Towards automated NLO tools

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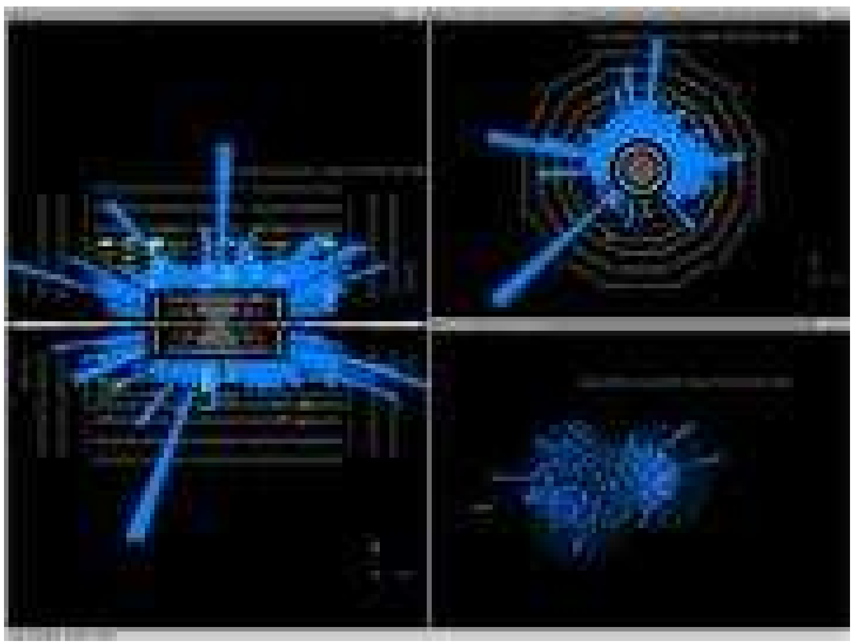
Grenoble, 30.06.09

The LHC

with LHC (or the Tevatron already ?) we are entering a

New Era in Particle Physics !



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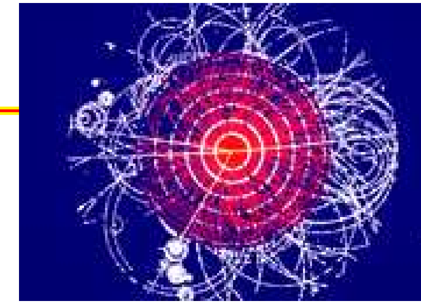
- 1 Terabyte of data every day
- ~ 1000 hadronic tracks in detector per event
  proton remnants or high energy interactions between quarks/gluons (QCD)

process	events/sec	
QCD jets $E_T > 150$ GeV	100	background
$W \to e\nu$	15	background
$t\bar{t}$	1	background
Higgs, $m_H \sim 130$ GeV	0.02	signal
gluinos, $m \sim 1$ TeV	0.001	signal

$\Rightarrow$ enormous backgrounds !

preparing for the LHC



- we might see very clear signatures
  e.g. 4 highly energetic leptons

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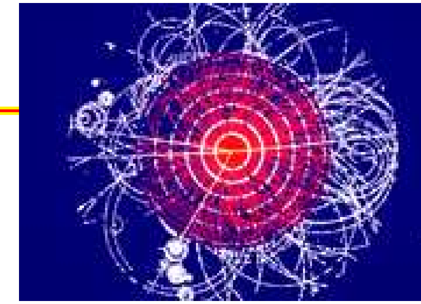

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  e.g. light Higgs boson $m_H \sim 120\,$GeV

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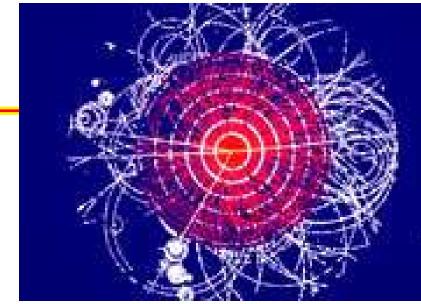

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  $\Rightarrow$ be prepared !

- we first have to "rediscover" the Standard Model, control jet energy scale, underlying event, ...
- maximal control of theory expectations
  for signals and backgrounds is required
- measuring the backgrounds is not always possible
  e.g. neutrinos in final state

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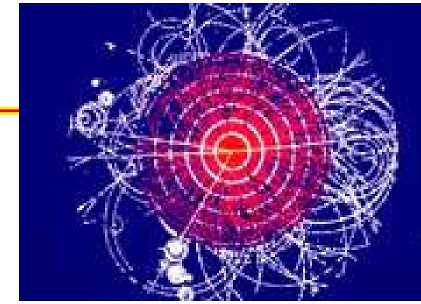

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- measuring the backgrounds is not always possible
  e.g. neutrinos in final state
- need to have **precise** theory predictions

LHC start-up phase

precise SM theory predictions needed for

- luminosity measurements, detector calibration
- understand lepton triggers, photon-, di-photon triggers
- b-tagging
- determination of jet energy scale
- PDF studies
- ...

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important processes:

W(Z) + n jets, γ + n jets, $\gamma\,\gamma$ (+ n jets), $t\,\bar{t}$, ...

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- determination of Higgs mass(es), width(s), spin, CP properties, couplings
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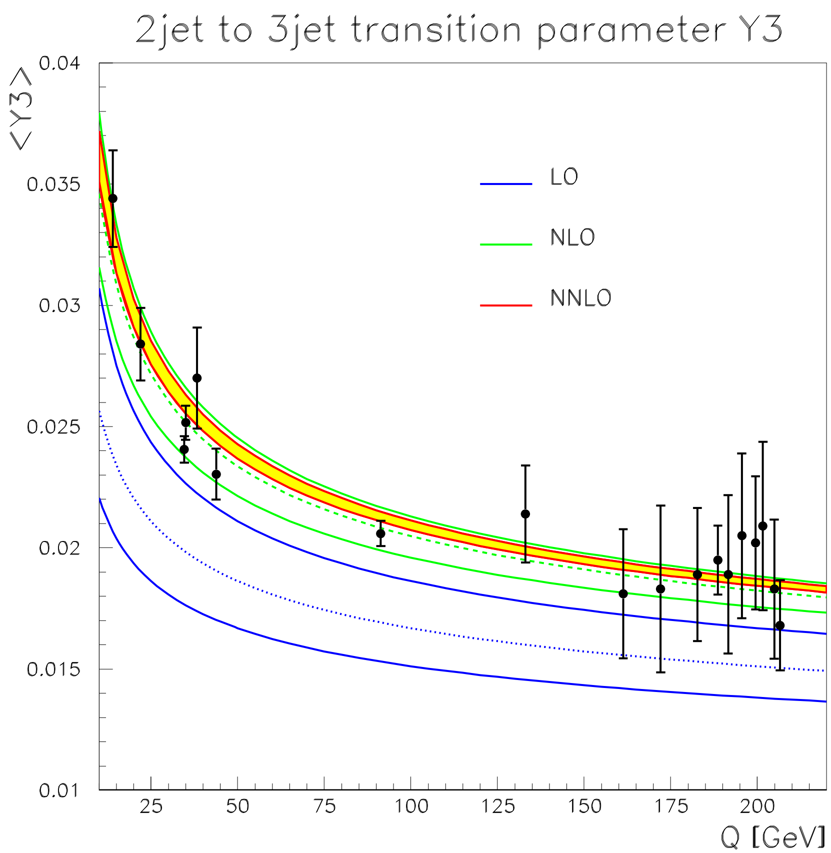
predictions based on Leading Order (LO) in perturbation theory are not sufficient in most cases

shortcomings of leading order predictions

- large renormalisation/factorisation scale dependence

$$\hat{\sigma} = \alpha_s^k(\mu)\ \left[\hat{\sigma}^{\rm LO} + \alpha_s(\mu)\,\hat{\sigma}^{\rm NLO}(\mu) + \alpha_s^2(\mu)\,\hat{\sigma}^{\rm NNLO}(\mu) + \ldots\right]$$

$$d\hat{\sigma}^{(n)}/d\ln(\mu^2) = \mathcal{O}(\alpha_s^{n+1})$$



example:

3-jet observable in e^+e^- annihilation

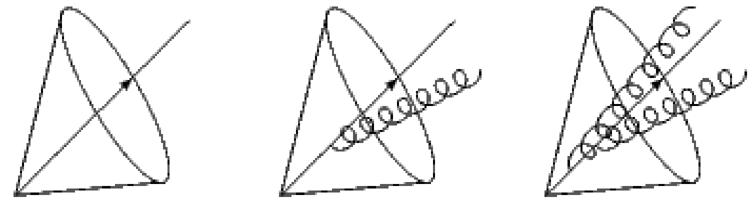
[A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, GH 08]

uncertainty bands:

$$M_Z/2 < \mu < 2\,M_Z$$

shortcomings of leading order predictions

- poor jet modelling



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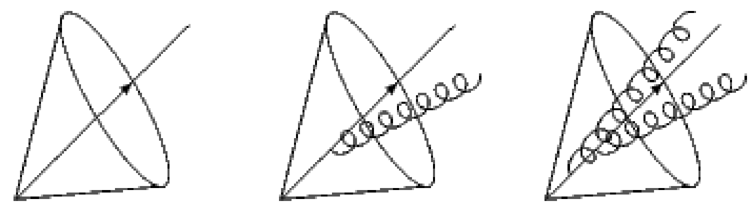


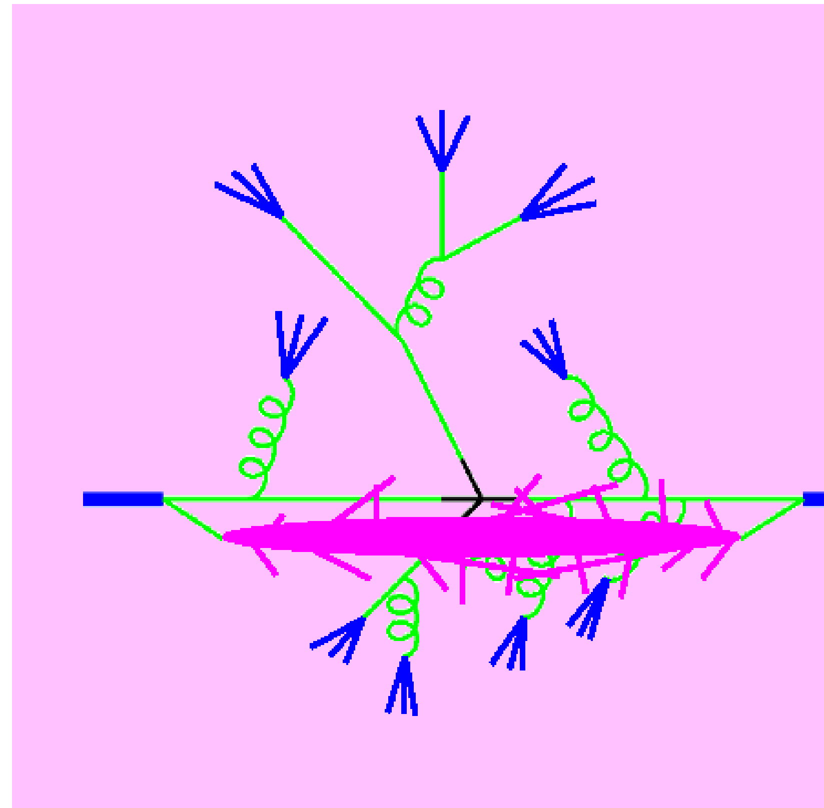
- cases where shapes of distributions are not well predicted by LO
  (new partonic processes become possible beyond LO)

- large sensitivity to experimental cuts

- . . .

generic event

1. hard interaction
   $\hat{\sigma} = \alpha_s^k \, \hat{\sigma}^{\mathrm{LO}} + \alpha_s^{k+1} \, \hat{\sigma}^{\mathrm{NLO}} + \dots$
   calculable order by order
   in perturbation theory
2. parton shower
   soft and collinear branching,
   treatment within perturbative
   QCD framework
3. hadronization
   non-perturbative models,
   fits to data
4. (underlying event)



theoretical description

- general purpose shower Monte Carlos
  (PYTHIA, HERWIG, ARIADNE, . . .)
  + resummation of leading logs
  + can describe hadronic final states, indispensable for realistic comparisons to data
  + large degree of automatisation
  - mainly based on soft/collinear branchings $\longrightarrow$ problems with large angle radiation
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- partonic NLO (NNLO, . . .) calculations, resummation
  + reduced scale dependence
  + can predict rates
  - very process specific
  - partonic final states

improvements

- combination of LO matrix elements with parton shower (Alpgen, Helac, Sherpa, Whizard, . . .)
  - matching non-trivial ! (CKKW, MLM, . . .)
  - powerful for description of shapes
  - cannot predict rates (virtual corrections missing)
- ideal case: combine virtues of parton shower and partonic higher order calculation

  programs like MC@NLO, POWHEG, Vincia, Nagy/Soper, . . .

  rapidly developing field, modularity desirable

  "standard" interfaces much discussed at Les Houches 2009

bottomline:

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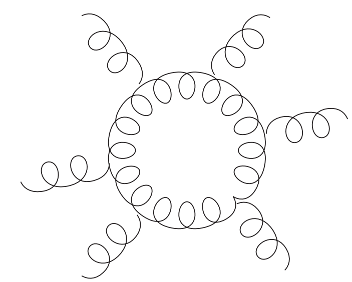
one-loop amplitudes



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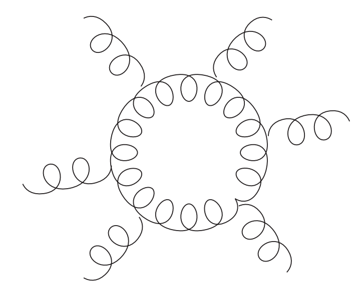
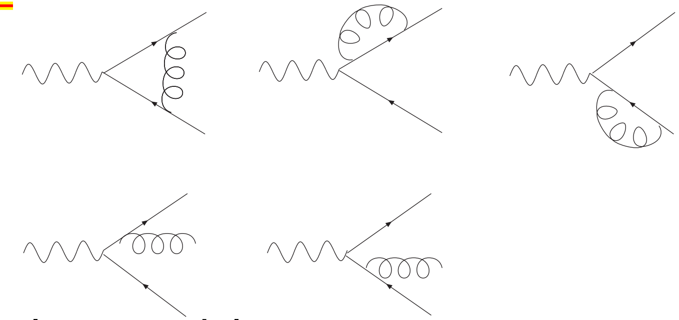


example for time scale to add one parton:

$pp \longrightarrow 2$ jets at NLO: Ellis/Sexton 1986

$pp \longrightarrow 3$ jets at NLO: Bern et al, Kunszt et al 1993-95

ingredients for m-particle observable at NLO



virtual part (one-loop integrals):

$$d\sigma^V = P_2/\epsilon^2 + P_1/\epsilon + P_0$$

real radiation part: soft/collinear emission of massless particles

$\Rightarrow$ need subtraction terms

$$\Rightarrow \int_{\text{sing}} d\sigma^S = -P_2/\epsilon^2 - P_1/\epsilon + Q_0$$

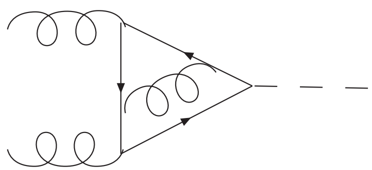
measurement function: $\sigma_m \sim |\mathcal{M}_n|^2 \, J_m^{(n)}$

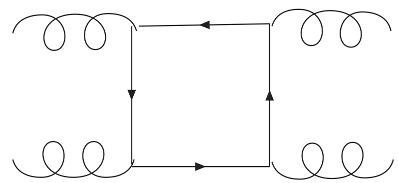
e.g. $J_m^{(n)}(p_1, \ldots, p_n)$ to form m jets from n partons

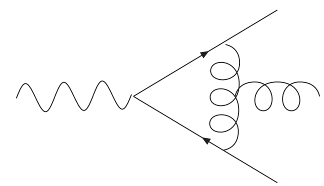
$$\sigma^{NLO} = \underbrace{\int_{m+1} \left[d\sigma^R - d\sigma^S \right]_{\epsilon=0}}_{\text{numerically}} + \underbrace{\int_m \left[\underbrace{d\sigma^V}_{\text{analytically}} + \underbrace{\int_{\text{s}} d\sigma^S}_{\text{analytically}} \right]_{\epsilon=0}}_{\text{numerically}}$$

Status NLO

- for scattering processes involving maximally 4 particles $(2 \to 1, 2 \to 2, 1 \to 3)$: many predictions beyond LO available







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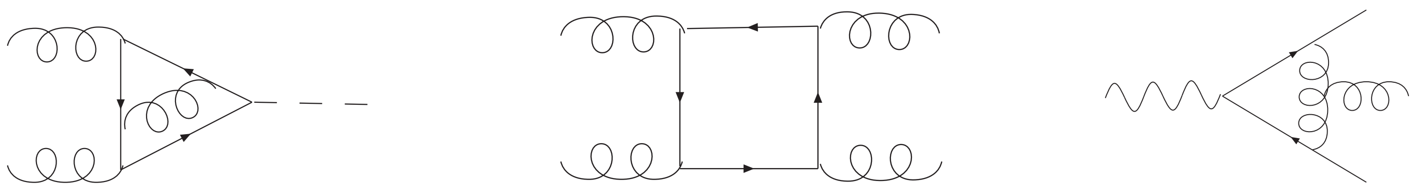
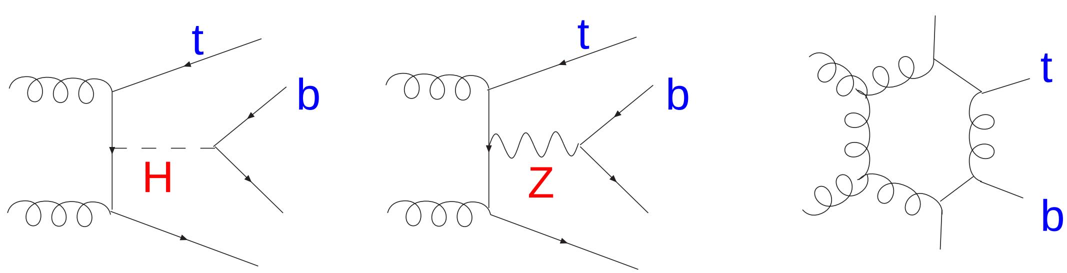


- LHC: most of the interesting processes lead to multi-particle $(2 \to 3, 4, \ldots)$ final states

$pp \to H + t\bar{t} \to b\bar{b}\,t\bar{t}$, $pp \to q\,q\,H \to W^+W^- + 2$ jets, …



N(N)LO wishlist for LHC Les Houches 07 (background only)

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to Z\,Z$ jet	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
4. $pp \to W\,W\,W$	SUSY trilepton
5. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, new physics
6. $pp \to V\,V + 2$ jets	VBF$\to H \to VV$
7. $pp \to V + 3$ jets	various new physics signatures
8. $pp \to b\bar{b}b\bar{b}$	H, SUSY searches
9. $\mathcal{O}(\alpha^2\alpha_s^3)\; gg \to WW$	EW sector
10. NNLO for $t\bar{t}$	SM benchmark, H couplings
11. NNLO to VBF, Z/γ+jet	Higgs couplings, SM benchmark

2009 status of NLO wishlist for LHC

$pp \to W\,W$ jet	Denner/Dittmaier/Kallweit/Uwer, Ellis/Campbell/Zanderighi
$pp \to Z\,Z$ jet	Binoth/Guillet/Karg/Kauer/Sanguinetti
$pp \to t\bar{t}\,b\bar{b}$	Bredenstein/Denner/Dittmaier/Pozzorini
$pp \to t\bar{t} + 2$ jets	
$pp \to Z\,Z\,Z$	Lazopoulos/Melnikov/Petriello, Hankele/Zeppenfeld
$pp \to V\,V\,V$	Binoth/Ossola/Papadopoulos/Pittau, Zeppenfeld et al.
$pp \to V\,V\,b\bar{b}$	
$pp \to V\,V + 2$ jets	VBF: Bozzi/Jäger/Oleari/Zeppenfeld, VBFNLO coll.
$pp \to W + 3$ jets	Blackhat coll., Ellis/Giele/Kunszt/Melnikov/Zanderighi*
$pp \to b\bar{b}b\bar{b}$	Binoth/Guffanti/Guillet/Reiter/Reuter
$pp \to t\,\bar{t}$ jet	Dittmaier/Uwer/Weinzierl
$pp \to t\,\bar{t}\,Z$	Lazopoulos/McElmurry/Melnikov/Petriello
$pp \to b\,\bar{b}\,Z\,,b\,\bar{b}\,W$	Febres Cordero/Reina/Wackeroth

• done (* leading colour only), • partial results

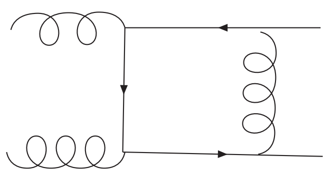
status of NNLO wishlist for LHC

NNLO for $pp \to WW$	Chachamis/Czakon/Eiras
NNLO for $pp \to t\bar{t}$	Czakon/Mitov/Moch/Uwer, Kniehl/Merebashvili/Körner/Rogal, Bonciani/Ferroglia/Gehrmann/Studerus
NNLO to VBF, Z/γ+jet	

- done
- virtual part done

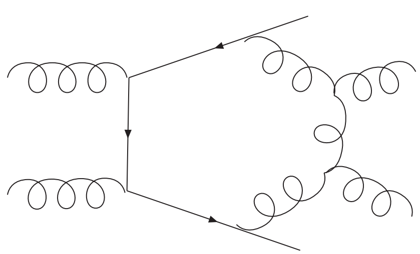
multi-particle production at NLO

bottleneck: virtual amplitudes



4-point integrals (boxes):

- known analytically
- invariants s, t, m_q



6-point integrals (hexagons):

- enormous complexity
- for analytic representation: express by integrals with < 5 legs
- direct numerical evaluation hampered by singularities

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  (pioneered by Passarino/Veltman)
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  needs extraction of poles beforehand
- unitarity-cut based ("string/twistor inspired")
  (pioneered by Bern, Dixon, Dunbar, Kosower ’94,
  Britto, Cachazo, Feng, Witten ’04,
  Ossola, Papadopoulos, Pittau ’06)
  abandons conventional use of (loop) Feynman diagrams

Automation

lots of progress recently !

- new tools based on numerical implementation of unitarity cuts

  Ossola/Papadopoulos/Pittau (CutTools),

  Berger/Bern/Dixon/Febres-Cordero/Forde/Gleisberg/Ita/Kosower/Maître (BlackHat),

  Ellis/Giele/Kunszt/Melnikov/Zanderighi (Rocket)

  Lazopoulos, Giele/Winter

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- new developments within methods based on Feynman diagrams
  Bredenstein/Denner/Dittmaier/Pozzorini,
  Hahn/Ilana/Rauch (FeynArts/FormCalc/LoopTools),
  Diakonidis/Fleischer/Gluza/Kajda/Riemann/Tausk,
  GOLEM, Passarino et al., Yuasa et al. (GraceNLO), . . .

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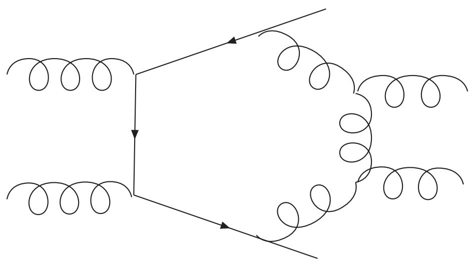
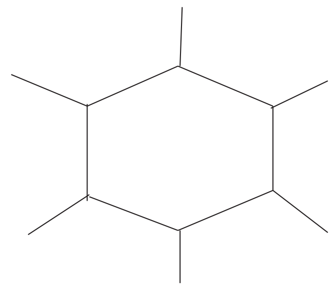
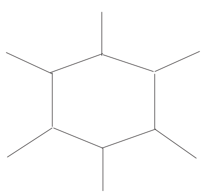
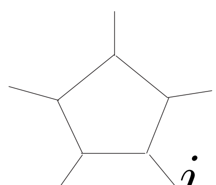
- automated subtraction for NLO real radiation
  Frederix/Gehrmann/Greiner, Hasegawa/Moch/Uwer, Tevlin/Seymour,
  Gleisberg/Krauss

algebraic reduction

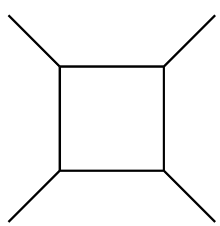
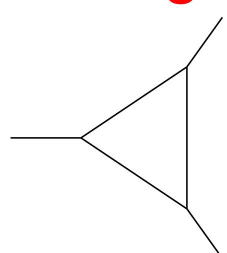
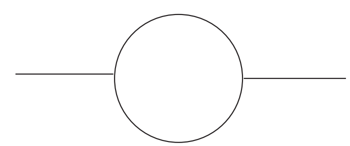
 →  + integrals with less legs

non-trivial tensor structure

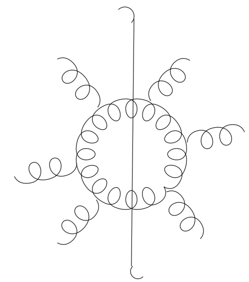
scalar 6-point function

 $= \sum_{i=1}^{6} b_i$  i … factorial growth in complexity !

reduction to set of basis integrals (4-, 3- and 2-point funcs.)

$\mathcal{A} = C_4$  $+ \, C_3$  $+ \, C_2$  $+ \, \mathcal{R}$

unitarity-based methods

$$\mathcal{A} = \sum_{\text{cuts}} \int d\,PS$$  $$+ \quad \mathcal{R}$$

- use analyticity structure to compose loop amplitudes from cuts

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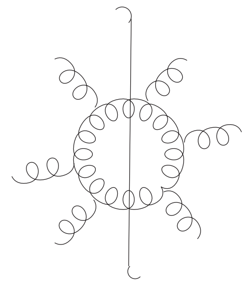
$$\mathcal{A} = \sum_{\text{cuts}} \int d\,PS$$  $$+ \quad \mathcal{R}$$

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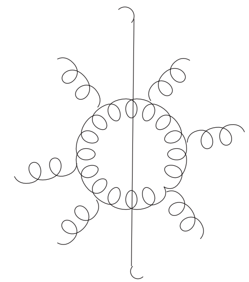
$$\mathcal{A} = \sum_{\text{cuts}} \int d\,PS$$  $$+ \quad \mathcal{R}$$

- use analyticity structure to compose loop amplitudes from cuts
- efficient for coefficients of boxes, triangles, bubbles
- lead to compact expressions
- obtaining rational terms $\mathcal{R}$ less straightforward
- most convenient for massless propagators

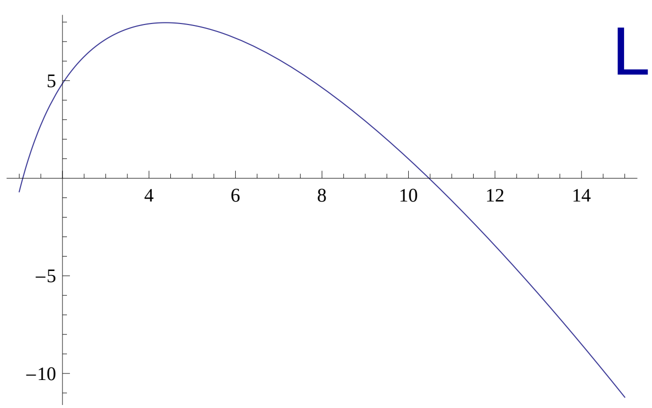
asymptotic complexity

- unitarity based methods:
  complexity of colour ordered amplitudes:

  $$\tau_{\text{tree}} \times \tau_{\text{cuts}} \sim N^4 \times \binom{N}{5} \xrightarrow{\text{N large}} N^9$$

- Feynman diagram reduction:

  $$\tau_{\text{diagrams}} \times \tau_{\text{form factors}} \sim 2^N \times \Gamma(N)$$



$\text{Log}(N^9/(\Gamma(N)\,2^N))$

The GOLEM project

GOLEM: General One-Loop Evaluator of Matrixelements

golem95 code: [Binoth, Guillet, GH, Pilon, Reiter '08]

- calculates form factors for tensor integrals up to rank six 6-point numerically
- master integrals valid for all kinematic regions, public version only massless internal particles so far
- no restriction on masses of external particles
- also contains one-dimensional integral representations for all tensor boxes except box with all 4 legs off-shell $\Rightarrow$ allows efficient numerical evaluation

the GOLEM project

Golem project:

[Binoth, Cullen, GH, Guffanti, Guillet, Karg, Kauer, Lee, Pilon, Reuter, Reiter, Rodgers, Wells, . . .]

- include automated diagram generation
  [T.Reiter]

  combine with real radiation
  [collaboration with A.Guffanti, N.Kauer, J.Reuter, . . .]

- combine with parton shower
  [collaboration with F.Krauss, F.Siegert, M.Schönherr, . . .]

(advert break)

The PHOX Family

NLO Monte Carlo programs (partonic event generators) to calculate cross sections for the production of large-p_T photons, hadrons and jets

http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

P. Aurenche, T. Binoth, M. Fontannaz, J.Ph. Guillet, GH, E. Pilon, M. Werlen

- DIPHOX

  $h_1\, h_2 \to \gamma\ \gamma\ + X$, $h_1\, h_2 \to \gamma\ h_3\ + X$, $h_1\, h_2 \to h_3\, h_4\ + X$

- JETPHOX

  $h_1\, h_2 \to \gamma$ jet $+ X$, $h_1\, h_2 \to \gamma\ + X$

  $h_1\, h_2 \to h_3$ jet $+ X$, $h_1\, h_2 \to h_3 + X$

- EPHOX

  $\gamma\, p \to \gamma$ jet $+ X$, $\gamma\, p \to \gamma\ + X$

  $\gamma\, p \to h$ jet $+ X$, $\gamma\, p \to h + X$

- TWINPHOX

  $\gamma\, \gamma \to \gamma$ jet $+ X$, $\gamma\, \gamma \to \gamma\ + X$

semi-numerical reduction

[Binoth, Guillet, GH, Kauer, Pilon, Schubert, Reiter '05 - '09]

combine virtues of numerical and algebraic methods:

- do tensor reduction semi-numerically
- reduce to scalar integrals to use analytic expressions where inverse determinants are harmless $\Rightarrow$ fast
- switch to numerical evaluation of boxes, triangles otherwise

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- formalism valid for massive and massless particles, arbitrary number of legs
- rational parts $\mathcal{R}$ are for free!
  complexity of expressions greatly reduced
  if $\mathcal{R}$ is projected out

reduction algorithm schematically

diagram generation (e.g. QGRAF, FeynArts, GRACEfig)

$\Downarrow$

$$A = \sum_i C_i^{\mu_1 \ldots \mu_r} \, I_{\mu_1 \ldots \mu_r}$$

$\Downarrow$

$$A = \sum_{\{l\}} f_l(p_i \cdot p_j, p_i \cdot \epsilon_j, \epsilon_i \cdot \epsilon_j) \, \{A_{\{l\}}^{N,r}, B_{\{l\}}^{N,r}, C_{\{l\}}^{N,r}\}$$

(Lorentz invariants $\times$ form factors)

$\Downarrow$

golem95

$\swarrow$ $\searrow$

numerical evaluation | reduction to scalar integrals

$\searrow$ $\swarrow$

numbers (Laurent series in ϵ)

form factor representation

$$I_N^{n,\,\mu_1\dots\mu_r}(S) =$$

$$\sum_{l_1\cdots l_r\in S} p_{l_1}^{\mu_1}\cdots p_{l_r}^{\mu_r}\; A^{N,r}_{l_1\dots,l_r}(S)$$

$$+ \sum_{l_1\cdots l_{r-2}\in S} \left[g^{\cdot\cdot}\, p^{\cdot}_{l_1}\cdots p^{\cdot}_{l_{r-2}}\right]^{\{\mu_1\cdots\mu_r\}}\; B^{N,r}_{l_1\dots,l_{r-2}}(S)$$

$$+ \sum_{l_1\cdots l_{r-4}\in S} \left[g^{\cdot\cdot}\, g^{\cdot\cdot}\, p^{\cdot}_{l_1}\cdots p^{\cdot}_{l_{r-4}}\right]^{\{\mu_1\cdots\mu_r\}}\; C^{N,r}_{l_1\dots,l_{r-4}}(S)$$

form factor representation

$$I_N^{n,\,\mu_1\ldots\mu_r}(S) =$$

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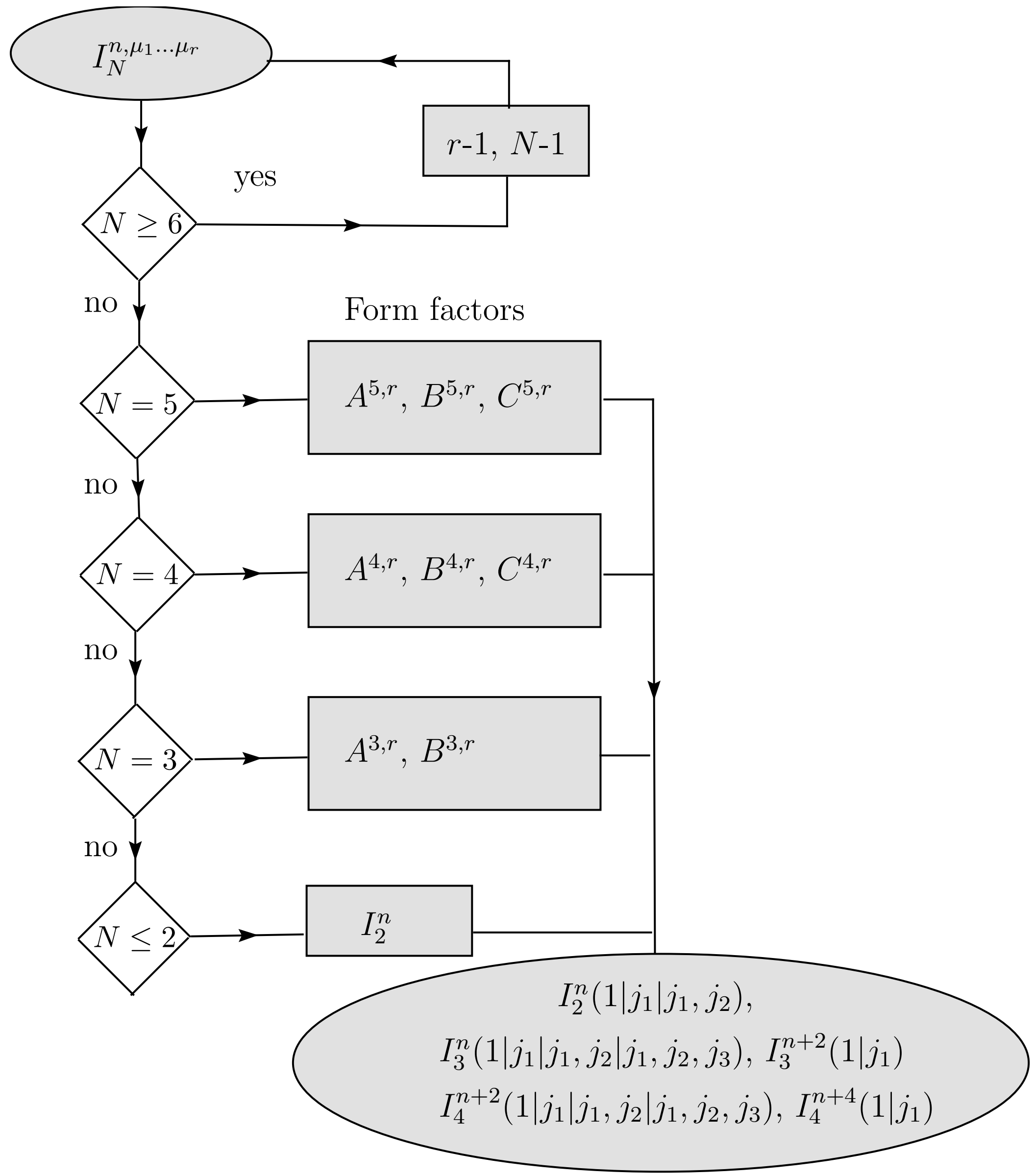
important: more than two metric tensors $g^{\mu\nu}$ never occur !

for $N \geq 6$: simultaneous reduction of rank r and number of legs N

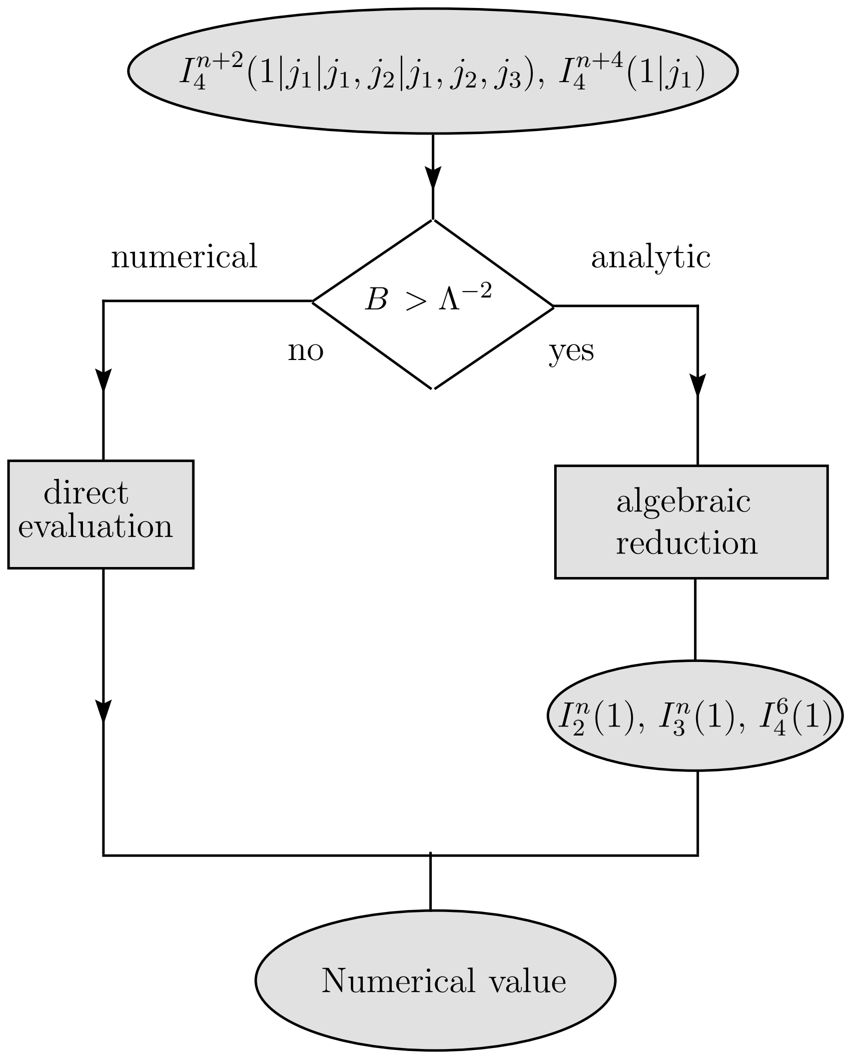
$$I_N^{n,\mu_1\ldots\mu_r}(S) \;=\; -\sum_{j\in S} \mathcal{C}_{j6}^{\mu_1}\, I_{N-1}^{n,\mu_2\ldots\mu_r}(S\setminus\{j\})$$

$$\mathcal{S}_{ij} \;=\; (r_i - r_j)^2 - m_i^2 - m_j^2$$

algebraic reduction



treatment of basis integrals



Gram determinants

- reduction $N \geq 5 \to N = 4$: inverse Gram determinants completely absent
- reduction of $N \leq 4$ tensor integrals: introduces spurious 1/det(G)

$$
\begin{aligned}
I_4^{n+2}(j_1;S) &= \frac{1}{B}\left\{ b_{j_1}\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j_2\in S} \mathcal{S}^{-1}_{j_1 j_2}\, I_3^n(S\setminus\{j_2\}) \right. \\
&\quad \left. -\frac{1}{2}\sum_{j_2\in S\setminus\{j_1\}} b_{j_2}\, I_3^n(j_1;S\setminus\{j_2\}) \right\} \\
I_4^{n+2}(j_1,j_2;S) &\sim \frac{1}{B^2}\;,\; I_4^{n+2}(j_1,j_2,j_3;S) \sim \frac{1}{B^3}\;\ldots \\
B &= \det(G)/\det(\mathcal{S})\,(-1)^{N+1} \\
\mathcal{S}_{ij} &= (r_i - r_j)^2 - m_i^2 - m_j^2 \quad ; \quad G_{ij} = 2\,r_i\cdot r_j
\end{aligned}
$$

Gram determinants

to avoid spurious 1/det(G) terms: do not reduce

golem95:

define dimensionless quantity $\hat{B} = B \times$ (largest entry of S)

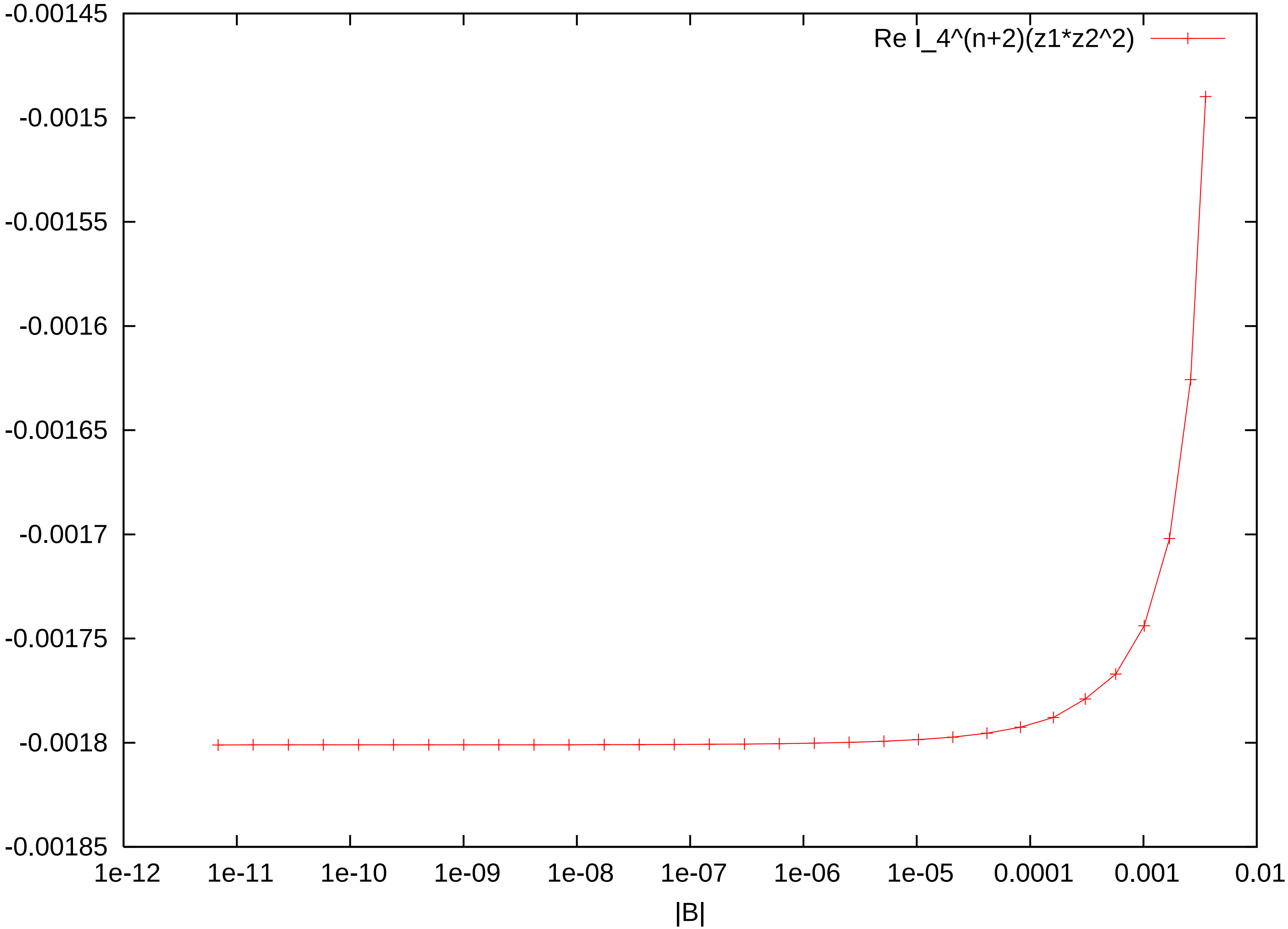
if $\hat{B} < \hat{B}^{\rm cut}$: switch to direct numerical evaluation

(default: $\hat{B}^{\rm cut} = 0.005$)

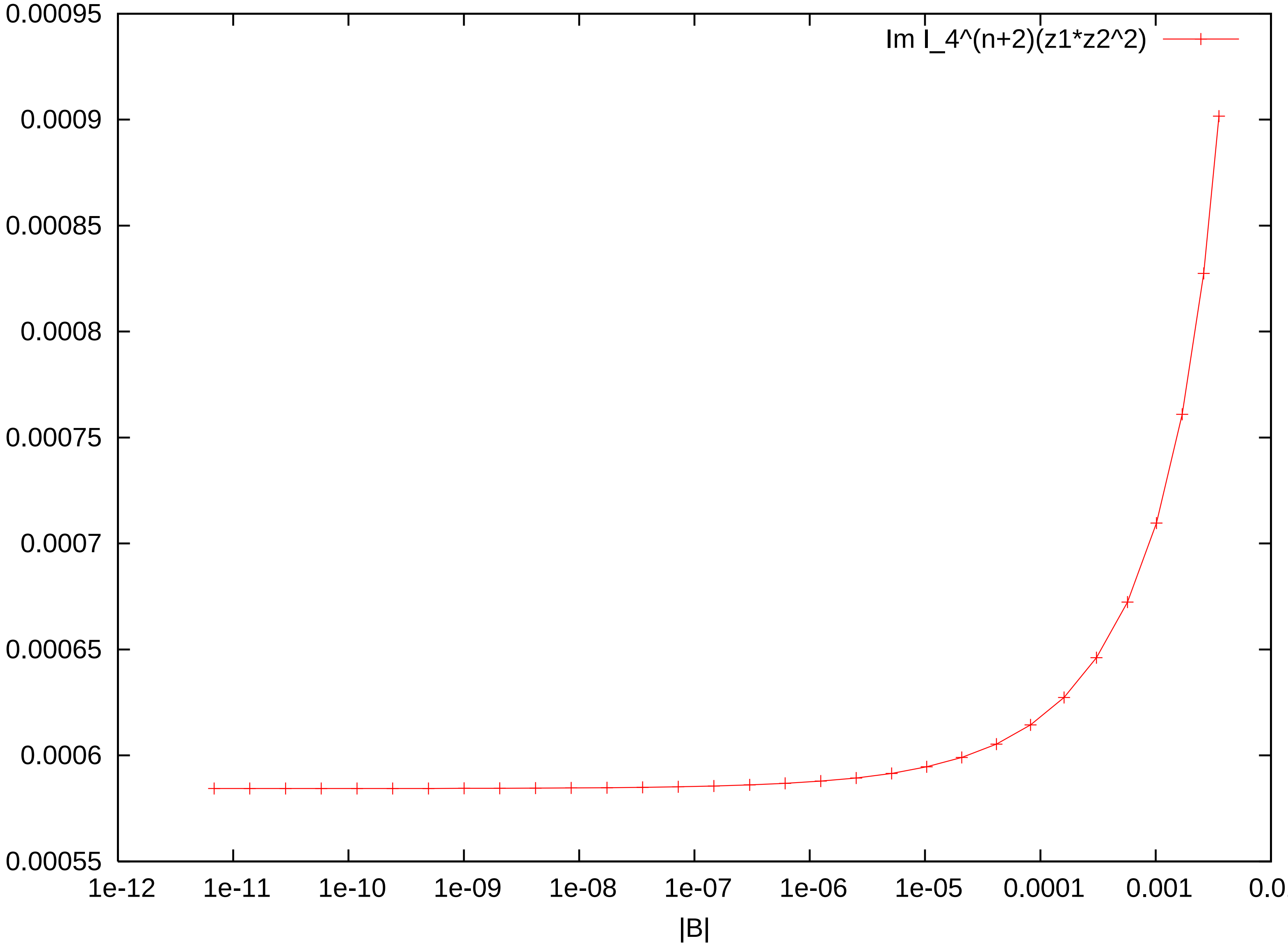
file demo_detg.f90 contains example where $\hat{B} \to 0$

in rank 3 box integral $I_4^{n+2}(1, 2, 2; S)$ with two massive legs

Real part for $B \to 0$

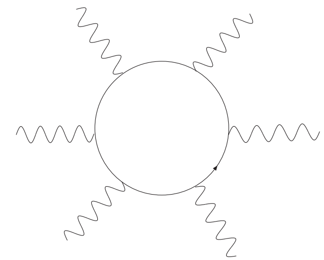


Imaginary part for B $\to$ 0

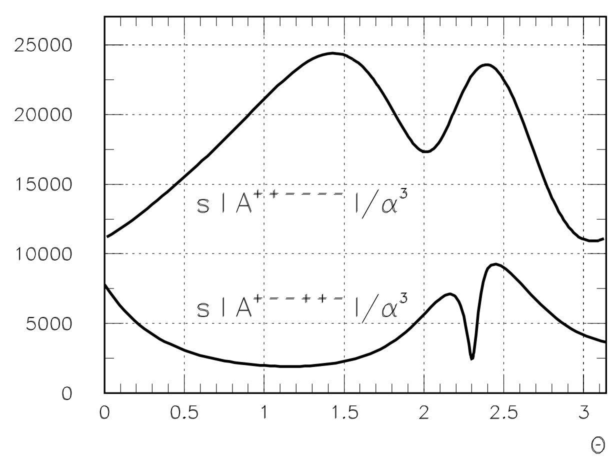


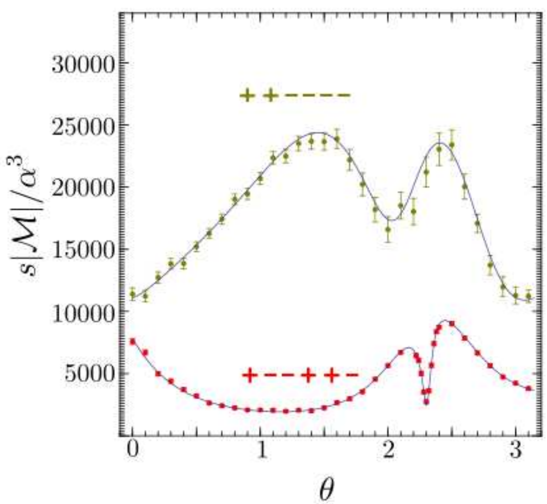
example six-photon amplitude

[Mahlon 94] (special helicity configurations only)
[Nagy, Soper 06; Gong, Nagy, Soper 08] (numerically)
[Binoth, Gehrmann, GH, Mastrolia 07]
[Ossola, Pittau, Papadopoulos 07] (numerically)
[Bernicot, Guillet 08]



- rational parts shown to be zero [Binoth, Guillet, GH 06]
- used both unitarity cuts and (analytical branch of) semi-numerical method



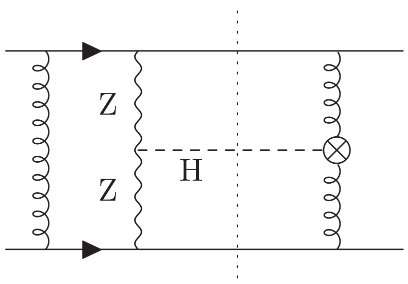


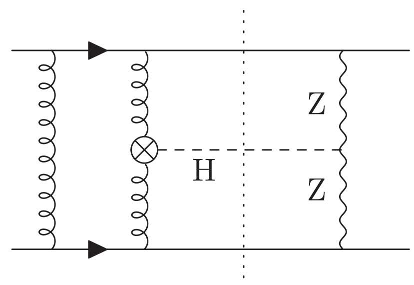
Gong, Nagy, Soper

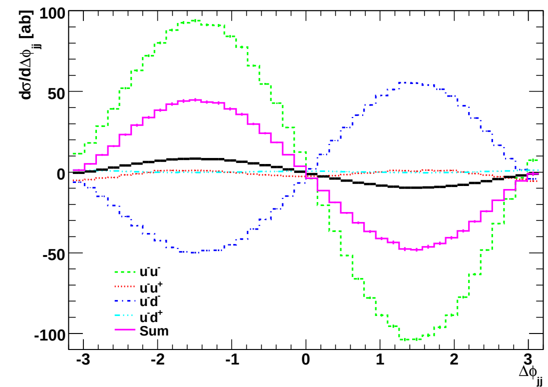
amplitudes with several mass scales

- semi-numerical approach does best
- example: one-loop interference between vector-boson fusion and gluon fusion in Higgs+2 jet production

  [Andersen, Binoth, GH, Smillie 07]







- investigate impact of interference on extraction of HZZ coupling from Higgs+2jet events

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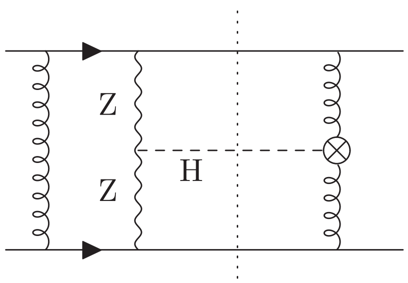
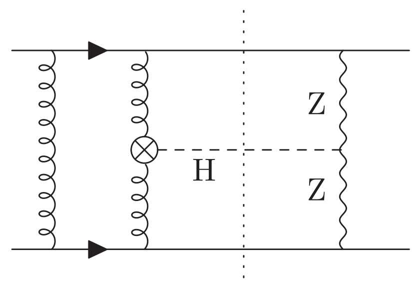
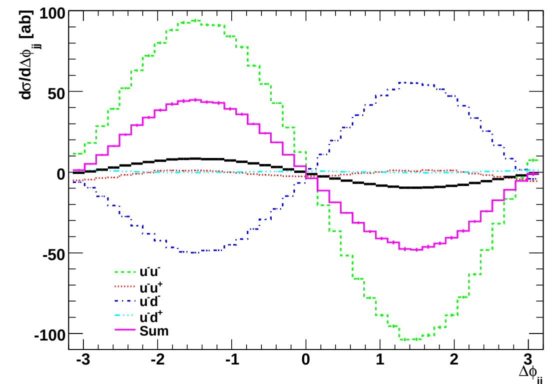






- investigate impact of interference on extraction of HZZ coupling from Higgs+2jet events

  $\longrightarrow$ no significant effect

$pp \longrightarrow b\bar{b}b\bar{b}$ one-loop amplitude

$q\bar{q} \longrightarrow b\bar{b}b\bar{b}$ completed

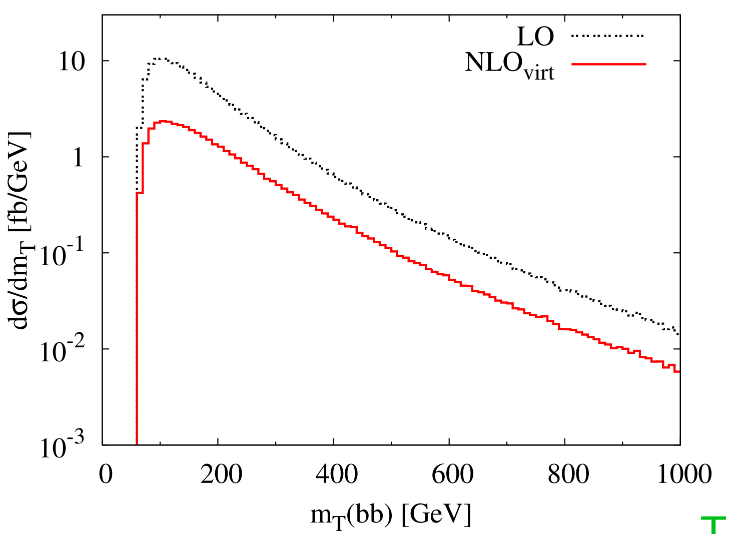
[Ph.D.thesis of Thomas Reiter]

$gg \longrightarrow b\bar{b}b\bar{b}$ virtual part completed

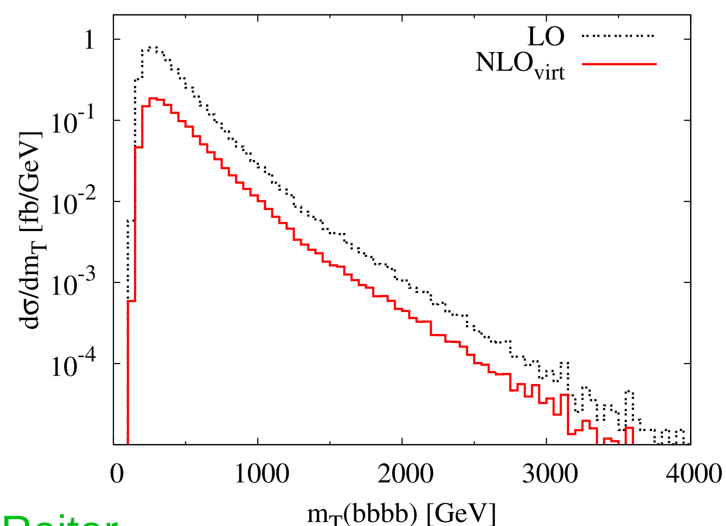
[Binoth, Guillet, Reiter]

results for finite combination

$|\mathcal{A}_{\mathrm{LO+NLOvirt}}|^2$ − UV counterterms − IR Catani-Seymour operators

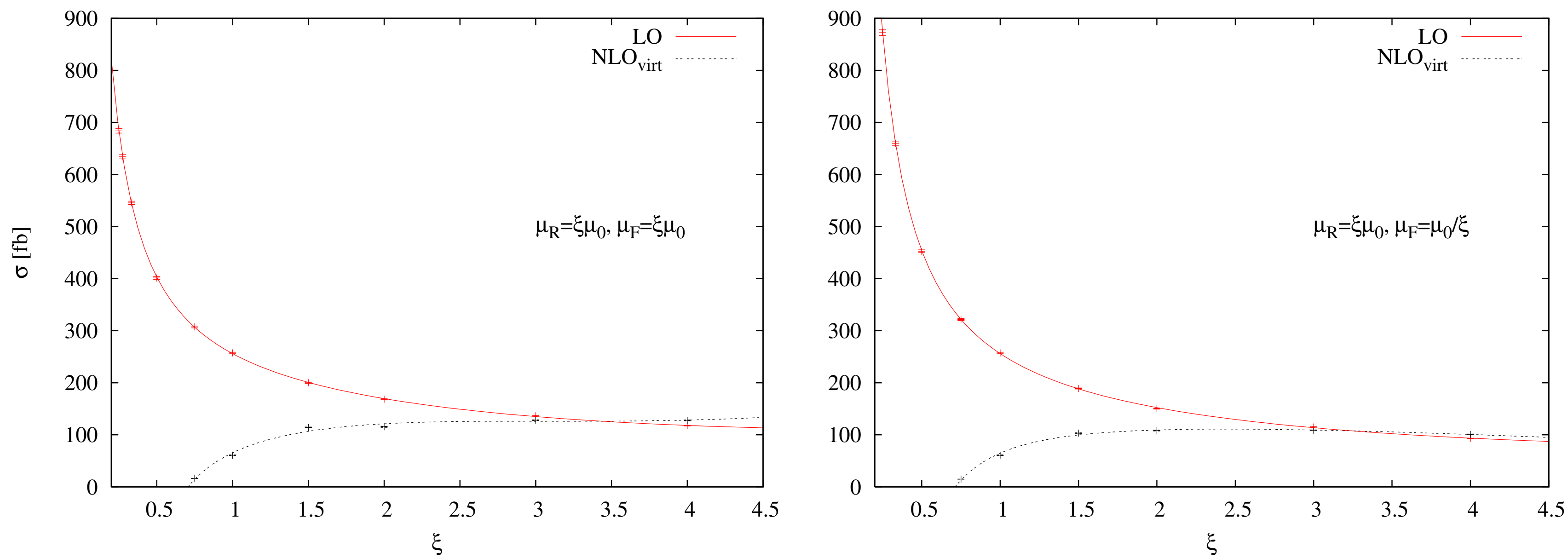


T.Reiter



scale variations

default scale choice $\mu_0 = \mu_R = \mu_F = \sum_{i=1}^{4} p_T^{(i)}/4$



T.Reiter

inclusion of full real radiation will lead to further compensation of $\alpha_s \log(\mu_F^2)$

Summary

- in order to discover and understand "New Physics" at TeV colliders: $\Rightarrow$ need accuracy beyond LO

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- in order to discover and understand "New Physics" at TeV colliders: $\Rightarrow$ need accuracy beyond LO
- we have powerful tools for high precision predictions

  GOLEM approach:
  - very flexible due to optional reduction or numerical evaluation of tensor integrals
  - good numerical behaviour as small inverse Gram determinants can be avoided
  - efficient extraction of IR singularities
  - high level of automation
  - combination with parton shower in progress
  - publicly available at http://lappweb.in2p3.fr/lapth/Golem/golem95.html