Gravitational waves from twisted light

Based on arXiv:2309.04191

Main collaborators:

Eduard Atonga, Ramy Aboushelbaya and Peter Norreys (Univ. of Oxford) , Aurélien Barrau (LPSC Grenoble), Chunshan Lin (Jagiellonian Univ.)

The action of gravity on light is well known but the converse

- i.e. the way light acts as a source of gravity -

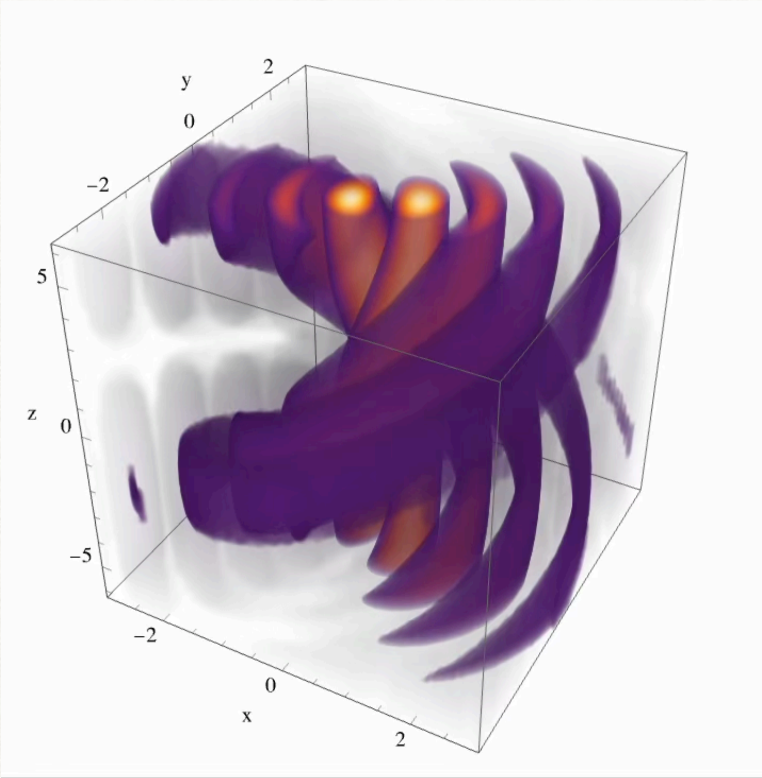
remains, to a large extent, unexplored.

On twisted light

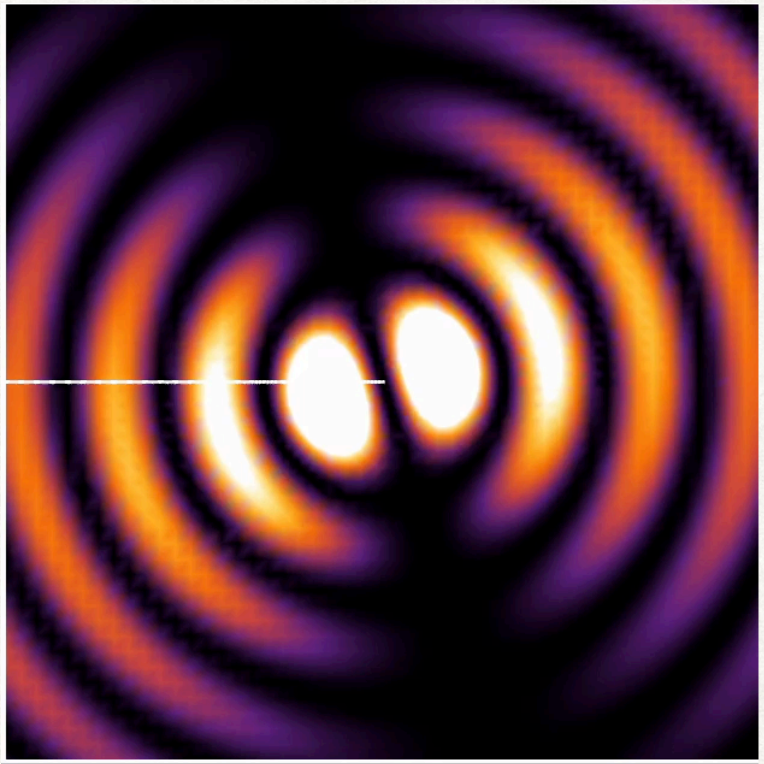
- **Light can carry orbital angular momentum (OAM)**

  The OAM is characterized by an azimuthal phase dependance of the electromagnetic field.

$$E(x^\mu) \propto e^{il\phi}, l \in \mathbb{Z}$$



l = 1 twisted light propagating along the z-axis

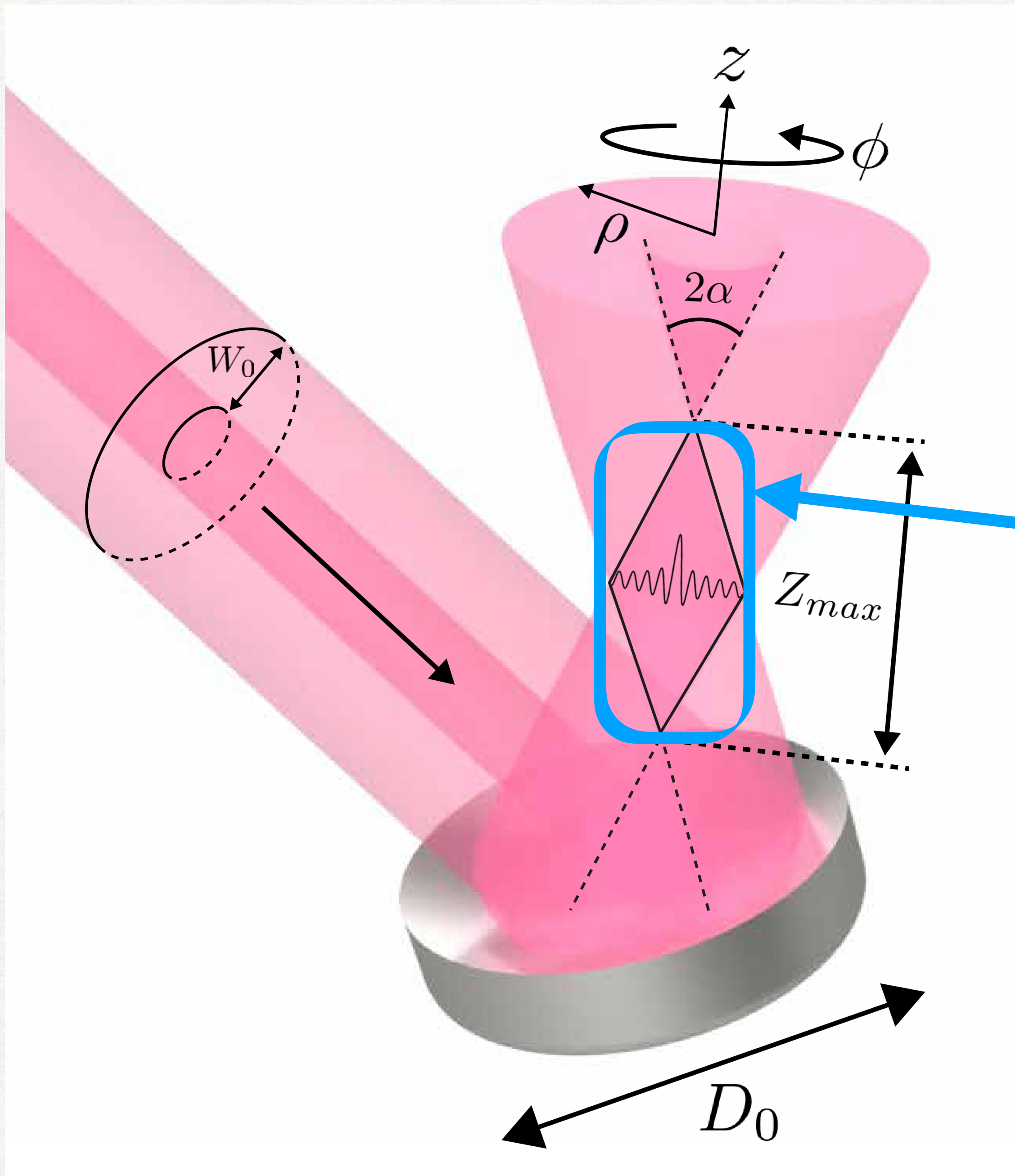


Projection onto the azimuthal plane

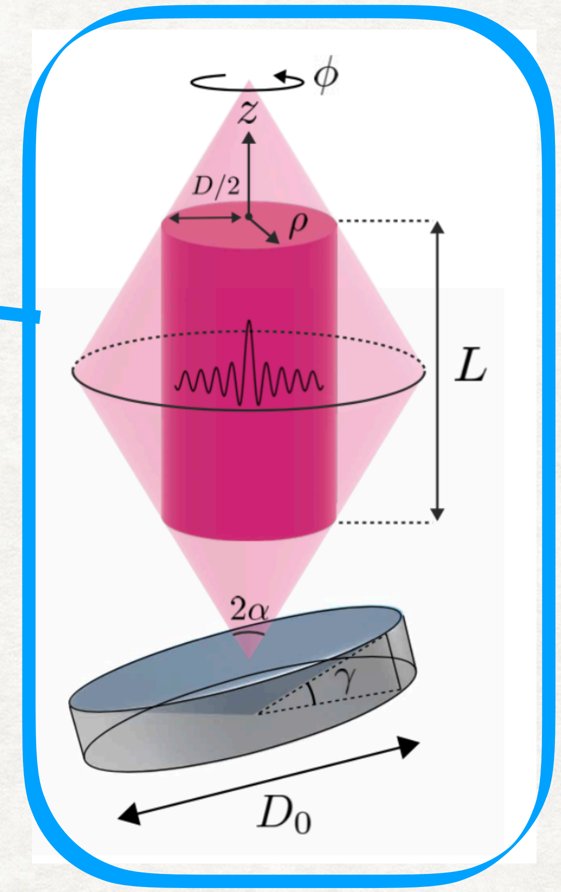
- **Different types of twisted beams:** - Laguerre-Gaussian beams

  - Bessel beams

Generation of Bessel modes



Region over which twisted light is generated



Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k} J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k} J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$
$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$
$$E_z = 0$$

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k} J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin\left(\omega t - k_z z + (l-1)\phi\right)\right]$$
$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos\left(\omega t - k_z z + (l-1)\phi\right)\right]$$
$$E_z = 0$$

$$B_x = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$
$$B_y = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$
$$B_z = \sin(\alpha)\frac{E_0}{c}J_l(\beta\rho)\cos(\omega t - k_z z + l\phi)$$

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k}J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$
$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$
$$E_z = 0$$

$$B_x = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$
$$B_y = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$
$$B_z = \sin(\alpha)\frac{E_0}{c}J_l(\beta\rho)\cos(\omega t - k_z z + l\phi)$$

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k}J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_z = 0$$

ω : **Pulse frequency**

$$B_x = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_y = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_z = \sin(\alpha)\frac{E_0}{c}J_l(\beta\rho)\cos(\omega t - k_z z + l\phi)$$

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k} J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_z = 0$$

$$B_x = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_y = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_z = \sin(\alpha)\frac{E_0}{c}J_l(\beta\rho)\cos(\omega t - k_z z + l\phi)$$

ω : **Pulse frequency**

α : **Half-cone angle**

Characterization of the Bessel modes

- **Bessel modes can be modeled as an infinite superposition of plane waves:**

$$\vec{E}(\vec{x},t) = \frac{1}{2\pi}\int_0^{2\pi} i^l E_0 e^{i(k_\mu(\phi)x^\mu + l\phi)}\hat{n}(\phi)d\phi$$

$\phi \equiv \arctan(y/x)$
Azimuthal position on the conical surface

- **Bessel mode E-field orbital angular momentum**

$$\vec{J} = \iint \frac{\epsilon_0 E_0}{c}\frac{l}{k}J_l^2(\beta\rho)\hat{z}dS$$

l : OAM parameter
J_l : Bessel functions

- **Fields components**

$$E_x = \frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_y = -\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$E_z = 0$$

$$B_x = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\cos(\omega t - k_z z + (l+1)\phi) + J_{l-1}(\beta\rho)\cos(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_y = \cos(\alpha)\frac{E_0}{2}\left[J_{l+1}(\beta\rho)\sin(\omega t - k_z z + (l+1)\phi) - J_{l-1}(\beta\rho)\sin(\omega t - k_z z + (l-1)\phi)\right]$$

$$B_z = \sin(\alpha)\frac{E_0}{c}J_l(\beta\rho)\cos(\omega t - k_z z + l\phi)$$

ω : **Pulse frequency**

α : **Half-cone angle**

β : **Beam waist**

Associated spacetime deformations

- **<u>Stress-energy tensor of the system</u>** *E. Atonga, K.M, R. Aboushelbaya, A. Barrau et. al.. 2309.04191*

$$T^{\mu\nu} = \begin{pmatrix} u & \vec{N}/c \\ \vec{N}/c & -\sigma_{i,j} \end{pmatrix}$$

$$u = \frac{\epsilon_0 c}{2}(E^2 + c^2 B^2)$$

EM field energy density

$$\vec{N} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Poynting vector

$$\sigma_{ij} = \epsilon_0 c(E_i E_j + c^2 B_i B_j) - u\delta_{i,j}$$

Maxwell tensor

Associated spacetime deformations

- **Stress-energy tensor of the system** *E. Atonga, K.M, R. Aboushelbaya, A. Barrau et. al.. 2309.04191*

$$T^{\mu\nu} = \begin{pmatrix} u & \vec{N}/c \\ \vec{N}/c & -\sigma_{i,j} \end{pmatrix}$$

$$u = \frac{\epsilon_0 c}{2}(E^2 + c^2 B^2)$$

EM field energy density

$$\vec{N} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Poynting vector

$$\sigma_{ij} = \epsilon_0 c(E_i E_j + c^2 B_i B_j) - u\delta_{i,j}$$

Maxwell tensor

- **Associated spacetime deformations**

In the compact source approximation:

$$\bar{h}^{\mu\nu}(t, \vec{x}) = \frac{1}{r}\frac{4G}{c^4}\int_{\mathcal{V}} T^{\mu\nu}\left(t - \frac{r}{c} + \frac{\vec{x'} \cdot \hat{n}}{c}, \vec{x'}\right) d^3x'$$

Associated spacetime deformations

- **Stress-energy tensor of the system** *E. Atonga, K.M, R. Aboushelbaya, A. Barrau et. al.. 2309.04191*

$$T^{\mu\nu} = \begin{pmatrix} u & \vec{N}/c \\ \vec{N}/c & -\sigma_{i,j} \end{pmatrix}$$

$$u = \frac{\epsilon_0 c}{2}(E^2 + c^2 B^2)$$

EM field energy density

$$\vec{N} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

Poynting vector

$$\sigma_{ij} = \epsilon_0 c(E_i E_j + c^2 B_i B_j) - u\delta_{i,j}$$

Maxwell tensor

- **Associated spacetime deformations**

In the compact source approximation:

$$\bar{h}^{\mu\nu}(t, \vec{x}) = \frac{1}{r}\frac{4G}{c^4}\int_{\mathcal{V}} T^{\mu\nu}\left(t - \frac{r}{c} + \frac{\vec{x'} \cdot \hat{n}}{c}, \vec{x'}\right) d^3x'$$

- **Projection into the TT gauge**

$$\bar{h}^{TT}_{ij} = \Lambda(\hat{n})_{ij,kl}\bar{h}^{kl}$$

Spacetime deformations in the TT gauge

$$\bar{h}_{\mu\nu}^{TT} = \bar{h}_{\mu\nu}^{D,TT} + \bar{h}_{\mu\nu}^{ZZ,TT} + \bar{h}_{\mu\nu}^{+,TT} + \bar{h}_{\mu\nu}^{\times,TT}$$

Spacetime deformations in the TT gauge

$$\bar{h}^{TT}_{\mu\nu} = \bar{h}^{D,TT}_{\mu\nu} + \bar{h}^{ZZ,TT}_{\mu\nu} + \bar{h}^{+,TT}_{\mu\nu} + \bar{h}^{\times,TT}_{\mu\nu}$$

$$\bar{h}^{D,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \bar{h}_D$$

$$\bar{h}^{ZZ,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \bar{h}_{ZZ}$$

$$\bar{h}^{+,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cos^2(\theta)\left[\cos^2(\theta)+1\right] & 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] \\ 0 & 0 & -\frac{1}{2}\left[1+\cos^2(\theta)\right] & 0 \\ 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] & 0 & \frac{1}{2}\sin^2(\theta)\left[\cos^2(\theta)+1\right] \end{pmatrix} \bar{h}_+$$

$$\bar{h}^{\times,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2(\theta) & 0 \\ 0 & \cos^2(\theta) & 0 & -\sin(\theta)\cos(\theta) \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \end{pmatrix} \bar{h}_\times .$$

Spacetime deformations in the TT gauge

$$\bar{h}^{TT}_{\mu\nu} = \bar{h}^{D,TT}_{\mu\nu} + \bar{h}^{ZZ,TT}_{\mu\nu} + \bar{h}^{+,TT}_{\mu\nu} + \bar{h}^{\times,TT}_{\mu\nu}$$

$$\begin{aligned}
\bar{h}_{+} &\equiv 2\bar{h}^{(2)}_{+} + \bar{h}^{(l+1)}_{+} + \bar{h}^{(l-1)}_{+} \ , \\
\bar{h}_{\times} &\equiv 2\bar{h}^{(2)}_{\times} + \bar{h}^{(l+1)}_{\times} + \bar{h}^{(l-1)}_{\times} \ , \\
\bar{h}_{XZ} &\equiv \bar{h}^{+,(1)}_{XZ} + \bar{h}^{-,(1)}_{XZ} + \bar{h}^{+,(2l+1)}_{XZ} + \bar{h}^{-,(2l-1)}_{XZ} \\
\bar{h}_{YZ} &\equiv \bar{h}^{+,(1)}_{YZ} - \bar{h}^{-,(1)}_{YZ} + \bar{h}^{+,(2l+1)}_{YZ} - \bar{h}^{-,(2l-1)}_{YZ}
\end{aligned}$$

$$\bar{h}^{D,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \boxed{\bar{h}_D}$$

$$\bar{h}^{ZZ,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \boxed{\bar{h}_{ZZ}}$$

$$\bar{h}^{+,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cos^2(\theta)\left[\cos^2(\theta)+1\right] & 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] \\ 0 & 0 & -\frac{1}{2}\left[1+\cos^2(\theta)\right] & 0 \\ 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] & 0 & \frac{1}{2}\sin^2(\theta)\left[\cos^2(\theta)+1\right] \end{pmatrix} \boxed{\bar{h}_{+}}$$

$$\bar{h}^{\times,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2(\theta) & 0 \\ 0 & \cos^2(\theta) & 0 & -\sin(\theta)\cos(\theta) \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \end{pmatrix} \boxed{\bar{h}_{\times}} \, .$$

Spacetime deformations in the TT gauge

$$\bar{h}^{TT}_{\mu\nu} = \bar{h}^{D,TT}_{\mu\nu} + \bar{h}^{ZZ,TT}_{\mu\nu} + \bar{h}^{+,TT}_{\mu\nu} + \bar{h}^{\times,TT}_{\mu\nu}$$

$$\bar{h}^{D,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\left[1-\cos(2\theta)\right] & 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] & 0 & \sin^2(\theta)\left[1-cos(2\theta)\right] \end{pmatrix} \bar{h}_D$$

$$\bar{h}^{ZZ,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\left[1-\cos(2\theta)\right] & 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] & 0 & \sin^2(\theta)\left[1-cos(2\theta)\right] \end{pmatrix} \bar{h}_{ZZ}$$

$$\bar{h}^{+,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cos^2(\theta)\left[\cos^2(\theta)+1\right] & 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] \\ 0 & 0 & -\frac{1}{2}\left[1+\cos^2(\theta)\right] & 0 \\ 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] & 0 & \frac{1}{2}\sin^2(\theta)\left[\cos^2(\theta)+1\right] \end{pmatrix} \bar{h}_+$$

$$\bar{h}^{\times,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2(\theta) & 0 \\ 0 & \cos^2(\theta) & 0 & -\sin(\theta)\cos(\theta) \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \end{pmatrix} \bar{h}_\times .$$

$$\begin{aligned} \bar{h}_+ &\equiv 2\bar{h}^{(2)}_+ + \bar{h}^{(l+1)}_+ + \bar{h}^{(l-1)}_+ \ , \\ \bar{h}_\times &\equiv 2\bar{h}^{(2)}_\times + \bar{h}^{(l+1)}_\times + \bar{h}^{(l-1)}_\times \ , \\ \bar{h}_{XZ} &\equiv \bar{h}^{+,(1)}_{XZ} + \bar{h}^{-,(1)}_{XZ} + \bar{h}^{+,(2l+1)}_{XZ} + \bar{h}^{-,(2l-1)}_{XZ} \\ \bar{h}_{YZ} &\equiv \bar{h}^{+,(1)}_{YZ} - \bar{h}^{-,(1)}_{YZ} + \bar{h}^{+,(2l+1)}_{YZ} - \bar{h}^{-,(2l-1)}_{YZ} \end{aligned}$$

$$\begin{aligned} \bar{h}_D &\equiv \frac{1}{2}\bar{h}_0(r)\sin^2(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \ , \\ \bar{h}_{ZZ} &\equiv 2\bar{h}_0(r)\left[1+\cos^2(\alpha)\right]\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right] \\ &\quad \times\cos(\psi_q) \ , \\ \bar{h}^{(Q)}_+ &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \ , \\ \bar{h}^{(Q)}_\times &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \ , \\ \bar{h}^{\pm,(s)}_{XZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \\ \bar{h}^{\pm,(s)}_{YZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \\ \bar{h}_N &\equiv 2\bar{h}_0(r)\cos(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \ . \end{aligned}$$

Spacetime deformations in the TT gauge

$$\bar{h}^{TT}_{\mu\nu} = \bar{h}^{D,TT}_{\mu\nu} + \bar{h}^{ZZ,TT}_{\mu\nu} + \bar{h}^{+,TT}_{\mu\nu} + \bar{h}^{\times,TT}_{\mu\nu}$$

$$\bar{h}^{D,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\left[1-\cos(2\theta)\right] & 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] & 0 & \sin^2(\theta)\left[1-cos(2\theta)\right] \end{pmatrix} \boxed{\bar{h}_D}$$

$$\bar{h}^{ZZ,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\left[1-\cos(2\theta)\right] & 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\left[\cos(2\theta)-1\right] & 0 & \sin^2(\theta)\left[1-cos(2\theta)\right] \end{pmatrix} \boxed{\bar{h}_{ZZ}}$$

$$\bar{h}^{+,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cos^2(\theta)\left[\cos^2(\theta)+1\right] & 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] \\ 0 & 0 & -\frac{1}{2}\left[1+\cos^2(\theta)\right] & 0 \\ 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] & 0 & \frac{1}{2}\sin^2(\theta)\left[\cos^2(\theta)+1\right] \end{pmatrix} \boxed{\bar{h}_+}$$

$$\bar{h}^{\times,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2(\theta) & 0 \\ 0 & \cos^2(\theta) & 0 & -\sin(\theta)\cos(\theta) \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \end{pmatrix} \boxed{\bar{h}_\times}$$

$$\begin{aligned} \bar{h}_+ &\equiv 2\bar{h}^{(2)}_+ + \bar{h}^{(l+1)}_+ + \bar{h}^{(l-1)}_+ \ , \\ \bar{h}_\times &\equiv 2\bar{h}^{(2)}_\times + \bar{h}^{(l+1)}_\times + \bar{h}^{(l-1)}_\times \ , \\ \bar{h}_{XZ} &\equiv \bar{h}^{+,(1)}_{XZ} + \bar{h}^{-,(1)}_{XZ} + \bar{h}^{+,(2l+1)}_{XZ} + \bar{h}^{-,(2l-1)}_{XZ} \\ \bar{h}_{YZ} &\equiv \bar{h}^{+,(1)}_{YZ} - \bar{h}^{-,(1)}_{YZ} + \bar{h}^{+,(2l+1)}_{YZ} - \bar{h}^{-,(2l-1)}_{YZ} \end{aligned}$$

$$\begin{aligned} \bar{h}_D &\equiv \frac{1}{2}\bar{h}_0(r)\sin^2(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \ , \\ \bar{h}_{ZZ} &\equiv 2\bar{h}_0(r)\left[1+\cos^2(\alpha)\right]\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right] \\ &\quad \times\cos(\psi_q) \ , \\ \bar{h}^{(Q)}_+ &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \ , \\ \bar{h}^{(Q)}_\times &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \ , \\ \bar{h}^{\pm,(s)}_{XZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \\ \bar{h}^{\pm,(s)}_{YZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \\ \bar{h}_N &\equiv 2\bar{h}_0(r)\cos(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \ . \end{aligned}$$

$$\begin{aligned} \bar{h}_0(r) &\equiv \frac{4\pi\epsilon_0 c E_0^2 GL}{\beta^2 c^5 r} \ , \\ \psi_q(t,r) &\equiv 2\omega(t-r/c) + 2q(\phi-\pi/2) \ , \\ \Gamma_q(\theta) &\equiv \int_0^{\frac{D\beta}{2}} \tau J_q^2(\tau) J_{2q}\left(\frac{\omega\tau}{c\beta}\sin(\theta)\right) d\tau \ , \\ \Lambda^\pm_s(\theta) &\equiv \int_0^{\frac{D\beta}{2}} \tau J_l(\tau) J_{l\pm1}(\tau) J_s\left(\frac{\omega\tau}{c\beta}\sin(\theta)\right) d\tau \\ \eta(\theta) &\equiv \frac{\omega L}{c}\left[\cos(\theta)-\cos(\alpha)\right] \ , \end{aligned}$$

Spacetime deformations in the TT gauge

$$\bar{h}^{TT}_{\mu\nu} = \bar{h}^{D,TT}_{\mu\nu} + \bar{h}^{ZZ,TT}_{\mu\nu} + \bar{h}^{+,TT}_{\mu\nu} + \bar{h}^{\times,TT}_{\mu\nu}$$

$$\bar{h}^{D,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \bar{h}_D$$

$$\bar{h}^{ZZ,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos^2(\theta)\,[1-\cos(2\theta)] & 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] \\ 0 & 0 & \cos(2\theta)-1 & 0 \\ 0 & \cos(\theta)\sin(\theta)\,[\cos(2\theta)-1] & 0 & \sin^2(\theta)\,[1-cos(2\theta)] \end{pmatrix} \bar{h}_{ZZ}$$

$$\bar{h}^{+,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\cos^2(\theta)\left[\cos^2(\theta)+1\right] & 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] \\ 0 & 0 & -\frac{1}{2}\left[1+\cos^2(\theta)\right] & 0 \\ 0 & -\frac{1}{2}\sin(\theta)\cos(\theta)\left[\cos^2(\theta)+1\right] & 0 & \frac{1}{2}\sin^2(\theta)\left[\cos^2(\theta)+1\right] \end{pmatrix} \bar{h}_+$$

$$\bar{h}^{\times,TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \cos^2(\theta) & 0 \\ 0 & \cos^2(\theta) & 0 & -\sin(\theta)\cos(\theta) \\ 0 & 0 & -\sin(\theta)\cos(\theta) & 0 \end{pmatrix} \bar{h}_\times .$$

$$\begin{aligned} \bar{h}_+ &\equiv 2\bar{h}^{(2)}_+ + \bar{h}^{(l+1)}_+ + \bar{h}^{(l-1)}_+ , \\ \bar{h}_\times &\equiv 2\bar{h}^{(2)}_\times + \bar{h}^{(l+1)}_\times + \bar{h}^{(l-1)}_\times , \\ \bar{h}_{XZ} &\equiv \bar{h}^{+,(1)}_{XZ} + \bar{h}^{-,(1)}_{XZ} + \bar{h}^{+,(2l+1)}_{XZ} + \bar{h}^{-,(2l-1)}_{XZ} \\ \bar{h}_{YZ} &\equiv \bar{h}^{+,(1)}_{YZ} - \bar{h}^{-,(1)}_{YZ} + \bar{h}^{+,(2l+1)}_{YZ} - \bar{h}^{-,(2l-1)}_{YZ} \end{aligned}$$

$$\begin{aligned} \bar{h}_D &\equiv \frac{1}{2}\bar{h}_0(r)\sin^2(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) , \\ \bar{h}_{ZZ} &\equiv 2\bar{h}_0(r)\left[1+\cos^2(\alpha)\right]\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right] \\ &\quad \times\cos(\psi_q) , \\ \bar{h}^{(Q)}_+ &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) , \\ \bar{h}^{(Q)}_\times &\equiv \bar{h}_0(r)\sin^2(\alpha)\Gamma_Q(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) , \\ \bar{h}^{\pm,(s)}_{XZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) \\ \bar{h}^{\pm,(s)}_{YZ} &\equiv \frac{1}{4}\bar{h}_0(r)\sin(2\alpha)\Lambda^\pm_s(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\sin(\psi_q) \\ \bar{h}_N &\equiv 2\bar{h}_0(r)\cos(\alpha)\Gamma_l(\theta)\mathrm{Sinc}\left[\eta(\theta)\right]\cos(\psi_q) . \end{aligned}$$

$$\begin{aligned} \bar{h}_0(r) &\equiv \frac{4\pi\epsilon_0 c E_0^2 G L}{\beta^2 c^5 r} , \\ \psi_q(t,r) &\equiv 2\omega(t-r/c) + 2q(\phi-\pi/2) , \\ \Gamma_q(\theta) &\equiv \int_0^{\frac{D\beta}{2}} \tau J_q^2(\tau) J_{2q}\left(\frac{\omega\tau}{c\beta}\sin(\theta)\right) d\tau , \\ \Lambda^\pm_s(\theta) &\equiv \int_0^{\frac{D\beta}{2}} \tau J_l(\tau) J_{l\pm1}(\tau) J_s\left(\frac{\omega\tau}{c\beta}\sin(\theta)\right) d\tau \\ \eta(\theta) &\equiv \frac{\omega L}{c}\left[\cos(\theta)-\cos(\alpha)\right] , \end{aligned}$$

L : Interaction length

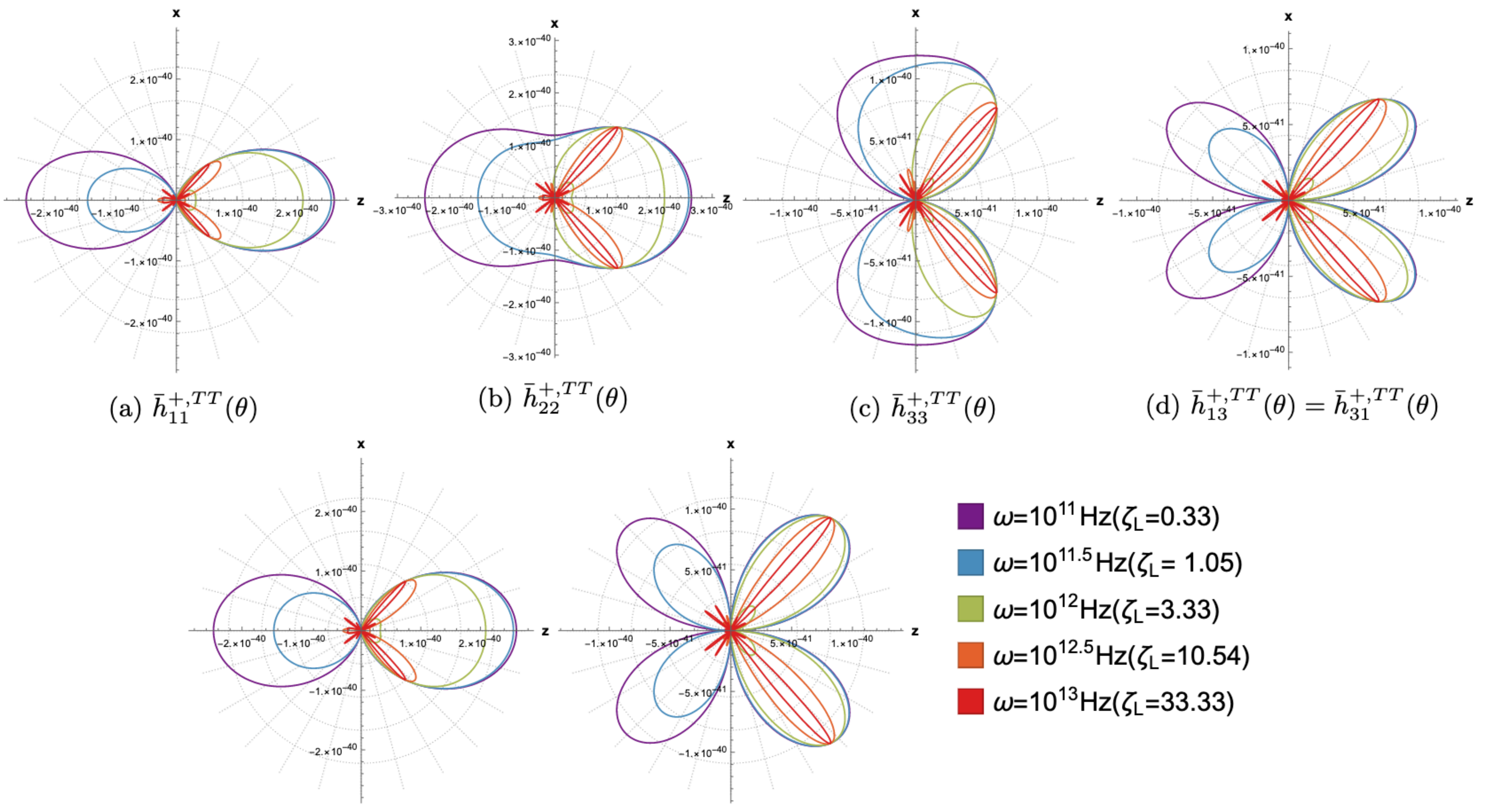
ω : Pulse frequency

α : Half-cone angle

β : Beam waist

D : Interaction region width

Spacetime deformations in the TT gauge

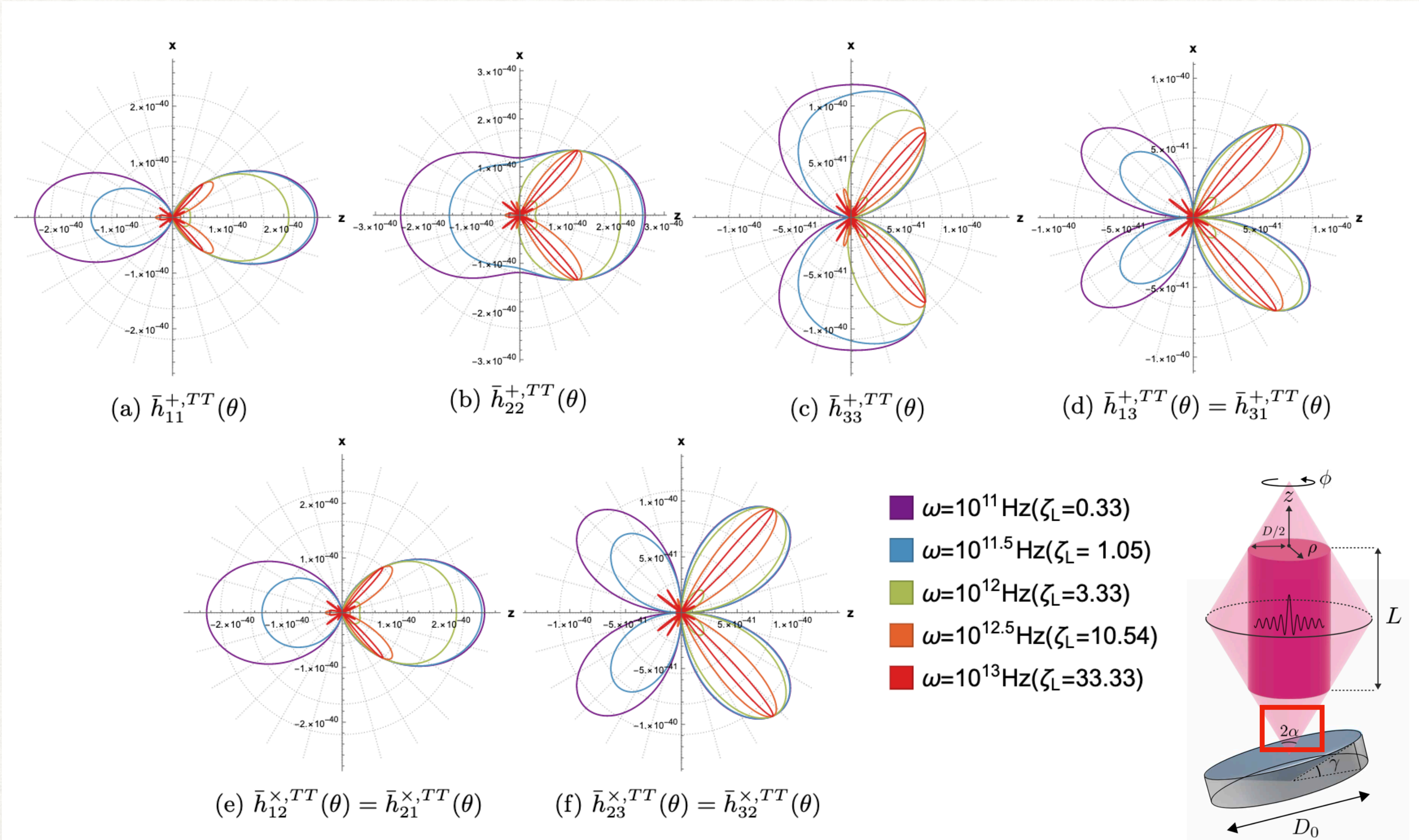


(a) $\bar{h}^{+,TT}_{11}(\theta)$ (b) $\bar{h}^{+,TT}_{22}(\theta)$ (c) $\bar{h}^{+,TT}_{33}(\theta)$ (d) $\bar{h}^{+,TT}_{13}(\theta) = \bar{h}^{+,TT}_{31}(\theta)$

(e) $\bar{h}^{\times,TT}_{12}(\theta) = \bar{h}^{\times,TT}_{21}(\theta)$ (f) $\bar{h}^{\times,TT}_{23}(\theta) = \bar{h}^{\times,TT}_{32}(\theta)$

When $\zeta_L \equiv \omega L / c > 1$ ⟶ **Beaming effect**

Spacetime deformations in the TT gauge



(a) $\bar{h}^{+,TT}_{11}(\theta)$ (b) $\bar{h}^{+,TT}_{22}(\theta)$ (c) $\bar{h}^{+,TT}_{33}(\theta)$ (d) $\bar{h}^{+,TT}_{13}(\theta) = \bar{h}^{+,TT}_{31}(\theta)$

(e) $\bar{h}^{\times,TT}_{12}(\theta) = \bar{h}^{\times,TT}_{21}(\theta)$ (f) $\bar{h}^{\times,TT}_{23}(\theta) = \bar{h}^{\times,TT}_{32}(\theta)$

When $\zeta_L \equiv \omega L/c > 1$ $\longrightarrow$ **Beaming effect towards the half-cone angle** α

Main characteristics of the GW signal

- **Frequency**

$$\omega_{GW} = 2\omega_{\text{pulse}} \simeq 8.55 \times 10^{14} Hz$$

For a NIF's 351 nm laser pulse

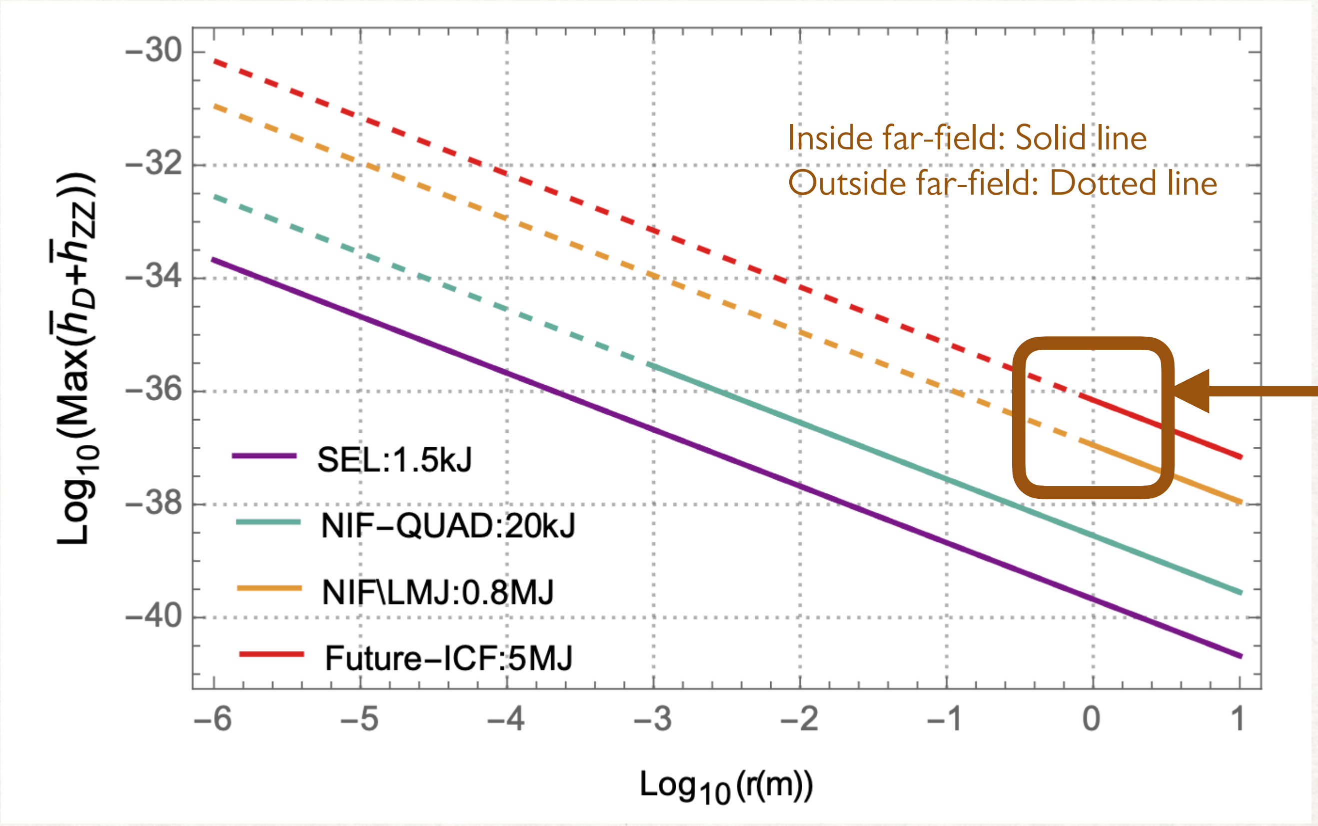
Main characteristics of the GW signal

- **Frequency**

$$\omega_{GW} = 2\omega_{\text{pulse}} \simeq 8.55 \times 10^{14} Hz$$

For a NIF's 351 nm laser pulse

- **Strain**



$h \sim 10^{-36}$ at best!

One meter away from the source

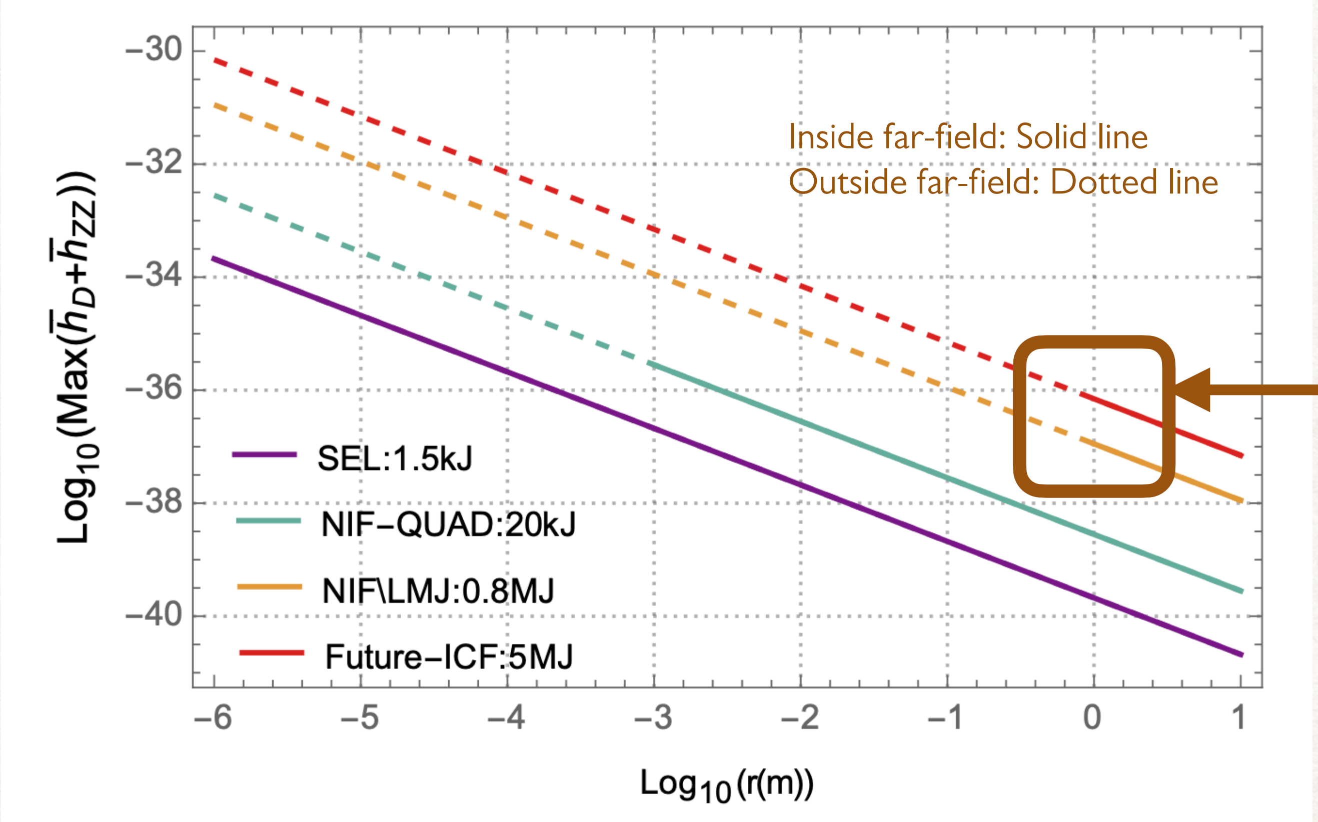
Main characteristics of the GW signal

- **Frequency**

$$\omega_{GW} = 2\omega_{\text{pulse}} \simeq 8.55 \times 10^{14} Hz$$

For a NIF's 351 nm laser pulse

- **Strain**



$h \sim 10^{-36}$ at best!

One meter away from the source

- **Power**

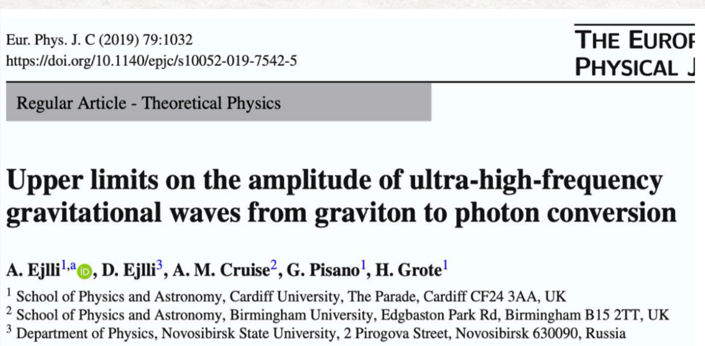
  - Excellent control on the direction of emission
  - $P \sim 9.4 \times 10^{-5}$ W.m^{-2} $> P_{GW15094} \sim 3.6 \times 10^{-6}$ W.m^{-2} *Powers evaluated at the detector location*

Gravitational waves from high-power twisted beams

In definitive

✓
- **The optical setup provides a very good control over the GWs properties**
  (frequency, direction of emission, polarisation states)
- **Subtle and unexpected effects do appear**
  (Beaming effect towards the half-cone angle alpha, ...)

✗
- **Generated strains are too low**


Eur. Phys. J. C (2019) 79:1032
https://doi.org/10.1140/epjc/s10052-019-7542-5

THE EUROP
PHYSICAL J

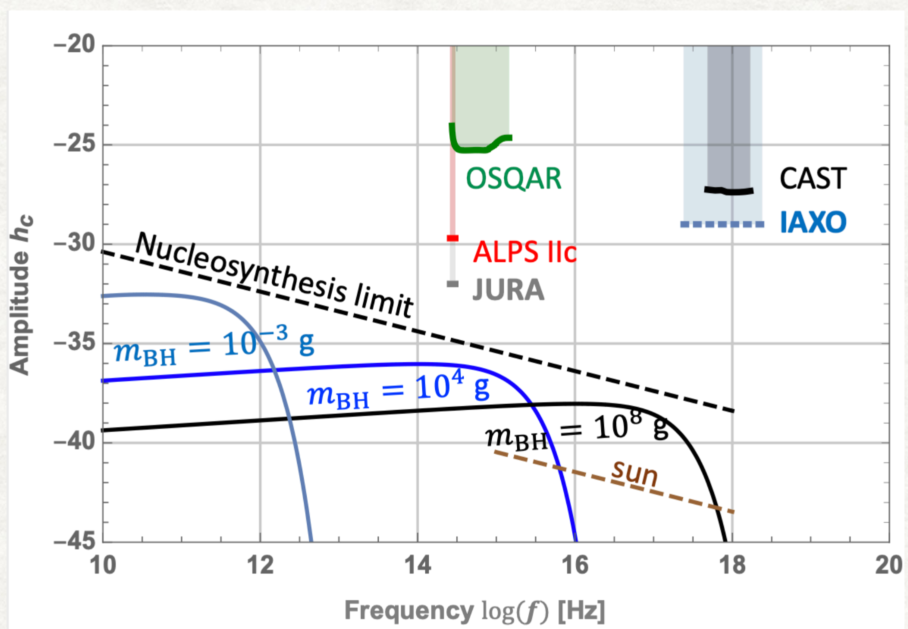
Regular Article - Theoretical Physics

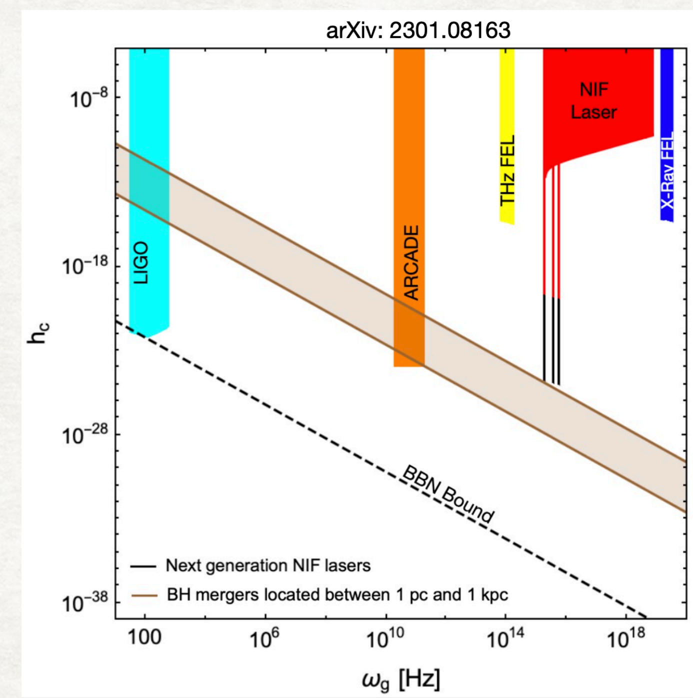
Upper limits on the amplitude of ultra-high-frequency gravitational waves from graviton to photon conversion

A. Ejlli[1,a], D. Ejlli[3], A. M. Cruise[2], G. Pisano[1], H. Grote[1]

[1] School of Physics and Astronomy, Cardiff University, The Parade, Cardiff CF24 3AA, UK
[2] School of Physics and Astronomy, Birmingham University, Edgbaston Park Rd, Birmingham B15 2TT, UK
[3] Department of Physics, Novosibirsk State University, 2 Pirogova Street, Novosibirsk 630090, Russia

Borrowed from Aldo's talk





Borrowed from Georgios's talk

Possibilities to boost the signal?

Beyond any possible application,
the question of the gravitational field created by light is, in itself,
fascinating and largely unexplored.
A lot remains to be understood.

Thank you!