Measuring SGWBs with PTAs

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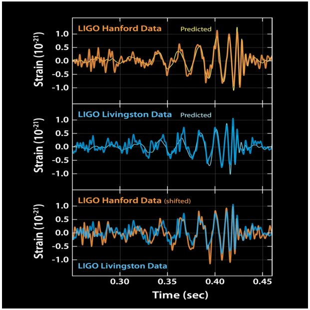
Overview

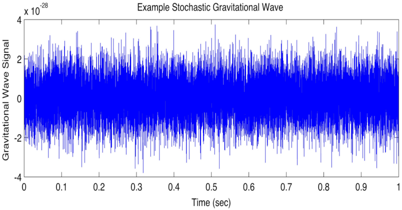
The dawn of GW astronomy

Gravitational Waves (GWs) are:
- Spacetime perturbations
- Almost free streaming
- The ultimate cosmological probe

There are two classes of signals:

- Deterministic signals
  - Typically from binaries
  - Time coherency
  - Localized in the sky
- Stochastic GW Backgrounds
  - Superposition of many signals
  - Early Universe processes
  - No time coherency
  - Diffuse signals

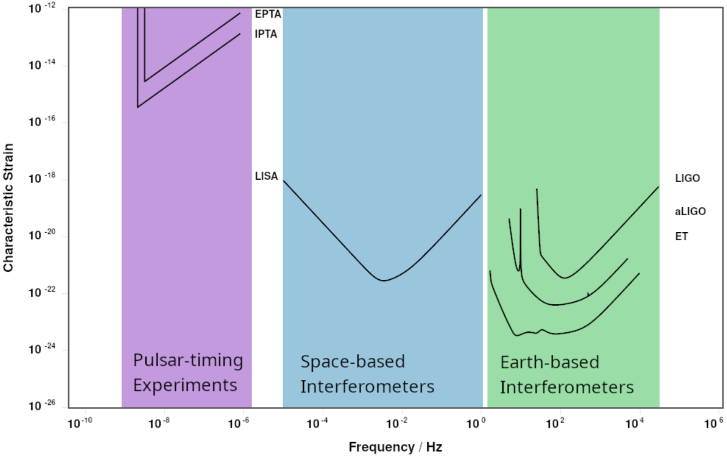




* Figures from https://www.ligo.org/detections/images/ligoGW150914signals-lg.jpg and https://www.ligo.org/science/GW-Overview/images/stochastic.jpg

Present and future GW detectors

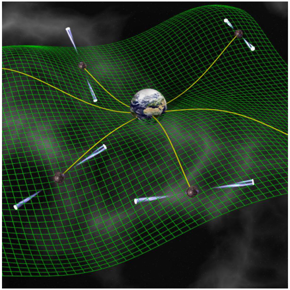
Different types of detectors will probe different frequency bands



The peak of the sensitivity roughly scales with the inverse of the arm length!

* Figure adapted from GWPlotter

First hints of SGWB from PTA experiments in 2020 ...



Pulsars are very precise clocks!

GWs might induce changes in the observed period

Error on the measurement performed with a single pulsar is huge

↓

Monitor many for error reduction

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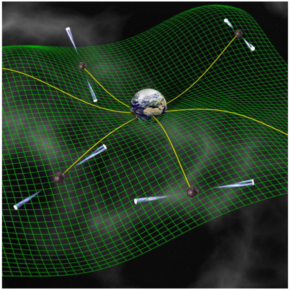

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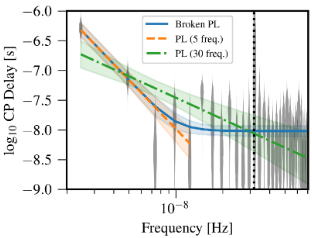
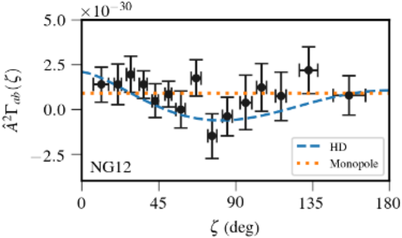




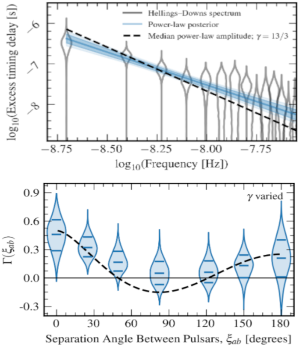
* Figures from https://www.lpc2e.cnrs.fr/en/projets/pta-2

NANOGrav Collaboration, Z. Arzoumanian et al., *Astrophys.J.Lett. 905 (2020) 2, L34*, ArXiv: 2009.04496.

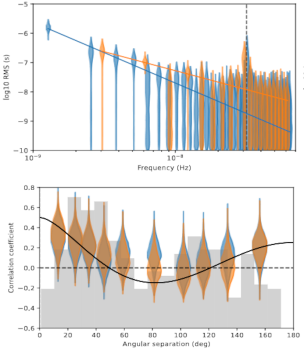
... significance now increasing ...

Better significance in the latest data release in late June !

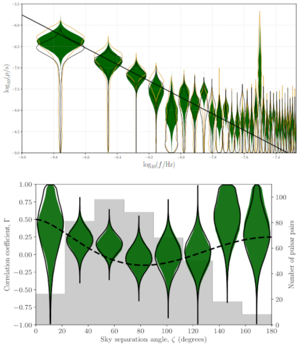
NANOGrav



EPTA and InPTA



PPTA

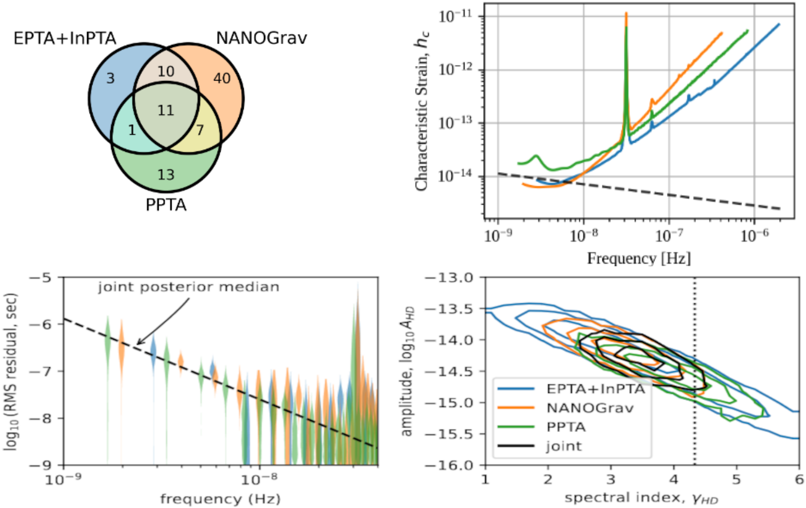


Not only significance for a common process but also for HD correlations!

* Figures from
NANOGrav collaboration, G. Agazie et al., Astrophys.J.Lett. 951 (2023) 1, L8, ArXiv: 2306.16213.
EPTA and InPTA Collaborations, J. Antoniadis et al., Astron.Astrophys. 678 (2023) A50, Astron.Astrophys. 678 (2023) A50, ArXiv: 2306.16214.
PPTA collaboration, Daniel J. Reardon et al., Astrophys.J.Lett. 951 (2023) 1, L6, ArXiv: 2306.16215.

... combining datasets might clarify.

Onging efforts to combine the date from the different collaborations



* Figures from
International Pulsar Timing Array, G. Agazie et al., ArXiv: 2309.00693.

Some general ingredients

Data $\tilde{d}$ (in frequency space) $\longrightarrow$ $\tilde{d} = \tilde{s} + \tilde{n}$

- For individual sources $\langle \tilde{s} \rangle \neq 0$
- For SGWBs $\langle \tilde{s} \rangle = 0$

- For noise $\langle \tilde{n} \rangle = 0$

For an isotropic SGWB $\longrightarrow$ $\langle h_\lambda(\vec{k})\, h^*_{\lambda'}(\vec{k}') \rangle \propto \delta_{\lambda\lambda'} P_h^\lambda(k) \delta(\vec{k} - \vec{k}')$

Assuming $\langle \tilde{s}\tilde{n} \rangle = 0$ and Gaussian signal and noise

$$\left\langle \tilde{d}^2 \right\rangle = \left\langle \tilde{s}^2 \right\rangle + \left\langle \tilde{n}^2 \right\rangle = \sum_\lambda \mathcal{R}_\lambda\, P_h^\lambda + N \equiv \mathcal{R}\left[P_h + S_n\right]$$

where we have introduced

- The (quadratic) response function of the instrument $\mathcal{R}$
- The (intensity of the) signal power spectrum P_h (in 1/Hz)
- The noise power spectrum N (in 1/Hz)
- The (square of the) Strain sensitivity S_n (in 1/Hz)

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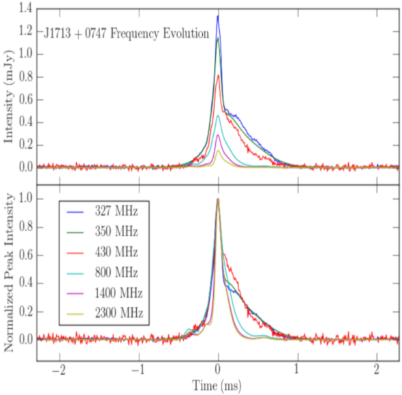
In order to compare with cosmological predictions it's customary to introduce

$$\Omega_{\rm GW} \equiv \frac{1}{3H_0^2 M_p^2} \frac{\partial \rho_{\rm GW}}{\partial \ln f} = \frac{4\pi^2}{3H_0^2} f^3 P_h \qquad \text{and} \qquad \Omega_n(f) = \frac{4\pi^2}{3H_0^2} f^3 S_n(f)\,,$$

where $H_0 \simeq h_0 \times 3.24 \times 10^{-18}\,{\rm Hz}$ is the Hubble parameter today.

Few details on the measurements

Each pulsar is monitored by one (or more) telescopes



- Data are collected every few days (sets the Nyquist, day is $\sim 10^{-5}$Hz)
- Several frequencies are monitored
- Combine many pulses per observation
- Repeat for **very** long time ($\gtrsim$ yrs) to go to very low frequencies ($f_{\rm yr} \sim 3.17 \times 10^{-8}$Hz)

Data are compared with some to timing model

$\downarrow$

The data used in the GW analysis are residuals after timing model subtraction

* Figure from https://nanograv.github.io/11yr_profile_variability/

Signal part

The time shift induced by a GW is

$$\Delta T_i(t) = \frac{\hat{p}_i^a \hat{p}_i^b}{2} \int_0^{L_i} \mathrm{d}s \, h_{ab}(t(s), \vec{x}(s)) \ ,$$

where $t(s) = t - L_i + s$ and $\vec{x}(s) = \vec{x}_0 + (L_i - s)\hat{p}_i$.

Expand h_{ab} in plane waves ($\tilde{h}_A(\vec{k})$ Fourier coefficients, e^A_{ab} polarization tensors)

$$\Delta T_i(t) = L_i \int \mathrm{d}\vec{k} \, \mathrm{e}^{-2\pi i \vec{k}\cdot\vec{x}_0} \sum_A \left[\mathrm{e}^{2\pi i k t} \mathcal{M}(\vec{k}, \hat{p}_i) \, \mathcal{G}^A(\hat{k}, \hat{p}_i) \, \tilde{h}_A(\vec{k}) + h.c.(-\vec{k}) \right]$$

where we have integrated in s and introduced

$$\mathcal{G}^A(\hat{k}, \hat{p}_i) \equiv \frac{\hat{p}_i^a \hat{p}_i^b}{2} \, e^A_{ab}(\hat{k}) \ , \qquad \mathcal{M}(\vec{k}, \hat{p}_i) \equiv -i \frac{1 - e^{-2i\pi k L_i (1 + \hat{k}\cdot\hat{p}_i)}}{2\pi k L_i (1 + \hat{k}\cdot\hat{p}_i)} \ .$$

For SGWB what we need is

$$\langle \Delta T_i(t) \Delta T_j(t) \rangle \propto L_i L_j \int \mathrm{d}k \, P_h(k) \int \mathrm{d}\hat{k} \, \sum_A \mathcal{M}(\vec{k}, \hat{p}_i) \mathcal{M}(\vec{k}, \hat{p}_j) \, \mathcal{G}^A(\hat{k}, \hat{p}_i) \mathcal{G}_A(\hat{k}, \hat{p}_j) \ .$$

Finally, we decompose the angularly-integrated quantity in two parts:

- A frequency-independent part, depending only on $\hat{p}_i \cdot \hat{p}_j$ (HD!)
- A frequency-dependent part, depending on the GW's frequency vs. L_i, L_j

Noise part

Many noise sources can affect our data:

(SC = spatially correlated, CH = Chromatic, aCH = achromatic)

Noise type	**Freq. shape**	**Space correlations**
Irregularities in the period	aCH	no
Orbital irregularities	aCH	no
Pulse jitter	might be CH,	no
Changes in the pulse profile	CH	no
ISM effects	CH	no
Solar system ephemeris	aCH	yes
Clock Errors	aCH	yes
...	???	???

Again, for SGWB, what we care for is something like:

$$\langle n_i(t) n_j(t') \rangle = \delta_{ij} P_n + \delta_{ij} P_{i,n} + P_{n,ij} + \ldots$$

Off-diagonal terms ($P_{n,ij}$) might in principle be there but even if they do, should (hopefully) not behave as HD!!

A likelihood for the time residuals

Schematically we express the time residuals as

$$\delta t = M\epsilon + Fa + n$$

where M is the design matrix, F is the Fourier design matrix and n is WN.

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Write a Gaussian likelihood for WN as

$$\ln \mathcal{L} = -\frac{1}{2}\ln\left[\det(2\pi C)\right] - \frac{1}{2}(\delta t - M\epsilon - Fa)C^{-1}(\delta t - M\epsilon - Fa)$$

where $C \equiv \langle nn \rangle$ is the WN covariance matrix.

Assume ϵ and a to be Gaussian (with variance X and φ, respectively)

$$\boxed{\ln \mathcal{L} = -\frac{1}{2}\ln\left[\det(2\pi N)\right] - \frac{1}{2}\delta t N^{-1}\delta t}$$

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Works well but it's quite slow!

A simplified likelihood and Fisher approximation

Can we make this much faster to do some forecasts??

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$$\ln \mathcal{L} = \sum_f -\frac{1}{2} \ln \left[\det(2\pi C^f_{IJ})\right] - \frac{1}{2} \tilde{d}^f_I C^{-1}_{IJ} \tilde{d}^f_J$$

where $\tilde{d}^f_I$ are the data for pulsar I and C^f_{IJ} is the covariance.

As before, we express C_{IJ} as signal + noise

$$C_{IJ} = P_{n,IJ} + R_{IJ} P_h$$

with P_h, $P_{n,IJ}$ signal and noise power spectra and R_{IJ} response.

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It's easy to show that the Fisher information matrix reads

$$F_{\alpha\beta} \equiv T_d \int_{f_{\rm min}}^{f_{\rm max}} \mathcal{C}_{IJ} \, \mathcal{C}_{JI} \, \frac{\partial \ln P_h}{\partial \theta_\alpha} \frac{\partial \ln P_h}{\partial \theta_\beta} \mathrm{d}f$$

where θ are the parameters and $\mathcal{C}_{IK} = \bar{C}^{-1}_{IJ} R_{JK}$ with $\bar{C}_{IJ} \equiv C_{IJ}/P_h$.

In collaboration with S. Babak, M. Falxa, G. Franciolini

Comparison with the standard approach (preliminary)

Is this method consistent with the standard approach??

Let's consider a PL signal $\longrightarrow$ $\Omega_{\rm GW} h^2 = 10^{\alpha_{\rm PL}} \left(\frac{f}{f_{\rm yr}}\right)^{n_T}$

and test against current data +
generated time residuals for future configurations using state-of-art codes
and run the full analysis (run time $\sim$ day vs. $\lesssim$ sec for Fisher)

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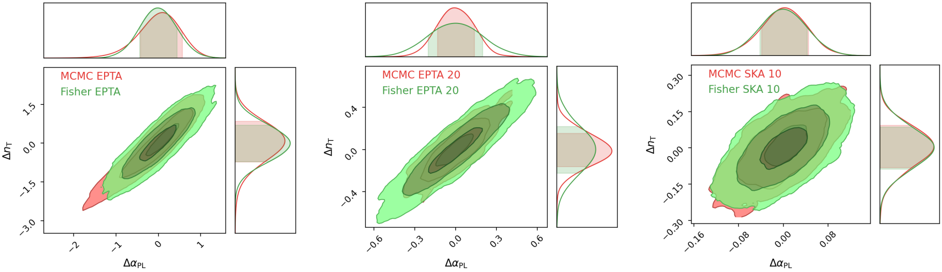
Current EPTA | Simulated EPTA 20 | Simulated SKA 10



The results match quite well!

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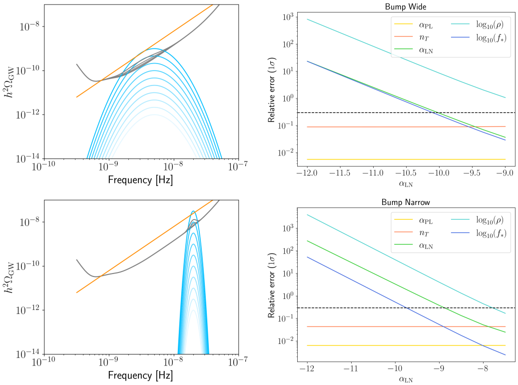

Obviously the louder the signal, the better we measure it!

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Conclusions and future perspectives

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- Present techniques to analyze PTA data are accurate but not super fast
- An approximate description might be much faster (and reasonably good)
- This approach might be quite useful to make forecasts!
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Future perspectives:

- Extensive tests for validation (different templates??)
- Application to a set of early Universe models (PTs, CSs, SIGWB, ...)
- Better model the noise components to improve accuracy (if needed)
- Extensions (e.g., to forecast the accuracy of the angular reconstruction?)
- ...

Last Slide

The end

Thank you for your attention