Neutron Centrifugal States, Whispering Gallery and short-range forces

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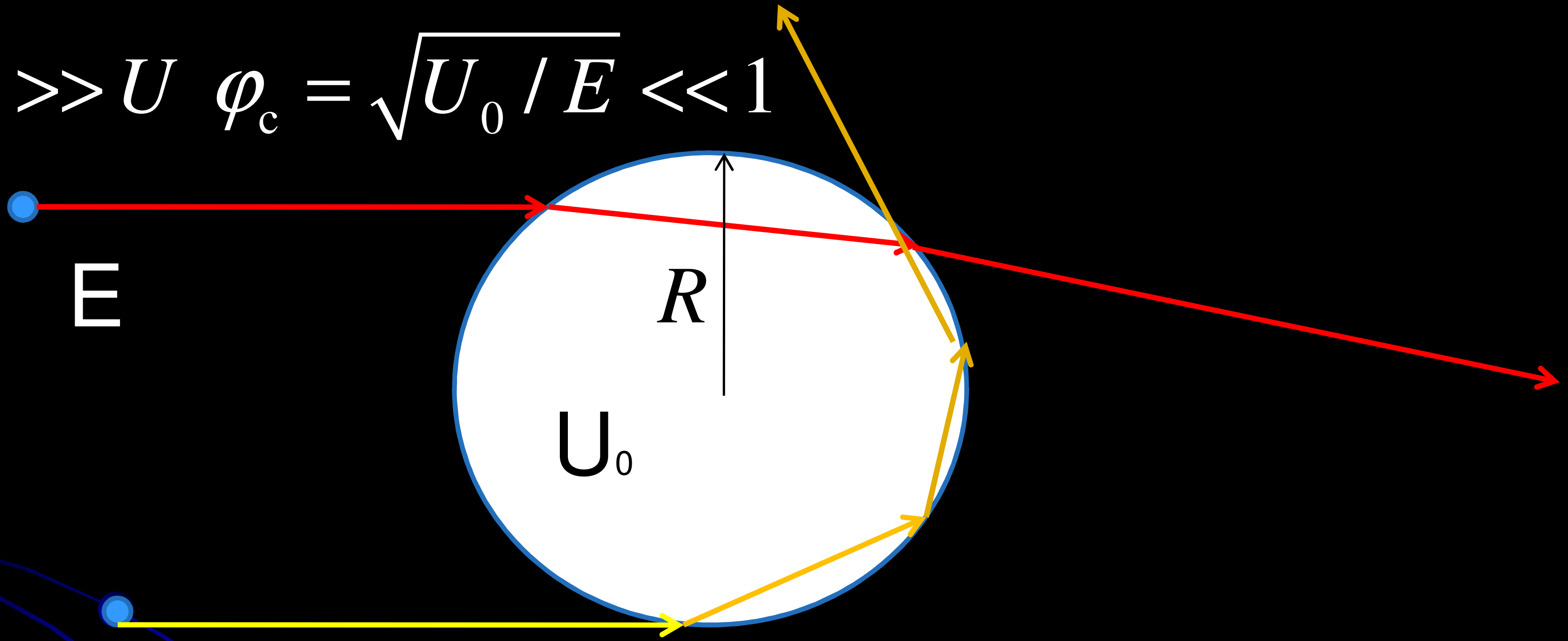
Plan of the talk

- Neutron scattering on a cylinder-whispering gallery effect
- Centrifugal states as they appear in neutron scattering
- Short-range forces effect

Neutron scattering on a cylinder

$$E \gg U \quad \varphi_c = \sqrt{U_0 / E} \ll 1$$



$$\varphi \gg \varphi_c$$

Scattering amplitude

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial\rho^2}-\frac{1}{2M\,\rho^2}\left(\frac{\partial^2}{\partial\varphi^2}+\frac{1}{4}\right)-U_0\Theta(\rho-R)-\frac{p^2}{2M}\right]\Phi(\rho,\varphi)=0$$

$$\Phi(\rho,\varphi)=\sum_{\mu=-\infty}^{\infty}\chi_{|\mu|}(\rho)\exp(i\mu\varphi) \qquad \mu_0 = MvR/h \approx 5*10^8$$

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial\rho^2}+\frac{1}{2M\,\rho^2}\left(\mu^2-\frac{1}{4}\right)-U_0\Theta(\rho-R)-\frac{p^2}{2M}\right]\chi_\mu(\rho)=0$$

$$\chi_\mu(\rho\to 0)=0$$

$$\chi_\mu(\rho\to\infty)=i^\mu\sqrt{\frac{2}{\pi p}}\sin(p\rho+\delta_{|\mu|}-\pi/2(\mu-1/2))$$

$$f(\varphi)=\frac{-i\sqrt{h}}{\sqrt{2\pi p}}\sum_{\mu=-\infty}^{\infty}(\exp(2i\delta_\mu)-1)\exp(i\mu\varphi)$$

Centrifugal states

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial z^2} - U_0\Theta(z) + \frac{\mu^2}{2MR^2}(1-2z/R) - E\right]\chi_\mu(z) = 0$$

$$z = \rho - R$$

$$\mu_0 = MvR/h \approx 5*10^8$$

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial z^2} - U_0\Theta(z) - \frac{Mv^2}{R}z - \varepsilon_\mu\right]\chi_\mu(z) = 0$$

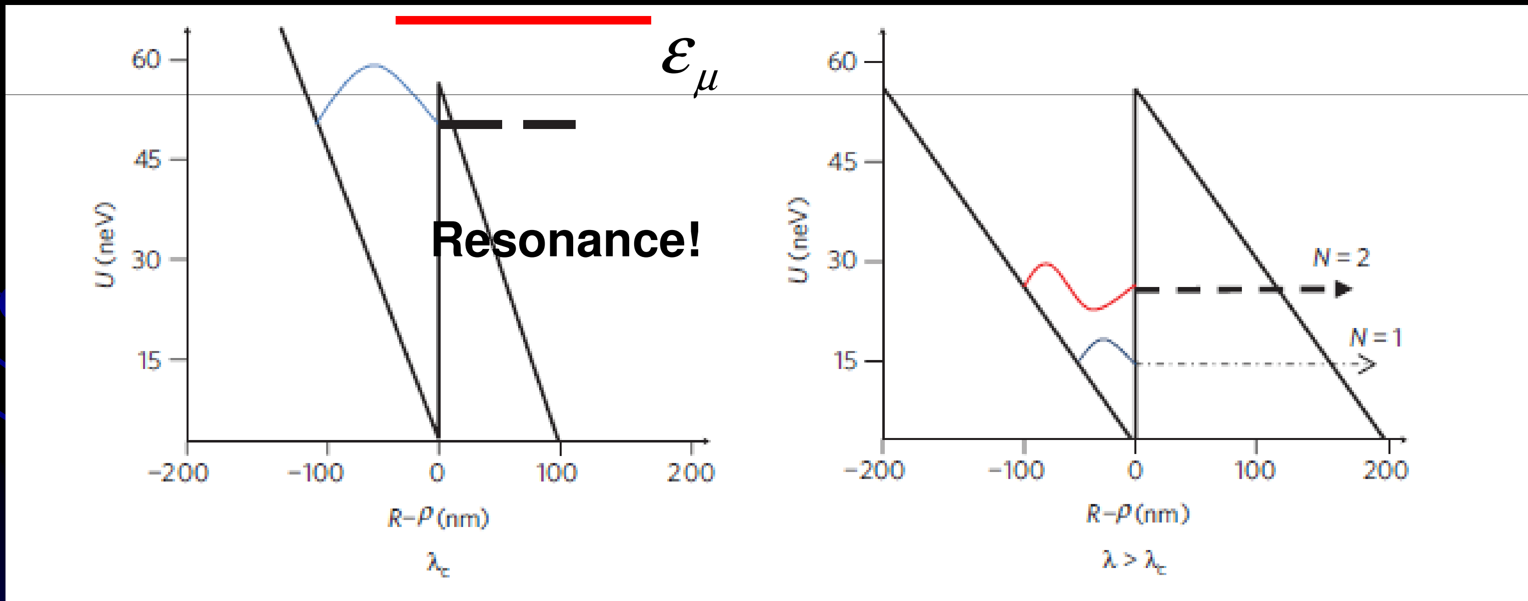
$$\varepsilon_\mu = \mu_0(\mu_0 - \mu)/(MR^2)$$

Here are centrifugal states

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial z^2}-U_0\Theta(z)-\frac{Mv^2}{R}z-\varepsilon_\mu\right]\chi_\mu(z)=0$$

$$\frac{l_0}{R}\approx 1.6\ 10^{-5}$$

$$\varepsilon_\mu=\mu_0(\mu_0-\mu)/(MR^2)$$



$$l_0=\left(\frac{\hbar^2R}{2M^2v^2}\right)^{1/3}\approx 40\ \text{nm}\quad \varepsilon_0=\left(\frac{\hbar^2Mv^4}{2R^2}\right)^{1/3}\approx 15\text{neV}\quad g_{eff}=\frac{v^2}{R}\sim 10^7 g$$

QUASISTATIONARY STATES

$$\tilde{\chi}_\mu(z) = \begin{cases} Ai(-z/l_0 - \varepsilon_\mu/\varepsilon_0) & \text{z} \le 0 \\ Bi(-z/l_0 - U_0/\varepsilon_0 - \varepsilon_\mu/\varepsilon_0) + iAi(-z/l_0 - U_0/\varepsilon_0 - \varepsilon_\mu/\varepsilon_0) & \text{z>0} \end{cases}$$

Outgoing wave solution

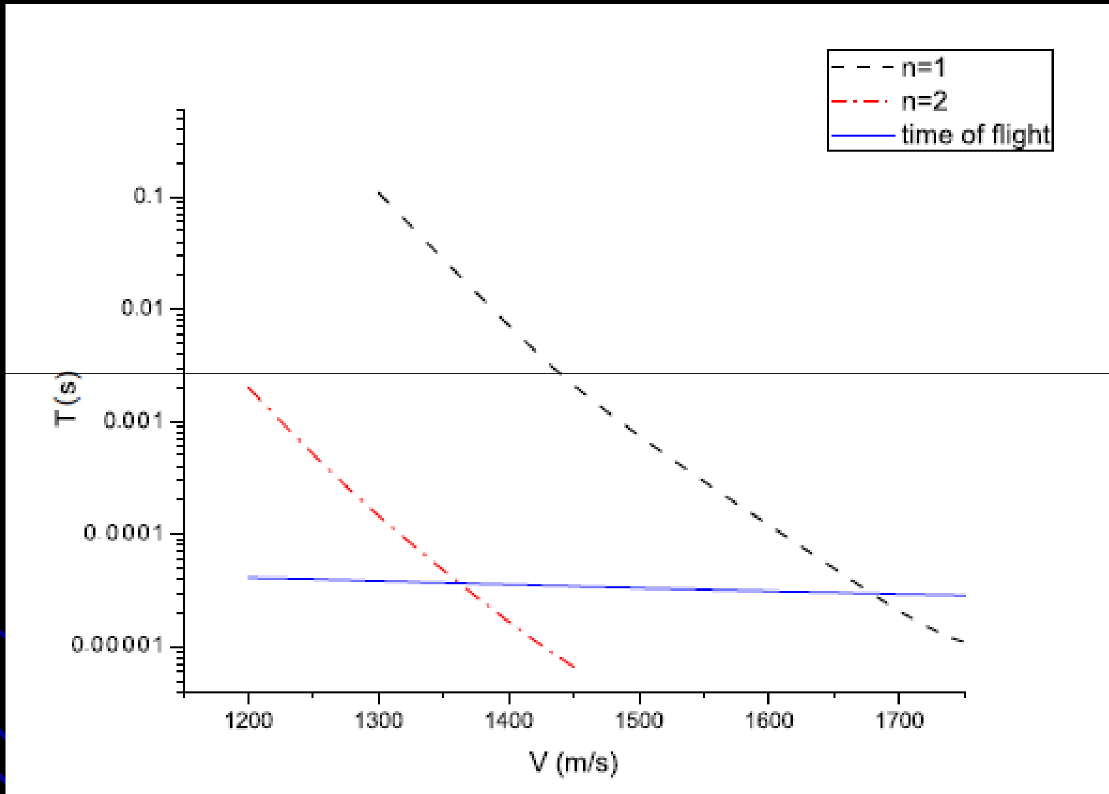
Complex Energy levels

$$h(\mu_n) = Bi'(U_0/\varepsilon_0 - \mu_n)Ai(-\mu_n) - Bi(U_0/\varepsilon_0 - \mu_n)Ai'(-\mu_n)$$

$$g(\mu_n) = Ai(U_0/\varepsilon_0 - \mu_n)Ai'(-\mu_n) - Ai'(U_0/\varepsilon_0 - \mu_n)Ai(-\mu_n)$$

$$h(\mu_n) + ig(\mu_n) = 0$$

Lifetimes



$$\Gamma \sim \exp\left(-\frac{4}{3}\left(\frac{U_0-\varepsilon_\mu}{\varepsilon_0}\right)^{3/2}\right)$$

How to see in neutron scattering?

Physical solution

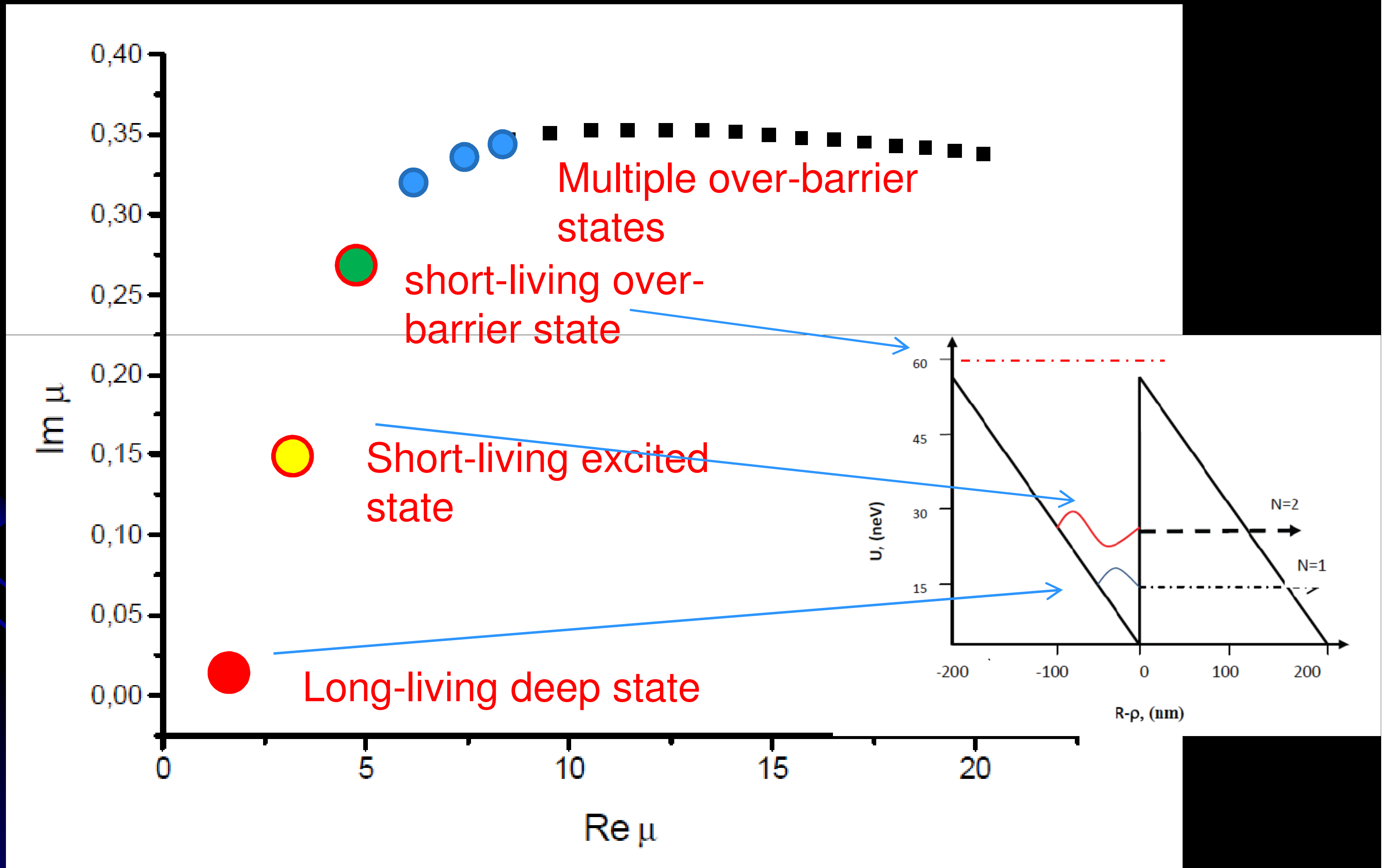
$$\chi_\mu(\rho) \to \exp(-ik\rho) - S_\mu \exp(ik\rho)$$

Poles of S-matrix as a function of complex μ (Regge poles)

$$S_\mu = \frac{h(\mu) - ig(\mu)}{h(\mu) + ig(\mu)} \equiv \exp(2i\delta_\mu)$$

$$f(\varphi) = \frac{-i\sqrt{h}}{\sqrt{2\pi p}} \sum_{\mu=-\infty}^{\infty} (\exp(2i\delta_\mu) - 1) \exp(i\mu\varphi)$$

Regge poles



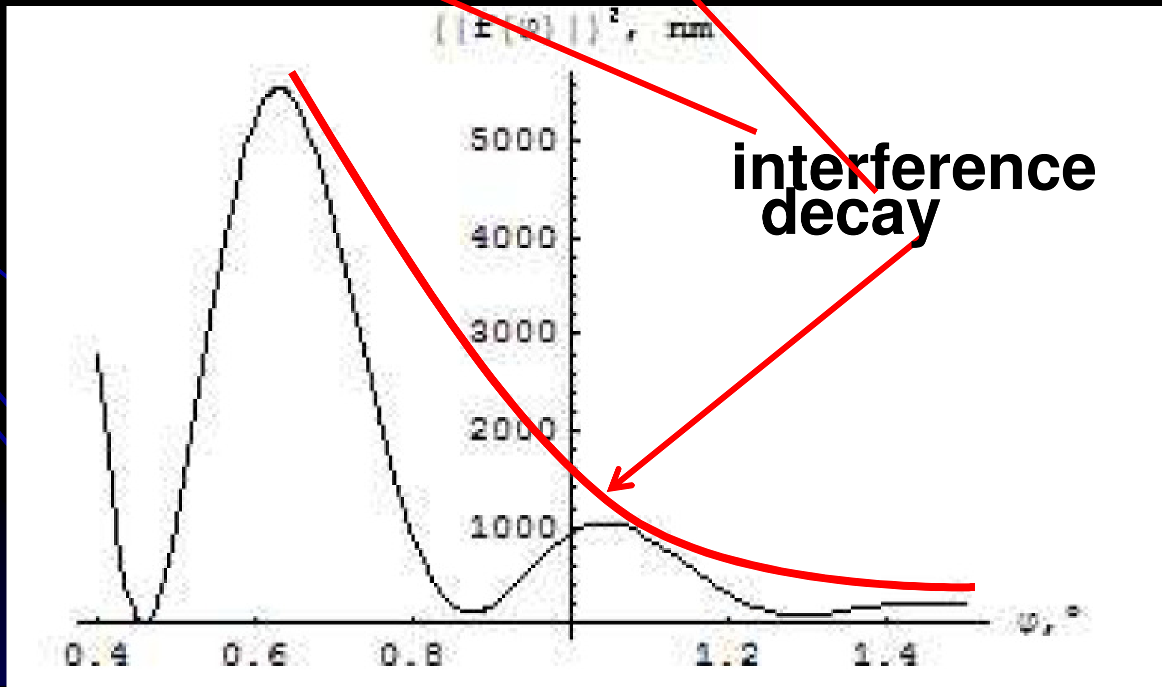
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0,00
Im μ
Multiple over-barrier states
short-living over-barrier state
Short-living excited state
Long-living deep state
0
5
10
15
20
Re μ
60
45
30
15
U, (neV)
N=2
N=1
-200
-100
0
100
200
R-ρ, (nm)

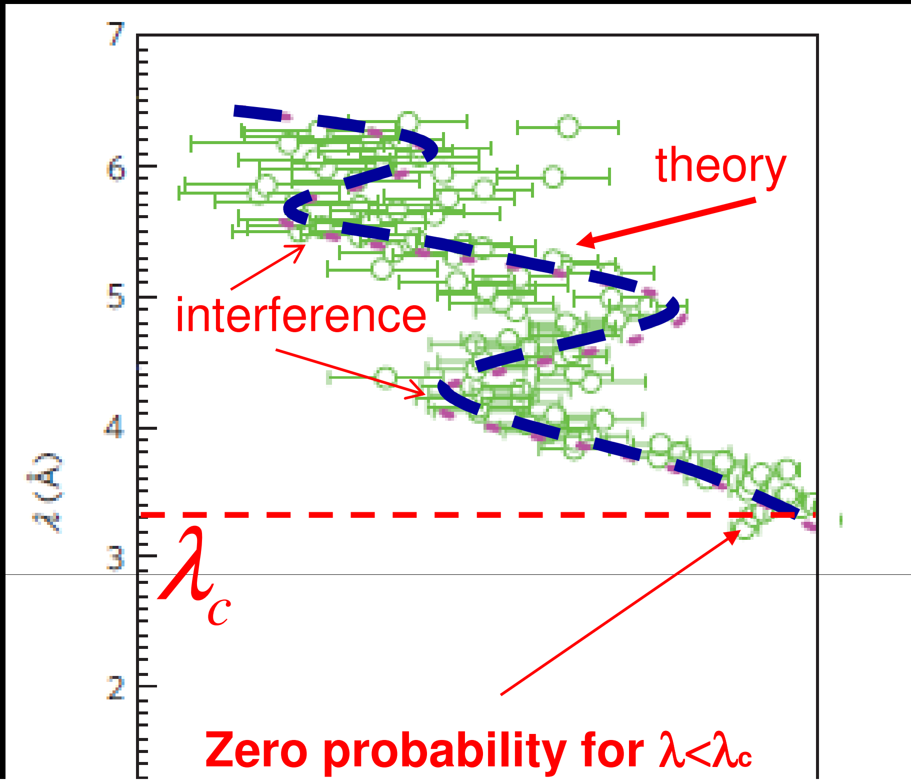
Scattering amplitude

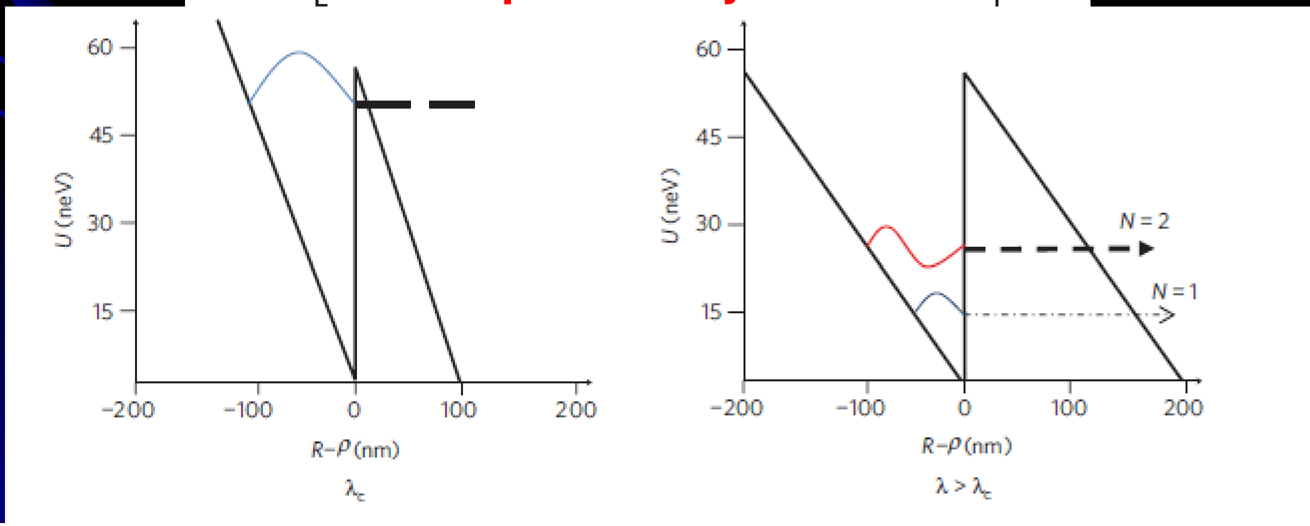
$$\mu_n = \mathrm{Re}\,\mu_n + i\gamma_n$$

$$f(\varphi) = -2i\left(\frac{\mu_0}{2}\right)^{1/3} \exp(i\mu_0\varphi)\sqrt{\frac{2\pi\hbar}{p}} \times$$

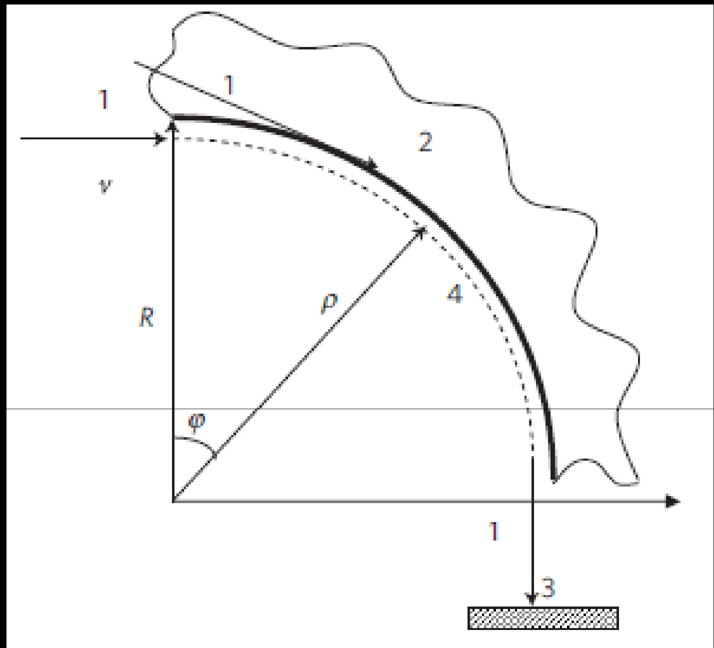
$$\sum_{n=1} \mathrm{Res}(S(\mu_n))\exp(-(\gamma_n + i\,\mathrm{Re}\,\mu_n)\varphi)$$




7
6
5
4
3
2
λ (Å)
theory
interference
λ_c
Zero probability for λ<λc


60
45
30
15
U (neV)
−200
−100
0
100
200
R−ρ (nm)
λc
N = 2
N = 1
λ > λc

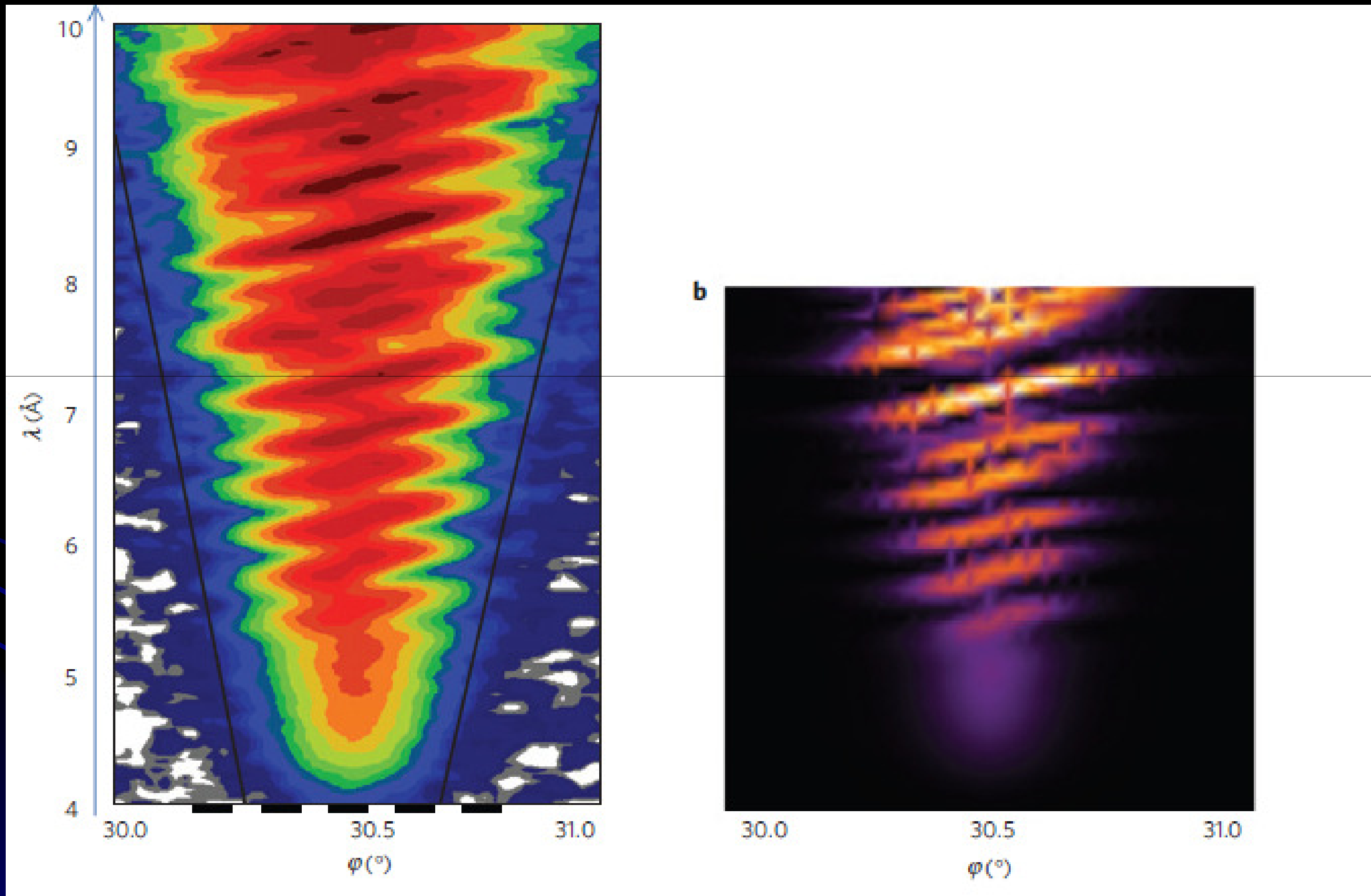
Centrifugal states Interference



$$\Phi(z,\varphi_0) \approx \sum_n C_n Ai(z-\mu_n)\exp(-\gamma_n\varphi_0)\exp(-i\,\mathrm{Re}\,\mu_n\varphi)$$

$$\left|f(k)\right|^2 = \sum_{n,m} C_n^* C_m \langle Ai_n|k\rangle\langle k|Ai_m\rangle \exp(-(\gamma_n+\gamma_m)\varphi)\exp(-i\varphi(\mathrm{Re}\,\mu_m - \mathrm{Re}\,\mu_n))$$

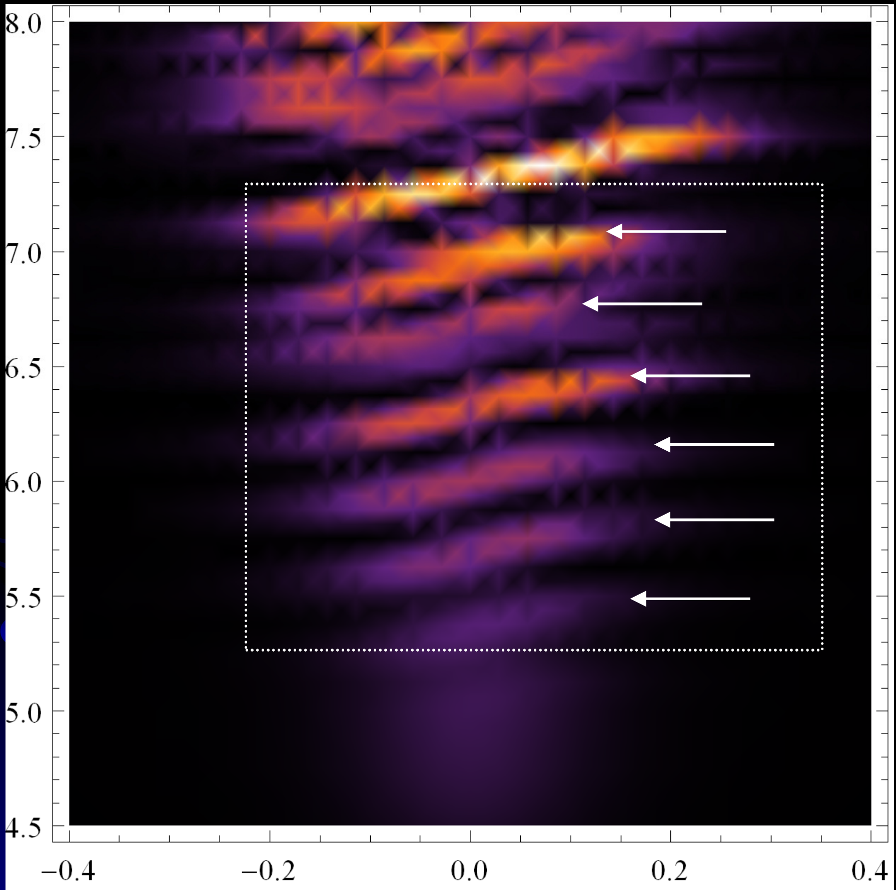
Deep Centrifugal states interference

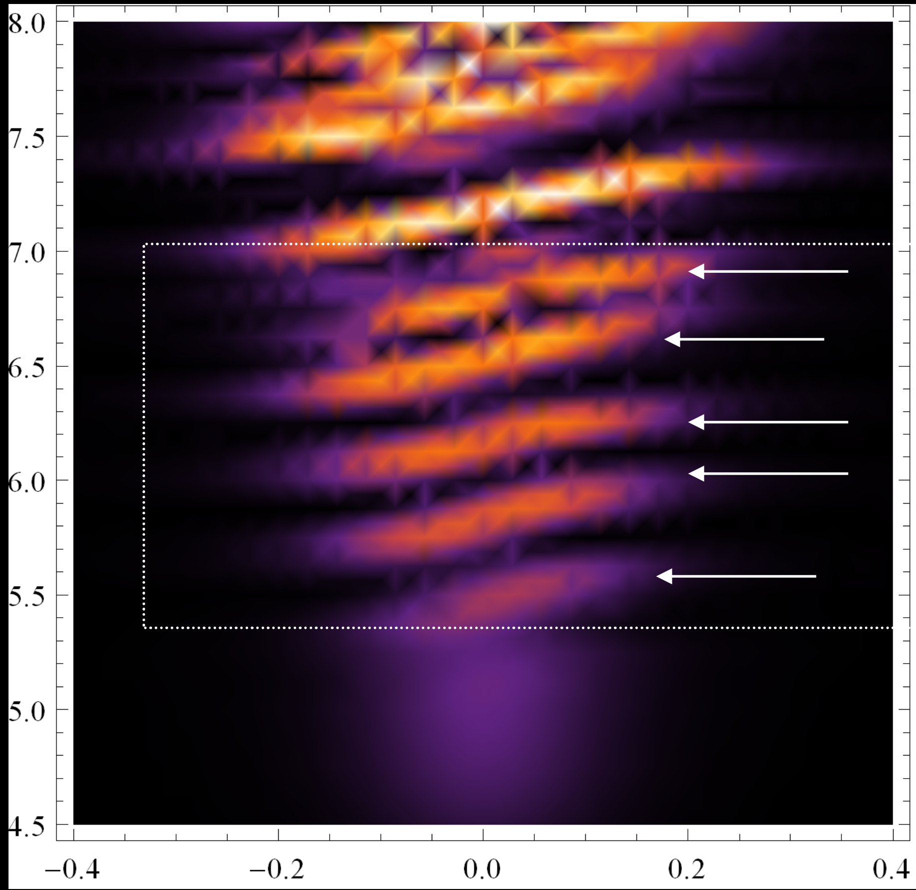


Sensitivity to additional interactions

$$U(z) = \frac{U_0}{1+\exp(-z/b)}$$

$$b \to 0 \; U(z) \to U_0 \Theta(z)$$

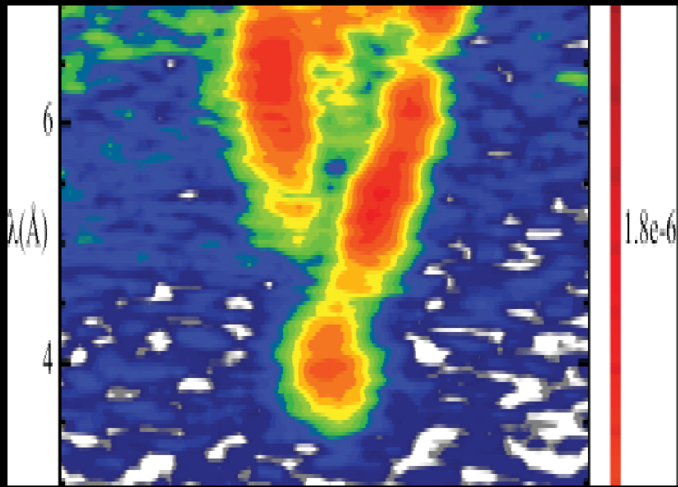


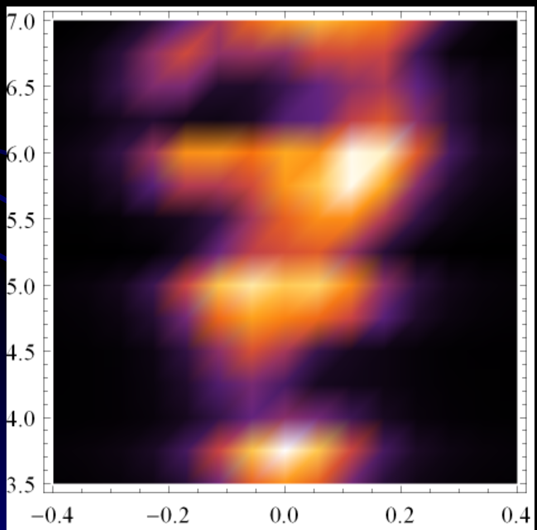


b=0

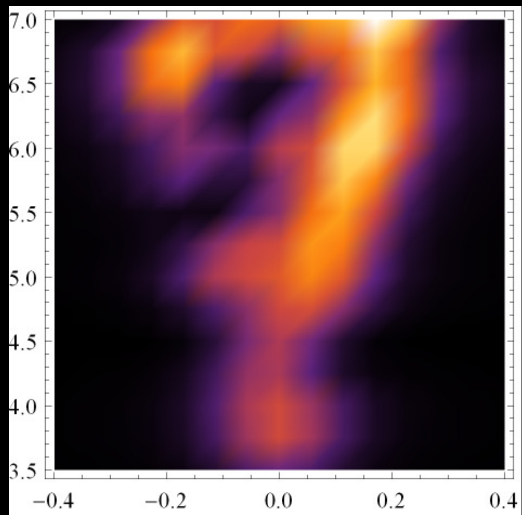
b=4nm

3.95 degree scattering





b=0



b=4nm

Some conclusions

- Centrifugal states- beautiful physics of matter waves – crossroad of different problems (from quantum bouncer to Regge poles)
- Example of precisely solvable problem
- Decay rate and interference - precisely measurable
- High sensitivity to modification of Fermi potential with nanometer range

Promising tool for studding extra-forces, surface effects, etc.