt{-g}R}_{S_{EH}} \underbrace{-m\int d\tau_1 - M\int d\tau_2}_{S_M}$$

Energy : $$E = \lim_{T\to+\infty} -\frac{1}{iT}\frac{Z(J)}{Z(0)} = \lim_{T\to+\infty} -\frac{1}{iT}\langle e^{\frac{i}{\hbar}S_M}\rangle$$

Corrections to Newton's potential

[Muzinich & Vokos ‘95]

Static source in gravity : $T^{\mu\nu}(x) = (M\delta^{(3)}(\vec{x}) + m\delta^{(3)}(\vec{x}-\vec{R}))\delta_0^\mu\delta_0^\nu\Pi(t) \quad \text{with} \quad m \ll M$

Total action : $S = \underbrace{-\frac{2}{\kappa^2}\int d^4x\sqrt{-g}R}_{S_{EH}} \underbrace{- m\int d\tau_1 - M\int d\tau_2}_{S_M}$

Energy : $E = \lim_{T\to+\infty} -\frac{1}{iT}\frac{Z(J)}{Z(0)} = \lim_{T\to+\infty} -\frac{1}{iT}\langle e^{\frac{i}{\hbar}S_M}\rangle$

At second order in G we have : $E = -\frac{GMm}{R} + \frac{G^2M^2m}{2R^2} - \frac{17G^2\hbar}{20\pi R^3}$

Newton potential

PN correction

Quantum correction

Non perturbative quantum black hole models

We focus on **metric** quantum black holes models, i.e. modified Schwarzschild metric :

-The Hayward metric [Hayward '05] : $ds^2 = \left(1 - \frac{2MGr^2}{r^3 + 2MG^2\hbar}\right) dt^2 - \frac{dr^2}{1 - \frac{2MGr^2}{r^3 + 2MG^2\hbar}} - r^2 d\Omega^2$

Non perturbative quantum black hole models

We focus on **metric** quantum black holes models, i.e. modified Schwarzschild metric :

-The Hayward metric [Hayward ‘05] : $ds^2 = \left(1 - \frac{2MGr^2}{r^3 + 2MG^2\hbar}\right) dt^2 - \frac{dr^2}{1 - \frac{2MGr^2}{r^3+2MG^2\hbar}} - r^2 d\Omega^2$

-Polymer black hole models : [Ashtekar Olmedo & Singh ‘18, Bodendorfer Mele & Munch ‘19, Ben Achour Lamy Liu & Noui ‘19, Alonso Bardaji Brizuela & Vera ‘21 …]

Non perturbative quantum black hole models

We focus on **metric** quantum black holes models, i.e. modified Schwarzschild metric :

-The Hayward metric [Hayward '05] : $ds^2 = \left(1 - \frac{2MGr^2}{r^3 + 2MG^2\hbar}\right) dt^2 - \frac{dr^2}{1 - \frac{2MGr^2}{r^3+2MG^2\hbar}} - r^2 d\Omega^2$

-Polymer black hole models : [Ashtekar Olmedo & Singh '18, Bodendorfer Mele & Munch '19, Ben Achour Lamy Liu & Noui '19, Alonso Bardaji Brizuela & Vera '21 …]

-Models inspired by string theory : [Nicolini Spallucci & Wondrak '19]

Static potential in GR

We consider a general **static and sph. sym.** metric : $ds^2 = g_{tt}(r)dt^2 - g_{rr}(r)dr^2 - r^2 d\Omega^2$

Lagrangian of a **test point particle of mass m** moving radially : $L = -m\frac{d\tau}{dt} = -m\sqrt{g_{tt} - v^2 g_{rr}}$

Static potential in GR

We consider a general **static and sph. sym.** metric : $ds^2 = g_{tt}(r)dt^2 - g_{rr}(r)dr^2 - r^2 d\Omega^2$

Lagrangian of a **test point particle of mass m** moving radially : $L = -m\frac{d\tau}{dt} = -m\sqrt{g_{tt} - v^2 g_{rr}}$

Hamiltonian : $H = m\frac{g_{tt}}{\sqrt{g_{tt} - v(p)^2 g_{rr}}} \xrightarrow{v=0} m\sqrt{g_{tt}}$

For the Schwarzschild metric : $H \approx m - \frac{GmM}{r} - \frac{G^2mM^2}{2r^2} \xrightarrow{r=R+MG} m - \frac{GmM}{R} + \frac{G^2mM^2}{2R^2}$

Masse energy

Newton potential

PN correction

Confrontation of the two approaches

We can now imagine a simple recipe to test modified Schwarzschild models :

-Expand $H = m\sqrt{g_{tt}}$ at order 2 in G.

-Write this expansion in the Harmonic gauge.

-Compare the PN and the quantum corrections to the graviton corrections.

Example : The Hayward metric

The Hayward metric is $ds^2 = \left(1 - \frac{2MGr^2}{r^3 + 2MG^2\hbar}\right) dt^2 - \frac{dr^2}{1 - \frac{2MGr^2}{r^3+2MG^2\hbar}} - r^2 d\Omega^2$

The Hamiltonian of a static test particle is $H = m - \frac{GmM}{r} - \frac{G^2mM^2}{2r^2} + \mathcal{O}(G^3)$

The area and harmonic radius are related by $r = R + MG + \mathcal{O}(G^2)$

We end up with $H = m - \frac{GmM}{R} + \frac{G^2mM^2}{2R^2} + \mathcal{O}(G^3)$

The graviton correction is not recover

Conclusion and outlook

-The EFT treatment of GR gives predictions at low energy that a non perturbative approach should recover.

-It is possible to see if a modified Schwarzschild metric reproduces one of these predictions, i.e. the graviton correction to Newton's potential.

-There is other predictions of the EFT treatment of GR (deviation angles [Bjerrum-Bohr, John F. Donoghue, Barry R. Holstein, Ludovic Planté, Pierre Vanhove]) which should impose more constraints on non perturbative models…