

# Induction signal characterization in the vertical drift TPC for the DUNE project

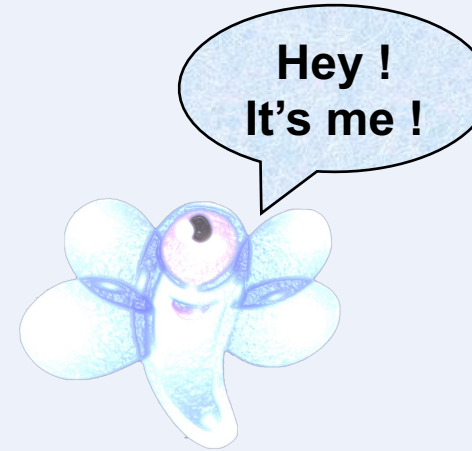
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L. ZAMBELLI – LAPP Annecy

D. DUCHESNEAU – LAPP Annecy

# Neutrino oscillation



- There are three leptonic flavors  $\nu_e, \nu_\mu, \nu_\tau$
- Neutrinos only interact by weak interaction → Small cross section
- **Neutrino oscillation:**
  - Assumes neutrino masses (SM predict massless for these ones)
  - Flavor eigenstates (which couple  $W^\pm, Z^0$ ) are different from mass eigenstates during their propagation

- **Flavor states** are a linear combination of mass states:  $|\nu_\alpha\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$  avec  $\begin{cases} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{cases}$

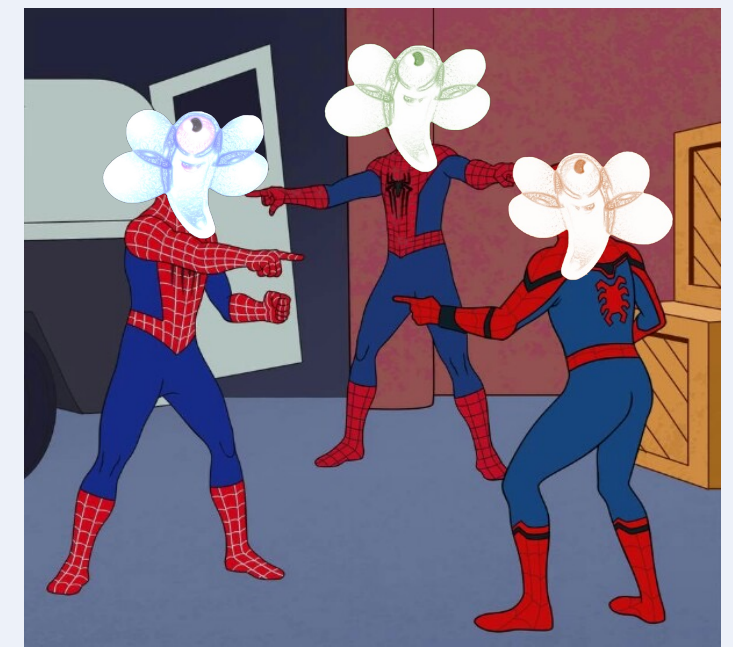
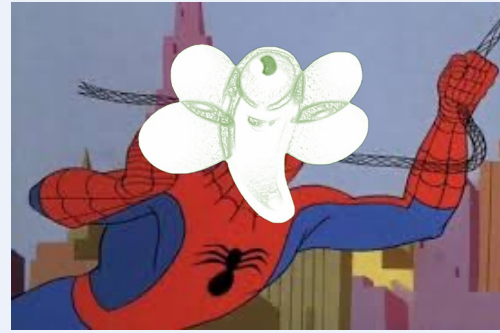
- **PMNS mixed matrix** (Pontecorvo-Maki-Nakagawa-Sakata):

$$U_{PMNS} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}}_{\substack{\text{Atmospheric} \\ \nu_\mu \leftrightarrow \nu_\tau}} \underbrace{\begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta_{CP}} & 0 & \cos \theta_{13} \end{pmatrix}}_{\substack{\text{Reactor \& accelerator} \\ \nu_\mu \leftrightarrow \nu_e}} \underbrace{\begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{Solar} \\ \nu_e \leftrightarrow \nu_X}}$$

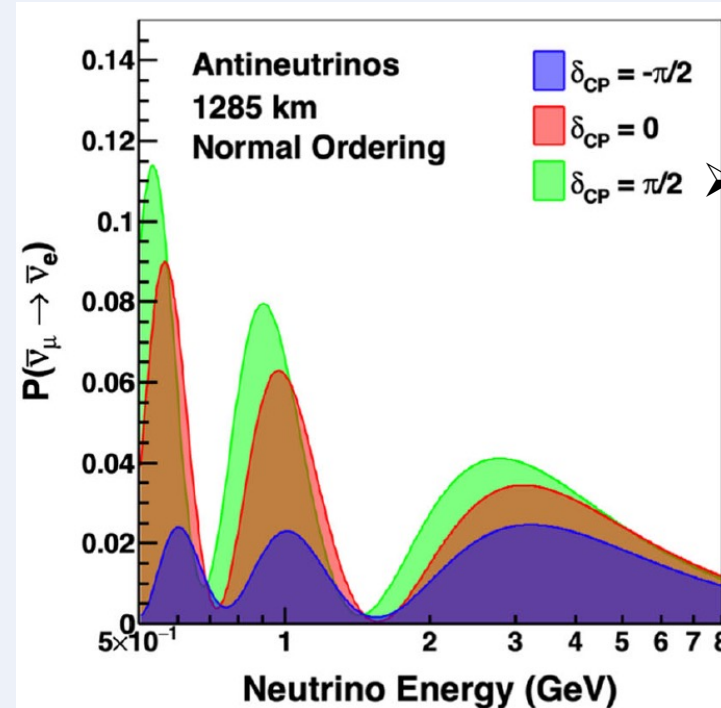
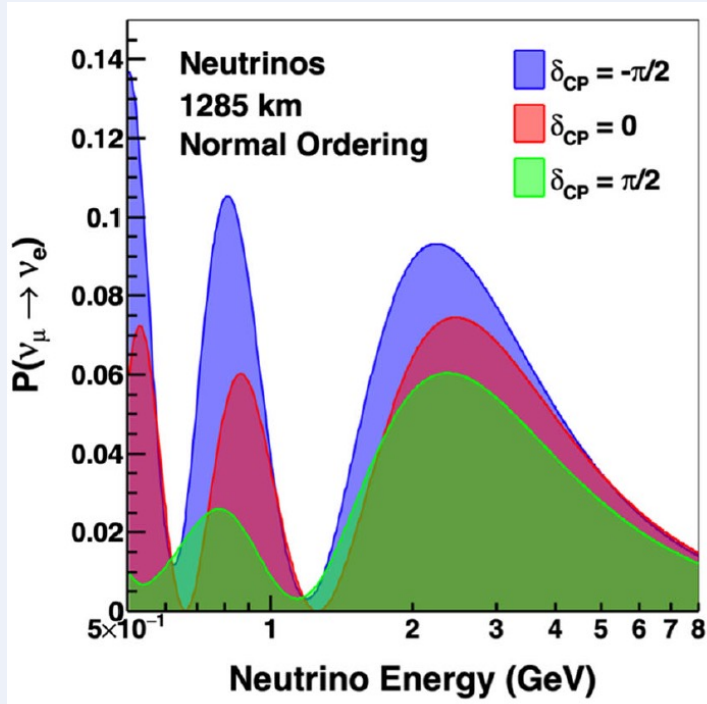
# Neutrino oscillation

➤ Probability of neutrino oscillations:

$$P(\nu_\alpha \rightarrow \nu_\beta) \equiv P(E_\nu, L)$$



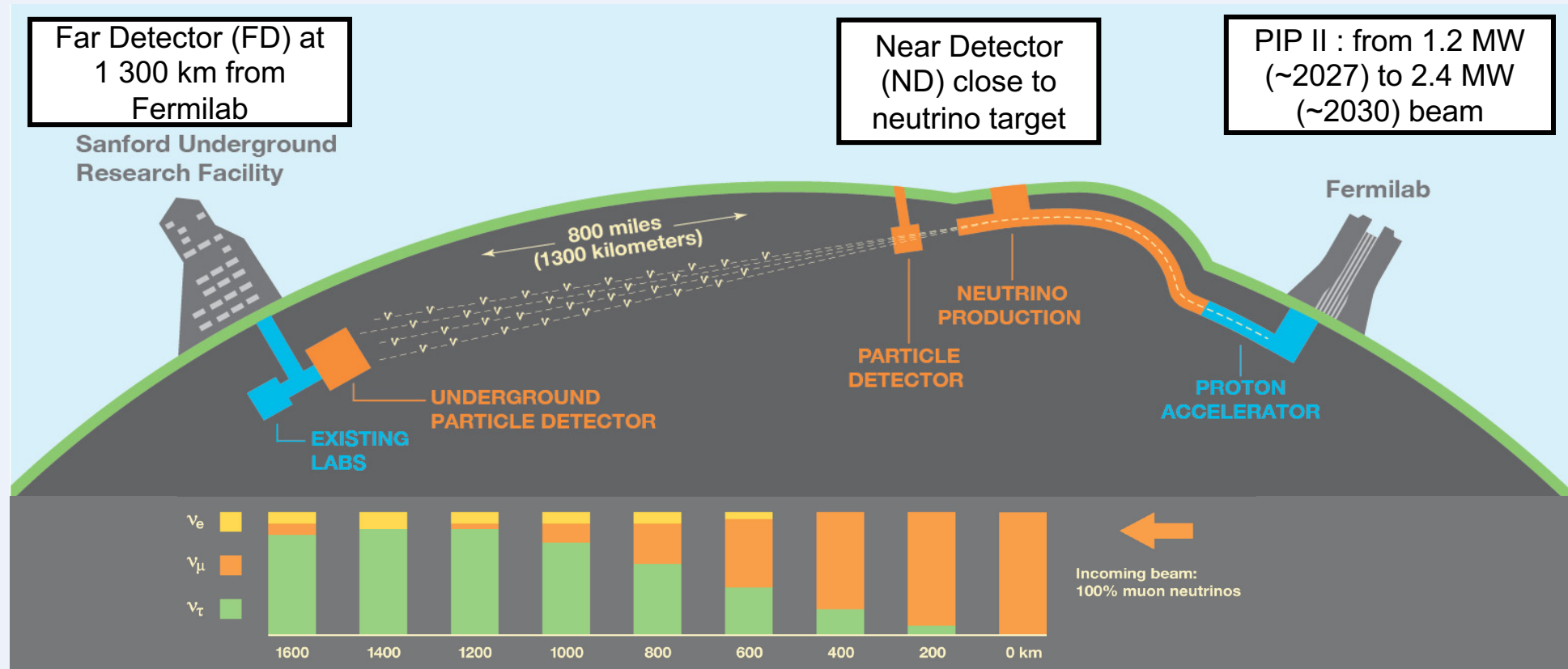
➤ Depends on 6 parameters:  $\Delta m_{31}^2$ ,  $\Delta m_{21}^2$ ,  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ ,  $\delta_{CP}$



➤ Unknown parameters:

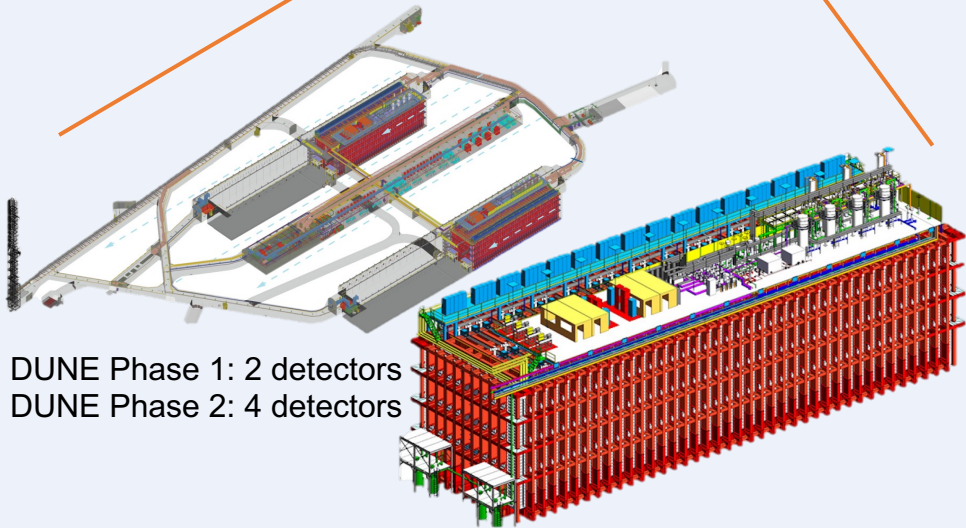
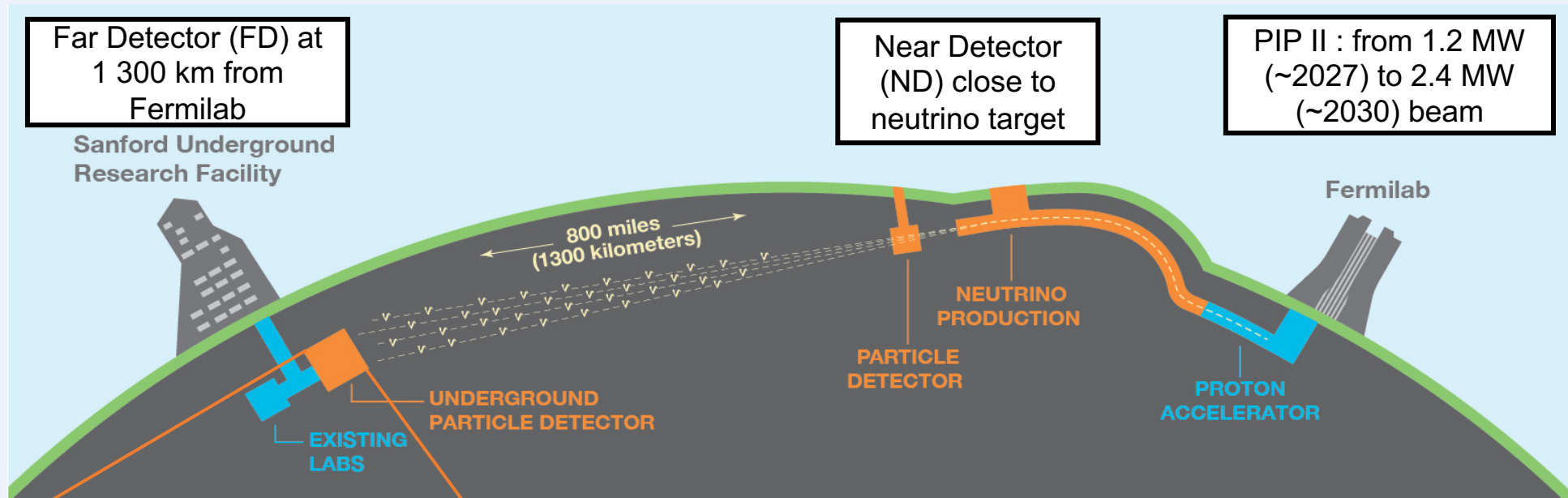
- Sign of  $|\Delta m_{31}^2| \rightarrow$  Mass ordering
- $\delta_{CP}$  value  $\rightarrow$  CP symmetry broken on lepton sector
- $\theta_{23}$  value and octant

# DUNE project overview





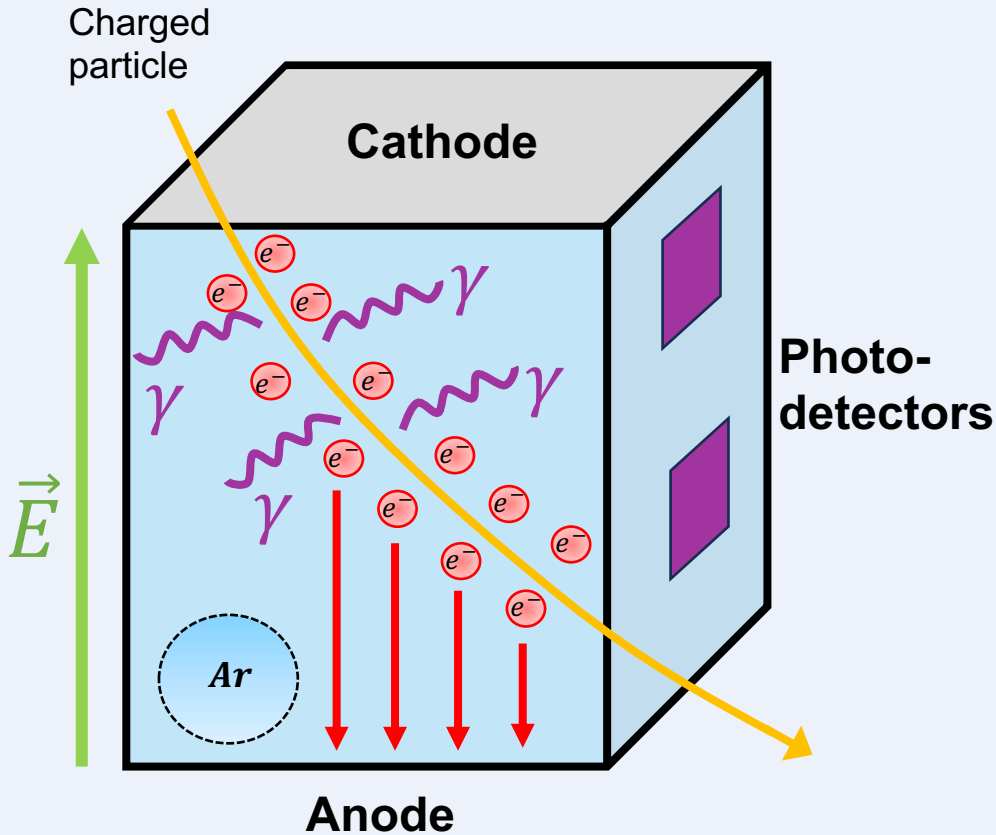
# DUNE project overview



- Located at 1 480 m depth at SURF
- 4 Liquid Argon Time-Projection Chamber (LArTPCs) modules with different designs
- The design of 2 modules has been chosen
- TPC active volume : 15 kt Liquid Argon

# LArTPC

## ➤ Liquid Argon:



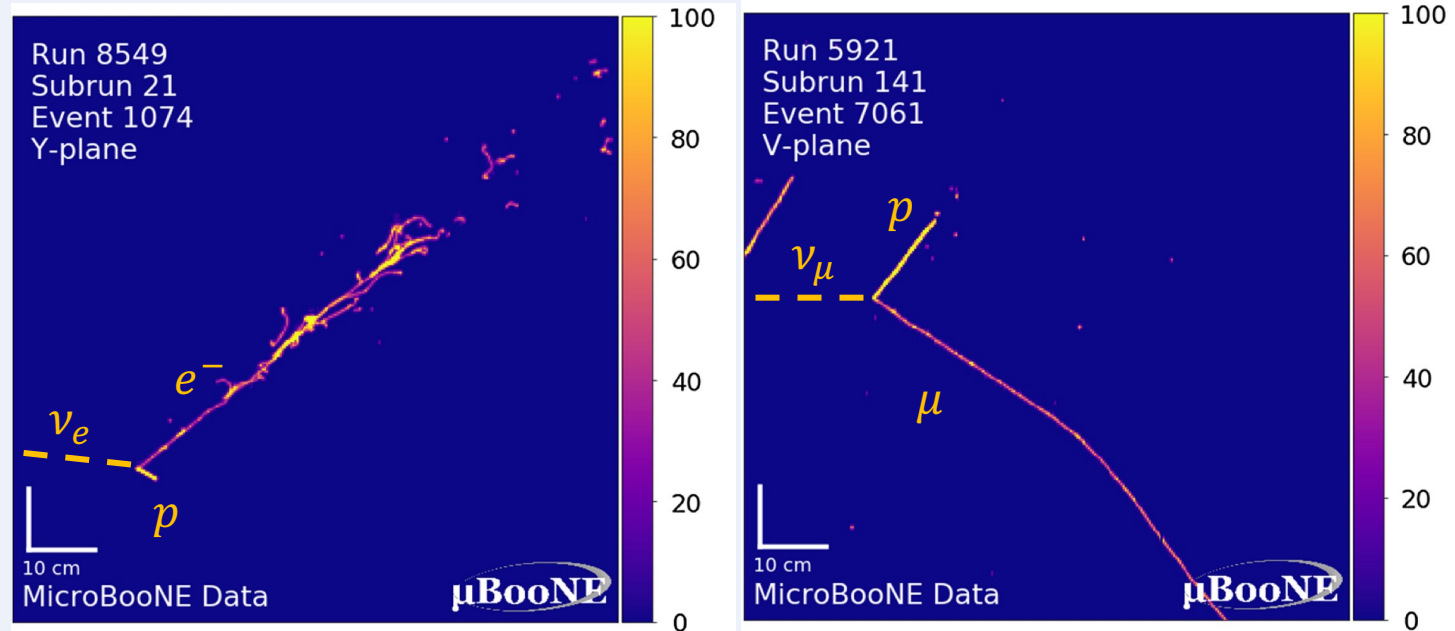
Charge signal

Light signal

## ➤ Time Projection Chamber:

- Segmented anodes used to collect charge signal
- $\tau_{drift} \gg \tau_{photon} \rightarrow$  light signal trigger detection
- Enables large volume
- High spatial resolution (few millimeters)

- Separate  $\nu_e / \nu_\mu$  events by track topology identification
- Calorimetry to have the energy of the primary neutrino



# Vertical drift design

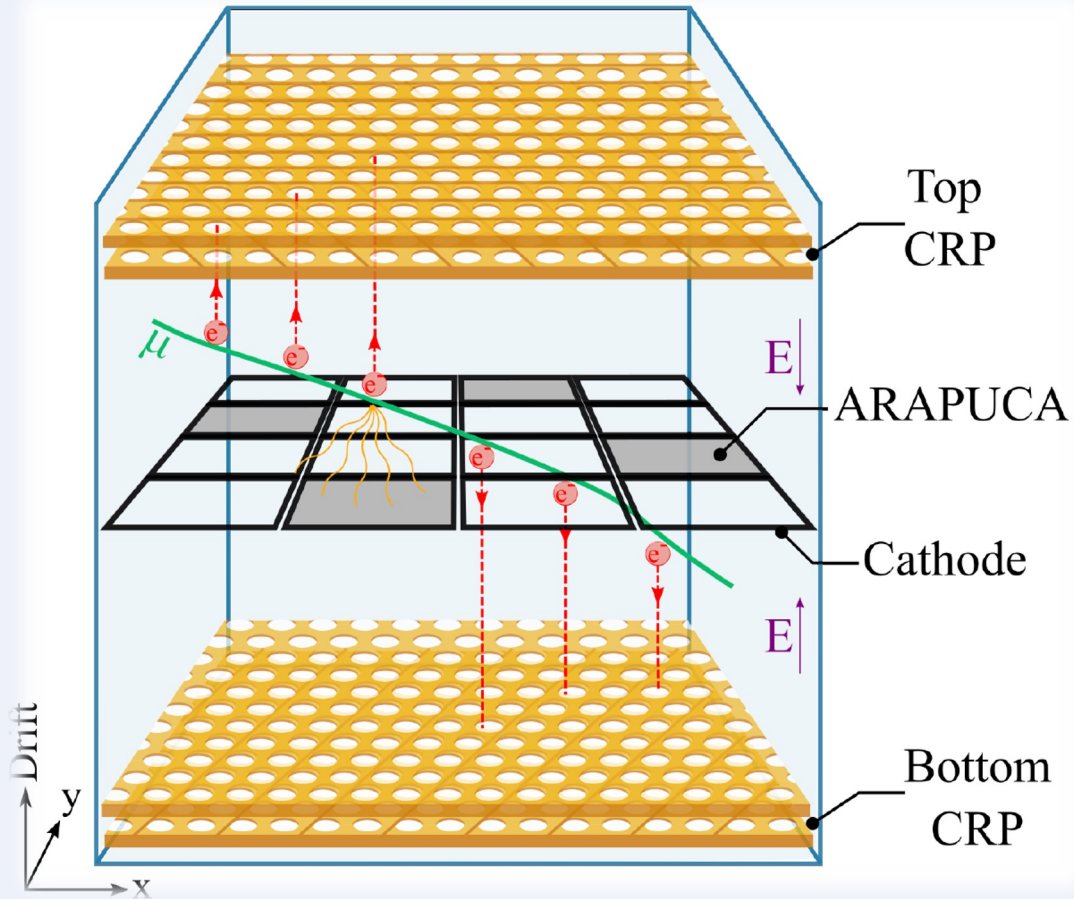


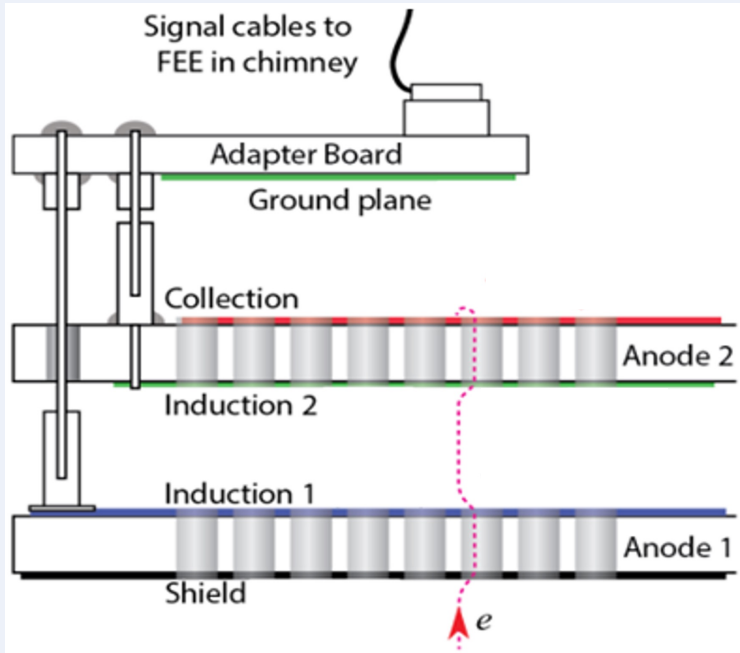
Diagram by L. Zambelli

- 2 volumes split by a cathode
  - Electric drift field:  $|\vec{E}| = 0.5 \text{ kV/cm}$
- X-ARAPUCA\* for light detection on the cathode
- The new perforated anode technology
  - Stack of 2 perforated Printed Circuit Boards (PCB)
  - Etched copper electrode strips on each PCB face
  - A sub-centimeter spatial resolution
  - Module called Charge-Readout Planes (CRP)  $\sim 3 \times 3 \text{ m}$
- DUNE Far detector at SURF:
  - 80 module CRPs for each top and bottom TPC
  - Half will be produced at Grenoble

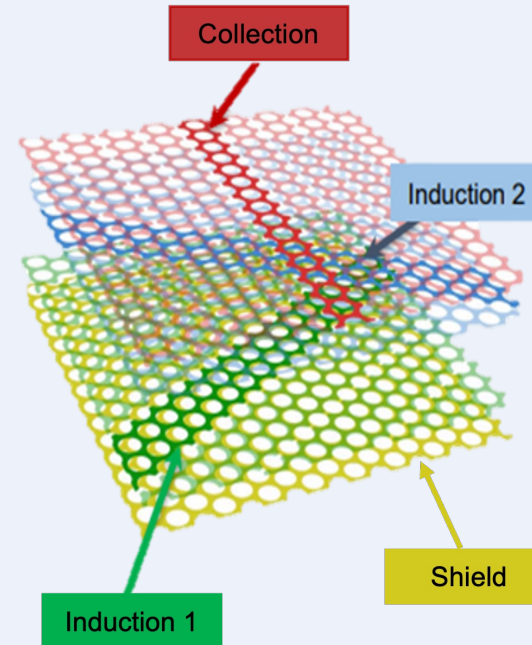
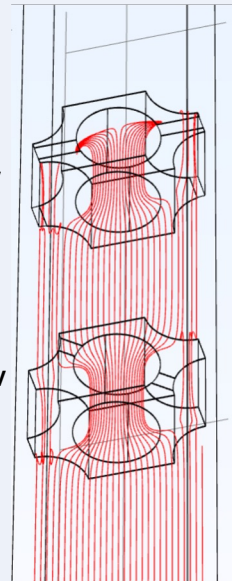
\* Light detector device

# The perforated anode technology

- Shield + 3 different charge readout layers:
  - **Induction 1** – strip orientation  $-30^\circ$  wrt beam axis
  - **Induction 2** – strip orientation  $+30^\circ$  wrt beam axis
  - **Collection** – strip orientation  $90^\circ$  wrt beam axis

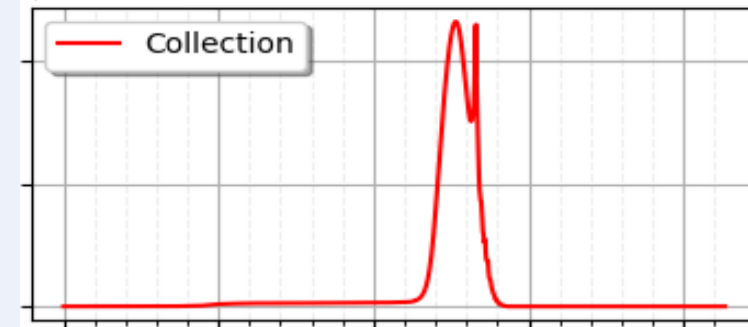
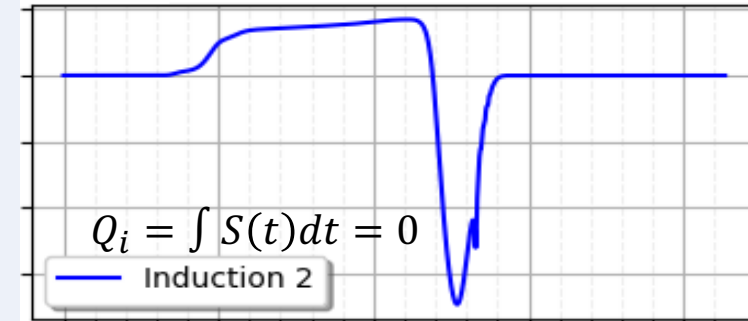
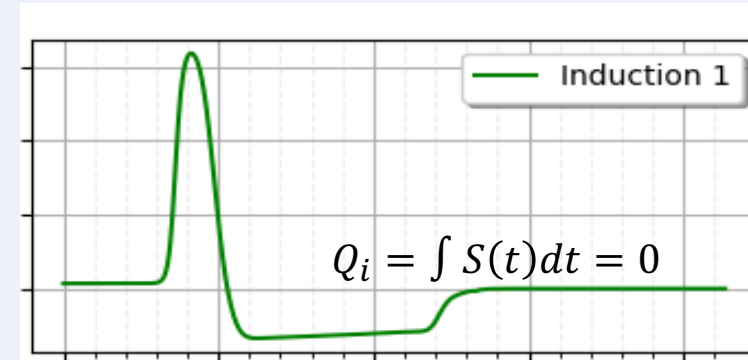


Collection : +1000V  
 Induction2 : 0V  
 Induction1 : -500V  
 Shield : -1500V



- **Induction views:**  
Bipolar signals
- **Collection view:**  
Unipolar signal

$$Q_{coll} = \int S(t)dt = N_e \times e$$





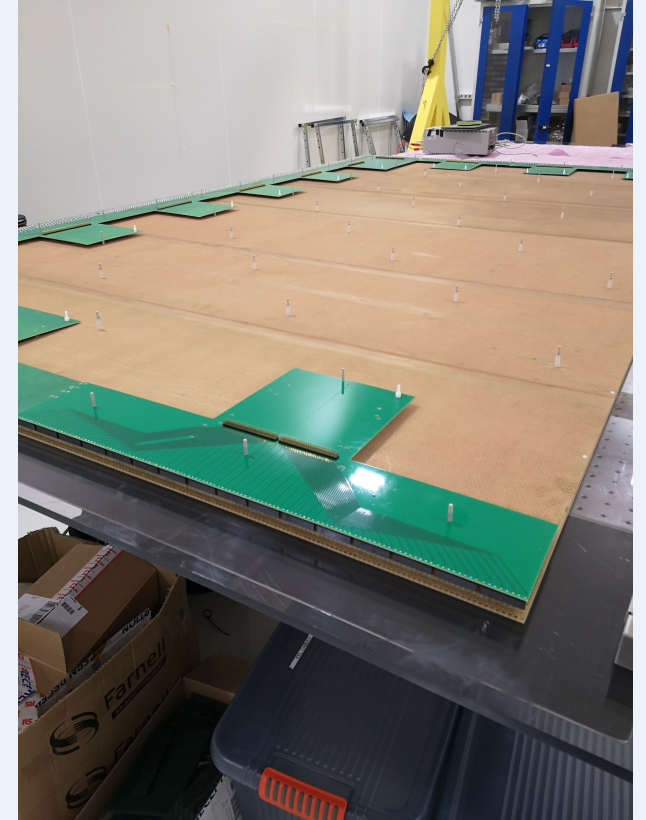
# Signal formation study on anodes

- **Problematic:**

- Use of new anode technology
- Important to know the deposited energy in the detector to measure the oscillation parameters
- Important to know to reconstruct the particle tracks from collected charge

- **My work:**

- Understanding the waveforms based on energy, track angle and position
- Understanding the charge lost in the anodes
- Estimate the different systematics
- **Study of induced signal formation on the anode**



CRP assembly at  
CERN

**AND GOD SAID**

$$i(t) = q \vec{E}_W \cdot \vec{v}_D$$

**AND THEN THERE WAS CHARGE SIGNAL**

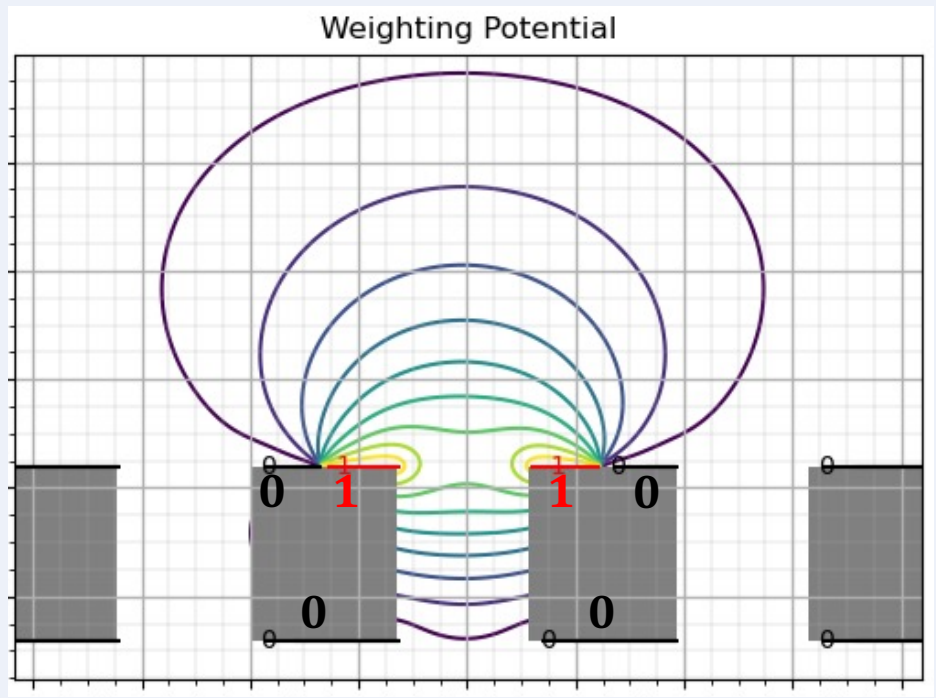


# Modeling signal formation

## ➤ Shockley-Ramo theorem:

$$i(t) = q \vec{E}_W \cdot \vec{v}_D$$

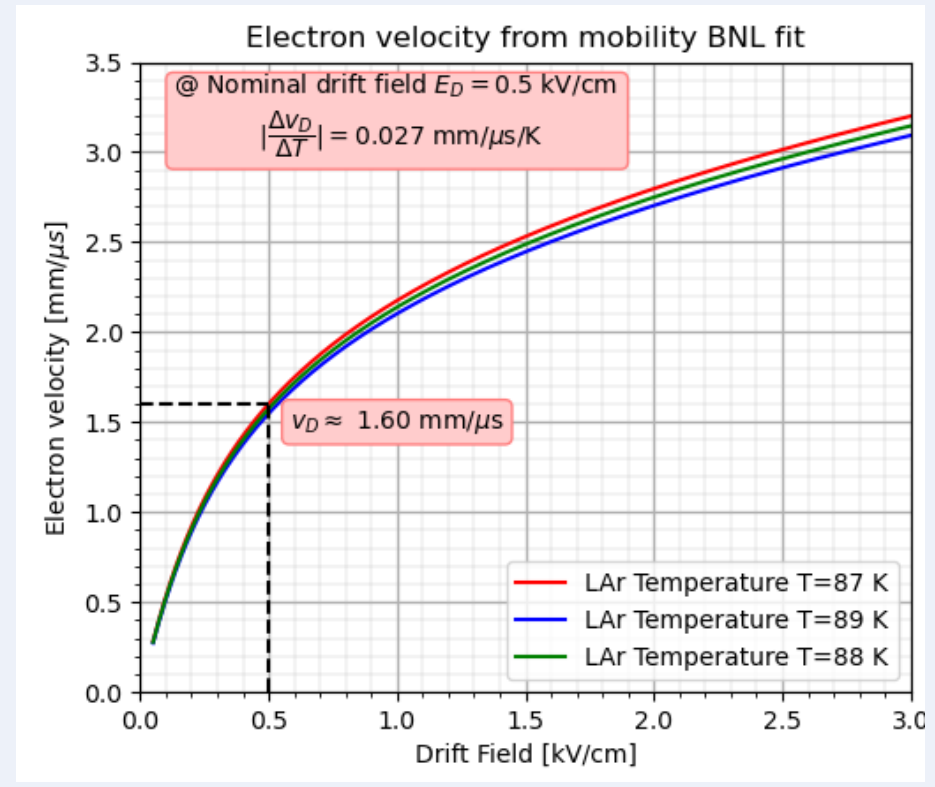
➤ Weighting Field  $\vec{E}_W$  is virtual field defined when the reading strip equal 1 V and all others fixed to 0 V



$\left\{ \begin{array}{l} \vec{E}_W: \text{ geometry factor} \\ \vec{v}_D: \text{ physics factor} \end{array} \right.$

## ➤ Drift velocity:

$$\vec{v}_D \equiv \vec{v}_D(\vec{E}_D, T)$$

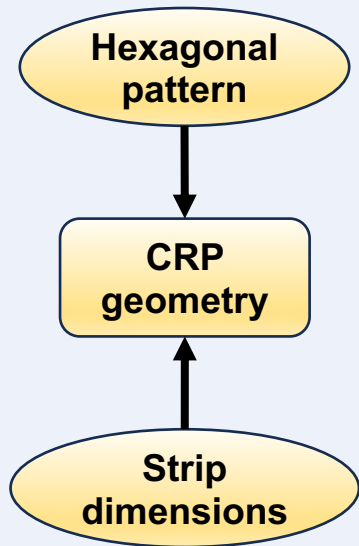


- The instantaneous current is induced by charge motion only
- Need simulation for complex geometry

<https://lar.bnl.gov/properties/trans.html>

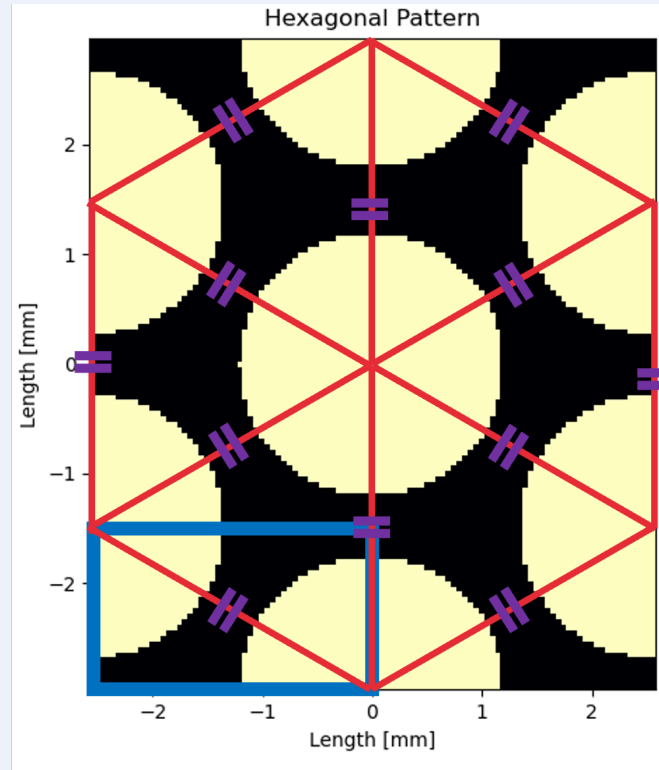
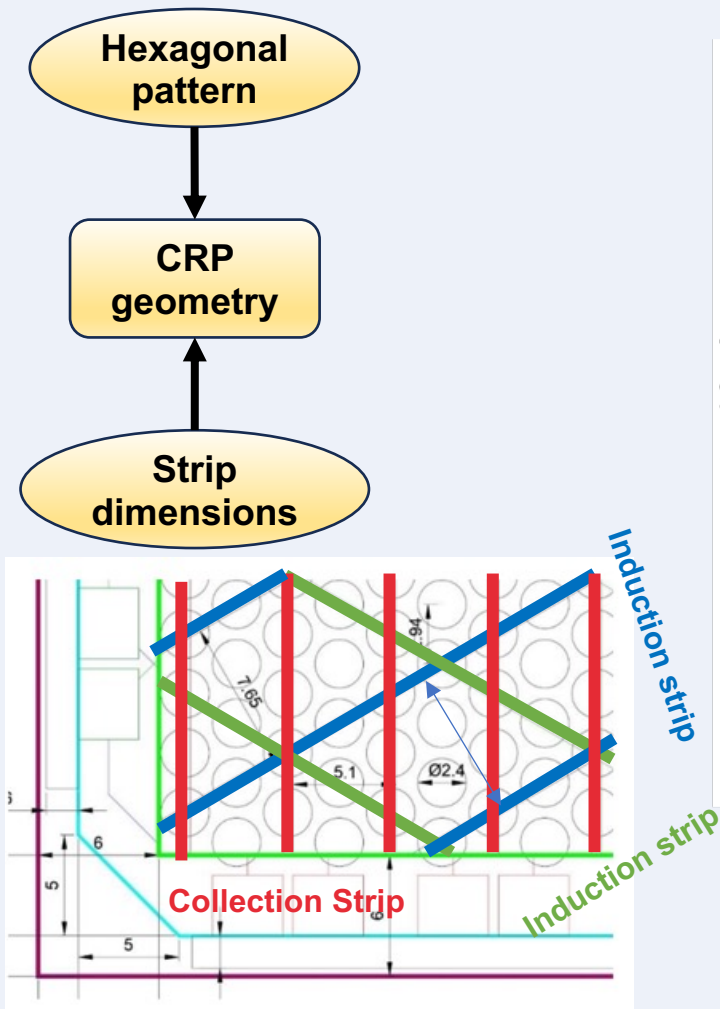
# Induced signal simulation

- Design of a simulation on Python

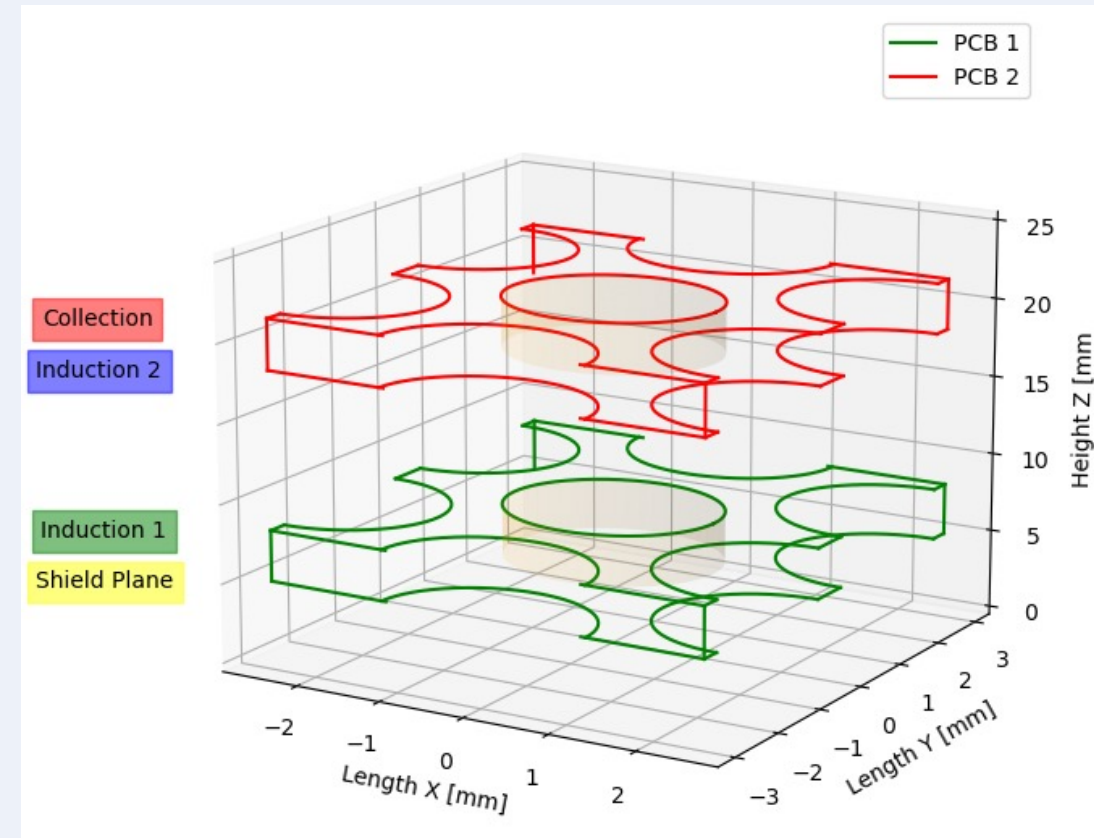


# Induced signal simulation

- Numerical simulation on Python
- Implement CRP geometry hexagonal pattern, strips, 2 Printed Circuit Boards, 4 equipotential planes

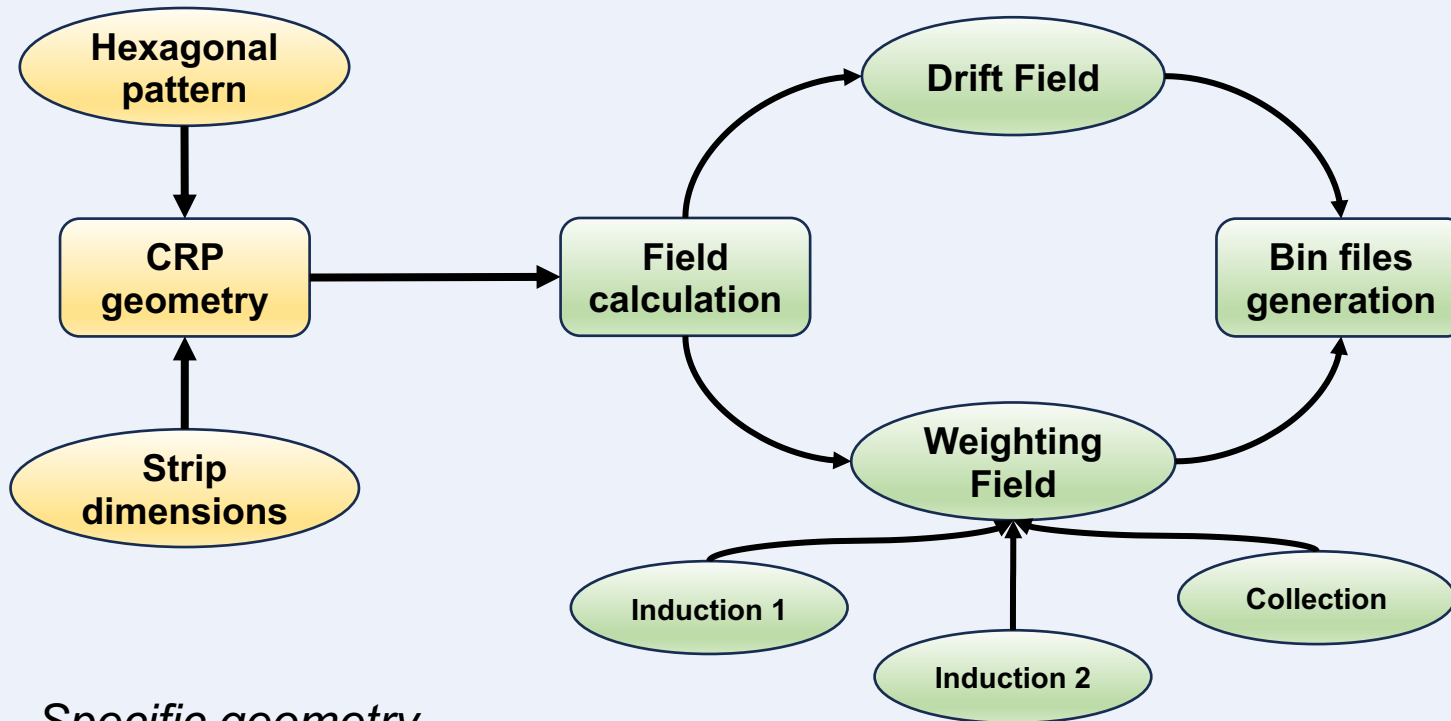


Step used:  $50 \mu m$



# Induced signal simulation

- Design of a simulation on Python



1. *Specific geometry setting up*

- Induced signal on the readout strips:

$$i(t) = q \overrightarrow{E}_W \cdot \overrightarrow{v}_D$$

- Weighting field  $\overrightarrow{E}_W$
- Drift Velocity Field:

$$\overrightarrow{v}_D = \mu(|\overrightarrow{E}_D|, T) \times \overrightarrow{E}_D$$

- Solving Laplace's equation in 3D:

$$\Delta\phi(x, y, z) = 0$$

- 3D Finite difference methods (FDM):

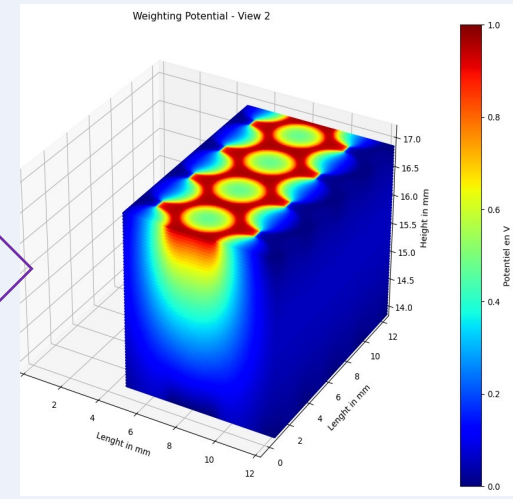
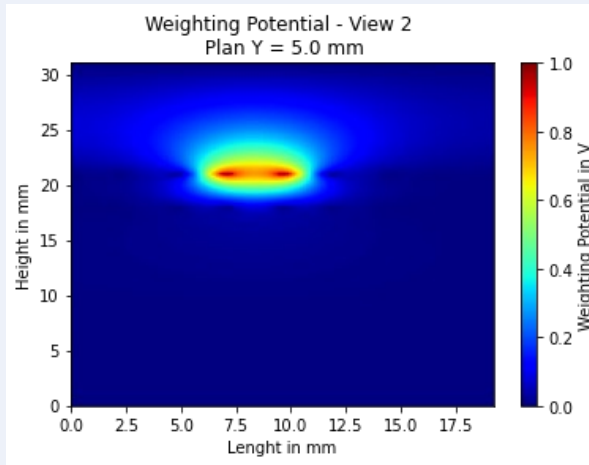
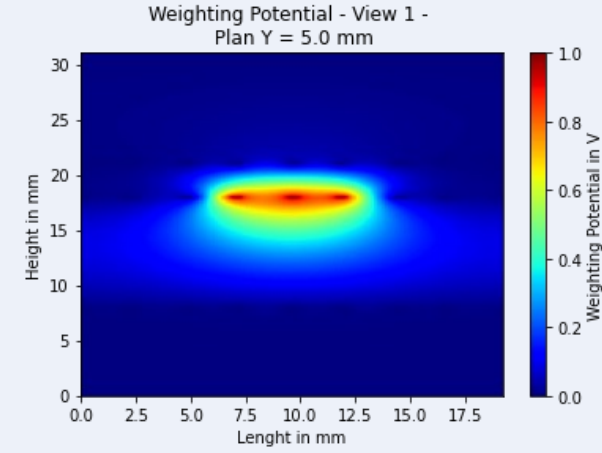
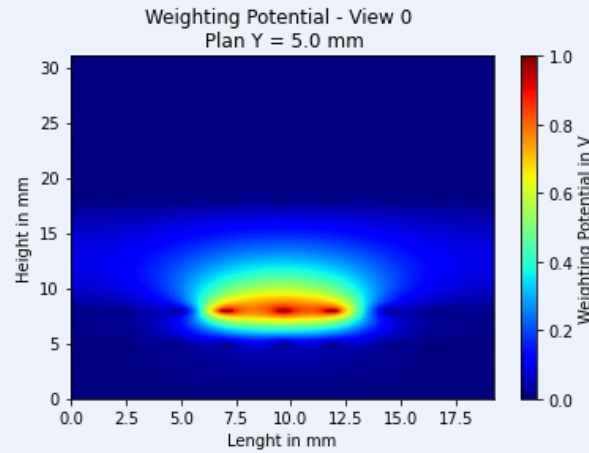
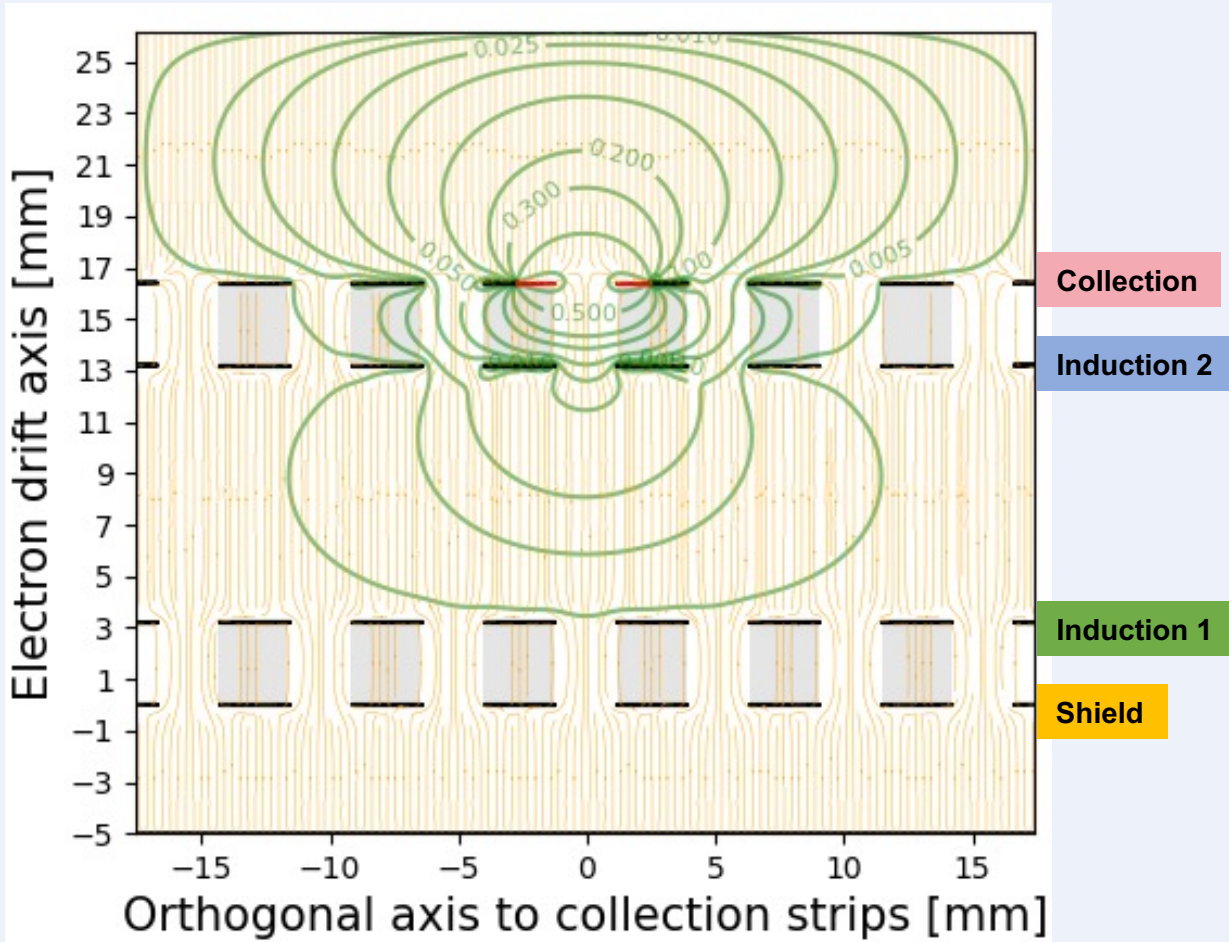
$$\phi_{i,j,k} = \frac{\phi_{i\pm 1,j,k} + \phi_{i,j\pm 1,k} + \phi_{i,j,k\pm 1}}{6}$$

- Iterative method, step by step

# Field result

## ➤ Weighting Potential

- 2D projection of drift field lines and weighting equipotential

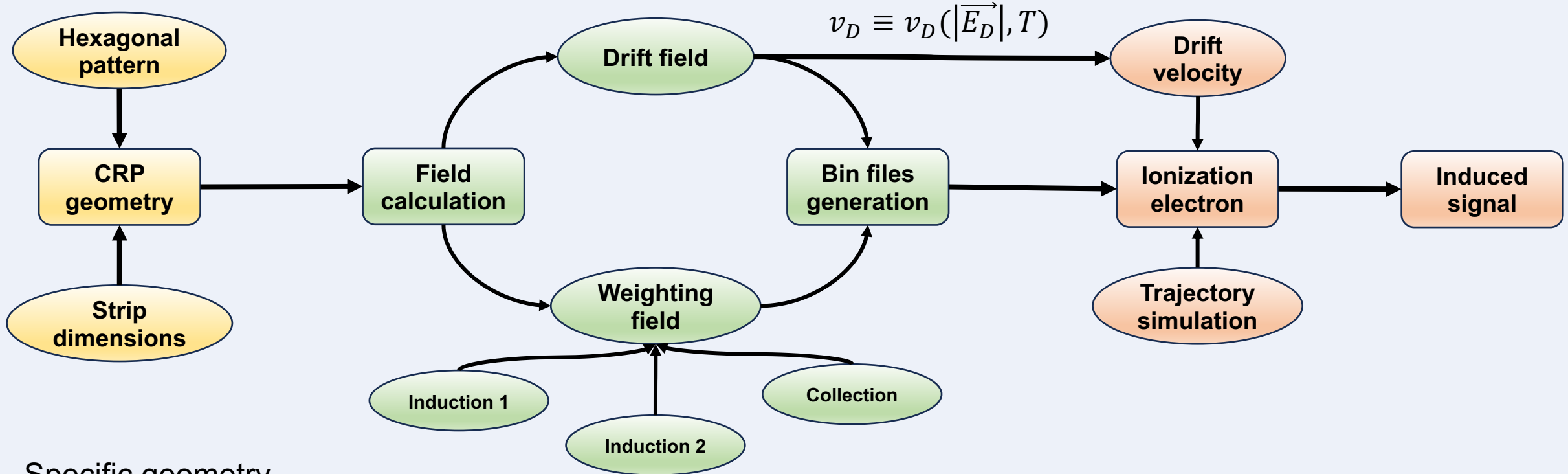


- Induced signal in the nearby strips

# Induced signal simulation

$$|\vec{E}| \in [0.5 ; 4] \text{ kV/cm}$$

➤ Design of a simulation on Python



1. Specific geometry setting up

2. Field generation with CRP geometry



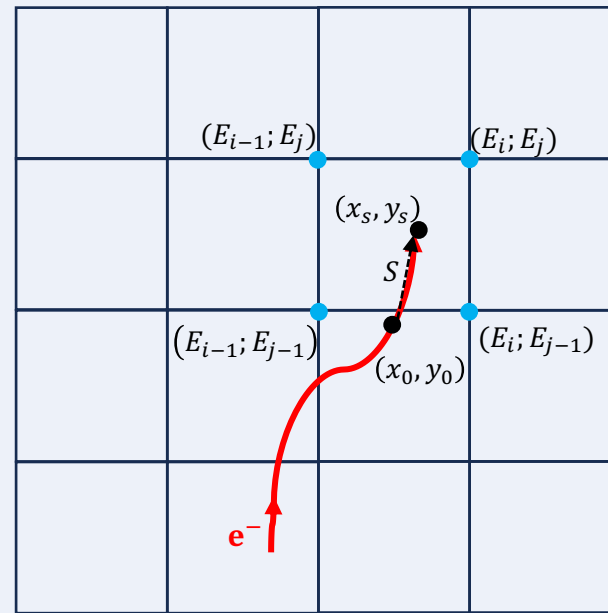
# Ionization electron generation

- Thermal electron
  - Trajectory follow drift field lines
- The electron trajectory is perfectly defined

- Runge Kunta like simulation

$$x_{i+1} = x_i + \frac{E_x^{interp}(x_i, y_i)}{|E(x_i, y_i)|} \times S$$

$$y_{i+1} = y_i + \frac{E_y^{interp}(x_i, y_i)}{|E(x_i, y_i)|} \times S$$

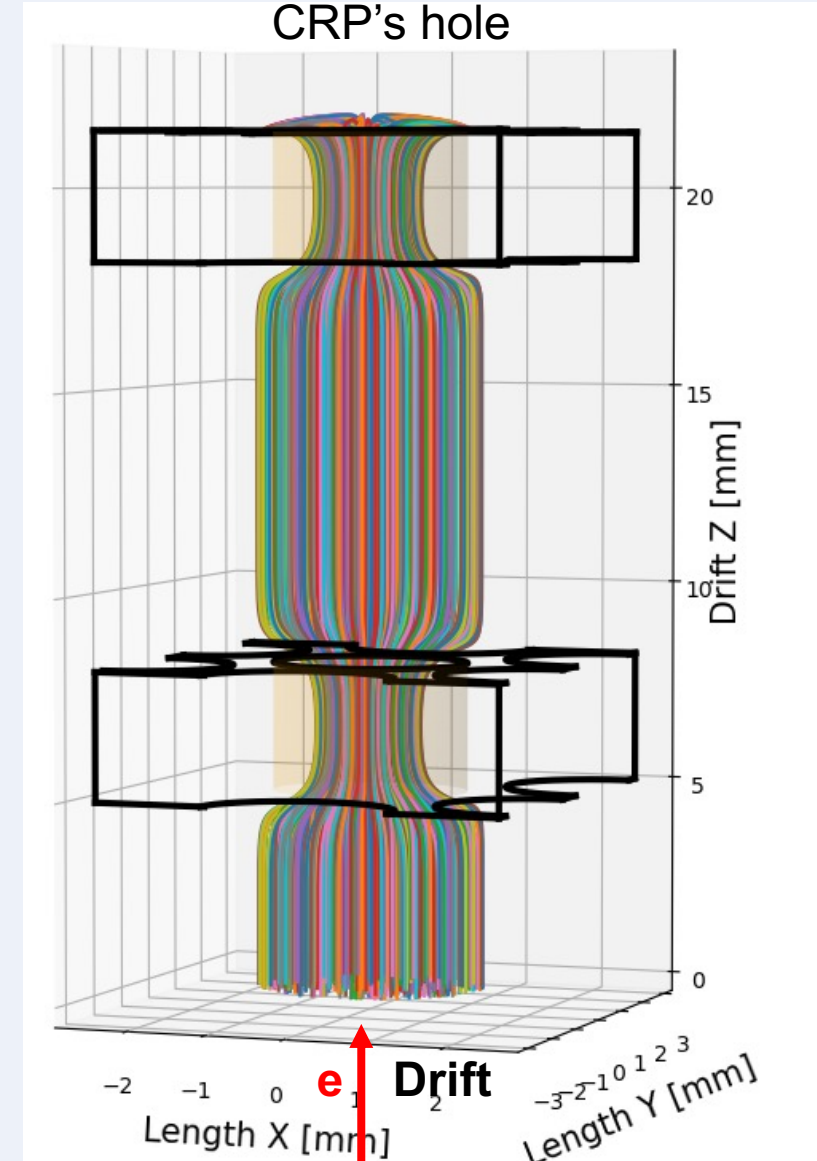


step = 50  $\mu$ m

- Drift simulation 5 mm

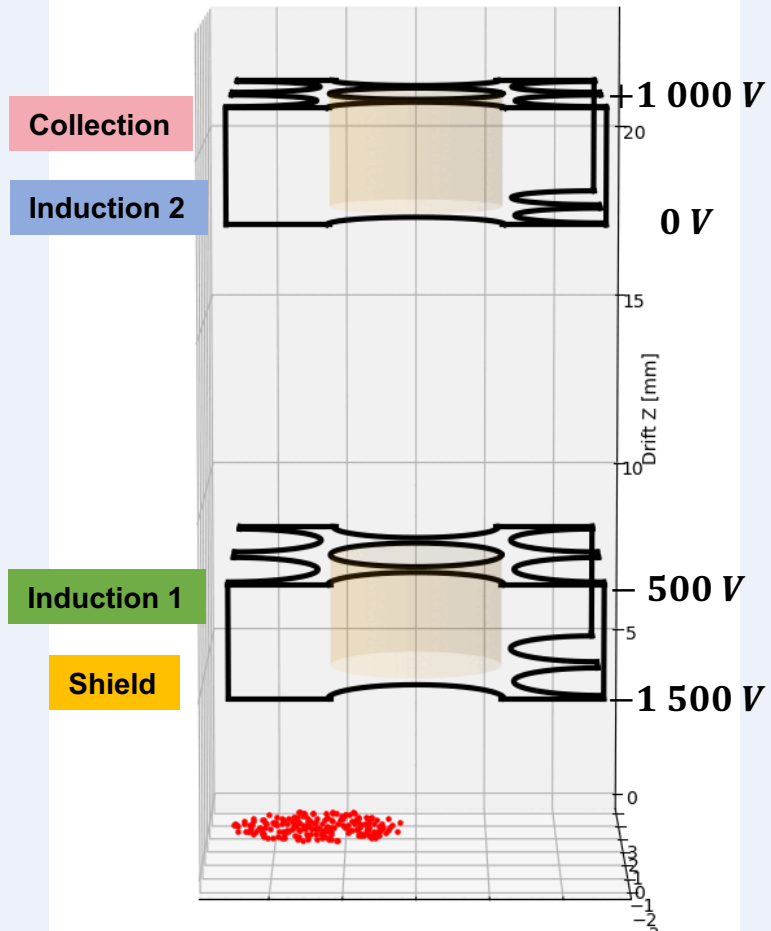
- Grid of the mesh used for the field calculation

- Electron trajectories passing through a CRP's hole

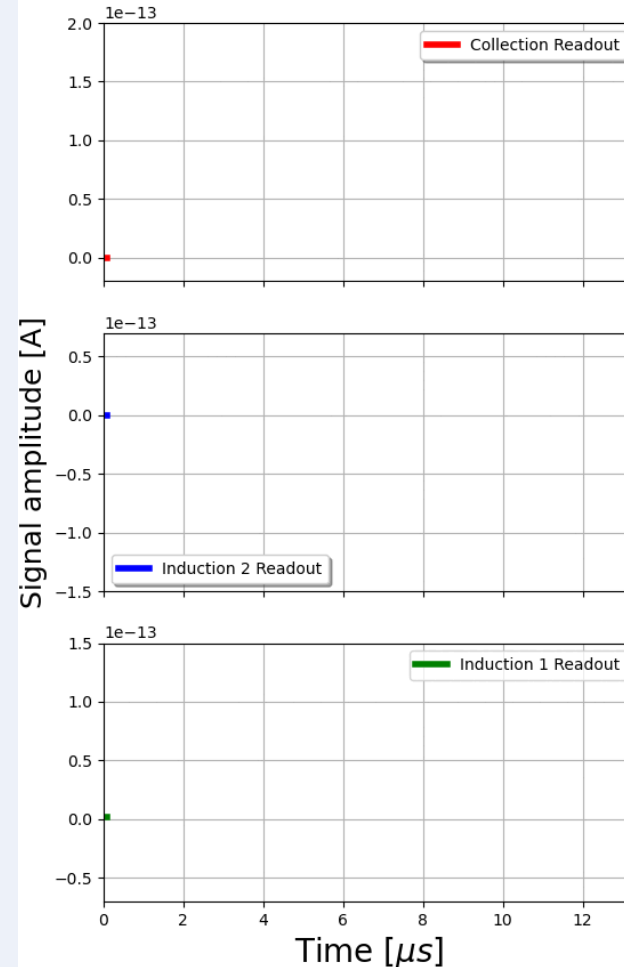


# Electron cloud simulation

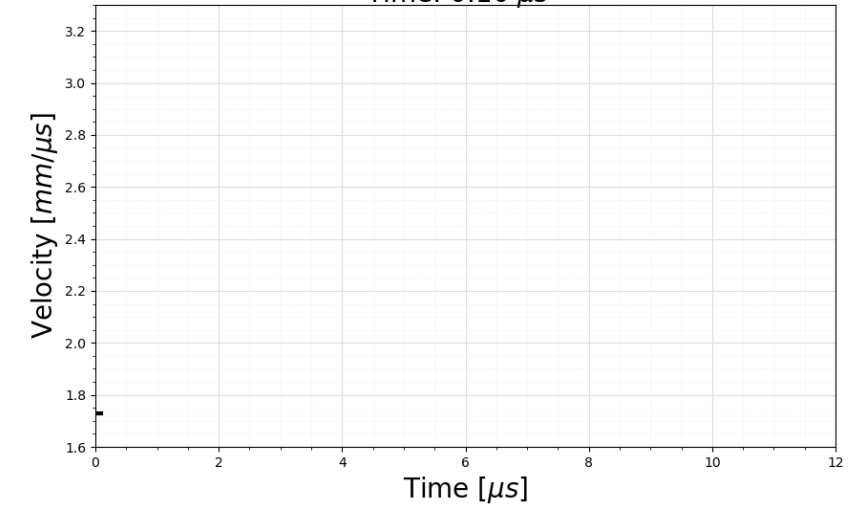
Electron cloud evolution in the anode  
Time: 0.10  $\mu\text{s}$



Signal on all views  
Time: 0.10  $\mu\text{s}$



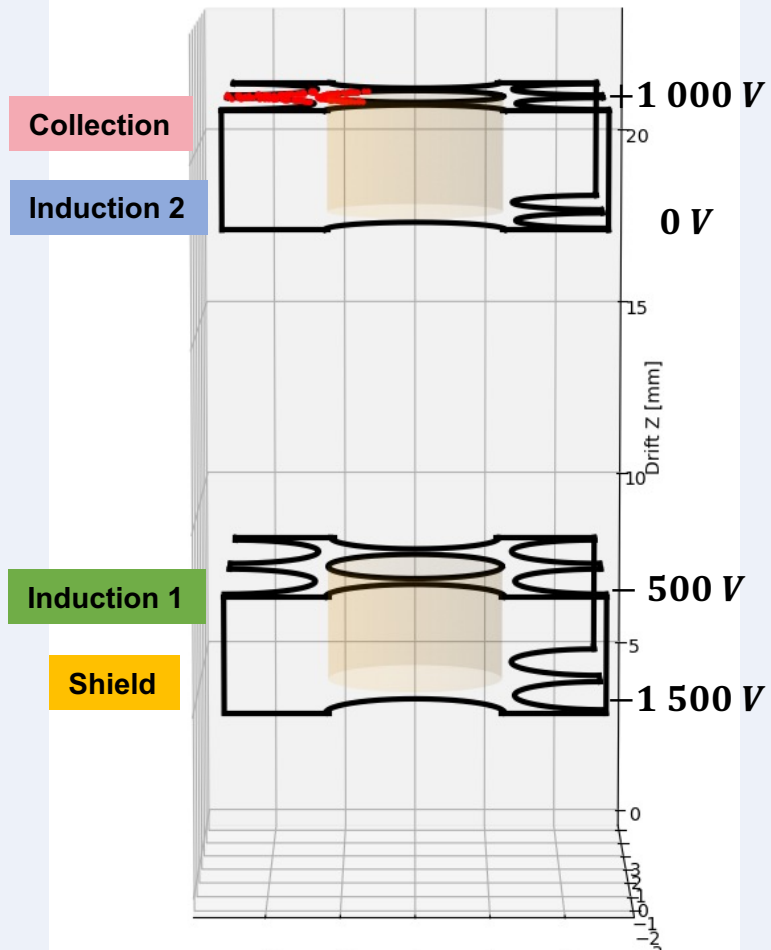
Average of cloud electron drift velocity  
Time: 0.10  $\mu\text{s}$



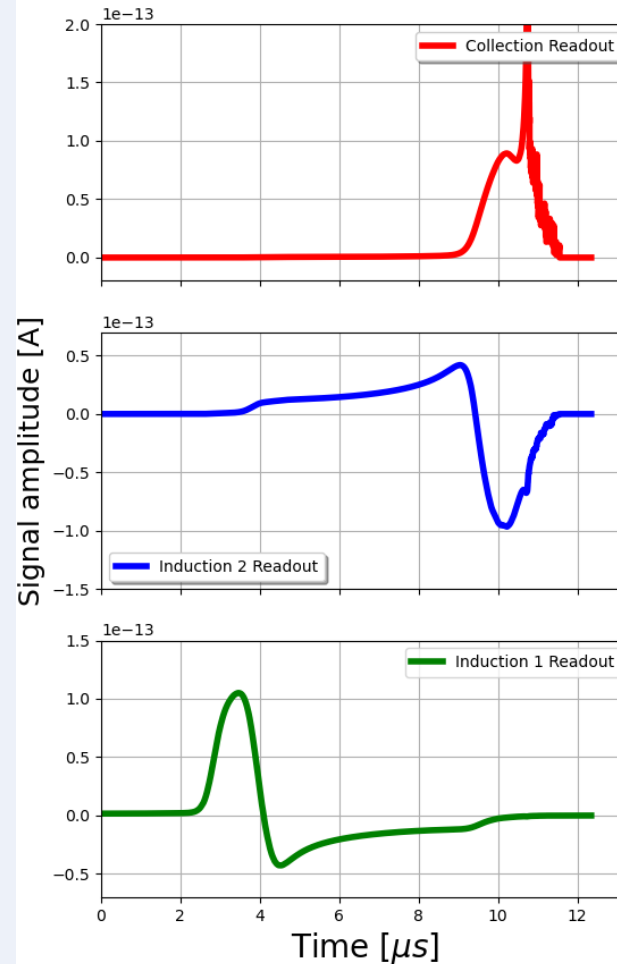
- Border effect near to the collection electrode
- The field takes  $\propto 1/r^2$  dependency which will induce a high frequency signal
- Electronic response will smooth the readout induced current

# Electron cloud simulation

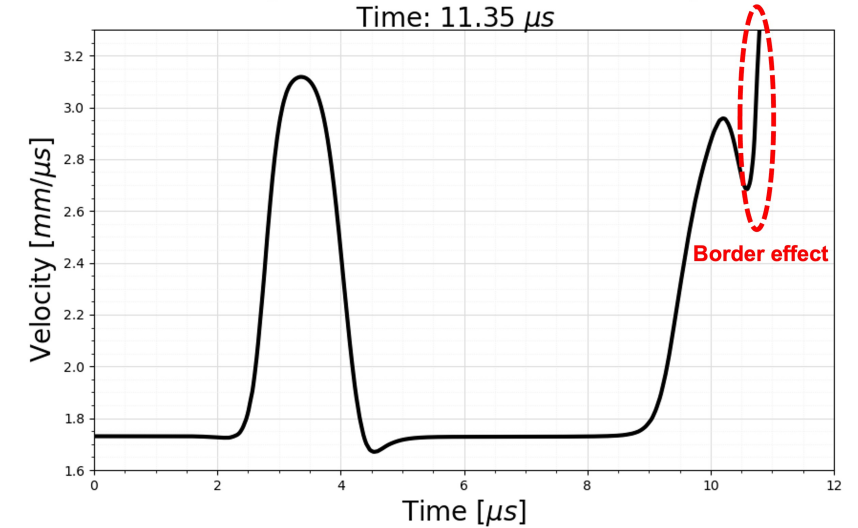
Electron cloud evolution in the anode  
Time: 12.35  $\mu\text{s}$



Signal on all views  
Time: 12.35  $\mu\text{s}$



Average of cloud electron drift velocity  
Time: 11.35  $\mu\text{s}$



- Border effect near to the collection electrode
- The field takes  $\propto 1/r^2$  dependency which will induce a high frequency signal
- Electronic response will smooth the readout induced current



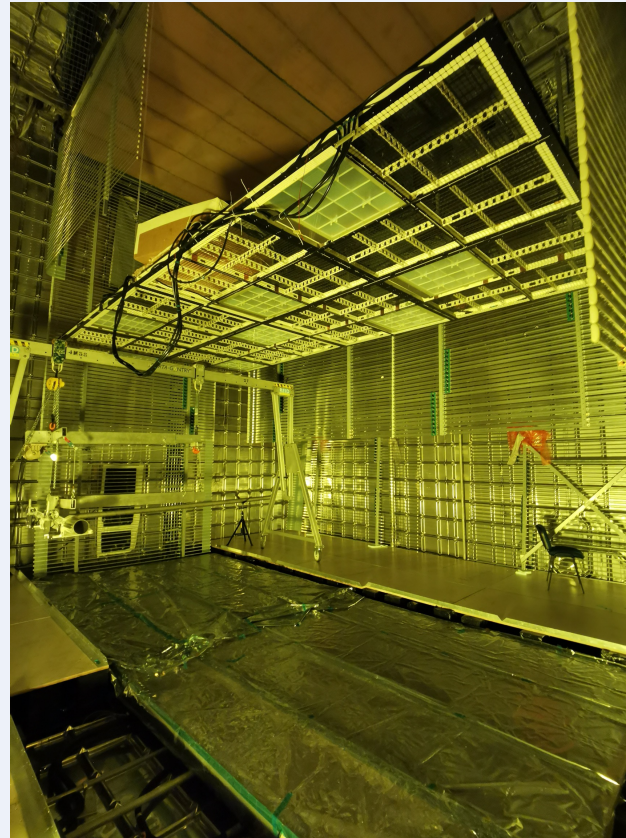
# Prototype at CERN

## ➤ ProtoDUNE Vertical Drift (VD):

- A prototype built at CERN to test the Vertical Drift technology at large scale.
- Data-taking should start in autumn 2024
- TPC size: 3.0 m (W) × 6.8 m (L) × 6.8 m (H) divided in two vertical drift volumes



2 Top CRP modules



2 Bottom CRP modules



- Vertical drift design at CERN
- Cryostat inside

## ➤ Coldbox:

- Liquid argon chamber for testing CRPs with cosmic rays at CERN
- 20 cm drift



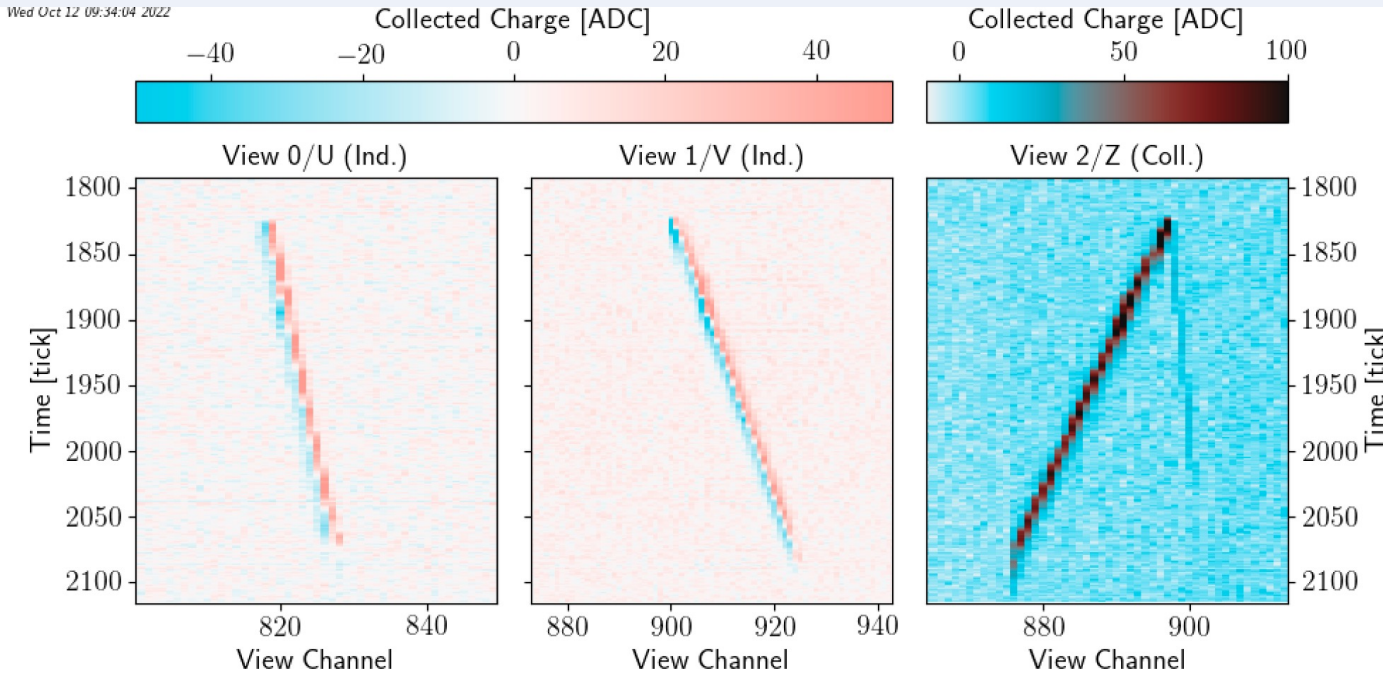
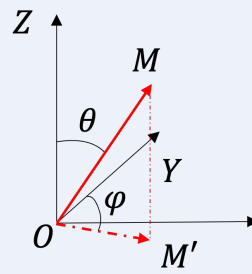
*Since 2022*

- Use coldbox data to compare with simulation

# Data extraction

## ➤ Event Display

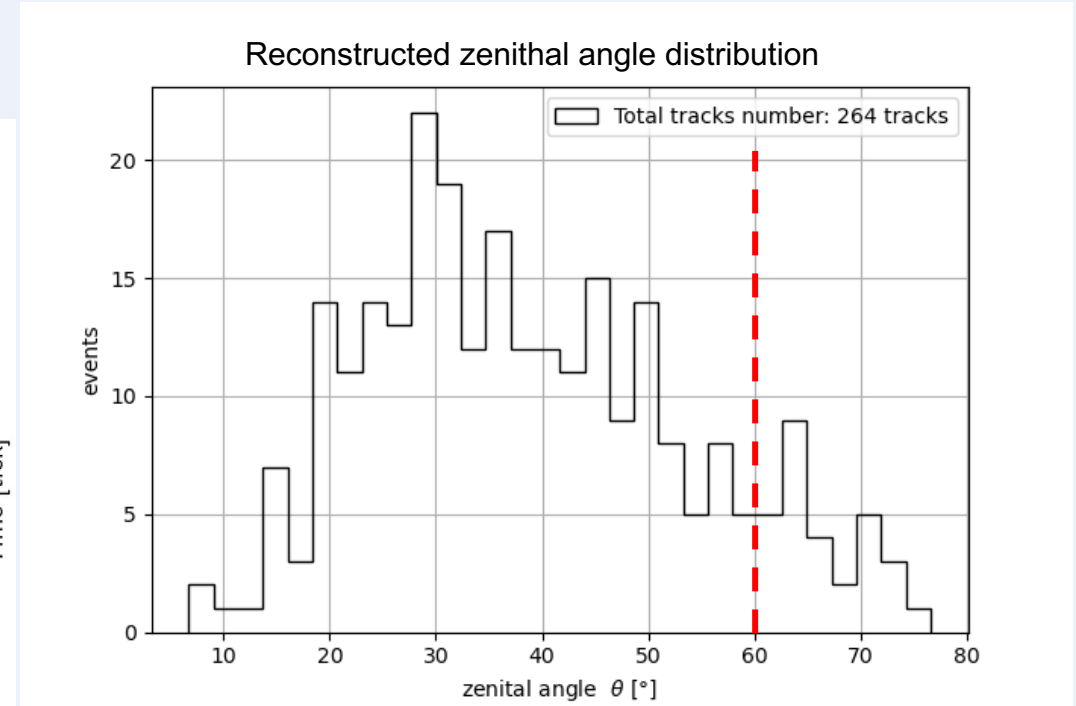
- An example of cosmic ray track from the Coldbox



- Bipolar signals induction view
- Muon events

## ➤ Coldbox's data cuts

- Tracks with reconstructed angle  $\theta > 60^\circ$
- $\varphi$  in the cone with  $45^\circ$  from the strip readout
- Track lengths  $> 20 \text{ cm}$



- Data reconstruction with LARDON (software developed by L. Zambelli)

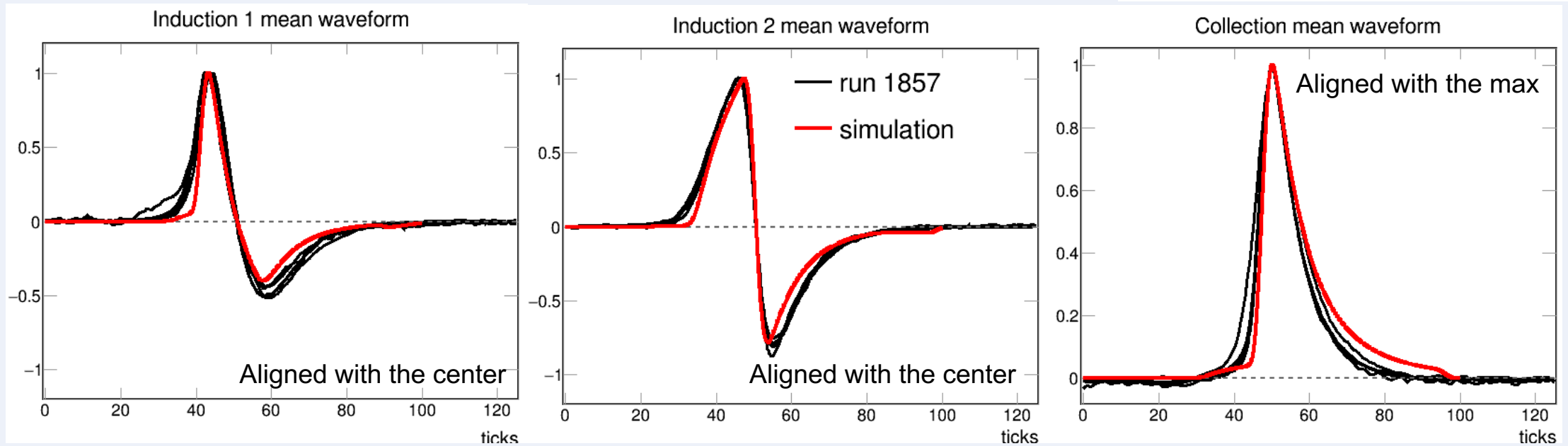
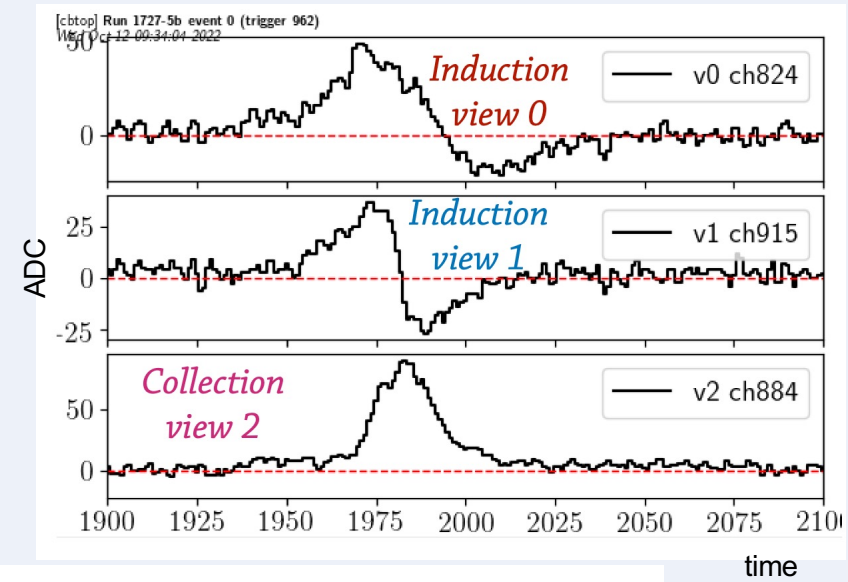
- **Select horizontal track to compare with simulation**



# Comparison data VS simulation

## ➤ Mean waveform

- Signals summation on each readout channel  
→ To reduce the incoherent noise
- Mean signal calculation for each track

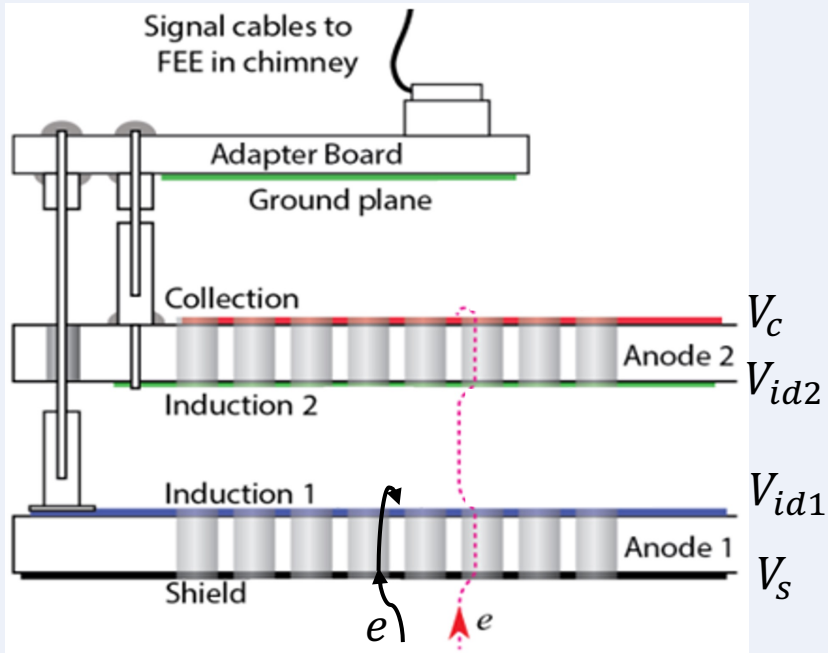


→ Simulated waveforms in agreement with data



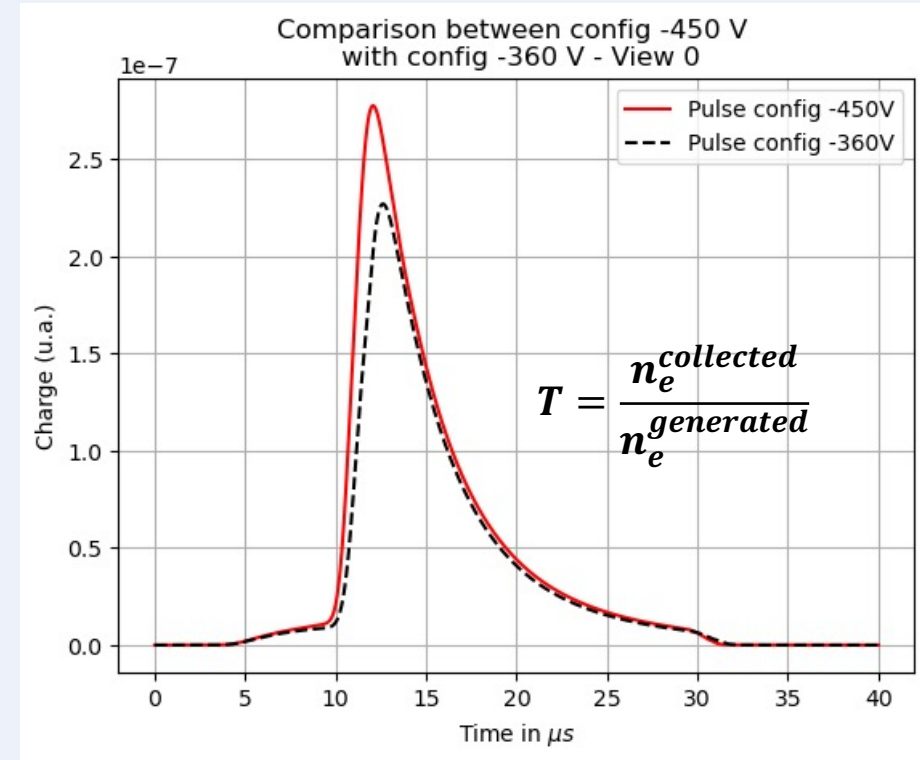
# CRP transparency study

- Work on signal amplitude to explore the different causes of charge loss
- Using two PCB can lead to loss of total CRP transparency:



- Some electrons are collected by the induction 1 plane
- Collected charge decreases

- The simulation has shown a significant effect between two bias configurations used in the coldbox tests:
  - Config 1:  $V_{id1} = -450\text{ V}$
  - Config 2:  $V_{id1} = -360\text{ V}$

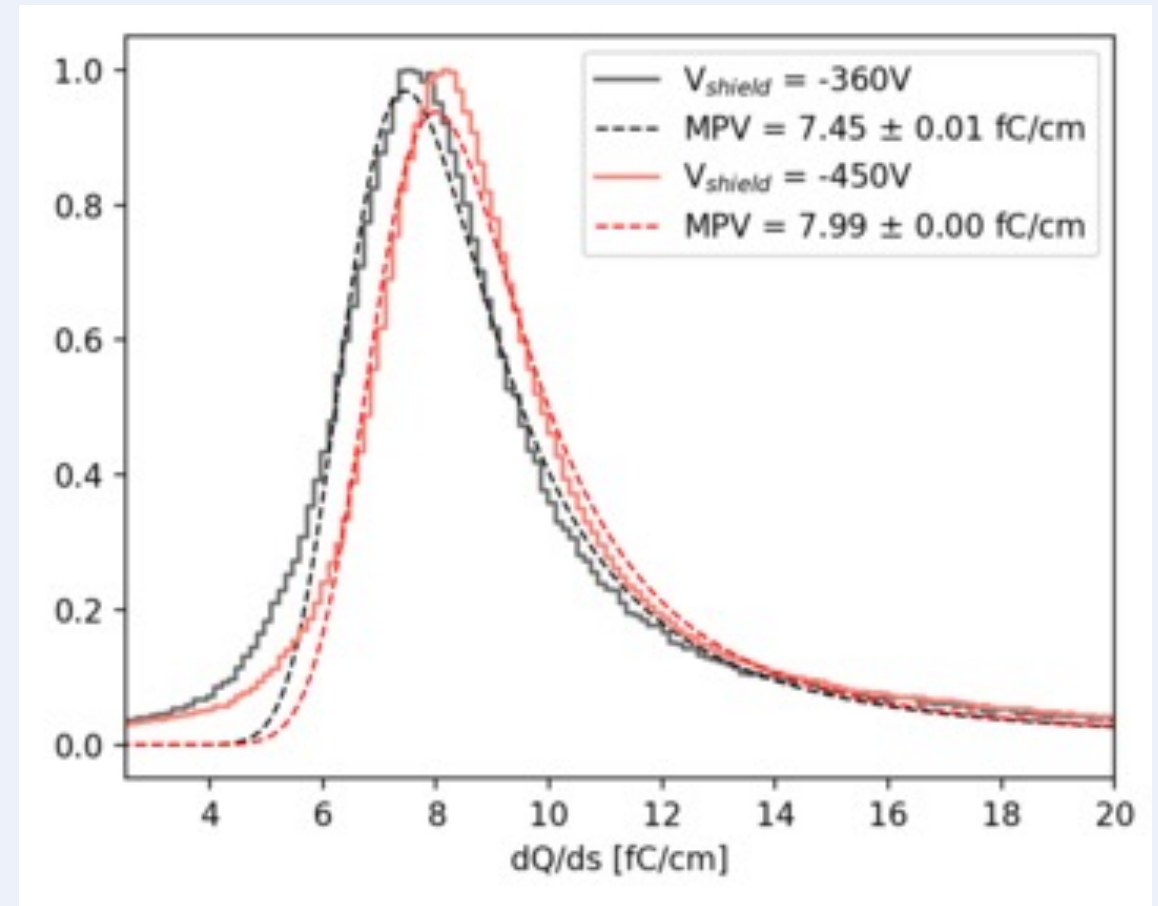


- Dependency on the bias voltage used

$$T_{sim} = \frac{Q_{tot}(-360)}{Q_{tot}(-450)} = 14\%$$

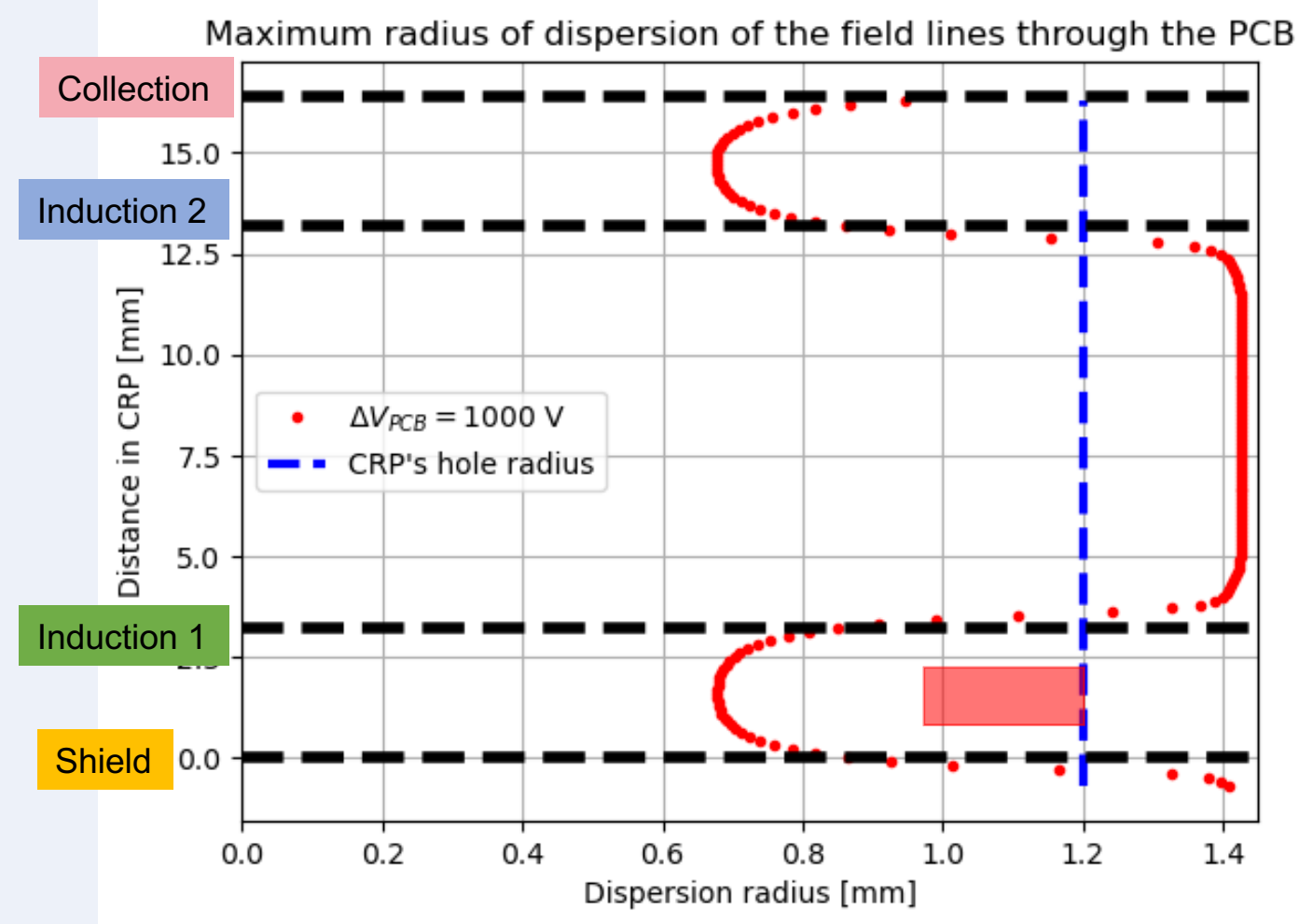
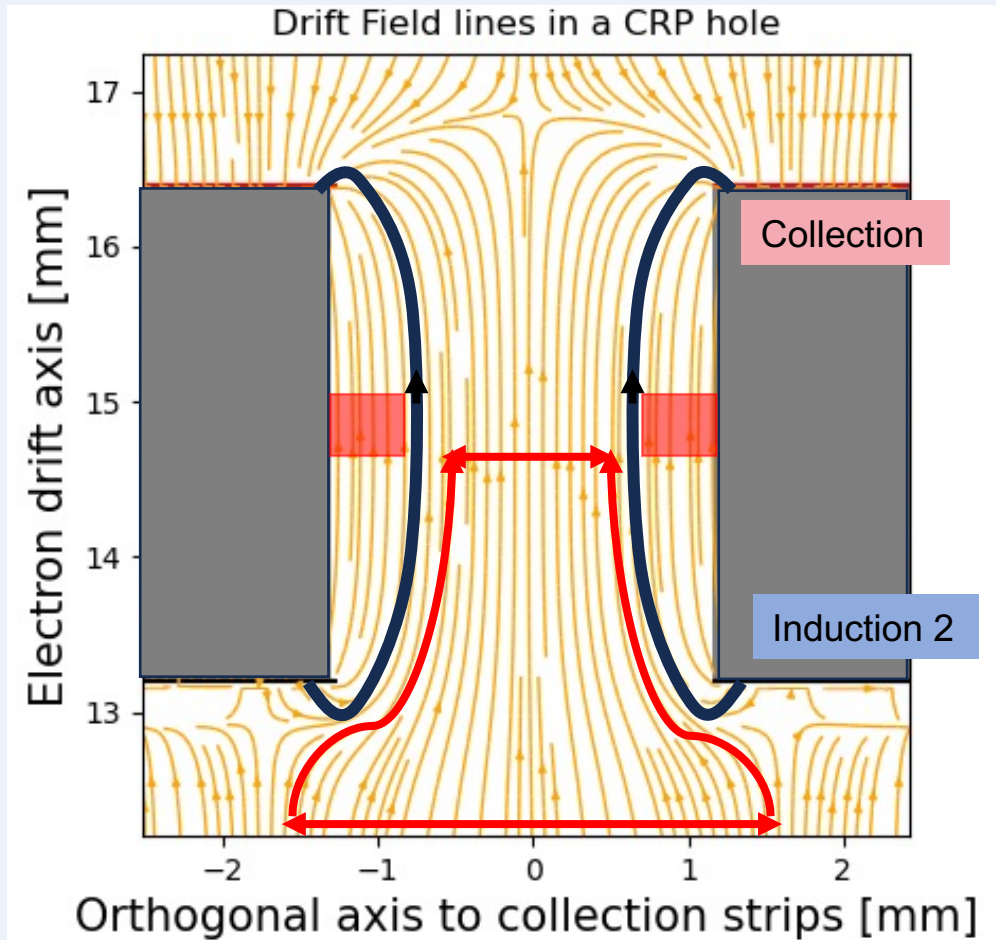
# CRP transparency study

- Coldbox's data with same configuration:
  - $dE/dx$  measured for muons
  - Comparison of the energy reconstruction
- Result:
  - **Increase of 6 % reconstructed energy for the "config -450 V"**
  - **A real effect but smaller than the simulation result**
- **Difference Simulation/Measurements need to be understood**



# Holes covering and loss charge

- Focalization of drift field lines in the CRP hole



- Field line too focused for loss charge → **Electron diffusion**

# Transverse electron diffusion

- Transverse electron diffusion could cause a loss of charge in the CRP

$$\frac{\partial n}{\partial t} = D_L \frac{\partial^2 n}{\partial z^2} + D_T \left( \frac{\partial^2 n}{\partial x^2} + \frac{\partial^2 n}{\partial y^2} \right) \quad \text{Fick's equation}$$

- Gaussian spatial distribution of the electrons over time
- Average diffusion length given by:

$$\sigma_{L,T} = \sqrt{2D_{L,T}t}$$

- Transverse diffusion coefficients measurement\*:

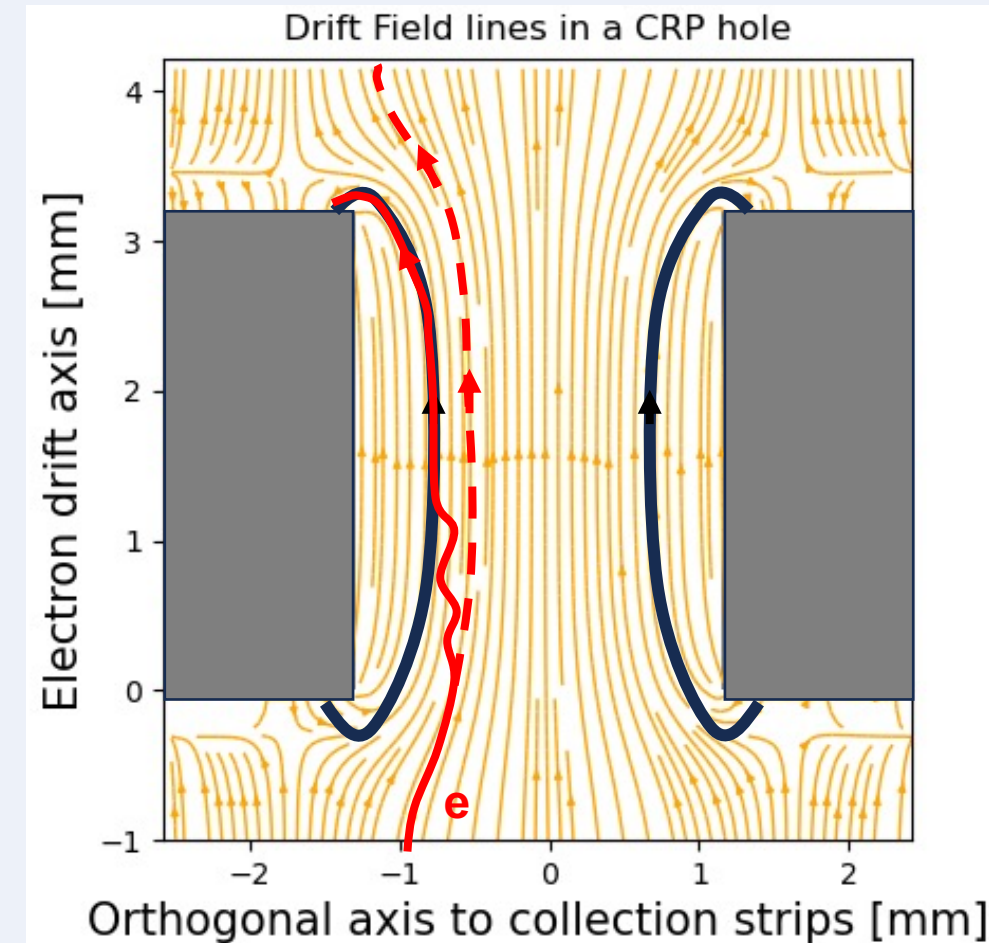
$$D_T = 13.2 \text{ cm}^2/\text{s}$$

- Maximal diffusion length for  $\sim 7 \text{ m}$  drift:

$$\sigma_T = 0.34 \text{ cm} < \text{strip length}$$

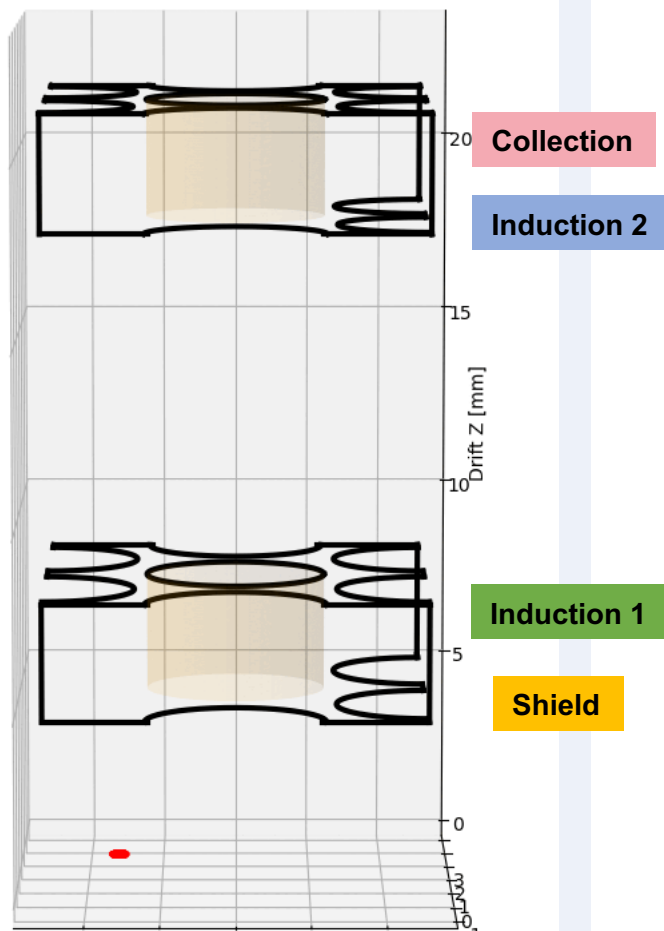
\* <https://lar.bnl.gov/properties/trans.html>

- Electron loss on Induction 1 due to the diffusion

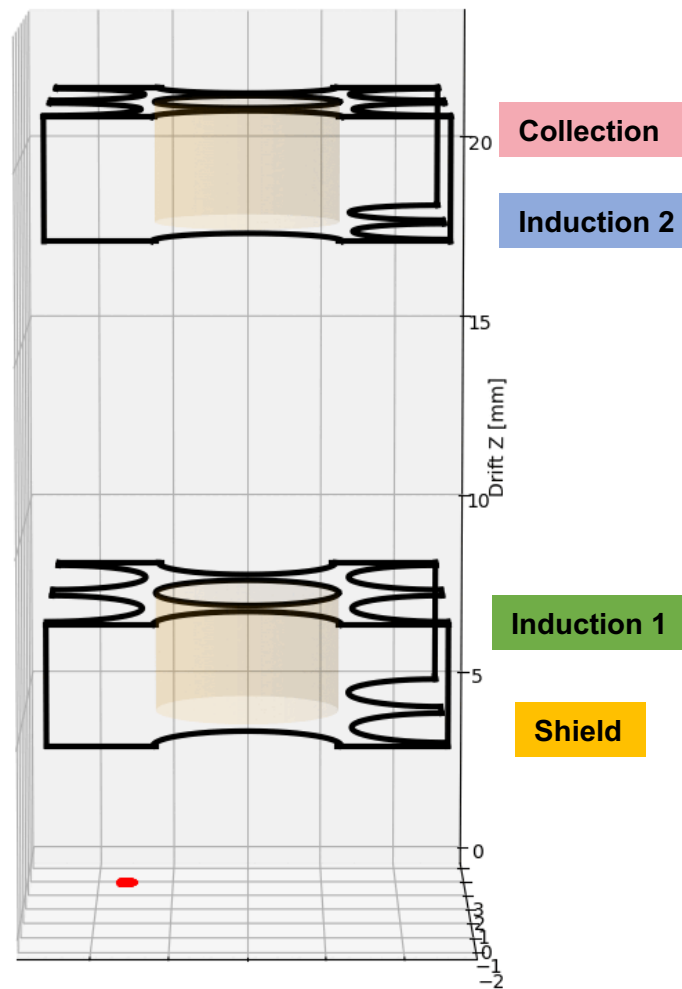


# Transverse electron diffusion

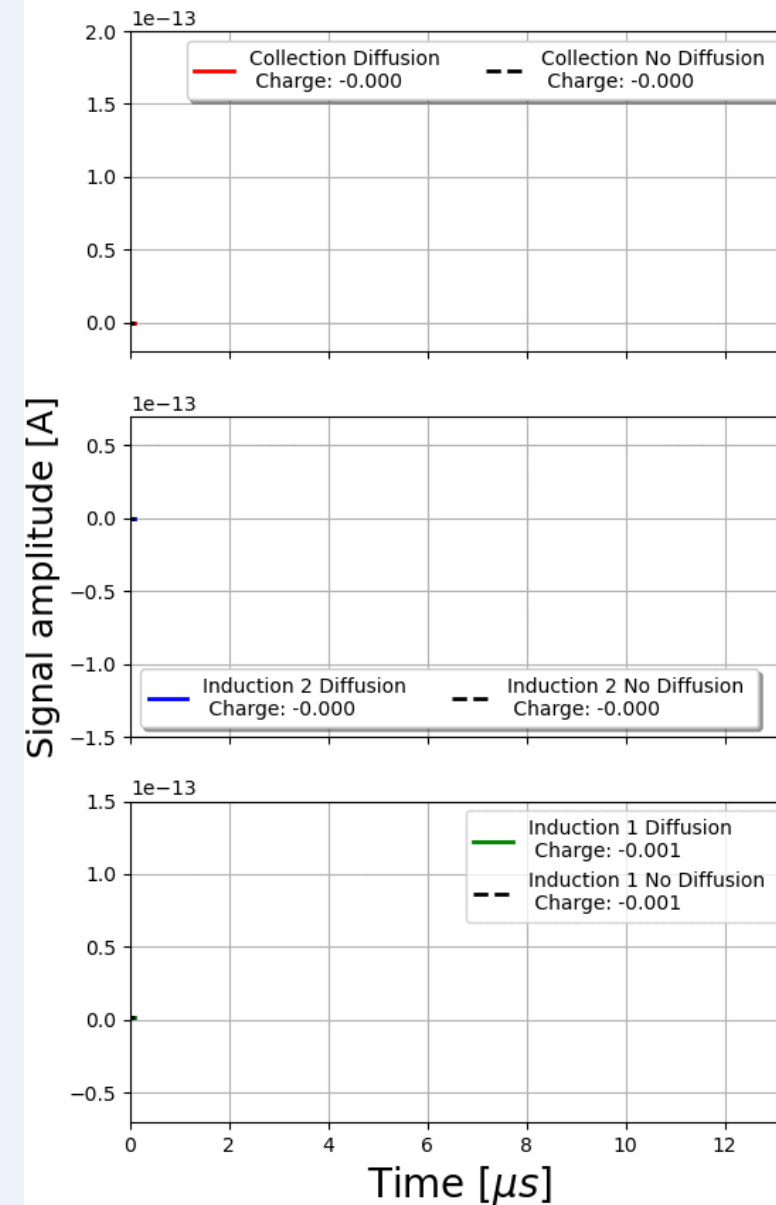
Electron cloud evolution no diffusion  
Time:  $0.10 \mu\text{s}$



Electron cloud evolution with transverse diffusion  
Time:  $0.10 \mu\text{s}$



Signal on all views  
Time:  $0.10 \mu\text{s}$

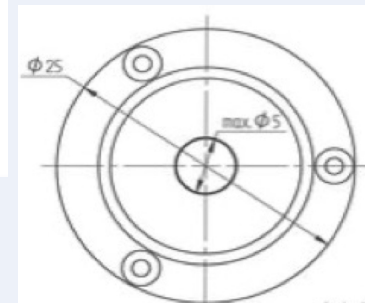
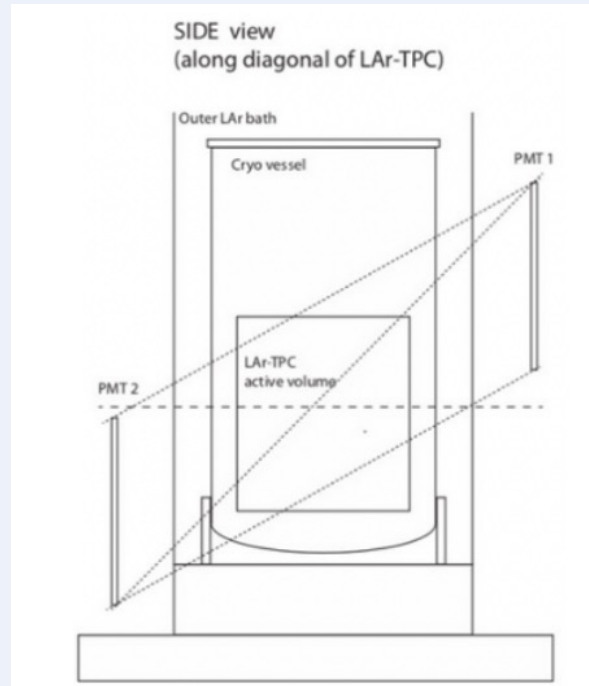
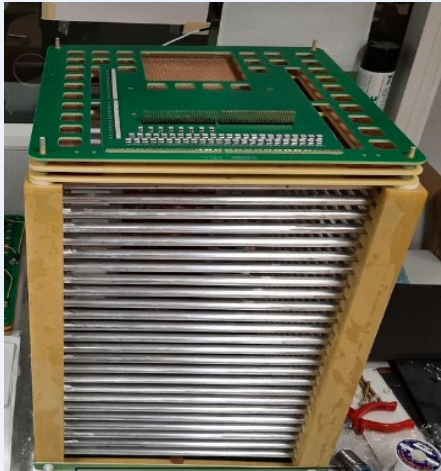




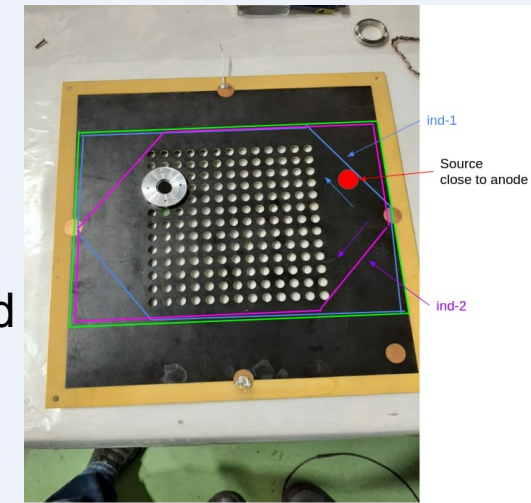
# R&D TPC 50 L detector

- Data-taken on R&D TPC at CERN last summer
- Each run with a different voltage bias

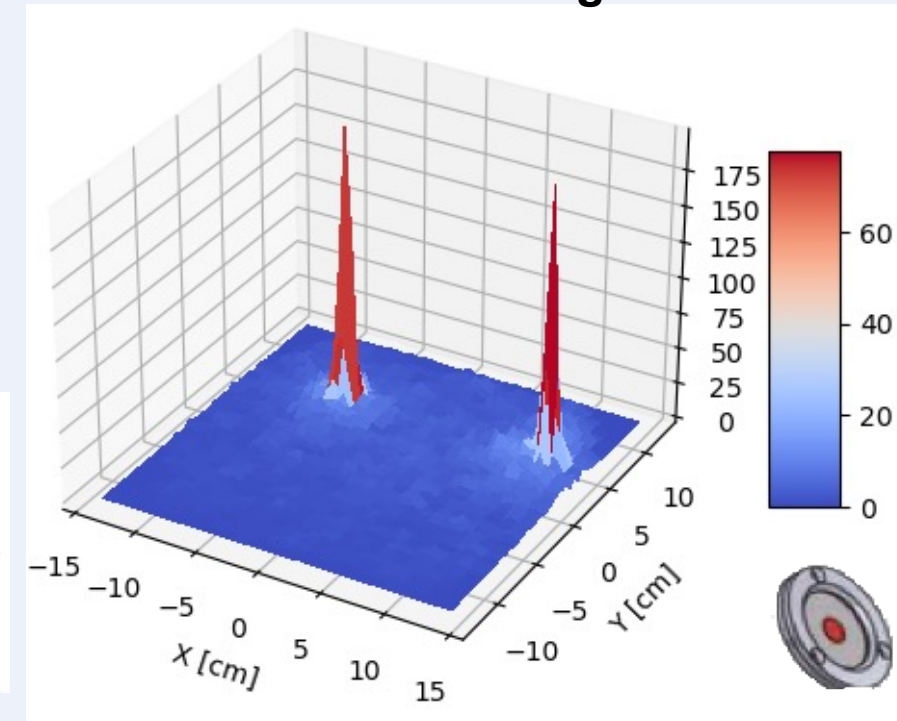
- $\sim 32 \times 32$  cm active area
- 52 cm drift
- Cosmic + random trigger



- 207 Bismuth sources
- Hexagonal active area
- Electron conversions around 1 MeV
- Range in liquid argon:  $\approx 5$  mm



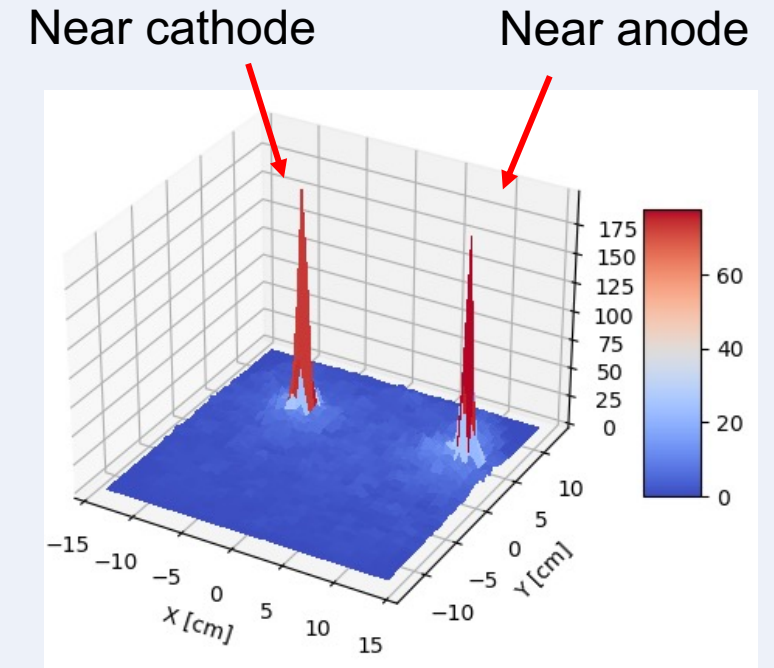
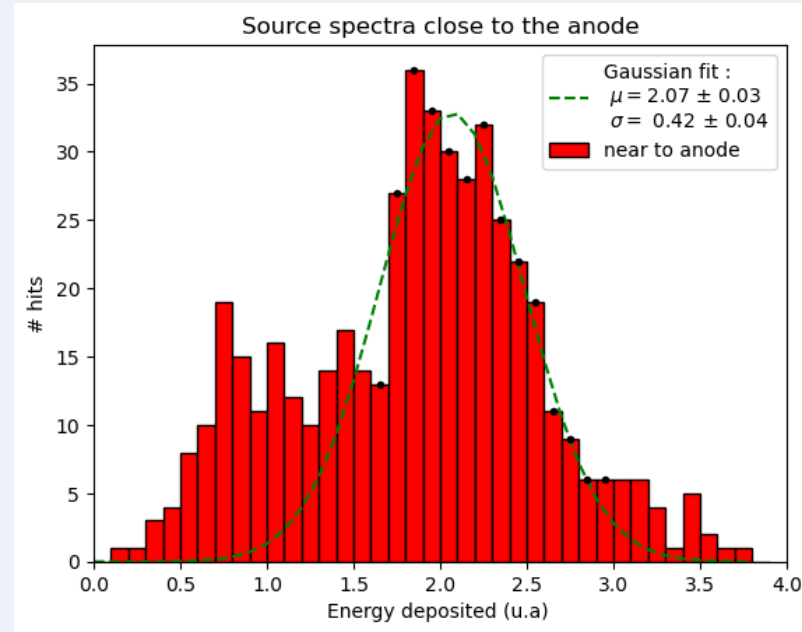
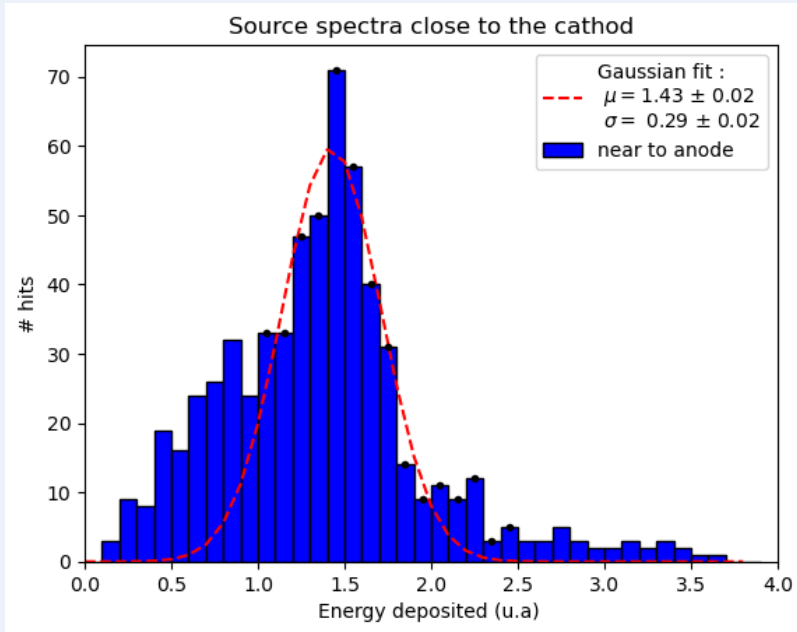
## Reconstructed single hits



- But not enough cosmic ray event



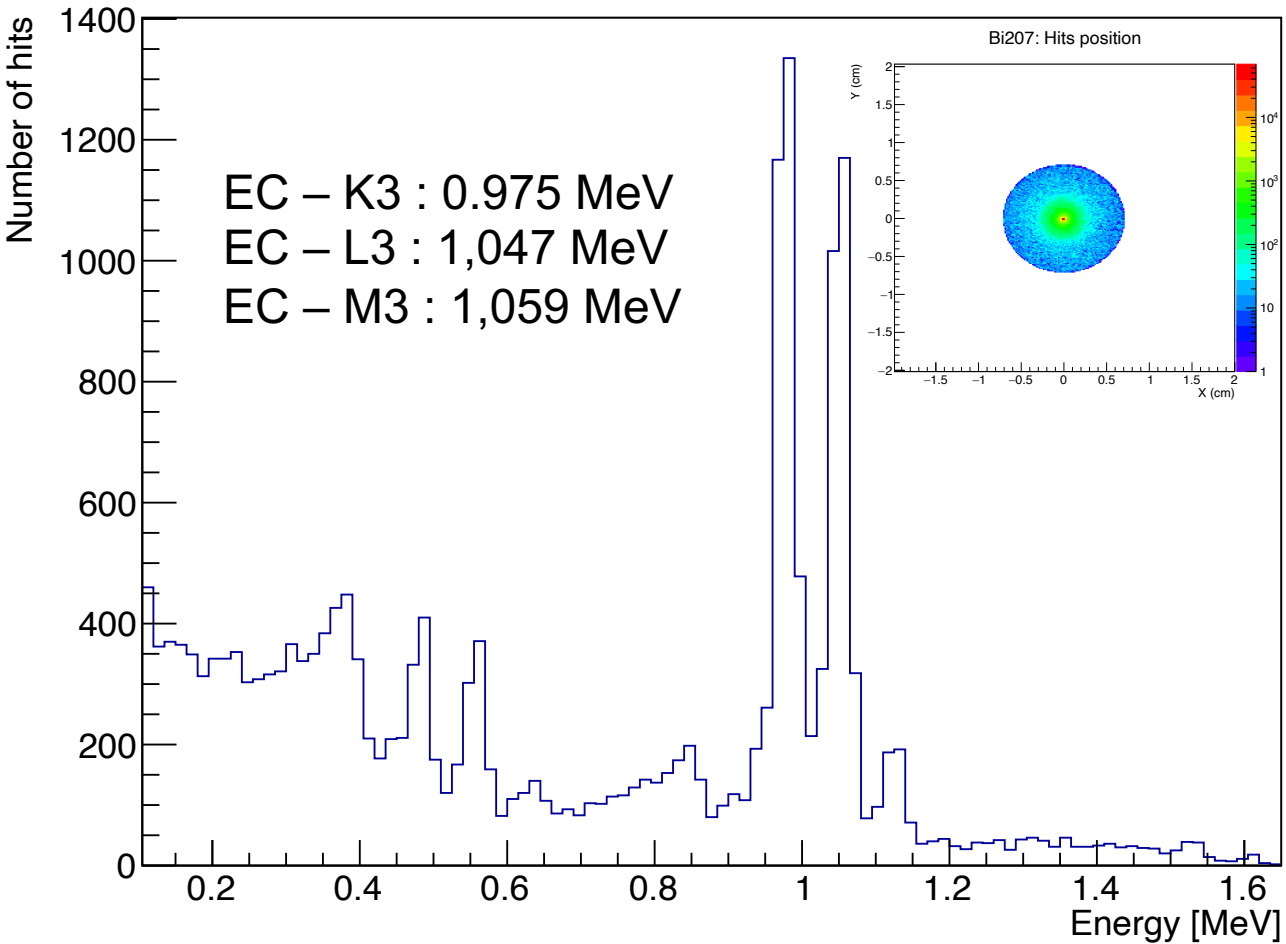
# Bi207 Reconstructed Spectra



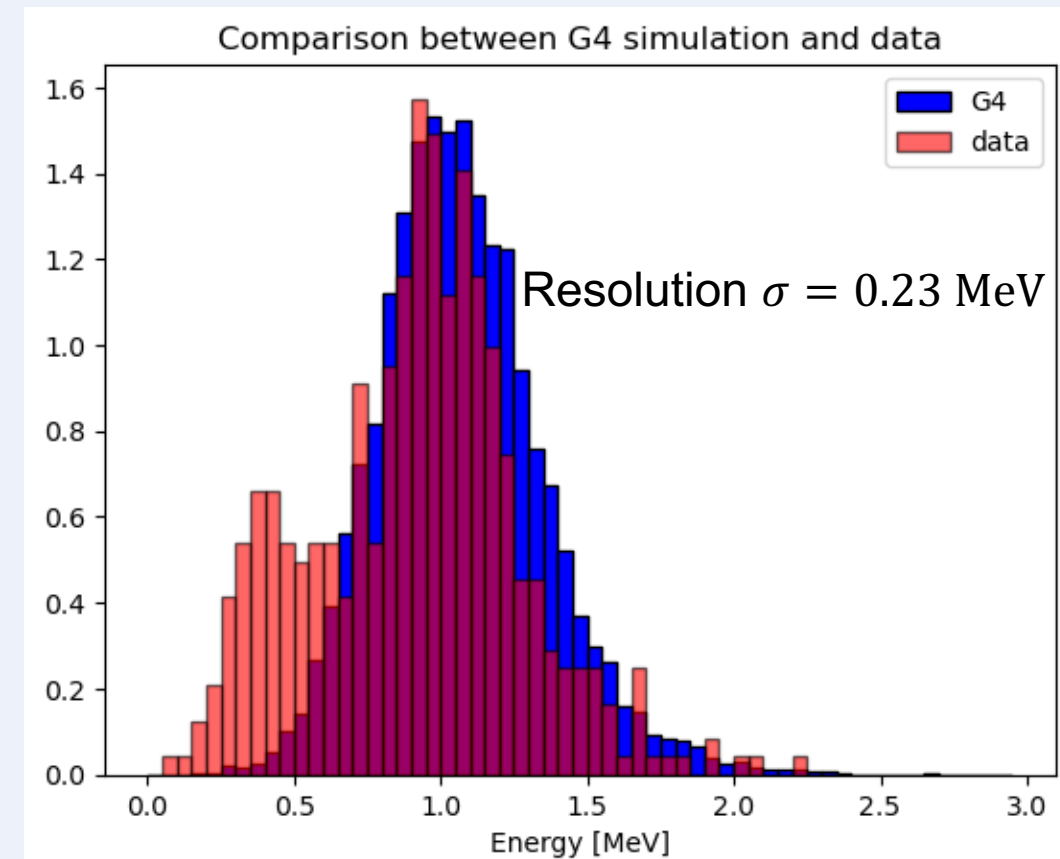
- Useful to calibrate detector with peak at 1 MeV
- Red is closer to the anode than blue
- Not enough single hit events

# G4 Simulation

Total deposited energy (e + photon),



Cut Geant4  $E > 0.8$  MeV



# Summary

## ➤ Work done:

- Numerical simulation conception to understand the formation of induction signals of all views
- Coldbox's data extraction → simulation is good agreement with this
- Get some data and start the analysis of 50 L R&D TPC
- Bismuth 207 events simulation Geant4 with 50 L and source geometry

## ➤ What's next ?

- Extend the simulation in a bigger volume → track angle studies and position in the TPC (boundary conditions)
- New run 50 L at soon with self-trigger for cosmic ray
- Keep working on the transverse electron diffusion and the impact on the loss charge
- Take part in the analyses of the ProtoDUNE VD at autumn 2024

# Back up

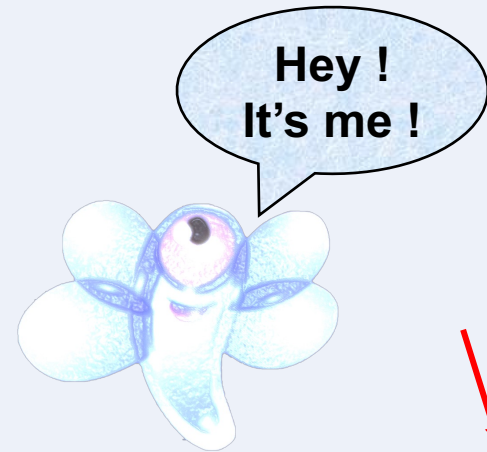


# Neutrino overview

- Chadwick (1914): Continuous  $\beta$  spectra
- Pauli (1930) proposed a neutral fermion called « neutrino »
- There are three lepton flavors  $\nu_e, \nu_\mu, \nu_\tau$
- Neutrino interact only by weak interaction  $\Rightarrow$  Only left-handed  
 $\Rightarrow$  Massless in Standard Model
- But neutrino need a mass to oscillate

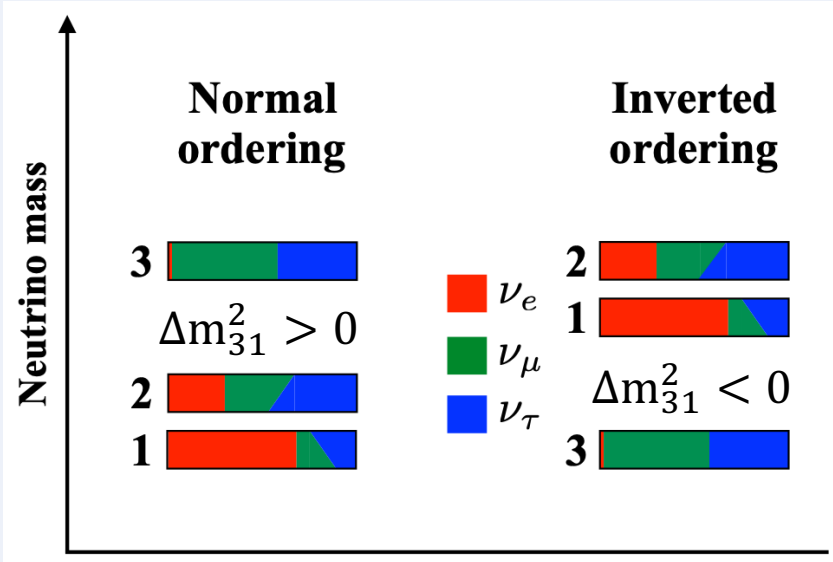
$$m_{\nu_e} < 1 \text{ eV}$$

$$\sum m_\nu > 0.06 \text{ eV}$$

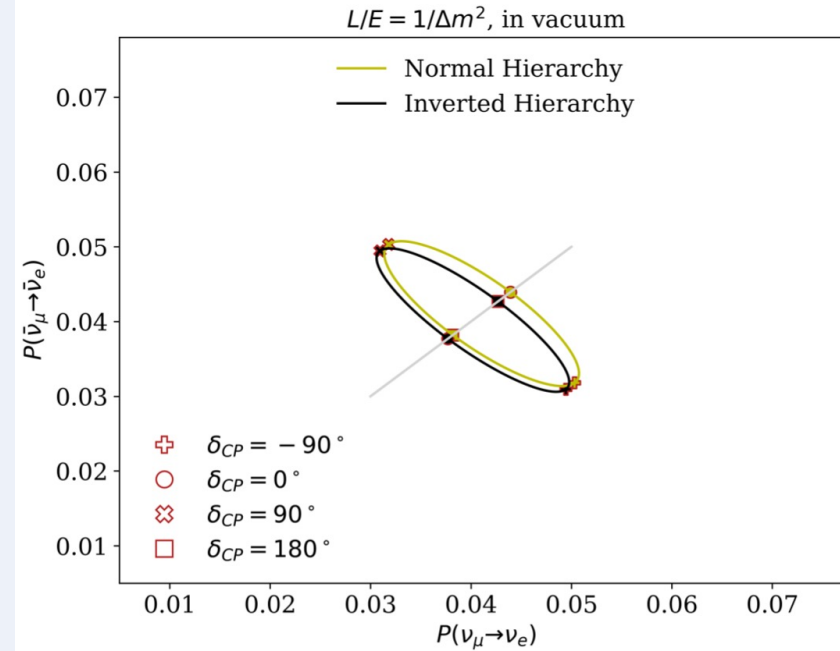


# Mass ordering

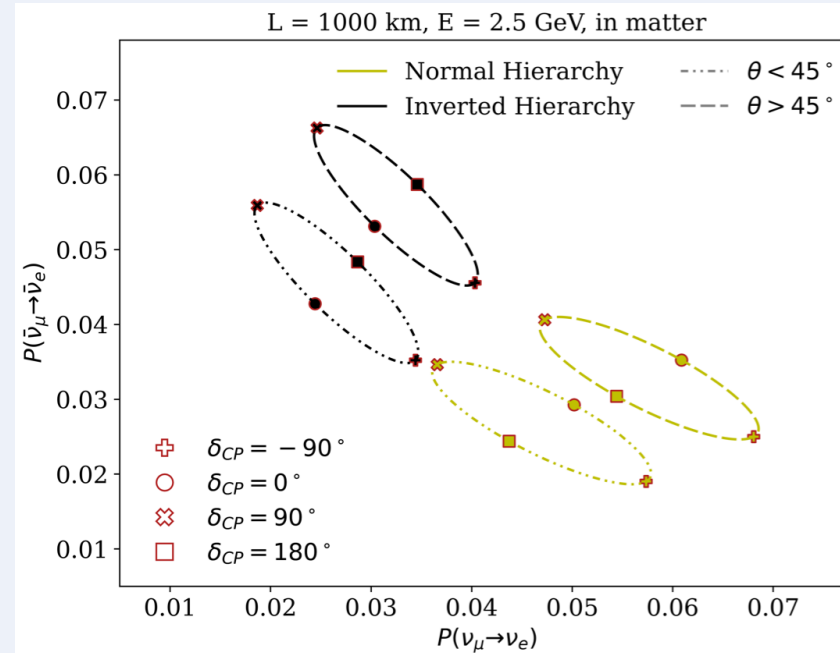
- Sign of  $|\Delta m_{31}^2| \rightarrow$  Mass ordering



Vacuum



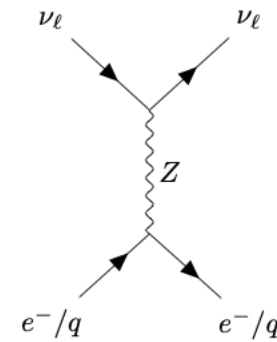
Matter



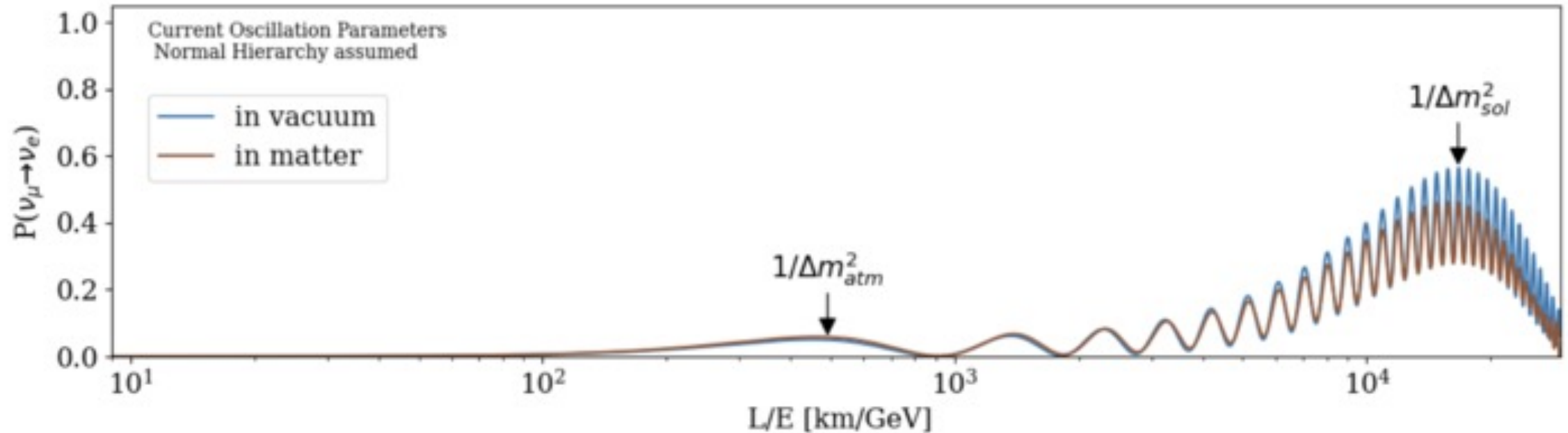
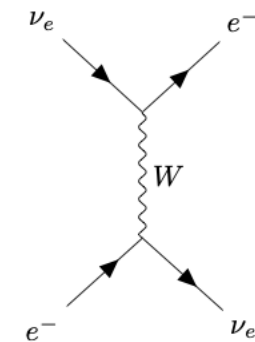
# Matter effect

- Neutrino oscillations are modified by matter effect
- Add an effective potential to the Hamiltonian

All neutrinos



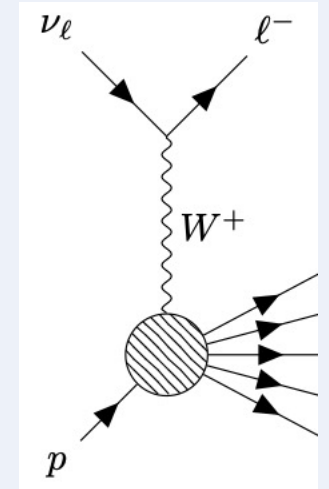
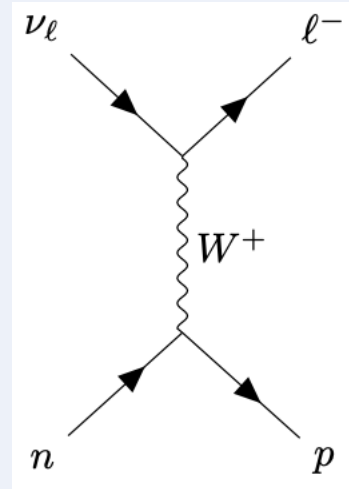
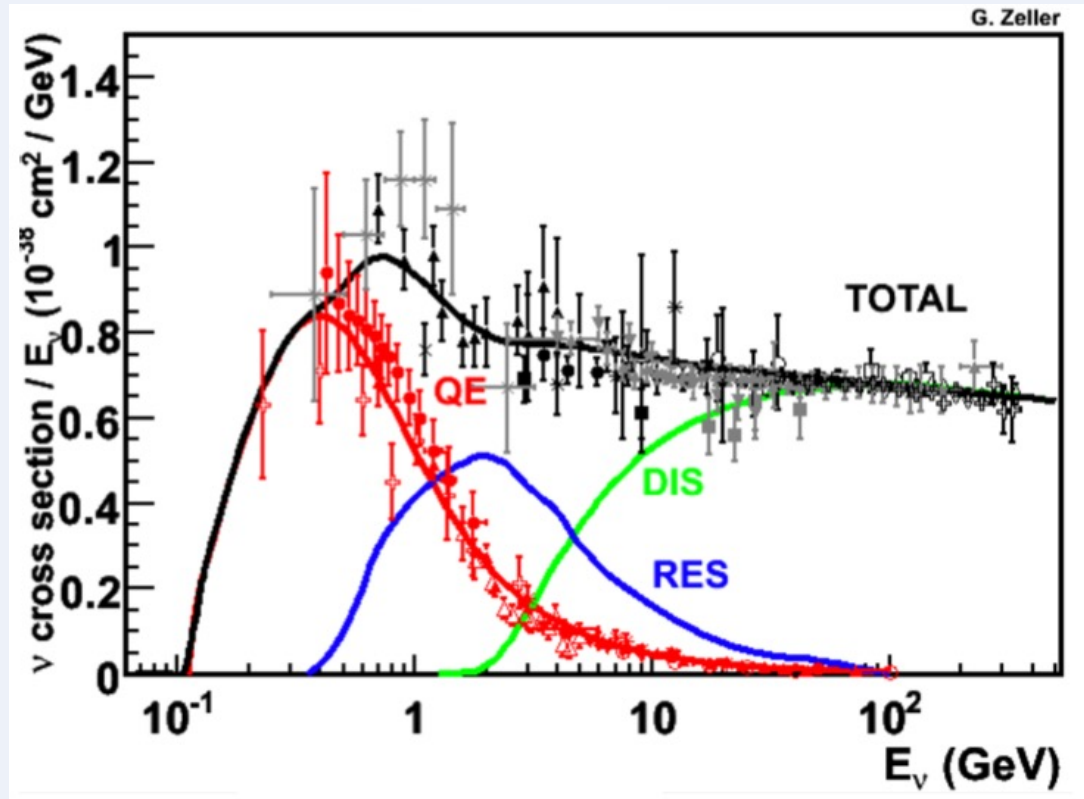
$\nu_e$  only



# Charged currents cross section

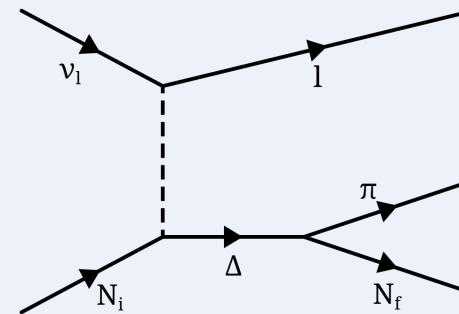
QE: Quasi elastic

DIS: Deep inelastic



Interaction with quarks

RES: Resonant

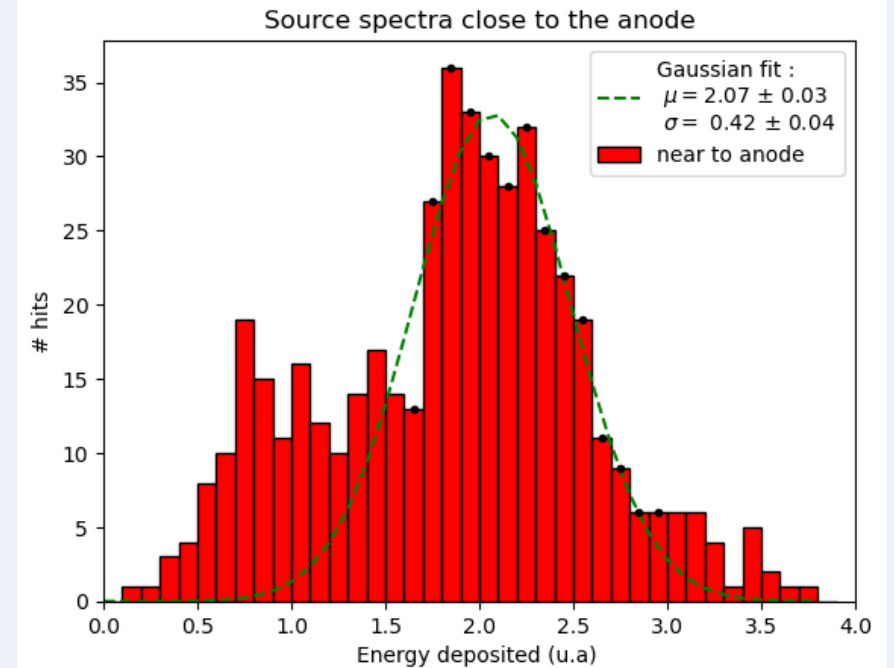
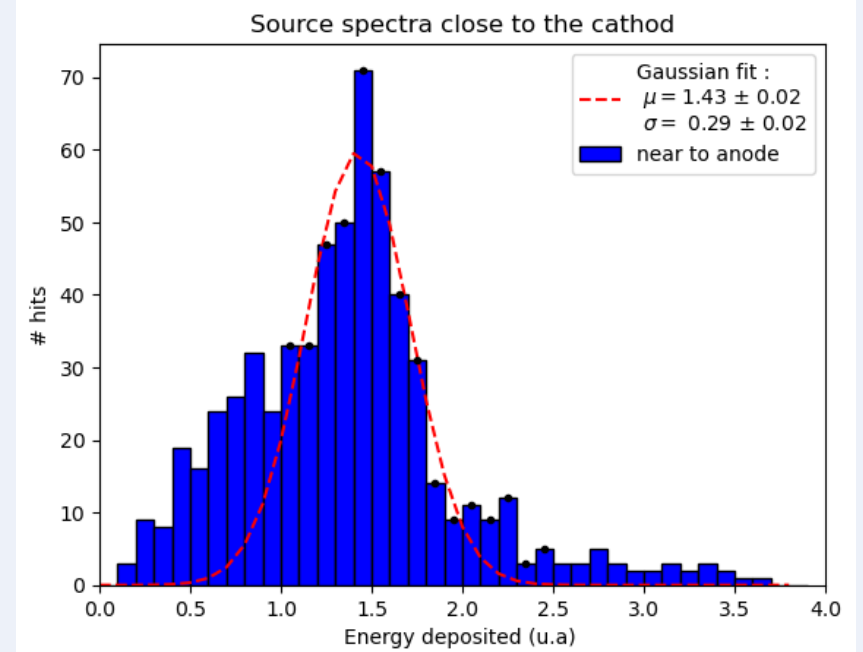
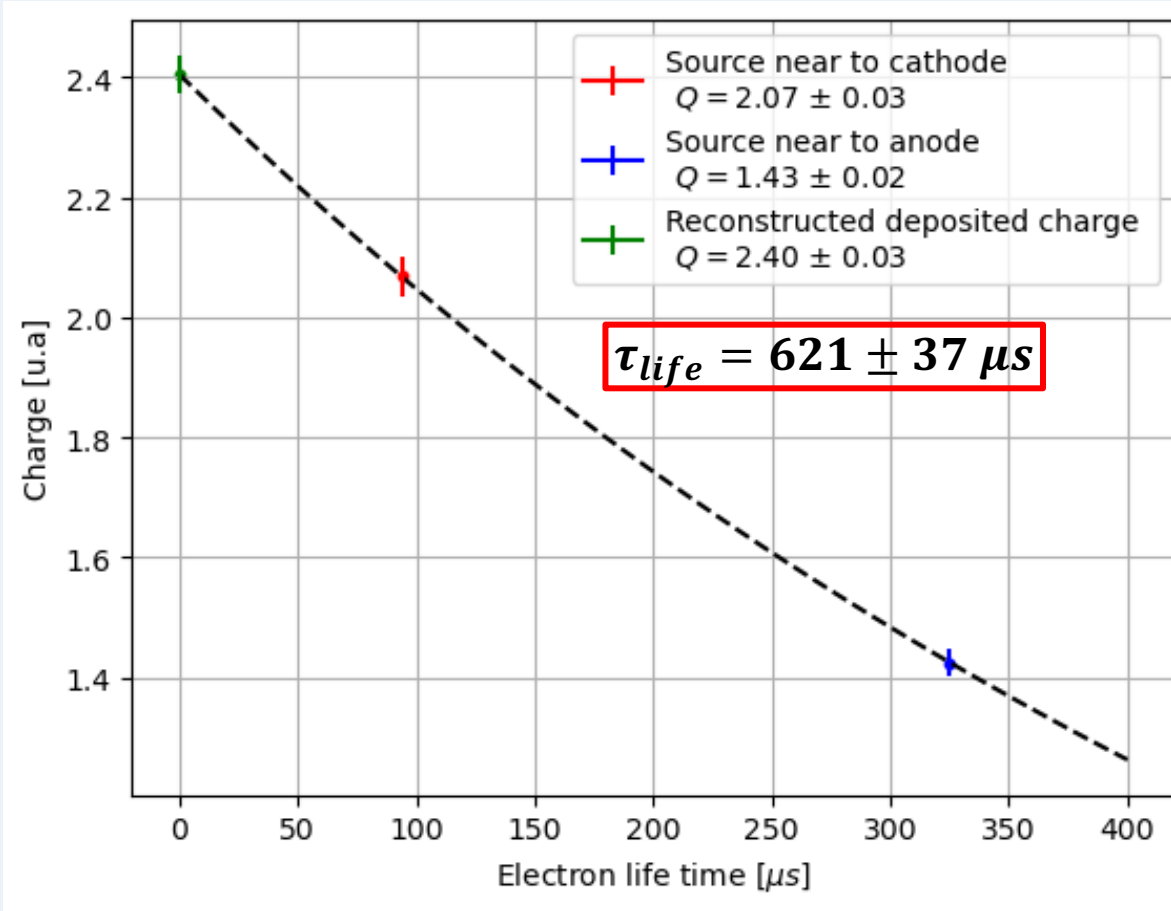




# Electron life time

- To reconstruct the charge, it is necessary to take into account impurities ( $N_2$ ,  $O_2$ , etc.):

$$\tau_{life} \approx \frac{300}{\rho(\text{impurities})}$$



# Boundary conditions (annexe ?)

## ➤ Mirror boundary conditions

$$\phi_{i,0,k} = \frac{\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,1,k} + \phi_{i,-1,k} + \phi_{i,j,k-1} + \phi_{i,j,k+1}}{6}$$

$$\phi_{i,-1,k} = \frac{\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,1,k} + \phi_{i,-1,k} + \phi_{i,j,k-1} + \phi_{i,j,k+1}}{6}$$

$$\phi_{i,j,0} = \frac{\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,1,k} + \phi_{i,-1,k} + \phi_{i,j,k-1} + \phi_{i,j,k+1}}{6}$$

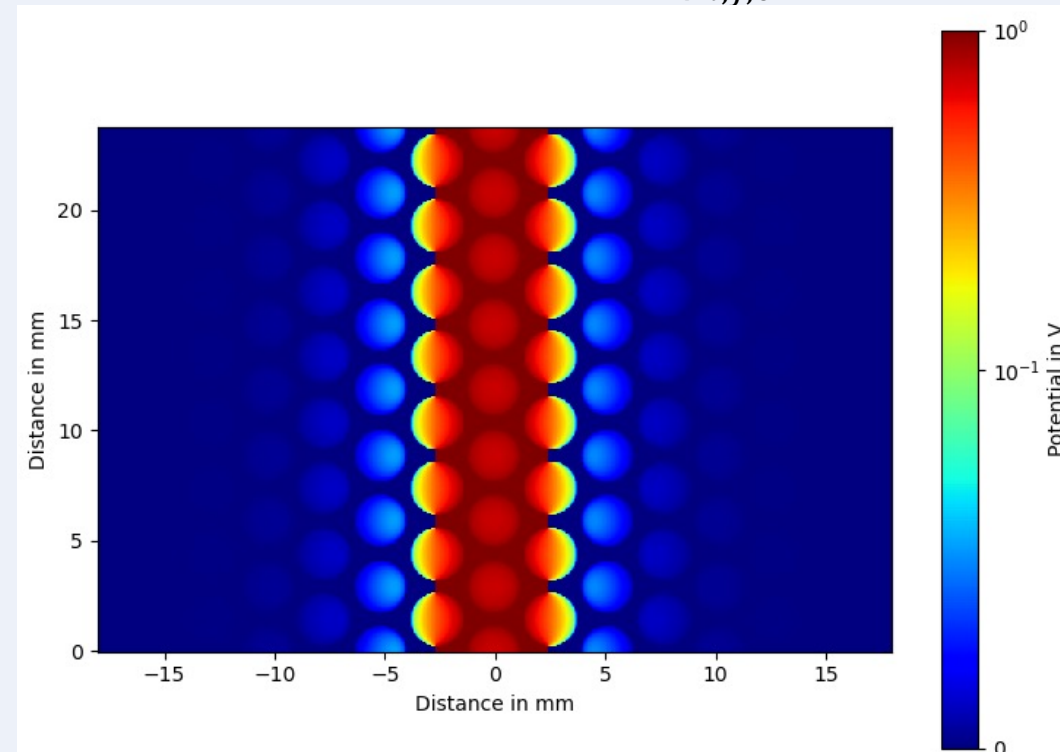
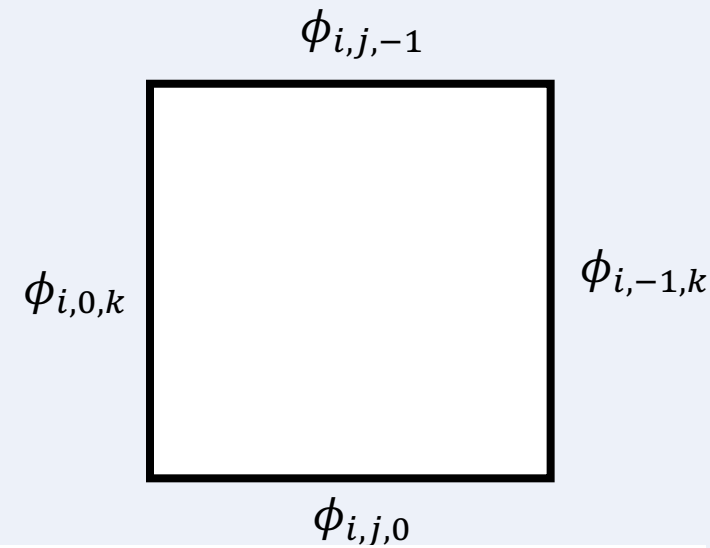
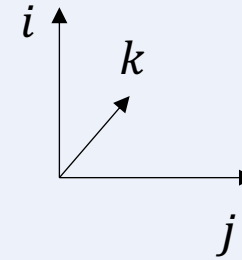
$$\phi_{i,j,-1} = \frac{\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,1,k} + \phi_{i,-1,k} + \phi_{i,j,k-1} + \phi_{i,j,k+1}}{6}$$

## ➤ Dirichlet boundary conditions

$$\phi_{i,0,k} = 0$$

$$\phi_{i,-1,k} = 0$$

Valid for a calculation windows  $\geq 7$  strips



# Ionization electron generation

- Thermal electron
  - Trajectory follow drift field lines

- Runge Kunta like simulation

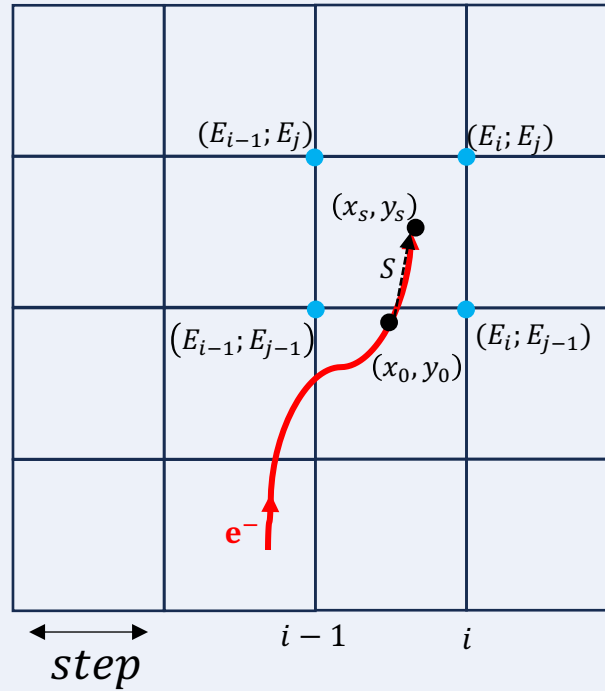
$$x_s = x_0 + \frac{E_x^{interp}(x_0, y_0)}{|E(x_0, y_0)|} \times S$$

$$y_s = y_0 + \frac{E_y^{interp}(x_0, y_0)}{|E(x_0, y_0)|} \times S$$

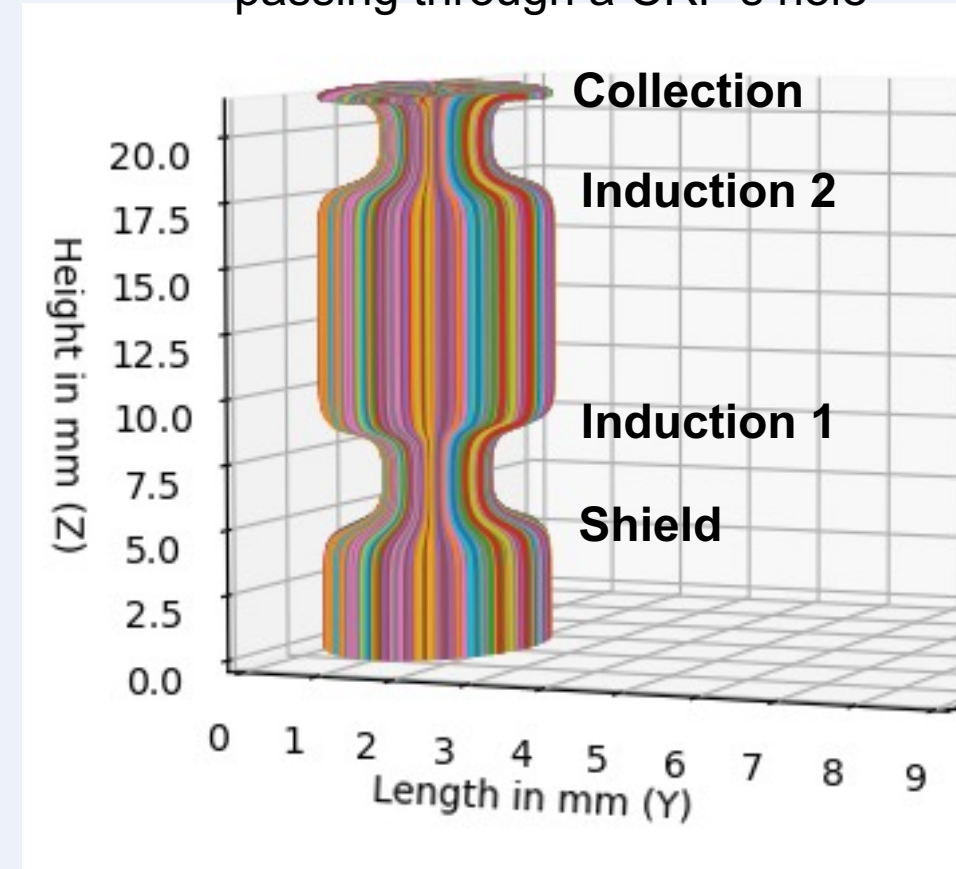
- Field interpolation

$$E_x^{interp} = E_{i-1} + \left( \frac{x_0}{step} - (i-1) \right) \times (E_i - E_{i-1})$$

- Drift simulation 5 mm



- Simulation of electron trajectories passing through a CRP's hole



# Drift velocity: Walkowiak Fit

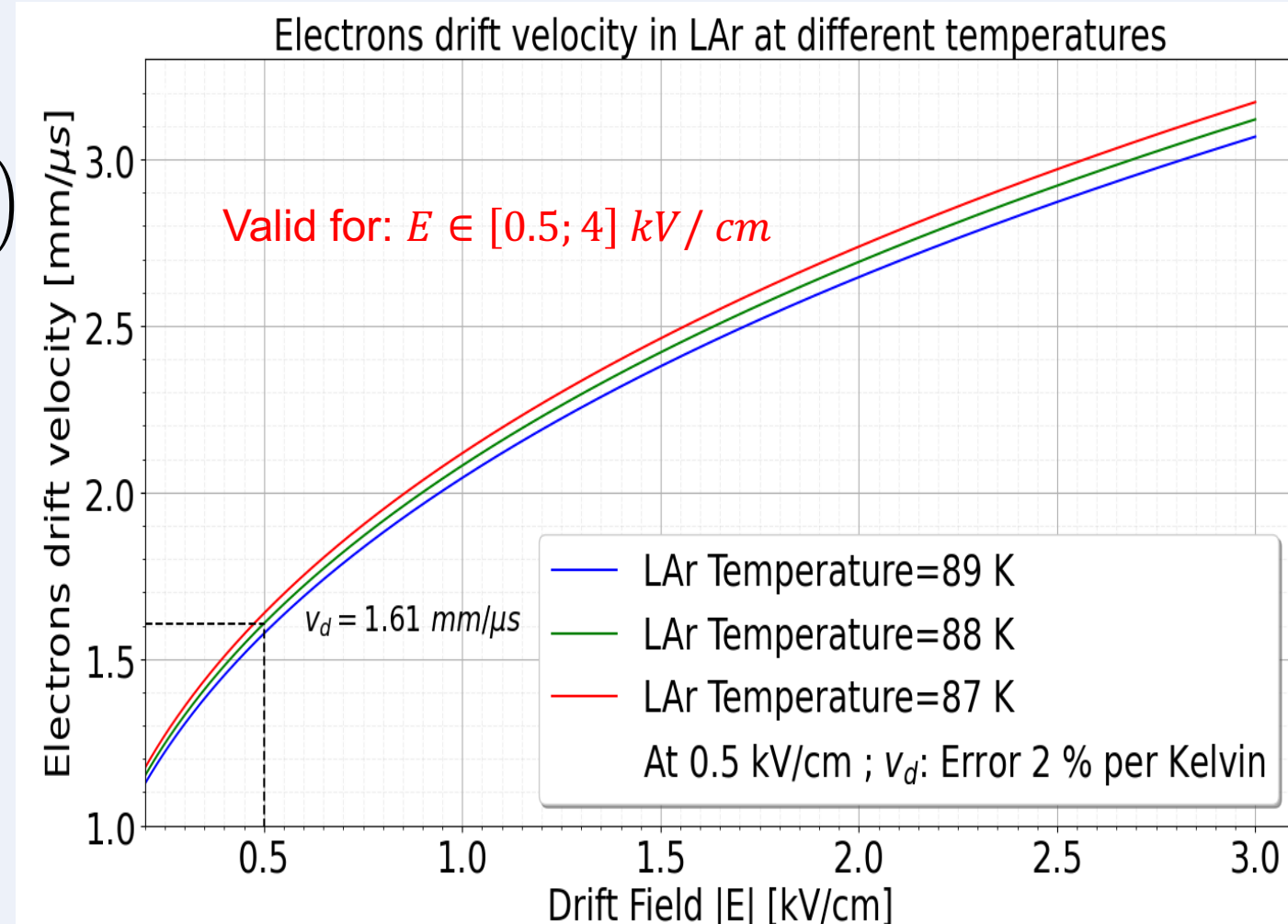
- Walkowiak fit (1999):

$$v_D \equiv v_D(|\vec{E}|, T)$$

$$= (P_1(T - T_0) + 1) \left( P_3|\vec{E}| \ln \left( 1 + \frac{P_4}{|\vec{E}|} \right) + P_5|\vec{E}|^{P_6} \right) + P_2(T - T_0)$$

With  $P_1, P_2, P_3, P_4, P_5$  and  $P_6$  fit parameters

$$\left\{ \begin{array}{l} P_1 = -0.01481 \pm 0.00095 \text{ K}^{-1} \\ P_2 = 0.0075 \pm 0.0028 \text{ K}^{-1} \\ P_3 = 0.141 \pm 0.023 \left( \frac{\text{kV}}{\text{cm}} \right)^{-1} \\ P_4 = 12.4 \pm 2.7 \left( \frac{\text{kV}}{\text{cm}} \right) \\ P_5 = 1.627 \pm 0.078 \left( \frac{\text{kV}}{\text{cm}} \right)^{-P_6} \\ P_6 = 0.317 \pm 0.021 \\ T_0 = 90.371 \text{ K} \end{array} \right.$$



W. Walkowiak. Drift velocity of free electrons in liquid argon 1999

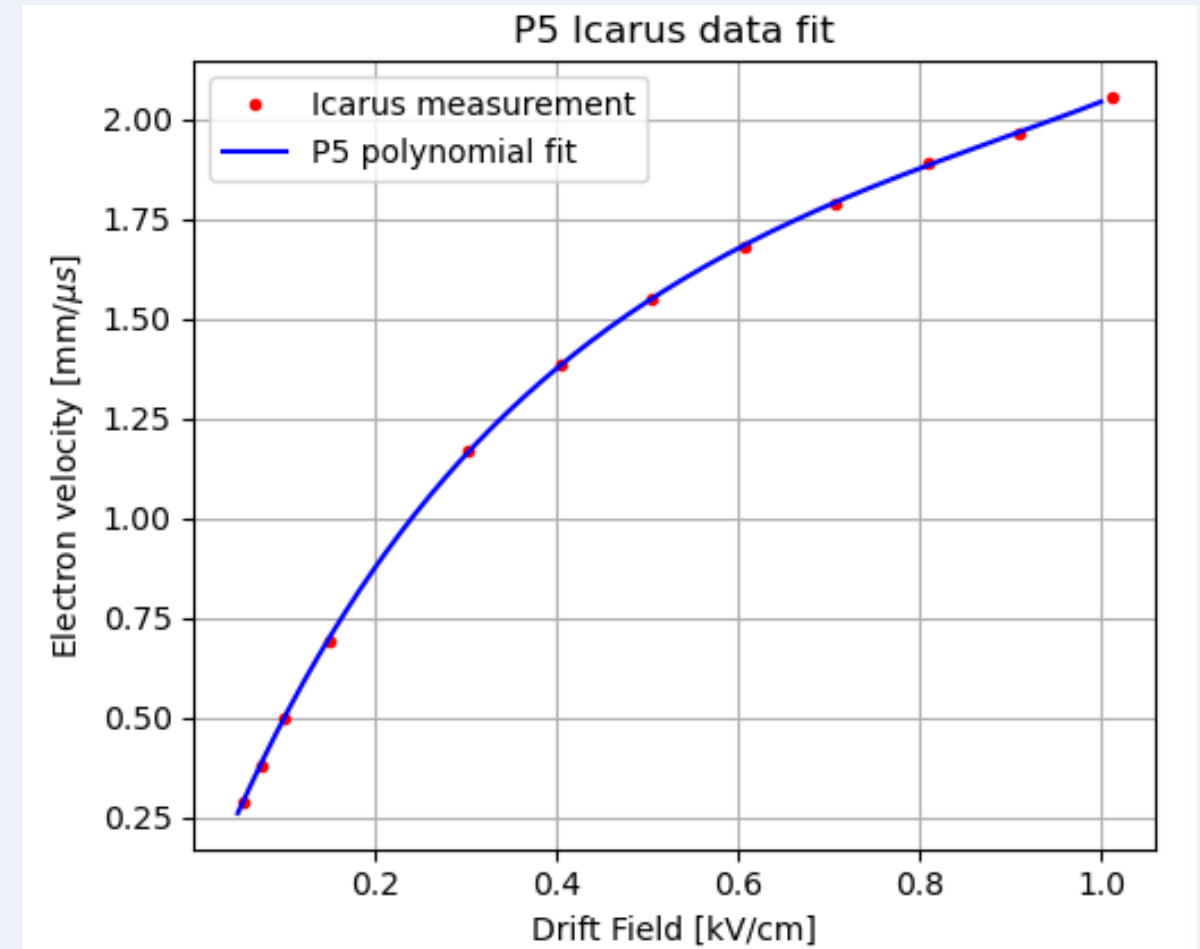


# Drift velocity: Icarus fit

- ICARUS detector using TPC technologie (2004)
- P5 Polynomial fit:

$$v_D(E, T = 89 K) = a + bE + cE^2 + dE^3 + eE^4 + fE^5$$

- Fit valid only:  $T = 89 K$



*ICARUS Collaboration, Analysis of the liquid argon purity in the ICARUS T600 TPC, 2004*

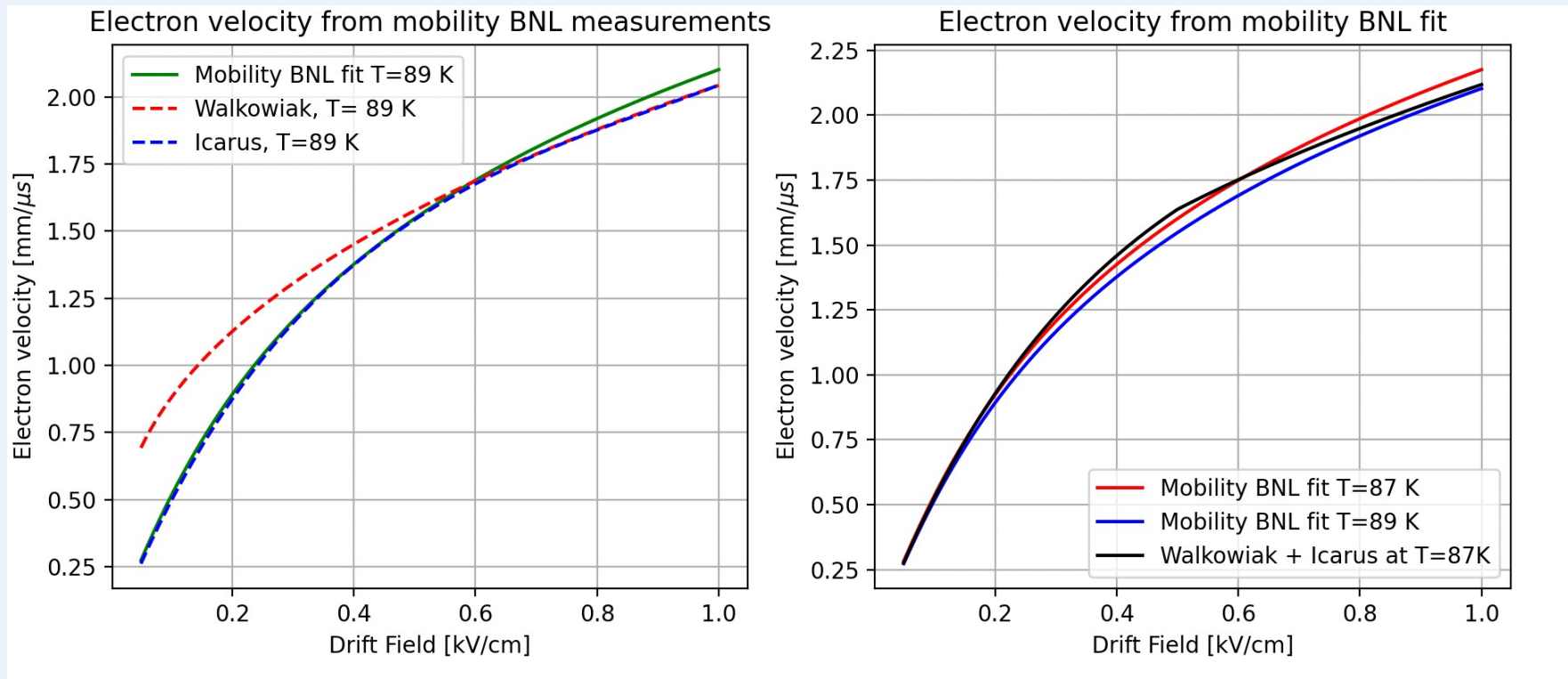
# Drift velocity: Brookhaven fit

- Global data fit scaled at  $T = 89 \text{ K}$

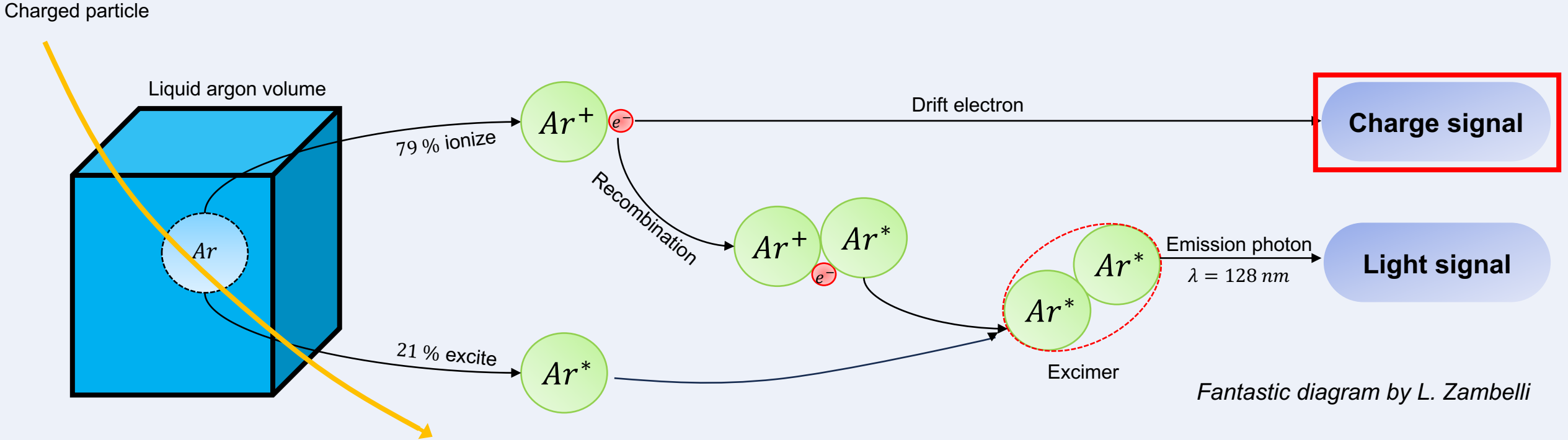
- Drift velocity:  $\vec{v}_D = \mu(|\vec{E}|, T) \vec{E}$  Avec:

$$\mu = \frac{a_0 + a_1 E + a_2 E^{3/2} + a_3 E^{5/2}}{1 + (a_1/a_0)E + a_4 E^2 + a_5 E^3} \left(\frac{T}{T_0}\right)^{-3/2}$$

$$\begin{cases} a_0 = 551.6 \\ a_1 = 7158.3 \\ a_2 = 4440.43 \\ a_3 = 4.29 \\ a_4 = 43.63 \\ a_5 = 0.2053 \end{cases}$$



# Principle of LArTPC detection



- Argon is chemically inert and dense
- Charged particles ionize (79 %) and excite (21 %) argon atoms.
- Scintillation light coming from argon de-excitation ( $\lambda = 128 \text{ nm}$ )
- Electrons of ionization drift to the anodes thanks to an electric field

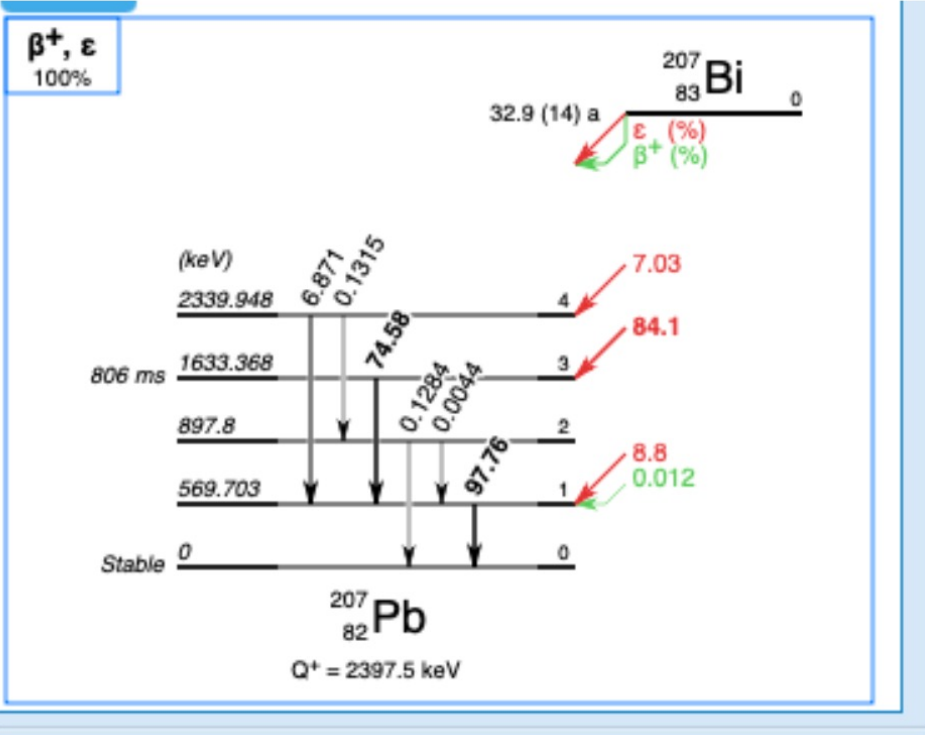
## ➤ LArTPC:

- Segmented anode used to collect charge signal
- $\tau_{drift} \gg \tau_{photon} \rightarrow$  light signal trigger detection

## Track and energy reconstruction

# BI207 sources

➤ Decay by electronic capture



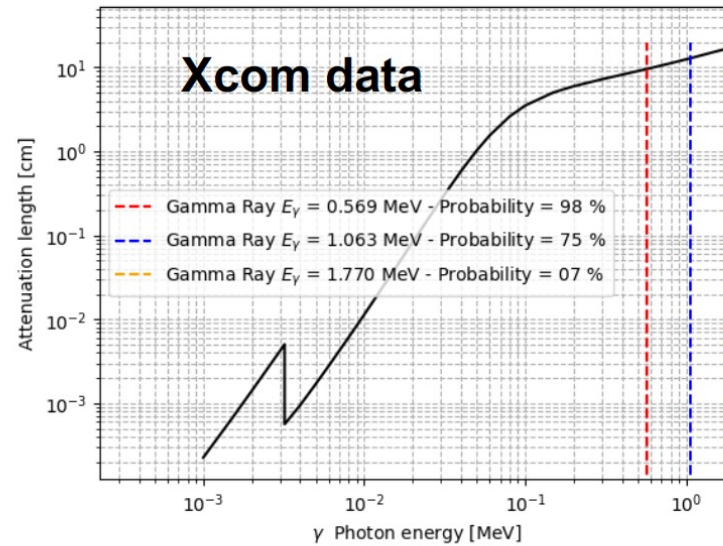
➤ Main  $\gamma$  rays:

- $\approx 570$  keV
- $\approx 1$  MeV
- $\approx 1.7$  MeV

➤ More complicated:

- Conversion Electron  $\approx 1$  MeV

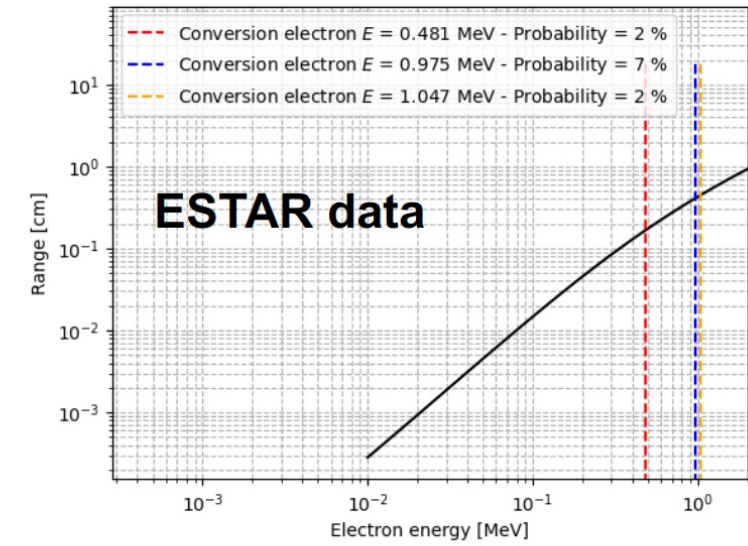
$\gamma$  attenuation length in liquid argon



- $\gamma$  attenuation length  $> 10$  cm

- Conversion electron range  $\approx 1$  cm

Electron range in liquid argon



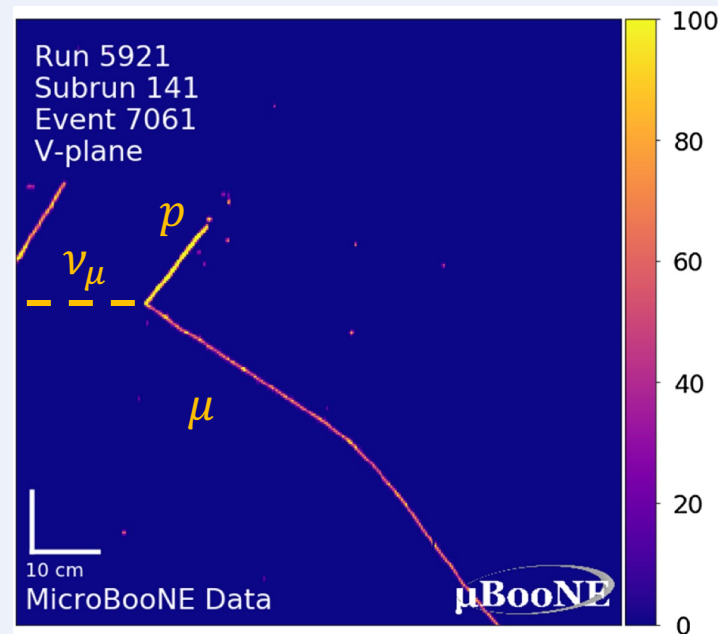
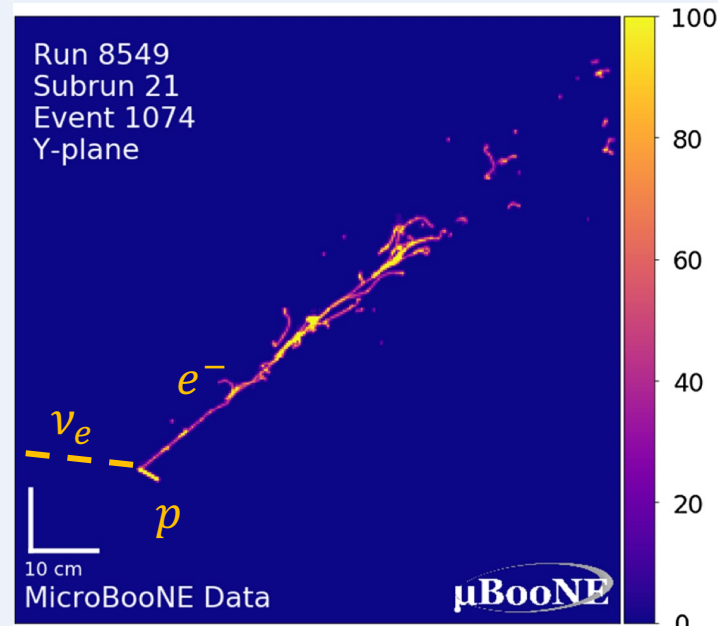
➤ Need to find Bi207 events from 50 L data

- Electron range very short

- Looking for only one signal from strips on all induction views  $\rightarrow$  called a single hits

# Why use TPC ?

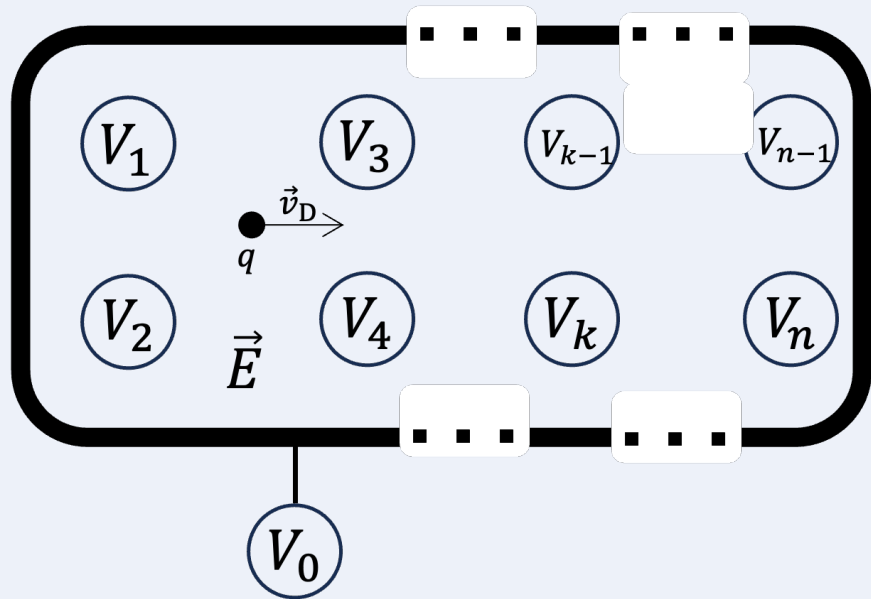
- Very important to discriminate muon and electron
  - At GeV-scale (1 – 10 GeV) neutrino interaction are dominated by quasi-elastic scattering processes
  - Charged current interaction gives a charged lepton in the final state used to tag the neutrino flavour
- $$\begin{cases} \nu_l + n \rightarrow l^- + p \\ \bar{\nu}_l + p \rightarrow l^+ + n \end{cases}$$
- Separate  $\nu_e / \nu_\mu$  events by track topology studies
  - DUNE need to measure the  $\nu_e / \nu_\mu$  rate





# Modeling signal formation

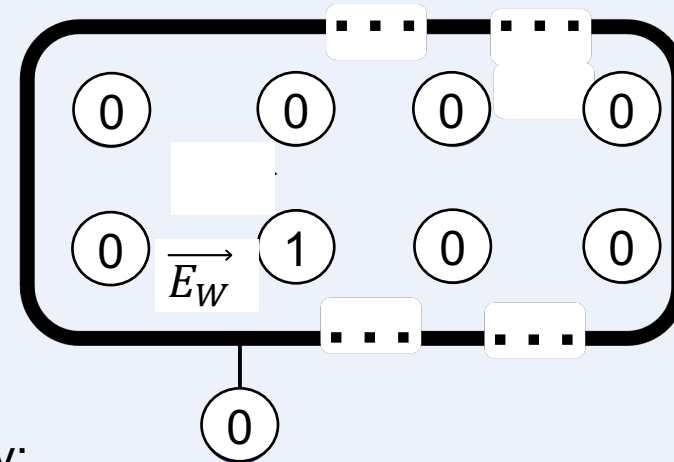
- General case – Shockley-Ramo theorem



- If electrodes get a fixed potential and the velocity of the charge particle is known, then the induced current  $i(t)$  on any electrode  $k$  is given by:

$$i_k(t) = q \vec{E}_w \cdot \vec{v}_D$$

- Weighting Field  $\vec{E}_w$  is virtual field that would exist in the case where the charge is removed, the reading strip equal 1 V and all others fixed to 0 V

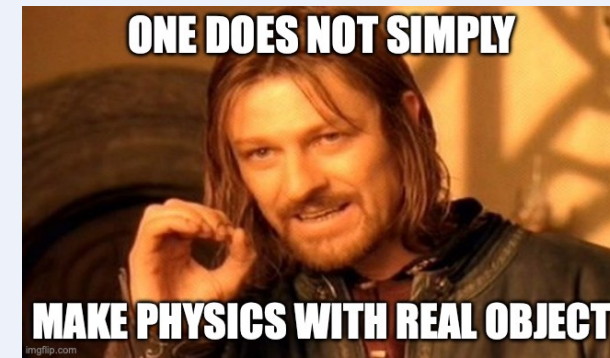


- Drift velocity:

$$\vec{v}_D(\vec{E}, T) = \mu(\vec{E}_D, T) \times \vec{E}_D$$

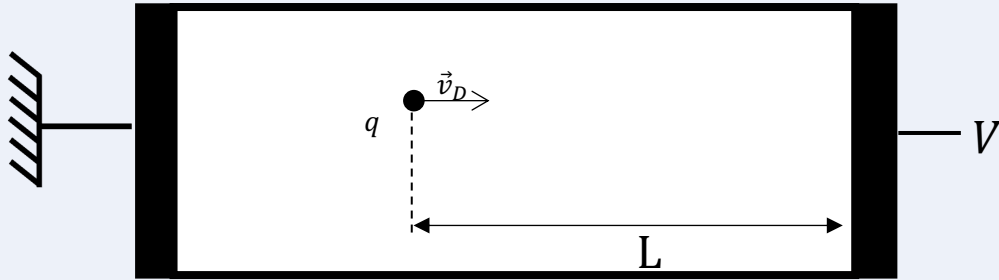
as a function of the **electron's mobility** in the medium.

$$\begin{cases} \vec{E}_w: \text{geometry factor} \\ \vec{v}_D: \text{physics factor} \end{cases}$$



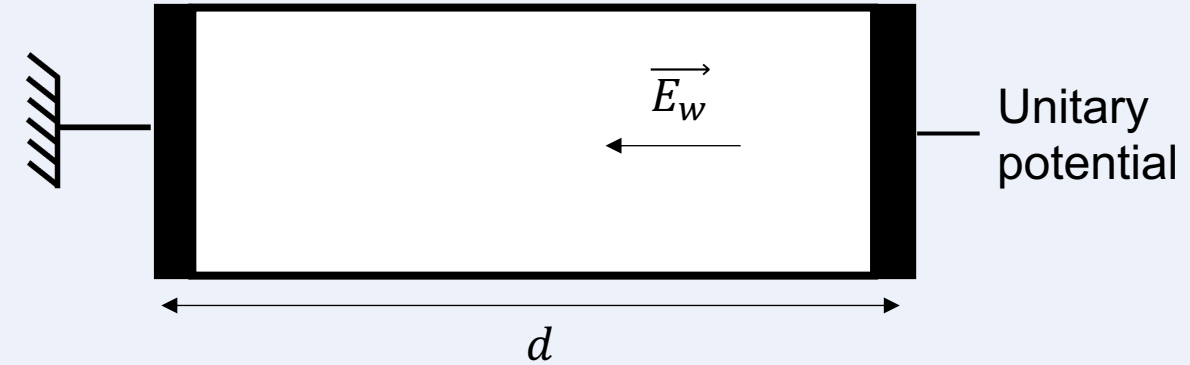
# Modeling signal formation

- The simplest case of charge signal generation

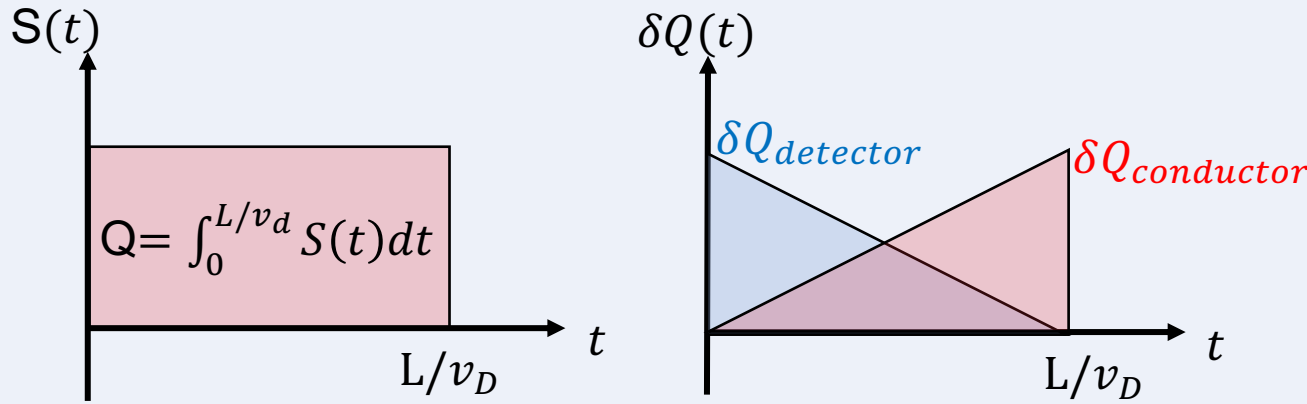


- Mirror effect to the partial charge induced  $\delta Q$  between the detector and the conductor

- The charge  $q$  is neutralized by its "mirror charge" when it is near the surface of another conductor



$$E_w = \frac{\Delta V}{d} = \frac{1}{d} \quad \Rightarrow \quad i(t) = q \times \frac{v_D}{d} = cste$$



- **The instantaneous current is induced by charge motion only**

- **Need simulation for complex geometry**

- Partial charge conservation  $\delta Q_{conductor} + \delta Q_{detector} = Q$