Sensitivities of Neutron Transport Modes with Monte-Carlo Methods

Presentation to the PhD Seminar

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Contents

- Introduction
- PhD context
- Progress and Results
- Conclusion and future work

"Old" News!

wnn
world nuclear news

CGN takes Taishan 1 offline following fuel failure

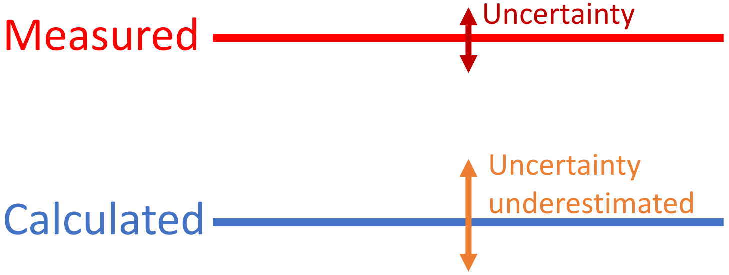

RÉPUBLIQUE FRANÇAISE

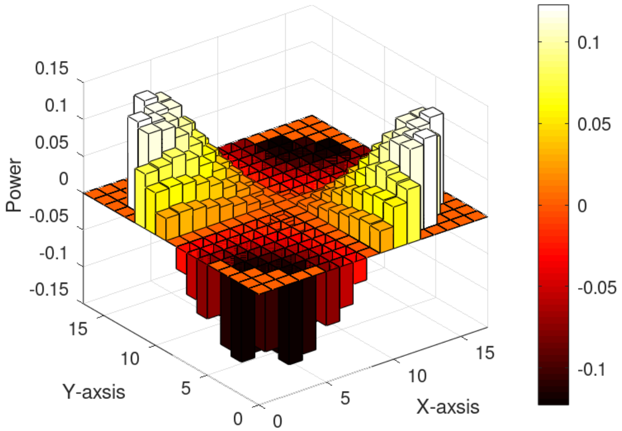
IRSN

RETOUR D'EXPÉRIENCE DES PROJETS D'EPR DANS LE MONDE

RÉPONSE À LA SAISINE DE LA CNDP DU 12 JUILLET 2022

- ✓ Discrepancies between the measured power distribution and the calculated values
- ✓ Power level fluctuations between the various core quadrants





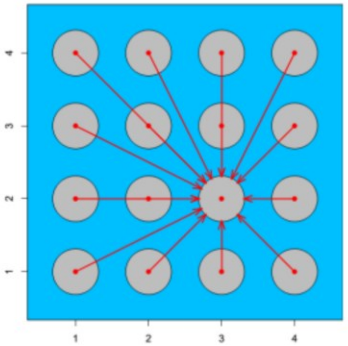
Neutron Transport

$$\frac{1}{v}\frac{d\psi}{dt} = -\hat{\Omega}.\nabla\psi - \Sigma_t\psi + \int_{E'=0}^{\infty}\int_{4\pi}\Sigma_s\left(E' \rightarrow E,\ \hat{\Omega}' \rightarrow \hat{\Omega}\right)\psi dE'\, d\hat{\Omega}' + \frac{\chi}{4\pi}\int_{E'=0}^{\infty}\int_{4\pi}\nu\Sigma_f\,\psi dE'\, d\hat{\Omega}' + S$$

Temporal change = Change due to streaming + Change due to collision + Source

- Deterministic codes – Diffusion equation
- Monte Carlo method
- Eigen value is calculated through an iterative process
- Large number of generations
- Fission Matrix

$$S_i = \frac{1}{k}\sum_{j=1}^{N} a_{i,j}S_j$$

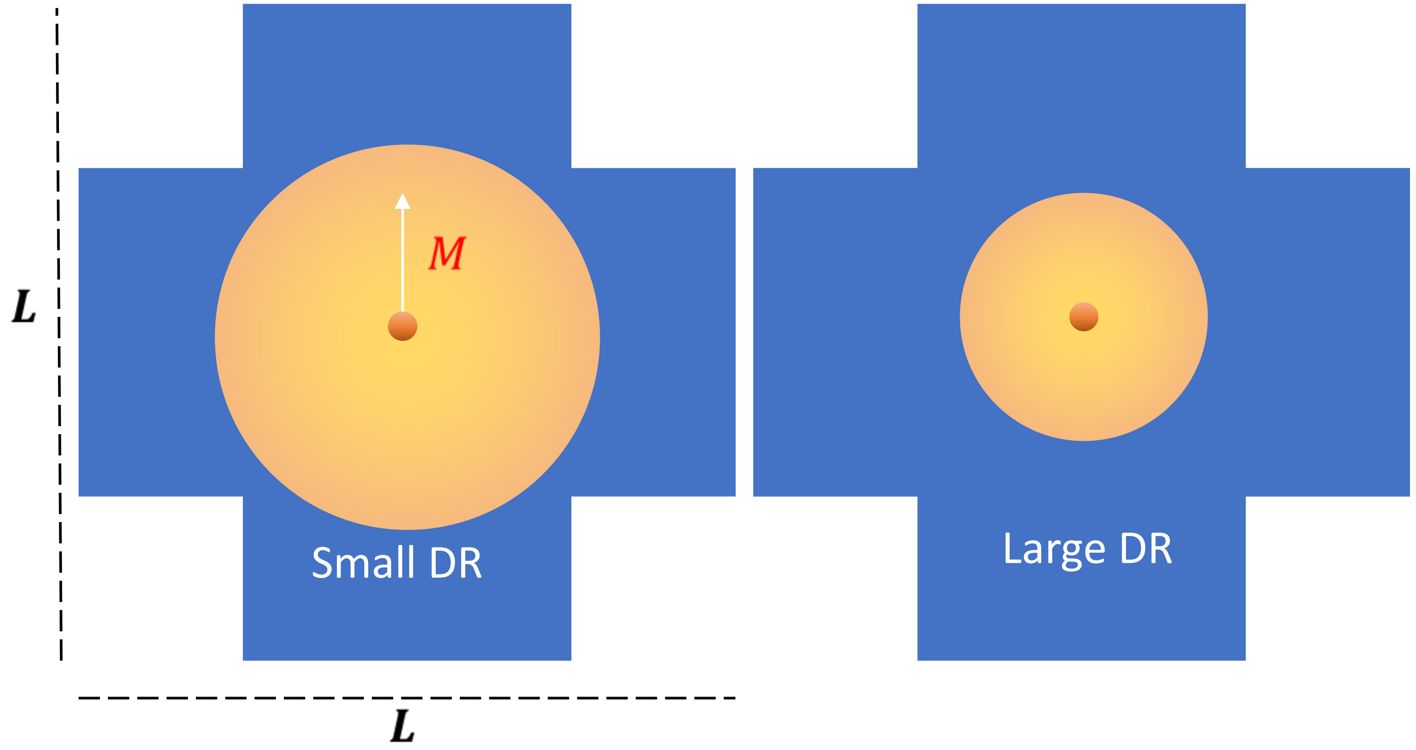


Adam Rau, Serpent User's group meeting, 2019.

$$\vec{F} = \frac{1}{k}A\vec{F}$$

Dominance Ratio

- DR measures the relation between reactor size and migration length
- In high DR cores, neutrons travels relatively less – see less of the core geometry
- More generations needed to better estimate
  - Keff
  - Power distribution
  - Importance of neutron



$$DR = \frac{k_1}{k_0} = \mathrm{F}(B_n{}^2, M^2) \qquad M^2 = \frac{D}{\Sigma_a} \qquad B_n{}^2 = \left(\frac{n\pi}{L}\right)^2$$

Sensitivity Computation

- Sensitivity refers to relative change in a response R due to a relative change in a perturbed quantity X.

$$S_X^R = \frac{\partial R/R}{\partial X/X}$$

- Direct method – reference
  - Changes in input parameters have to be chosen cautiously
  - Not practical in multiple response and multiple perturbations

$$S_X^R = \frac{X_{non-pert}\,(R_{pert} - R_{non-pert})}{R_{non-pert}(X_{pert} - X_{non-pert})}$$

- Perturbation theory
  - Sensitivity requires an importance function
  - It is obtained by the adjoint problem
  - Adjoint flux Represents the importance of neutrons in contributing to other parameters like power, flux, etc.

$$S_X^k = \frac{\mathrm{X}}{\mathrm{R}} \frac{\langle \psi^\dagger, \left(\frac{1}{k}\frac{\partial F}{\partial X} - \frac{\partial H}{\partial X}\right)\psi \rangle}{\langle \psi^\dagger, F\psi \rangle}$$

$$\boldsymbol{H}^\dagger \boldsymbol{\psi}^\dagger = \frac{\mathbf{1}}{\boldsymbol{k}}\, \mathbf{F}^\dagger \boldsymbol{\psi}^\dagger$$

Sensitivity in Monte Carlo Codes

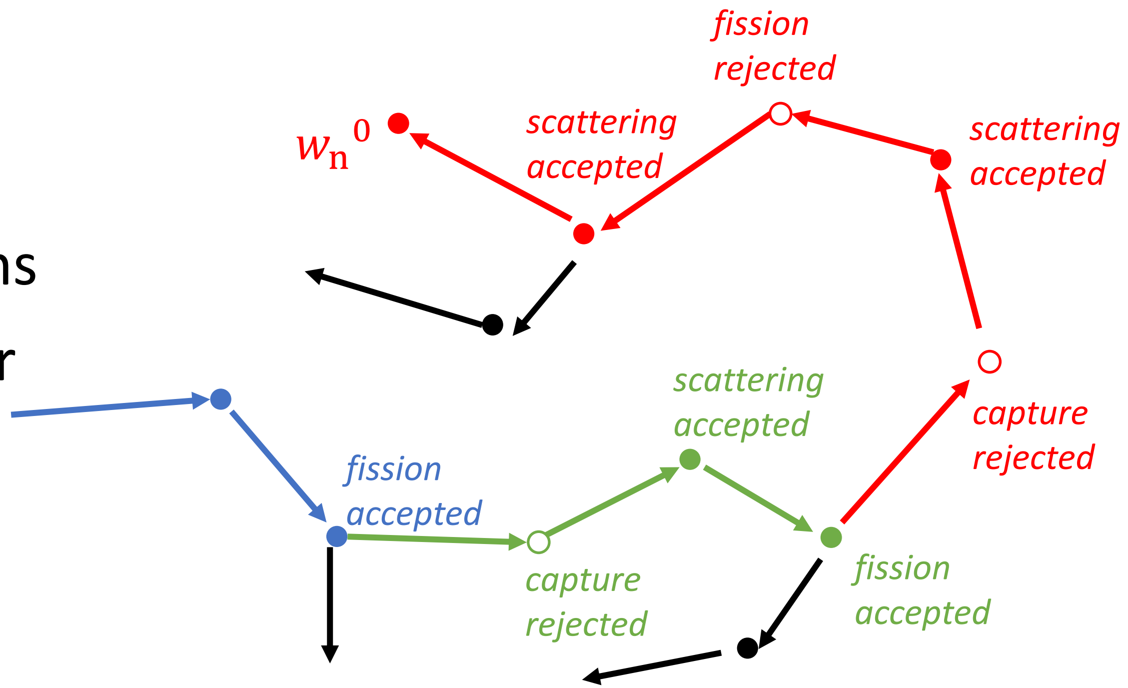
$$R = \sum \Sigma_r w_n l$$

l is track length over all trajectories

- Recording accepted and rejected collisions
- Weighing the importance of neutron over number of generations (ancestors)

$$\frac{\partial w_n / w_n}{\partial x / x} = \sum_{g=(\alpha-\lambda)}^{\alpha} \left({}^{(n,g)}ACC_x - {}^{(n,g)}REJ_x \right)$$

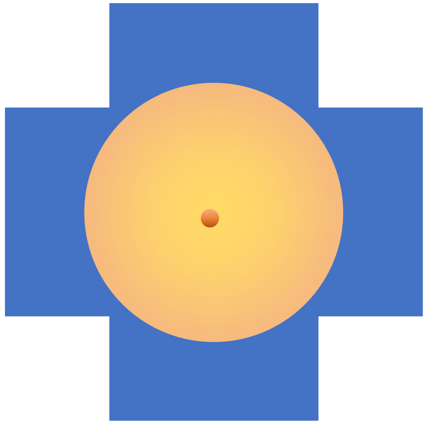
- Convergence of sensitivities depends on number of these latent generations
- More generations, more time, more memory



Source: Aufiero, M. et al. "A collision history-based approach to sensitivity/ perturbation calculations in the continuous energy Monte Carlo code SERPENT".

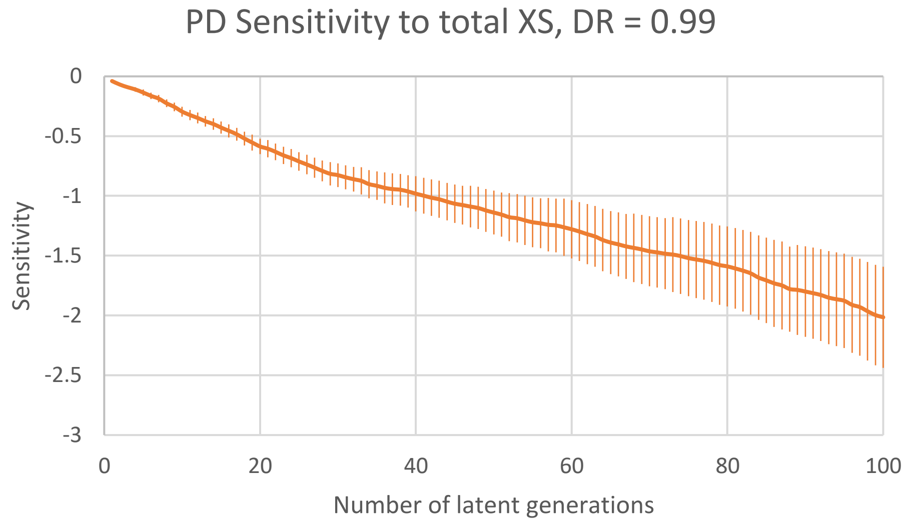
What does happen in large Cores?

- Most cases: sensitivity converges with 10-20 LatGen
- Non-convergence in large cores even at 100 LatGen
- Months of calculation time



N/Batches	LatGen	Isotopes	Memory (Gbyte)
60000	35	10	122
125000	40	10	120
70000	100	2	122

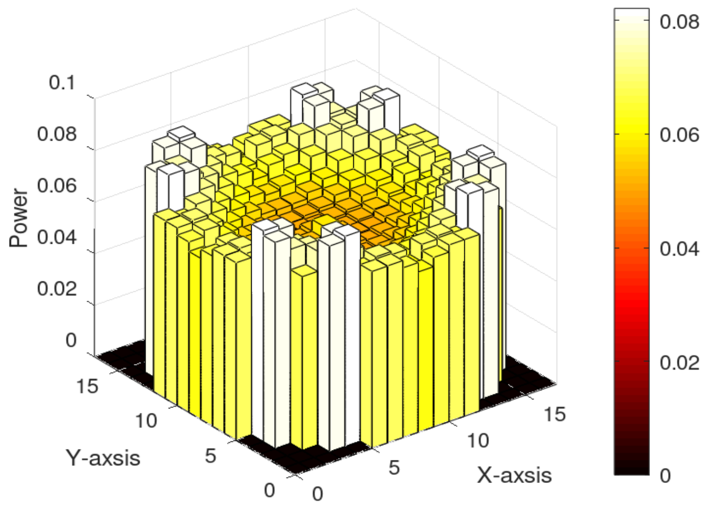
Lopez,P et al.(2021)



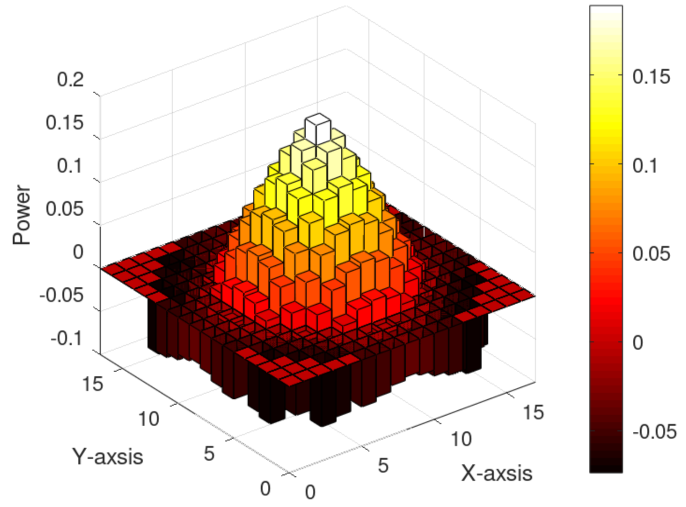
What does happen in large Cores?

- **What about Decomposition of eigenmodes to compute sensitivities?**
- Enhancing accuracy and precision
- Lowering computational resources

$$\frac{dS_0}{dX} = \sum_{i=0}^{\infty} S_i . \frac{S_i^{\dagger} \frac{dF}{dX} S_0}{k_0(1 - \frac{k_i}{k_0})}$$



Fundamental Mode



Perturbed Mode

Thesis Goals

Objectives

- Addressing the limitations associated with sensitivity analysis within Monte Carlo (MC) code
- Investigating the statistical convergence of sensitivities in reactor cores with high dominance ratios
- Using higher modes to calculate sensitivities

The PhD is part of SUDEC project which is part of NEEDS program.

The aim of SUDEC is to study the propagation of uncertainty in nuclear reactor Multiphysics calculations

- Burnup
- Neutronics-TH coupling

cnrs

cea

EDF

framatome

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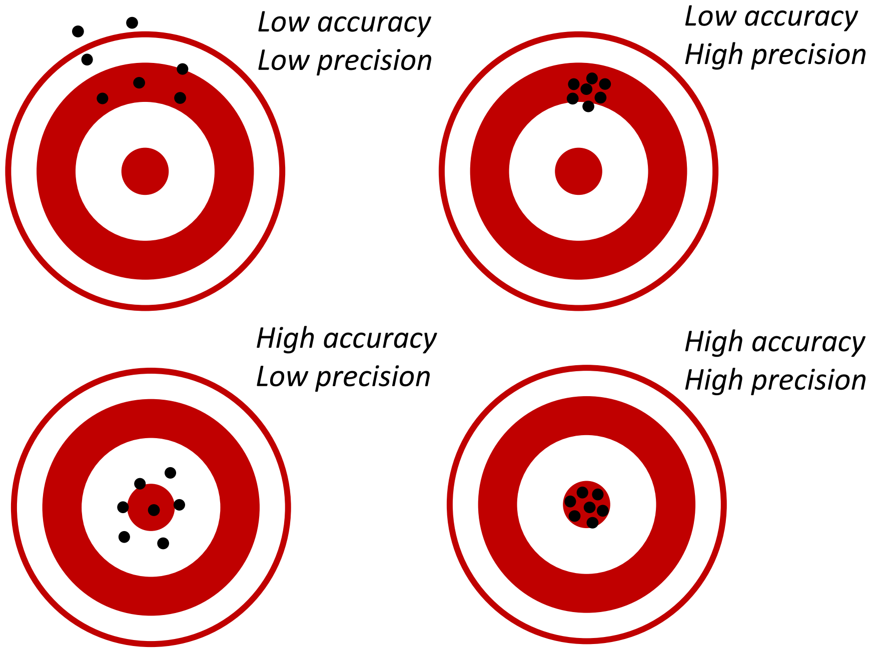
ET DE SÛRETÉ NUCLÉAIRE

ANDRA
Agence nationale pour la gestion des déchets radioactifs

orano

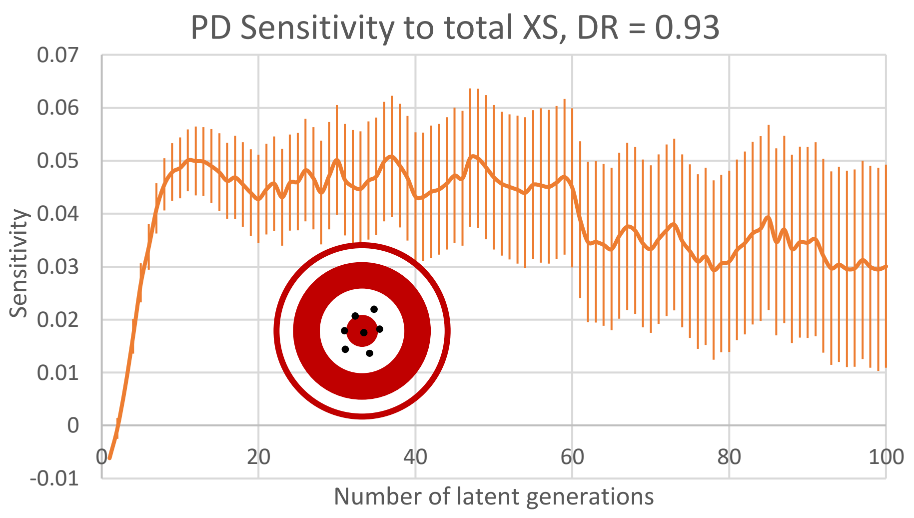
We don't know that we don't know!

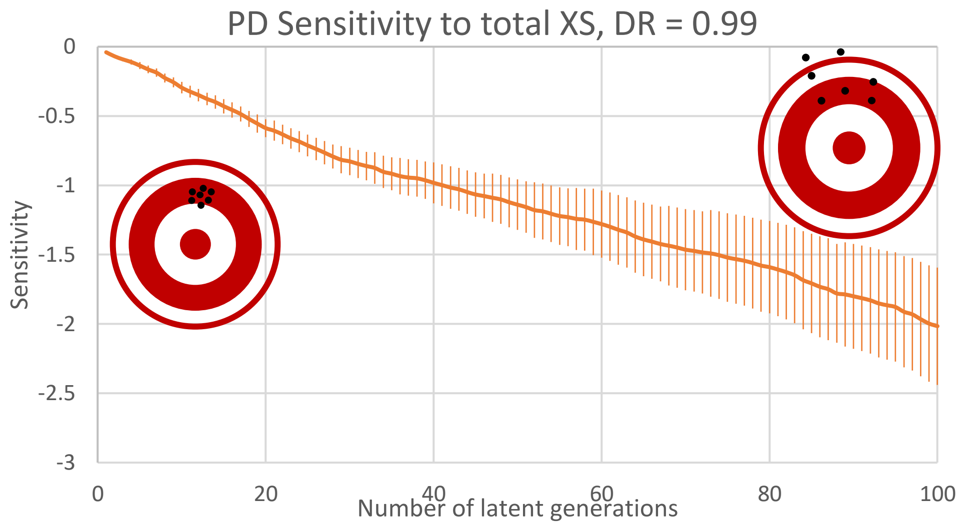
- In these large cores
  - We don't know if sensitivity is correct/accurate
  - We don't know if it is precise
- What to do?
  - Observing the statistical convergence of sensitivities
  - With number of latent generations
  - For multiple cases:
    - different keff
    - various locations where sensitivity is observed
    - various reactor configurations



1^{st} trial Results

- Alignment with existing literature for smaller reactor cores
- Sensitivities in larger cores are not converging and not well understood
- The increased relative variance with LatGen is observed

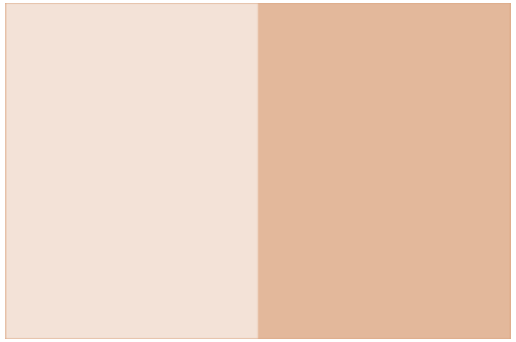




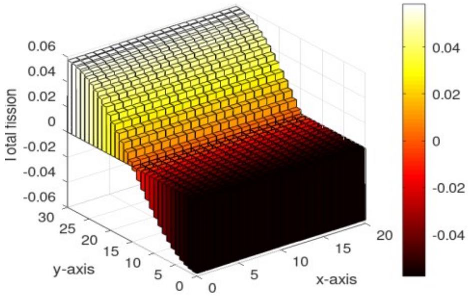
Computation of Sensitivity in Monte-Carlo codes: Investigation of Convergence for Reactor Cores with High Dominance Ratio, M.Rababah & A.Bidaud, Physor2024 conference.

Factors impacting convergence of sensitivities

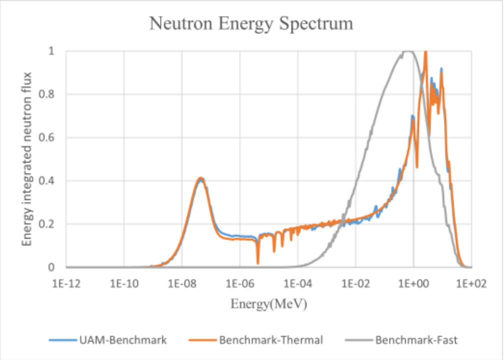
- We hypothesize that convergence depends not on size but on dominance ratio
- We designed a benchmark to test the factors that may impact convergence of sensitivities
  - Homogenous core with two distinct DR and neutron energy spectra
  - We perturb half the geometry and compute sensitivity of keff and PD to change in total cross-section and nubar (neutron fission yields)



Geometry plot of the parallelepiped, x-axis view

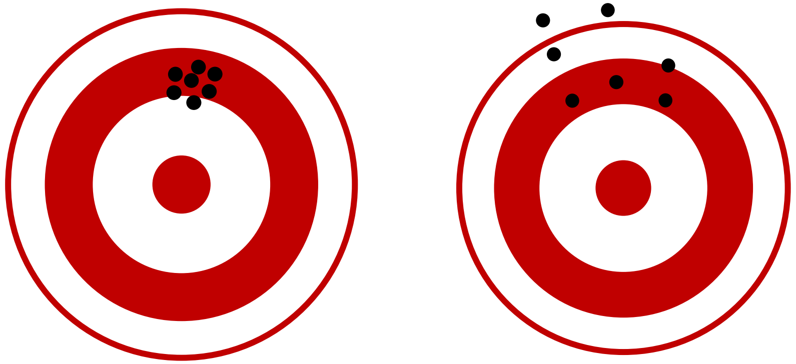


Perturbed mode



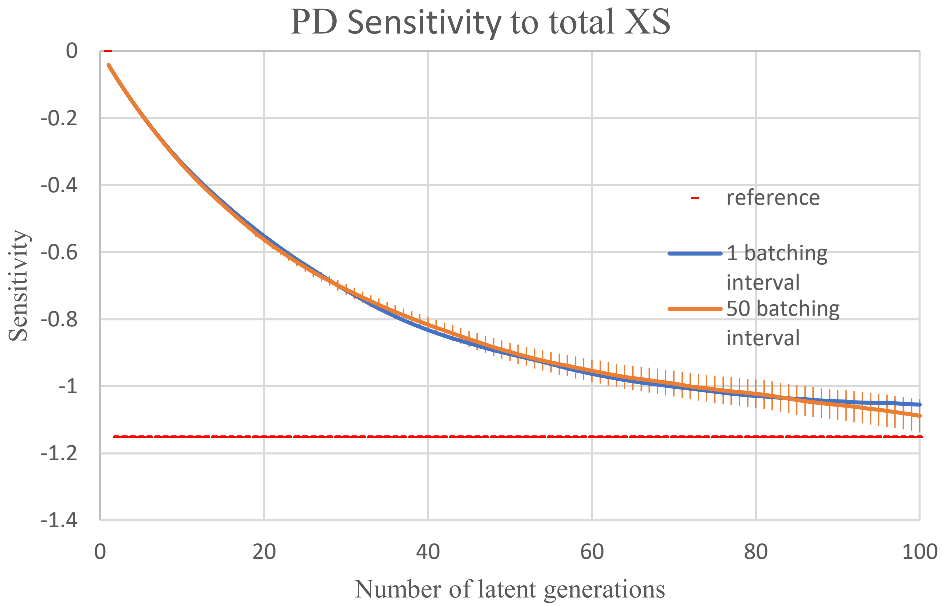
The effect of dominance ratio on the statistical convergence of sensitivity in Monte Carlo codes, M.Rababah & A.Bidaud, SNA&MC conf 2024. ***Submitted****.*

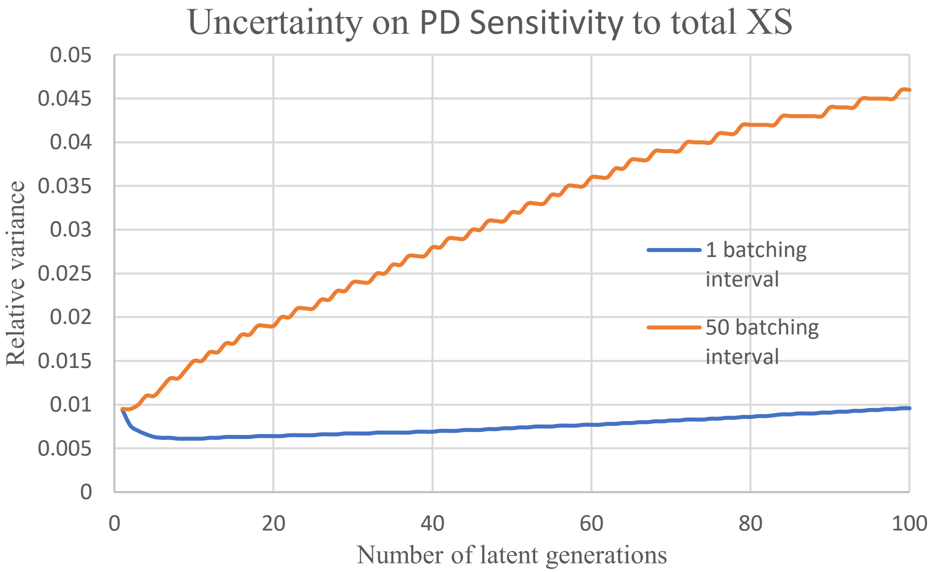
Can we know that we don't know?



Testing different options in Serpent

- Using different *"batching interval"*
- Statistics are divided in batches
- Expected value does not change but the statistical uncertainty does





Results – Higher modes

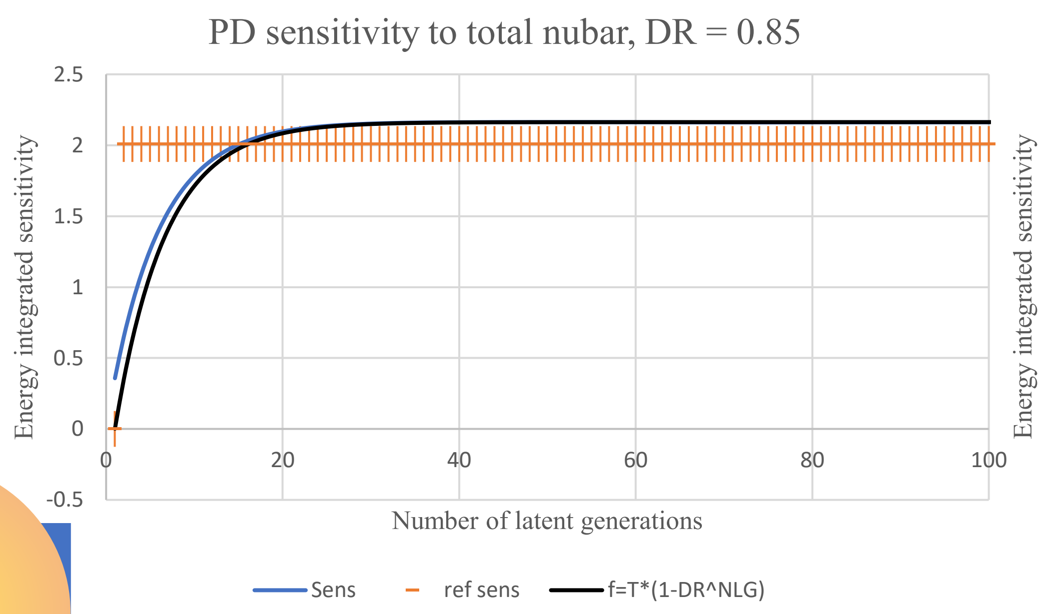
Computing sensitivities using different methods

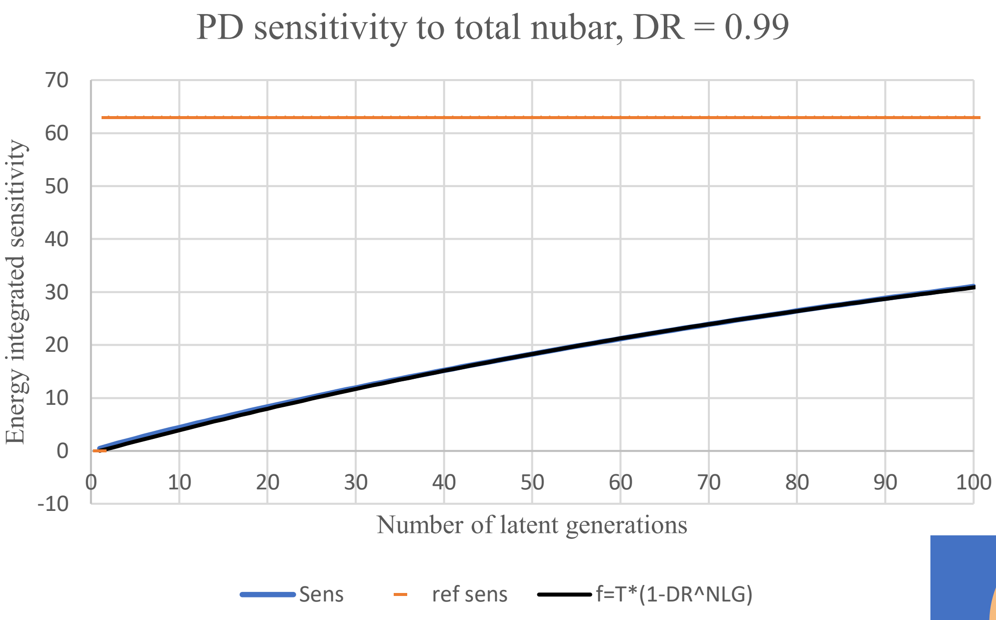
- Direct method – reference
- Monte Carlo Method - Serpent
- Modes decomposition
  - Sensitivity convergence depends on DR
  - Use of eigenmodes predict good results
  - Lowering computational cost: **Serpent – 21 days Vs Eigenmodes – few hours**

DR	Spectrum	ref-sensitivity	Serpent -Sensitivity at 100 latent generation	Eigenmodes formula
0.85	Thermal	2.01	2.16	2.52
	Fast	2.40	2.32	3.17
0.99	Thermal	63.02	31.02	71.90
	Fast	54.33	29.76	65.35

Results – Convergence behavior

- Regular convergence behavior with $1 - DR^{NLG}$
- Using $f = T(1 - DR^{NLG})$





Results – Beyond 100 LatGen

- Establishing a formula to predict number of LatGen at which sensitivity converges

$$number\ of\ recommended\ latent\ generations = \frac{ln(1 - X\%)}{ln(DR)}$$

- We need 570 LatGen to get 99% of ref-sens value

Latent generation	100	200	300	400	500	600
Sens-Serpent	31.02	45.66	52.68	61.04	61.84	62.98
Sens-reference				63.02		

- **Minimize users' efforts and lower computational cost**

Conclusion

- We have explored the sensitivity convergence issue and tested multiple factors that may affect them
  - Reactor size, energy spectrum, locations, keff, etc.
- We demonstrate that convergence of sensitivity depends on dominance ratio
- We developed formula to recommend number of latent generations – thus saving efforts and computation cost
- We demonstrated that using the modes can efficiently predict sensitivities – for certain cases

Future work

- Dominance ratio with statistical convergence of standard deviations
- Generalize the use of eigenmodes to compute sensitivities
- Expand the capabilities of Serpent

$$\frac{dS_0}{dX} = \sum_{i=0}^{\infty} S_i . \frac{S_i^{\dagger} \frac{dF}{dX} S_0}{k_0 (1 - \frac{k_i}{k_0})}$$

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Cooperation

- With SCK.CEN
- With MilanoMultiPhyiscs
- With CEA - Saclay

Milano Multiphysics
Smart solutions for complex problems

Questions

It is the mark of an educated mind to rest satisfied with the degree of precision which the nature of the subject admits and not to seek exactness where only an approximation is possible. – wikiquote.