Noncommutative quantum mechanics
and
GRANIT constraints on Standard Model extensions

Noncommutative Quantum Mechanics

GRANIT constraints on Unparticle inspired extensions of the Standard Model

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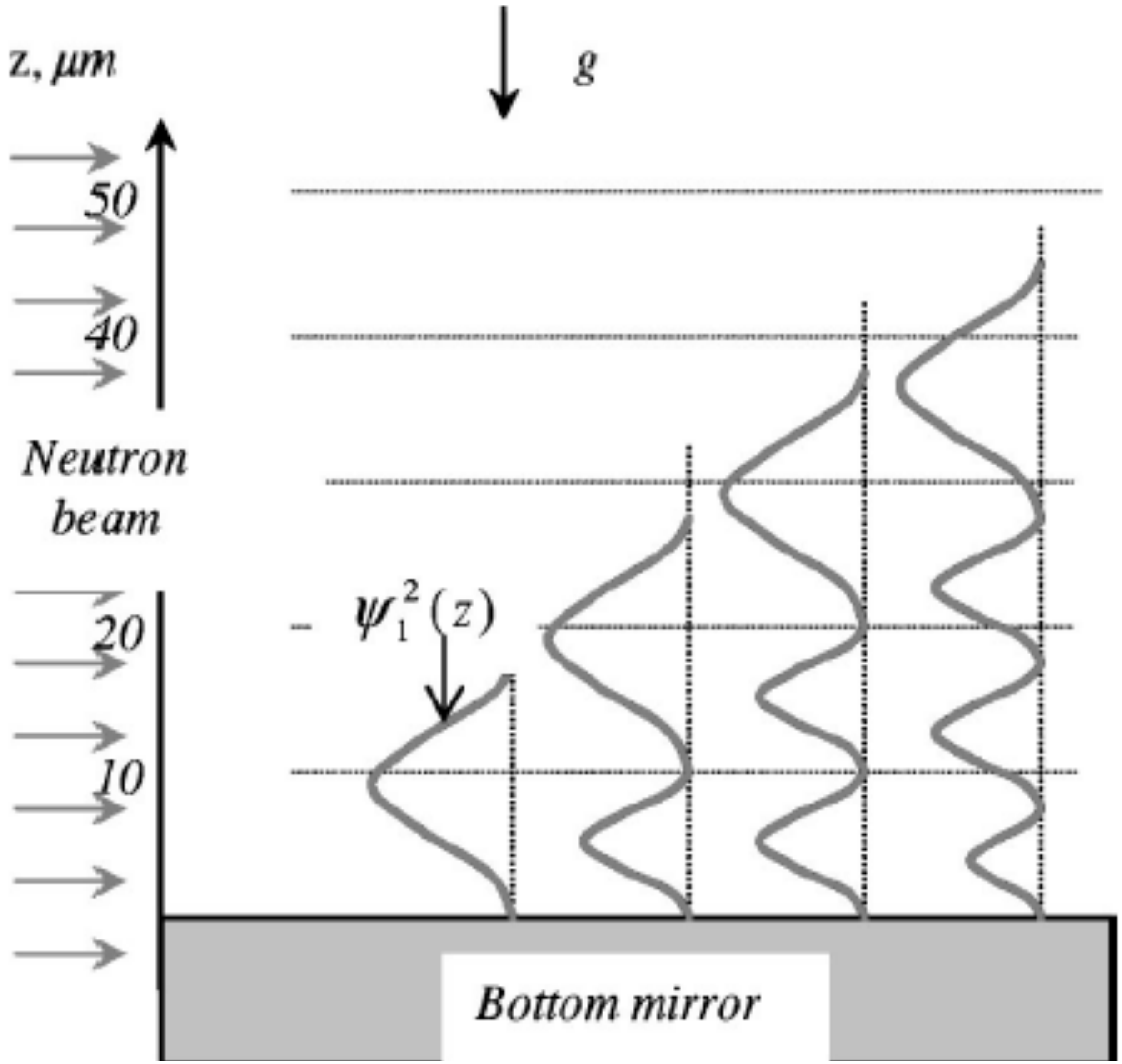
- **Gravitational Quantum Well (GQW)**
- **Noncommutative Geometry**
- **Noncommutative Quantum Mechanics**
- **Noncommutative Gravitational Quantum Well (NCGQW)**
- **The NCGQW Berry Phase and the Seiberg-Witten map**
- **Phase-space noncommutative Quantum Cosmology**
- **Phase-space noncommutative Black Holes**
- **Unparticle extensions of the Standard Model (SM)**
- **GRANIT constraints on unparticle extensions of the SM**

Gravitational Quantum Well (GQW)

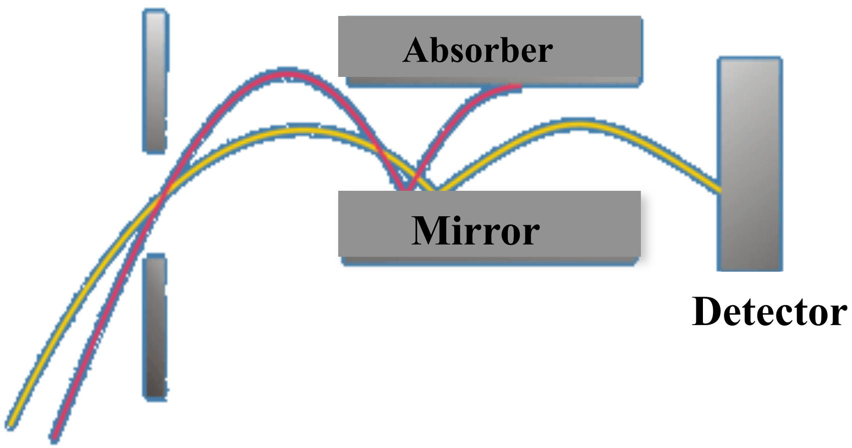
[Nesvizhevsky et al. 2002-2005]

- **Bound neutrons by gravity in the x-direction** ($\mathbf{g} = -g\mathbf{e_x}$)
- **Hamiltonian:** $H' = \frac{p_x'^2}{2m} + \frac{p_y'^2}{2m} + mgx'$
- **Wave function ~ Airy function:** $\psi_n(x') = A_n Ai(x')$
- **Classical turning points:** $x_n = E_n/mg$
- **Energy spectrum:** $E_n = -\left(\frac{mg^2\hbar^2}{2}\right)^{1/3}\alpha_n$
- **Experimental results:** $$x_1^{exp} = 12,2 \pm 1,8(syst.) \pm 0,7(stat.) \; (\mu\text{m}) \;,$$ $$x_2^{exp} = 21,6 \pm 2,2(syst.) \pm 0,7(stat.) \; (\mu\text{m}) \;.$$
- **Theoretical results:** $x_1 = 13,7\;\mu\text{m}$ $\quad x_2 = 24,0\;\mu\text{m}$

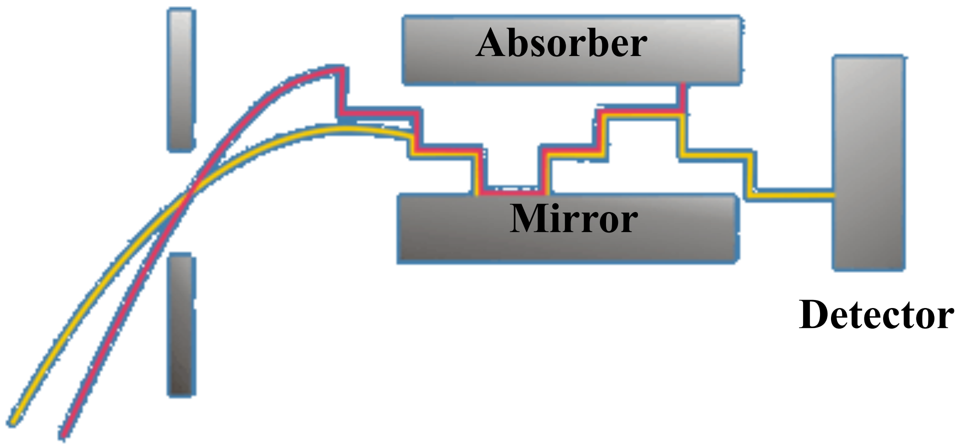


g
z, μm
50
40
Neutron
beam
20
10
$\psi_1^2(z)$
Bottom mirror

Classical Behaviour



Quantum Behaviour



Gravitational Quantum Well: Equivalence Principle and New Forces of Nature

[O.B., Nunes 2003]
[Abele, Baessler, Westphal 2003]
[Nesvizhevsky, Protasov 2004]

- **Equivalence Principle:** $$\frac{m_i - m}{m} = \frac{3}{2} m g^2 \hbar^2 \alpha_n^3 \frac{\Delta E}{E_n^4}$$

If $\Delta E = 10^{-18}\ eV$ **:** $$\frac{m_i - m}{m} < 1.9 \times 10^{-6}$$

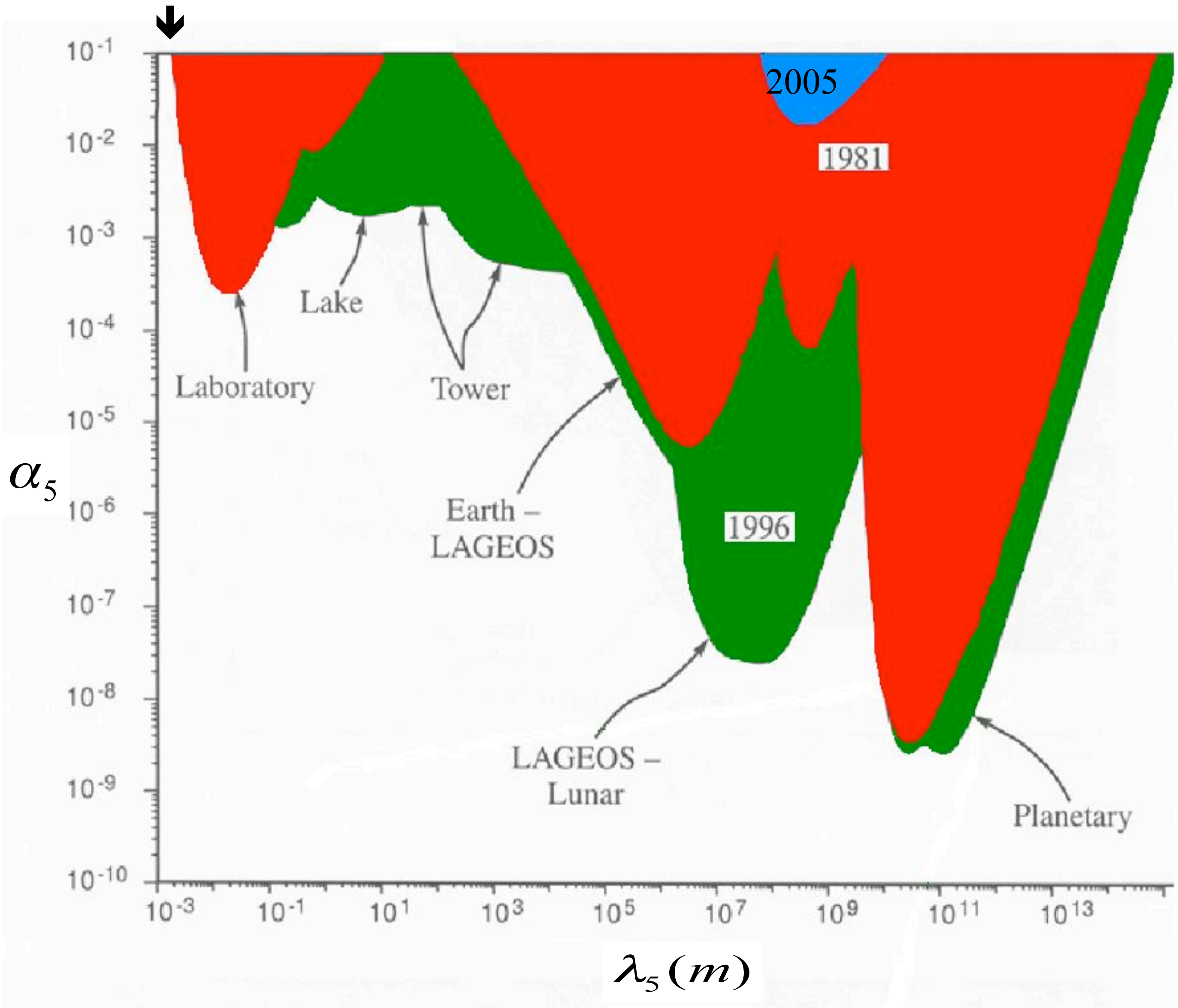
- **New force of Nature:**

Yukawa-type interaction between masses m_1 and m_2

$$V(r) = -\frac{G_\infty\ m_1\ m_2}{r}\ [1 + \alpha_5\ \exp(-r/\lambda_5)]$$

$$\vec{F}(r) = -\nabla V(r) = -\frac{G(r)\ m_1\ m_2}{r^2}\ \hat{\mathbf{r}}$$

$$G(r) = G_\infty [1 + \alpha_5\ (1 + r/\lambda_5)\ \exp(-r/\lambda_5)]$$



2005
1981
1996
Lake
Laboratory
Tower
Earth – LAGEOS
LAGEOS – Lunar
Planetary
α_5
$\lambda_5\,(m)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-3}
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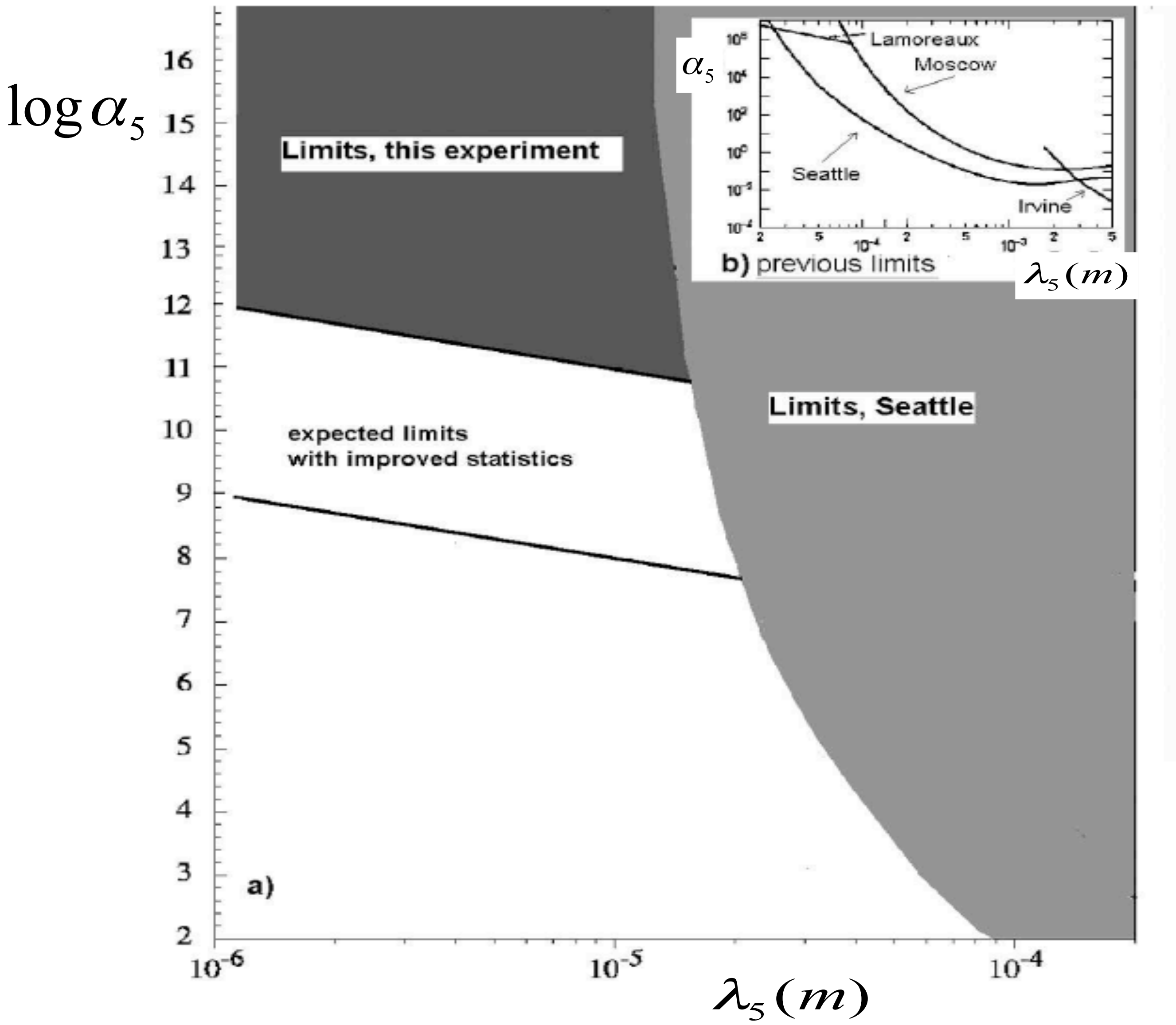
10^{11}
10^{13}



[*Abele, Baessler, Westphal 2003*]

Noncommutative Geometry

Space where the configuration variables satisfy

$$[x_\mu , x_\nu] = i\theta_{\mu\nu}$$

$\theta_{\mu\nu}$ - antisymmetric real matrix ($[\theta] = M^{-2}$)

- Snyder (& Heisenberg) (1947): suggestion to resolve the problem of ultraviolet infinities in quantum field theory

- Connes et al. (1994): necessary ingredient for Quantum Gravity

- Geometry at the Planck scale: $M_P = 1.2 \times 10^{19}$ GeV , $L=10^{-35}$ m

- Mathematical approach: differential structure of generic C* algebras

[Connes 1985]
[Woronowicz 1987]

Striking Motivation: String/M- theory

Noncommutative (NC) structure of spacetime arises when a non-vanishing background NS B-field is turned on. For instance, the end points of the open string obey, in the presence of a constant Bμν field, the commutation relation:

$$[x^{\mu}, x^{\nu}] = i\theta^{\mu\nu}$$

$\theta^{\mu\nu}$ - **antisymmetric real constant matrix ($[\theta] = M_P^{-2}$)**

[Shu 1999, Schomerus 1999, Seiberg, Witten 1999]

$\theta^{\mu\nu}$ - **provides a directionality to space-time for fixed inertial frames**

$$\Downarrow$$

Any NC theory violates 4-dimentional particle Lorentz invariance

[Carroll, Harvey, Kostelecký, Lane, Okamoto 2001]

- Field algebraic structure (Moyal Bracket of functions on R^4)

$$f(x) * g\ (x) = exp\left[\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}\right] f(y)g(z)|_{y=z=x}$$

which is associative, noncommutative and satisfies

$$\int f * g = \int g * f = \int f\ g$$

- NC Field Theory

$$S = \int \mathcal{L}[\phi] \ \Rightarrow \ S_{NC} = \int \mathcal{L}_*[\phi]$$

- MODEL: NC QED

$$\mathcal{L}_{\mathcal{NC}} = \tfrac{1}{2} i \overline{\hat{\psi}} * \gamma^{\mu} \overset{\leftrightarrow}{\hat{D}}_{\mu} \hat{\psi} - m \overline{\hat{\psi}} * \hat{\psi} - \frac{1}{4q^2} \hat{F}_{\mu\nu} * \hat{F}^{\mu\nu}$$

$$\hat{F}_{\mu\nu} = \partial_{\mu} \hat{A}_{\nu} - \partial_{\nu} \hat{A}_{\mu} - i[\hat{A}_{\mu}, \hat{A}_{\mu}]$$

$$\hat{D}_{\mu} \hat{\psi} = \partial_{\mu} \hat{\psi} - \hat{A}_{\mu} * \hat{\psi}$$

$$\hat{f} * \overset{\leftrightarrow}{\hat{D}}_{\mu} g \equiv \hat{f} * \hat{D}_{\mu} g - \hat{D}_{\mu} \hat{f} * \hat{g}$$

- Seiberg-Witten map at lowest order in $\theta^{\mu\nu}$

$$\hat{A}_\mu = A_\mu - \frac{1}{2}\theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu})$$

$$\hat{\psi} = \psi - \frac{1}{2}\theta^{\alpha\beta} A_\alpha \partial_\beta \psi$$

- NC QED to leading order in $\theta^{\mu\nu}$

$$\begin{aligned}\mathcal{L}_{\mathcal{NC}} &= \mathcal{L}_{\mathcal{QED}} - \frac{1}{8} iq\theta^{\alpha\beta} F_{\alpha\beta} \overline{\psi}\, \gamma^\mu \, \overleftrightarrow{D}_\mu \, \psi + \frac{1}{4} iq\theta^{\alpha\beta} F_{\alpha\mu} \overline{\psi}\, \gamma^\mu \, \overleftrightarrow{D}_\beta \, \psi \\ &+ \frac{1}{4} mq\theta^{\alpha\beta} F_{\alpha\beta} \overline{\psi} \psi \\ &- \frac{1}{2} q\theta^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} F^{\mu\nu} + \frac{1}{8} q\theta^{\alpha\beta} F_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}\end{aligned}$$

$$A_\mu \to q A_\mu \quad , \quad D_\mu \psi = \partial_\mu \psi - iq A_\mu \psi$$

Choosing $F_{\mu\nu} \rightarrow f_{\mu\nu} + F_{\mu\nu}$, where $f_{\mu\nu}$ is a constant background field and $F_{\mu\nu}$ a small dynamical fluctuation:

$$\mathcal{L}_{\mathcal{NC}} = \mathcal{L}_{\mathcal{QED}} + \frac{1}{2} i c_{\mu\nu} \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi - \frac{1}{4} k_{F\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

where

$$q_{eff} = (1 + \frac{1}{4} q f^{\mu\nu} \theta_{\mu\nu}) q$$

$$c_{\mu\nu} = -\frac{1}{2} q f_{\mu}^{\lambda} \theta_{\lambda\nu}$$

$$k_{F\alpha\beta\gamma\delta} = -q f_{\alpha}^{\lambda} \theta_{\lambda\gamma} \eta_{\beta\delta} + \frac{1}{2} q f_{\alpha\gamma} \theta_{\beta\delta} - \frac{1}{4} q f_{\alpha\beta} \theta_{\gamma\delta} - (\alpha \leftrightarrow \beta) - (\gamma \leftrightarrow \delta) + (\alpha\beta \leftrightarrow \gamma\delta)$$

Observational Bounds

- Birefringence of radiation from cosmological sources

$$\Downarrow$$

$$|k_F| < 10^{-28} \text{ and no significant bound on } \theta$$

- Clock comparison tests $\Rightarrow |\theta^{yz}|, |\theta^{zx}| < (10\, TeV)^{-2}$

[Carroll, Harvey, Kostelecký, Lane, Okamoto 2001]

Coupling to gravity?

[O.B., Guisado PRD 2002]

- Generalized Moyal Product for tensors T and W:

$$T * W\,(x) = \sum_{n=0}^{\infty} \frac{(i/2)^n}{n!} \theta^{\alpha_1\beta_1} \ldots \theta^{\alpha_n\beta_n} \left(T_{;\alpha_1\ldots\alpha_n}\right)\left(W_{;\beta_1\ldots\beta_n}\right)$$

where the semicolon denotes covariant derivative with Levi-Civitta connection and $\theta^{\alpha\beta}$ is a non-constant rank-2 antisymmetric tensor

- Massive scalar field coupled to Gravity

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left\{\nabla^\mu\Phi\mathcal{O}\nabla_\mu\Phi + m^2\Phi\mathcal{O}\Phi\right\}$$

- Equation of motion

$$\nabla^\mu\mathcal{O}\nabla_\mu\Phi - m^2\mathcal{O}\Phi = 0$$

where

$$\mathcal{O}W = \sum_{n=0}^{\infty}\frac{(-1/4)^n}{(2n)!}\left[\theta^{\alpha_1\beta_1}\ldots\theta^{\alpha_{2n}\beta_{2n}}\left(W_{;\beta_1\ldots\beta_{2n}}\right)\right]_{;\alpha_{2n}\ldots\alpha_1}$$

- NC potential

$$V_{NC}(\Phi) = \sum_{n=0}^{\infty}\frac{\lambda_n}{n!}\overbrace{\Phi*\ldots*\Phi}^{n\,factors}$$

⋆ Perturbative approach: $* \simeq 1 + \hat{*}$:

$$V_{NC}(\Phi) \equiv V(\Phi) + \left(\frac{V}{\Phi}\right)' (\Phi \hat{*} \Phi)$$

where $' = d/d\Phi$ and the first non-trivial contribution

$$\Phi \hat{*} \Phi = -\tfrac{1}{8}\theta^{\alpha_1\beta_1}\theta^{\alpha_2\beta_2}\left(\Phi_{;\alpha_1\alpha_2}\right)\left(\Phi_{;\beta_1\beta_2}\right)$$

Hence

$$-\frac{\delta S_{pot}}{\delta\Phi} = V' + \left(\frac{V}{\Phi}\right)'' (\Phi \hat{*} \Phi) - \tfrac{1}{4}\mathcal{F}[V, \Phi]$$

where

$$\mathcal{F}[V, \Phi] = \left[\left(\frac{V}{\Phi}\right)' \theta^{\alpha_1\beta_1}\theta^{\alpha_2\beta_2}\phi_{;\beta_1\beta_2}\right]_{;\alpha_2\alpha_1}$$

- **Stability Conditions based on the Positive Energy Theorem of GR**

O.B. PLB 1987; O.B. & Zarro, PLB 2009

- Einstein-Hilbert action is unchanged by noncommutativity:

$$R_{\alpha\beta} = -8\pi k \left[\frac{1}{2}\nabla_\alpha \Phi \mathcal{O} \nabla_\beta \Phi + g_{\alpha\beta} V_{NC}(\Phi)\right]$$

- Spatially flat Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

- Ansatz: $\theta^{\alpha\beta} = \theta^{\alpha\beta}(t)$

$$\theta^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

- **Set:** $\vec{E} = 0$ **and** $B^2 = \hat{B}^2 R^{-2\varepsilon}$

Slow-roll in Chaotic Inflation

- First order of perturbation theory in $\hat{B}^2$

$$\Phi = \phi + \hat{B}^2 \varphi \qquad R = a + \hat{B}^2 \chi$$

where ϕ and a are solutions of the unperturbed (commutative) problem.

- Onset of inflation and slow-roll regime occur once the following conditions are met

$$\frac{V'}{V} \leq \sqrt{48\pi} \quad , \quad \frac{V''}{V} \leq 24\pi$$

and therefore

$$\left|\dot{\phi}\right| \leq \sqrt{2} V^{1/2}$$

Chaotic Inflationary Potential:

$$V(\Phi) = \lambda\, v(\Phi)$$

where $\lambda \simeq 10^{-14}$, $v \leq 10^2$ and $\phi \simeq few M_P$.

* New noncommutative terms are propto $a^{4-2\varepsilon}$ and factors of **V** and $\dot{\phi}$

However, as the Universe expands exponentially, perturbation theory is meaningful only if $\varepsilon \geq 2$

$$\Downarrow$$

$$\varepsilon = 2$$

Otherwise noncommutativity cannot lead to any effects

It is shown that $\theta \sim a^{-2}$ and hence noncommutativity does not affect the inflationary dynamics

[O.B., Guisado 2002]

Noncommutative Quantum Mechanics (I)

[Chaichian, Sheikh-Jabbari, Tureanu 2000]
[Gamboa, Loewe, Rojas 2001; Nair, Polychronakos 2001]
[Ho, Kao 2002; Zhang 2004]

- Non-relativistic limit of NC quantum field theory (one-particle sector) corresponds to the Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\Delta\psi + V * \psi$$

where one can write the noncommutative part $V * \psi$ as

$$V\left(q'_i - \frac{p'_j \theta \varepsilon_{ij}}{2\hbar}\right)\psi(q')$$

through the "Seiberg-Witten" map.

Noncommutative Quantum Mechanics (II)

The "Seiberg-Witten" map is a linear non-canonical set of transformations

$$q_i = q'_i - \frac{\theta}{2\hbar}\varepsilon_{ij}p'_j \quad , \quad p_i = p'_i$$

that relates the NC variables which satisfy the NC algebra

$$[q_i, q_j] = i\theta\varepsilon_{ij} \quad , \quad [p_i, p_j] = 0 \quad , \quad [q_i, p_j] = i\hbar\delta_{ij}$$

with the variables that satisfy the Heisenberg-Weyl algebra of Quantum Mechanics

$$[q'_i, q'_j] = 0 \quad , \quad [p'_i, p'_j] = 0 \quad , \quad [q'_i, p'_j] = i\hbar\delta_{ij}$$

Noncommutative Gravitational Quantum Well (NCGQW)

[O.B., Rosa, Aragão, Castorina, Zappalà, Phys. Rev. D72 (2005), hep-th/0505064]

Consider the NC extension of QM in the phase space:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \,, \quad [p^\mu, p^\nu] = i\eta^{\mu\nu} \,, \quad [x^\mu, p^\nu] = i\hbar\delta^{\mu\nu}$$

$\longrightarrow$

$$[x, y] = i\theta \,, \quad [p_x, p_y] = i\eta \,, \quad [x_i, p_j] = i\hbar\delta_{ij}$$

4-dimensions | *2-dimensions*

$\sqrt{\theta}$ **and** $\sqrt{\eta}$ **set the fundamental scales of distance and momentum**

Ways to implement a noncommutative version of QM:

$$\begin{aligned} x &= x' - \frac{\theta}{\hbar} p'_y \\ y &= y' , \\ p_x &= p'_x , \\ p_y &= p'_y - \frac{\eta}{\hbar} x' \end{aligned}$$

or

$$\begin{aligned} x &= x' , \\ y &= y' + \frac{\theta}{\hbar} p'_x , \\ p_x &= p'_x + \frac{\eta}{\hbar} y' , \\ p_y &= p'_y . \end{aligned}$$

To lift the ambiguity consider the combination of the above:

$$\begin{aligned} x &= x' - \frac{\theta}{2\hbar} p'_y , \\ y &= y' + \frac{\theta}{2\hbar} p'_x , \\ p_x &= p'_x + \frac{\eta}{2\hbar} y' , \\ p_y &= p'_y - \frac{\eta}{2\hbar} x' , \end{aligned}$$

→

$$[x_i, p_j] = i\hbar \underbrace{\left(1 + \frac{\theta\eta}{4\hbar^2}\right)}_{\text{Effective Planck's constant}} \delta_{ij}$$

Effective Planck´s constant

Most general set of transformations:

[O.B., Rosa, Aragão, Castorina, Zappalà, Mod. Phys. Lett. A21(2006), hep-th/0509207]

$$x = \xi\left(x' - \frac{\theta}{2\hbar}p'_y\right), \qquad y = \xi\left(y' + \frac{\theta}{2\hbar}p'_x\right)$$

$$p_x = \xi\left(p'_x + \frac{\eta}{2\hbar}y'\right), \qquad p_y = \xi\left(p'_y - \frac{\eta}{2\hbar}x'\right)$$

That admits the 4-dimensional generalization:

$$x^\mu = \xi\left(x'^\mu - \frac{\theta^\mu{}_\nu}{2\hbar}p'^\nu\right), \qquad p^\mu = \xi\left(p'^\mu + \frac{\eta^\mu{}_\nu}{2\hbar}x'^\nu\right)$$

It then follows that:

$$[x, y] = i\xi^2\theta \equiv i\theta_{eff}, \qquad [p_x, p_y] = i\xi^2\eta \equiv i\eta_{eff}$$

$$[x_i, p_j] = i\xi^2\hbar\left(1 + \tfrac{\theta\eta}{4\hbar^2}\right)\delta_{ij} \equiv i\hbar_{eff}\delta_{ij}, \quad i = 1, 2 \ .$$

Hence, if one chooses $\xi = 1$ **:**

$$\theta_{eff} = \theta,\ \eta_{eff} = \eta,\ \hbar_{eff} = \hbar\left(1 + \frac{\theta\eta}{4\hbar^2}\right)$$

If on the other hand one chooses $\xi = (1 + \theta\eta/4\hbar^2)^{-1/2}$ **then:**

$$\theta_{eff} = \frac{\theta}{1 + \frac{\theta\eta}{4\hbar^2}},\ \eta_{eff} = \frac{\eta}{1 + \frac{\theta\eta}{4\hbar^2}},\ \hbar_{eff} = \hbar$$

That is, these models are mathematically equivalent.

Four-dimensional generalization

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \; ,$$
$$[p^\mu, p^\nu] = i\eta^{\mu\nu} \; ,$$
$$[x^\mu, p^\nu] = i\hbar\left(\delta^{\mu\nu} + \frac{\theta^{\mu\alpha}\eta^\nu{}_\alpha}{4\hbar^2}\right)$$

Non-diagonal

$$\hbar_{eff} = \hbar\left(1 + \frac{Tr[\theta\eta]}{4\hbar^2}\right)$$

* **Noncommutative Hamiltonian** ($C=[1-\xi_{NC}]^{-1}$, $\xi_{NC} = \eta\theta/4\hbar^2$):

$$H = \frac{\bar{p_x}^2}{2m} + \frac{\bar{p_y}^2}{2m} + mgx + \frac{C\eta}{2m\hbar}(x\bar{p_y} - y\bar{p_x}) + \frac{C^2}{8m\hbar^2}\eta^2(x^2 + y^2)$$

$$(\overline{p_x} = Cp_x, \overline{p_y} = Cp_y + \frac{m^2 g\theta}{2\hbar})$$

At first order in the noncommutative parameters:

$$H = H' + \frac{\eta}{2m\hbar}(xp_y - yp_x)$$

GQW data allow setting bounds on the fundamental momentum scale:

$$|\sqrt{\eta}| \lesssim 0,90\,\mathrm{meV/c} \qquad (n=1)\ ,$$
$$|\sqrt{\eta}| \lesssim 0,79\,\mathrm{meV/c} \qquad (n=2)\ .$$

Remarks:

Uncertainty Principle allows improving these bounds down to 1μeV/c

Second order terms are 6-7 orders of magnitude smaller than the first order ones

Bounds on the correction to Planck´s constant ($\hbar_{eff} = \hbar(1+\xi_{NC})$):

$$|\sqrt{\theta}| \lesssim 1\,\mathrm{fm} \quad \longrightarrow \quad \begin{array}{lll} |\xi_{NC}| \lesssim 5,2\times 10^{-24} & (n=1)\ , \\ |\xi_{NC}| \lesssim 4,0\times 10^{-24} & (n=2)\ . \end{array}$$

Fun with the NCGQW

- **Higher-order corrections? Doubtful!**

$$H_{\text{eff}} = mc^2 - \frac{\hbar^2}{2m}\Delta - mU - \frac{\hbar^4}{8m^3c^2}\Delta^2 + \frac{3\hbar^2}{2mc^2}(U\Delta + \nabla U\cdot\nabla) + \frac{mU^2}{2c^2} + \frac{3\hbar^2\Delta U}{4mc^2} + \mathcal{O}\left(\frac{1}{c^4}\right)$$

- **Berry phase? Non-trivial topological phase acumulated on a adiabatic evolution of the system around a closed path in the parameter space**

$$\Psi_n(\mathbf{x}) \to e^{\mathrm{i}\gamma_n(C)}\Psi_n(\mathbf{x})$$

$$\gamma_n(C) = \mathrm{i}\oint_C \langle n(\mathbf{R})|\nabla_{\mathbf{R}} n(\mathbf{R})\rangle \cdot \mathrm{d}\mathbf{R}$$

$|n(\mathbf{R})\rangle$ **-eigenstate**

Berry Phase and the Seiberg-Witten Map

[Bastos, O.B., PLA 2006]

- **Commutative GQW**

$$\gamma_n(C) = 0$$

- **NCGQW**

$$\Delta\gamma_n(C) \sim -i\left(\frac{2}{m^4 g\hbar^4}\right)^{2/3}\left(\frac{C\eta}{2}\right)^3 \Sigma_{n\neq m}\frac{1}{(\alpha_n-\alpha_m)^2}$$

$$C=[1-\xi_{NC}]^{-1},\ \xi_{NC}=\eta\theta/4\hbar^2$$

$$\gamma_n(C) = 0$$

Independently of the SW map

[Bastos, O.B., Dias, Prata, J. Math. Phys. 2008]

Unparticles I

- Physics beyond the Standard Model (SM) is required for various reasons: to endow neutrinos with masses, to solve the hierarchy problem, to unify all interactions, etc. Suggestions: SUSY, SUGRA, extra dimensions,…

- LHC era: clues and surprises for new physics …

- Can we have any other viable and testable alternative that one could look for?

Unparticles Georgi, PRL 2007, PLB 2007

- Unparticles represent a new possibility for the physics of a hidden sector that couples to the SM through higher dimensional operators

- This extension assumes that a scale invariant sector manfests itself at low energies

Un-particles ⟷ Un-conventional particles associated with this sector

Unparticles II

- SM is not scale invariant, but some interesting extensions are: N=4 Super Yang-Mills theory, Conformal Field Theory, …

- Banks-Zaks theory for the N massless vector quarks exhibits non-trivial scale invariance in the infrared for suitable combinations of flavours and colours. There might exist other gauge theory candidates …

Banks & Zaks, NPB 1982

- Can such a hidden sector have any effect on the low energy phenomena?

Fox et al, PRD 2007

- The very high-energy theory contains SM fields and fields of a theory with a non-trivial IR fixed point, the BZ fields, then:

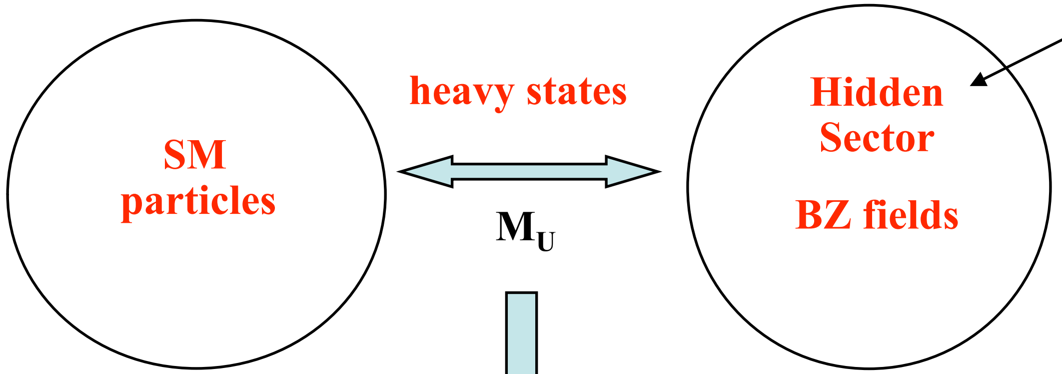
- Below a scale M_U there are non-renormalizable couplings involving both SM and BZ fields suppressed by powers of M_U

$$\frac{1}{M_U^{d_{UV}+n-4}} O_{UV}\, O_{SM}$$

Unparticles III

Georgi, PRL 2007, PLB 2007

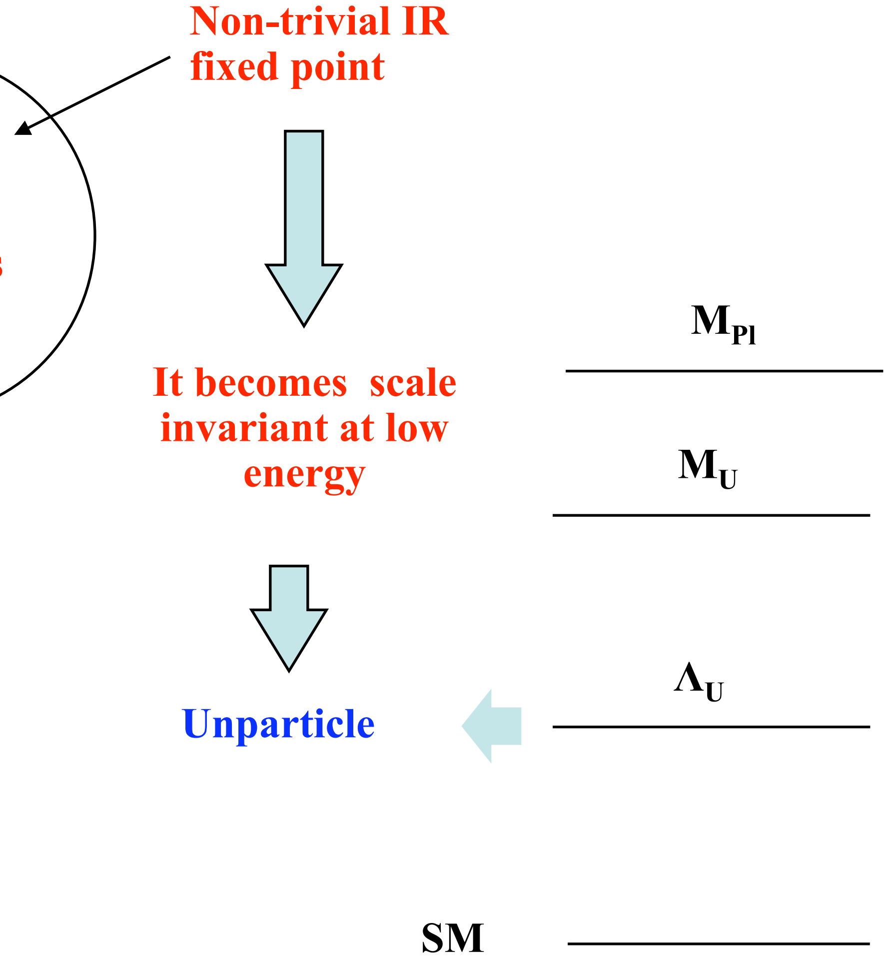
Effective field theory



Integrating out the heavy states, below the scale M_U

$$\frac{1}{M_U^{d_{UV}+n-4}} O_{UV}\, O_{SM}$$

Λ_U is usually assumed to be TeV scale



Unparticles IV

• Below Λ_U the hidden sector becomes scale invariant and operators O_{UV} mutate into an unparticle operator O_U with non-integer scaling dimension d_u. The coupling of the field operators can be generically written as

$$\frac{\Lambda_U^{d_{UV}-d_u}}{M_U^{d_{UV}+n-4}} O_U \, O_{SM}$$

• Operator O_U can be a scalar, a vector, a tensor or even a spinor

• It is shown that phase space $d\Phi(d_u)$ for an unparticle operator of dimension d_u is the same as the phase space for $n=d_u$ massless invisible particles. Thus, $d\Phi(d_u)$ is proportional to

$$A_{d_u} = \frac{16\pi^{5/2}}{(2\pi)^{2d_u}} \frac{\Gamma(d_u+\frac{1}{2})\Gamma(d_u-\frac{1}{2})}{\Gamma(2d_u)}$$

Unparticle with scale dimension d_u looks like d_u invisible massless particles

Georgi, PRL 2007, PLB 2007

Unparticles V

- Collider signatures and other phenomenological aspects have been investigated as well as astrophysical and cosmological constraints

  Cheung et al, PRL2007, PRD2007; Luo & Zhu, PLB 2008; Chen & Geng, PRD 2007; Ding & Yan, PRD 2007; Liao, PRD 2007; Aliev et al, PLB 2007, Li & Wei, PLB 2007, Lu et al, PRD 2007, Fox et al, PRD 2007; Greiner, PLB 2007, Chen & He, PRD 2007, Kikuchi and Okada, PLB 2008, Delgado et al, JHEP 2007, Anchordoqui & Goldberg, PLB 2008, Davoudiasl, PRL 2007, MCDonald, JCAP 2009, Hannestad et al, PRD 2007, Das, PRD 2007, Freitas & Wyler, JHEP 2007,…

- The exchange of unparticles can give rise to long range forces and hence deviations from the usual inverse square law (ISL) due to the anomalous scaling dimension of the unparticle propagator

  Liao & Liu, PRL 2007
  Deshpande et al, PLB 2008
  Goldberg & Nath, PRL 2008

- One investigates the deviations from the ISL due to tensor and vector particle exchange

Long range forces from tensor unparticles

- If O_U is a rank-two tensor it can couple to the stress-energy tensor $T^{\mu\nu}$

$$\frac{1}{M_\star \Lambda_U^{d_u-1}} \sqrt{g}\, T_{\mu\nu} O_U^{\mu\nu}$$

$$M_\star = \Lambda_U \left(\frac{M_U}{\Lambda_U}\right)^{d_{UV}}$$

$$G_N = 6.7\times 10^{-39}\ GeV$$

$$M_{Pl} = 1.22\times 10^{19}\ GeV$$

- This interaction generates the potential

$$V_u(r) = -G_N \frac{m_1 m_2}{r} \left(\frac{R_G}{r}\right)^{2d_u-2}$$

Goldberg & Nath, PRL 2008

where the characteristic length scale for which "ungravity" interactions become significant is defined to be

$$R_G = \frac{1}{\Lambda_U}\left(\frac{M_{Pl}}{M_\star}\right)^{\frac{1}{d_u-1}} C(d_u)^{\frac{1}{2d_u-2}} \qquad C(d_u) = \frac{2}{\pi^{2d_u-1}} \frac{\Gamma(d_u+\frac{1}{2})\Gamma(d_u-\frac{1}{2})}{\Gamma(2d_u)}$$

Long range forces from vector unparticles

- If one considers the coupling between a vector unparticle and a baryonic (or leptonic) current

$$\frac{\lambda}{\Lambda_U^{d_u-1}} J_\mu O_U^\mu$$

- Then

$$V_u(r) = \frac{\lambda^2 N_1 N_2 \tilde{C}(d_u)}{\Lambda_U^{2d_u-2}} \frac{1}{r^{2d_u-1}}$$

Deshpande et al, PLB 2008

where $N_{1,2}$ are the total number of baryons (leptons) of the two interacting objects and

$$\tilde{C}(d_u) = \frac{1}{2\pi^{2d_u}} \frac{\Gamma(d_u + \frac{1}{2})\Gamma(d_u - \frac{1}{2})}{\Gamma(2d_u)}$$

Long range forces from vector unparticles

- Combining with the gravitational potential one can write

$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 - \left(\frac{\tilde{R}_G}{r}\right)^{2d_u - 2}\right]$$

with

$$\tilde{R}_G = \frac{1}{\Lambda_U}\left(\frac{\lambda M_{Pl}}{\Lambda_U}\right)^{\frac{1}{d_u - 1}} \tilde{C}(d_u)^{\frac{1}{2d_u - 2}}$$

- In both cases one has a potential of the same form, the vector one being repulsive, the tensor one attractive:

$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 \pm \left(\frac{R_G}{r}\right)^{2d_u - 2}\right]$$

Varying G and Big Bang Nucleosynthesis (BBN)

- The production of light elements (^{2}H, ^{3}He, ^{4}He and ^{7}Li) in BBN is the result of the efficiency of the weak interaction reactions ($p + e \leftrightarrow n + \nu$ and related processes) and nuclear reactions which build light nuclei from neutrons and protons in the expanding Universe

- The value of the gravitational coupling determines the expansion rate of the Universe and thus the relevant time scales for the production of light elements

- Thus, if one assumes that the gravitational coupling at the time of BBN is different from its present value, then the light element abundances will be different with respect to the standard BBN predictions

- BBN is a good probe of a putative variation of G, since is a fairly early event in the history of the Universe

- Although observations agree fairly well with the standard BBN scenario, there is still some room for the variation of the gravitational coupling

- Given the large statistical and systematic errors, typical constraints on the variation of G are down to a few percent

$$-0.036 \leq \frac{\Delta G}{G} \equiv \left[\frac{G(r) - G_N}{G_N}\right] \leq 0.086$$

at 95% C.L.

Iocco et al, Phys. Rep. 2009

Varying G, BBN and unparticles

- One investigates the limits on different energy scales that can be derived using the bounds on the variation of the gravitational coupling *G*

$$V(r) = -G_N \frac{m_1 m_2}{r}\left[1 \pm \left(\frac{R_G}{r}\right)^{2d_u - 2}\right] \quad \Rightarrow \quad \left|\frac{\Delta G}{G}\right| = (2d_u - 1)\left(\frac{R_G}{r}\right)^{2d_u - 2}$$

- First one needs to estimate a typical distance **r** between particles interacting through the new interaction at the time of BBN

- The typical distance between the interacting particles should be smaller or of the same order of magnitude as their mean free paths λ_p and λ_n

- It turns out that during this epoch neutrons and protons have mean free paths of the same order of magnitude

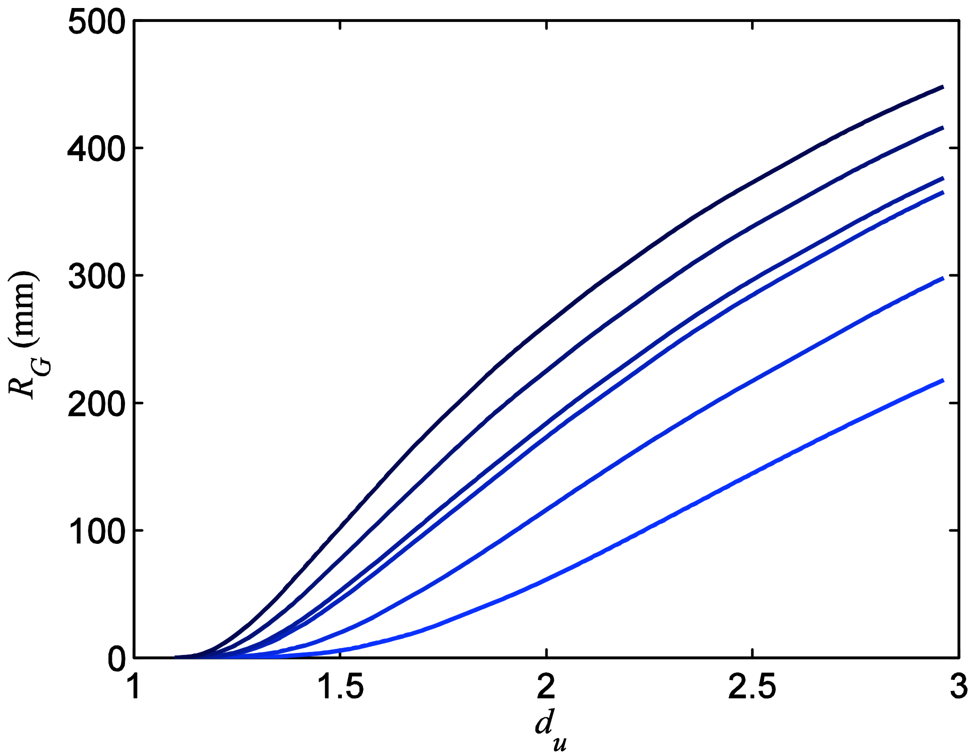
$$\boxed{\lambda_p \sim \lambda_n \sim \lambda = 1\,m}$$

O.B., F. Nunes, PLB 1999

Applegate et al, PRD 1987

Varying G, BBN and unparticles

Upper bound on R_G as function of d_u



$$\left|\frac{\Delta G}{G}\right| \leq 0.2,\ 0.15,\ 0.1,\ 0.086,\ 0.036,\ 0.01$$

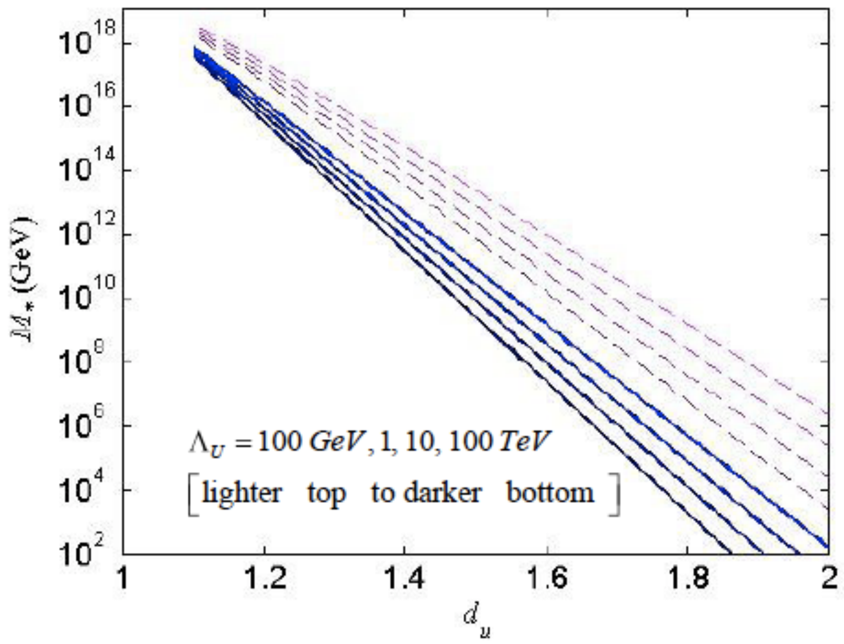
(from top to bottom)

$$\left|\frac{\Delta G}{G}\right| = (2d_u - 1)\left(\frac{R_G}{r}\right)^{2d_u-2}$$

$d_u, \Lambda_U, M_\star$ $\qquad$ d_u, Λ_U, λ

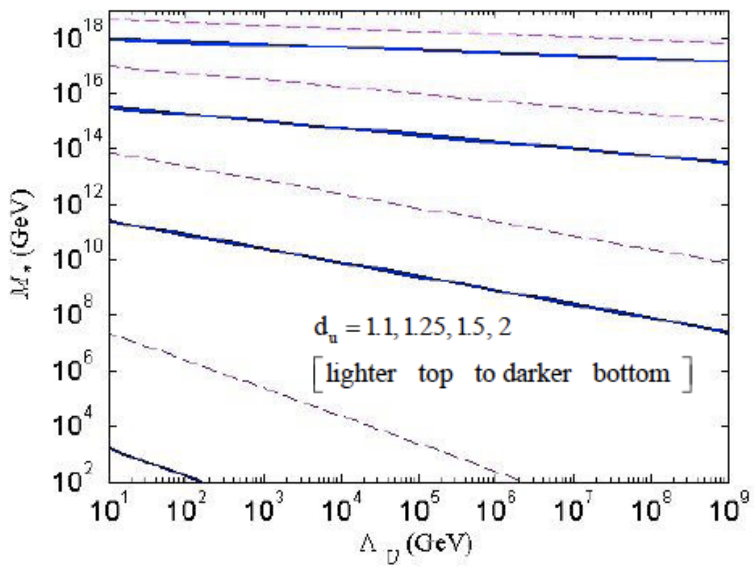
O.B., N. Santos, PRD 2009

Constraints on tensor unparticles



$$\frac{\Delta G}{G} = (2d_u - 1)\left(\frac{R_G}{r}\right)^{2d_u-2} \leq 0.086$$

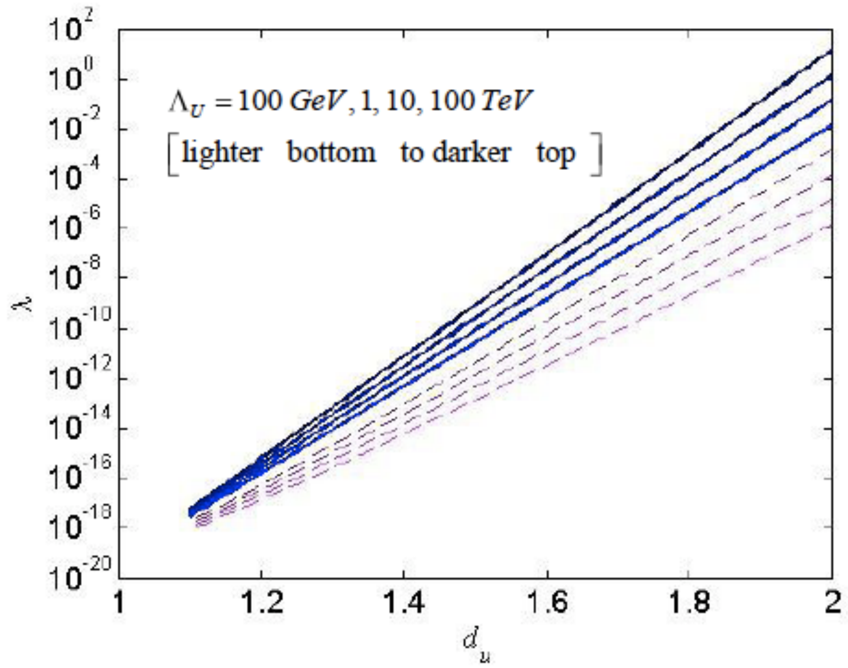
$$R_G = \frac{1}{\Lambda_U}\left(\frac{M_{Pl}}{M_\star}\right)^{\frac{1}{d_u-1}} C(d_u)^{\frac{1}{2d_u-2}}$$



Allowed region (above the curves):
-- BBN bounds (solid lines)
-- ISL violation data (dashed lines)

O.B., N. Santos, PRD 2009

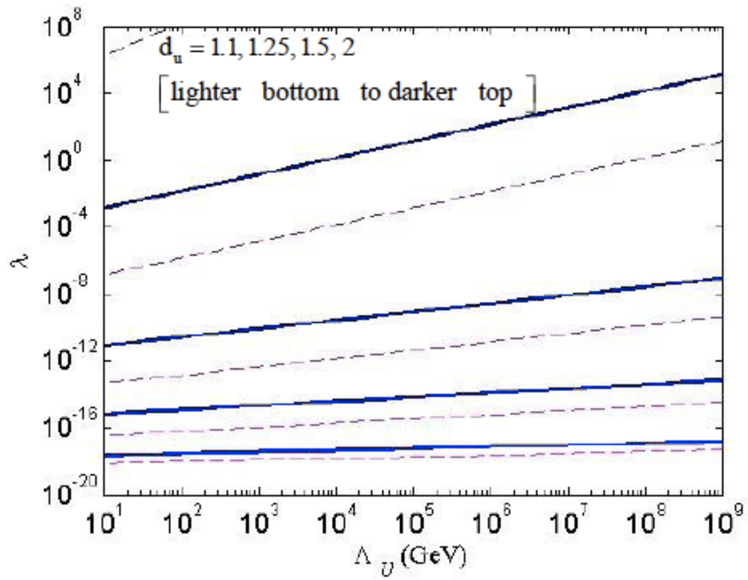
Constraints on vector unparticles



$$\frac{\Delta G}{G} = -(2d_u - 1)\left(\frac{\tilde{R}_G}{r}\right)^{2d_u - 2} \geq -0.036$$

$$\tilde{R}_G = \frac{1}{\Lambda_U}\left(\frac{\lambda M_{Pl}}{u}\right)^{\frac{1}{d_u - 1}} \tilde{C}(d_u)^{\frac{1}{2d_u - 2}}$$

Allowed region (below the curves) :
-- BBN bounds (solid lines)
-- ISL violation data (dashed lines)



O.B., N. Santos, PRD 2009

Constraints on unparticles

- One finds that the BBN bounds are less stringent than the laboratory ones that search for violations of the ISL

- However, for d_u close to unity, the BBN bounds are comparable. The difference between BBN and laboratory bounds becomes more visible for larger values of d_u

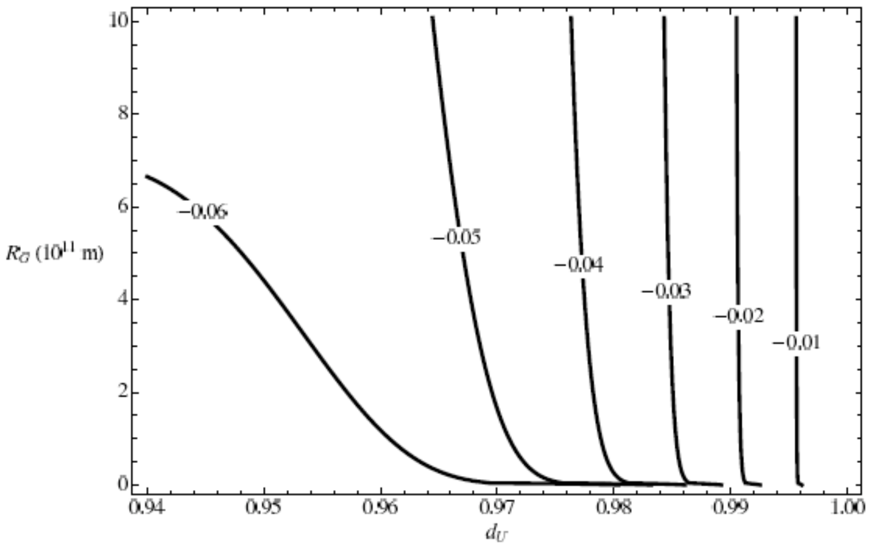
	d_u	1.1	2
BBN	M_* (GeV)	$\geq 6.04 \times 10^{17}$	≥ 15.9
	λ	$\leq 3.54 \times 10^{-18}$	$\leq 1.34 \times 10^{-1}$
Lab	M_* (GeV)	$\geq 2.83 \times 10^{18}$	$\geq 2.36 \times 10^{5}$
	λ	$\leq 1.17 \times 10^{-18}$	$\leq 1.40 \times 10^{-5}$

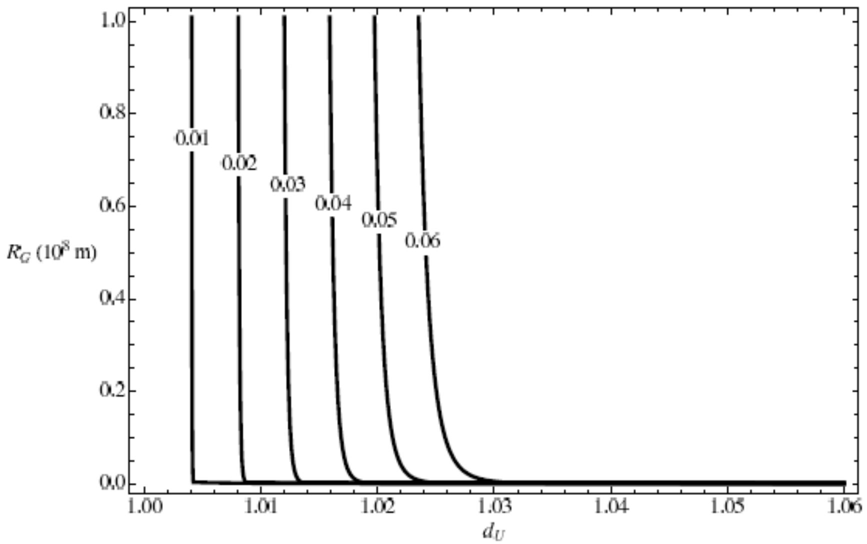
Bounds for $\Lambda_U = 1$ TeV

O.B., N. Santos, PRD 2009

Stellar stability constraints (ungravity inspired)

- Stellar Stability: effect on Sun's central temperature ($\Delta T/T$)





O.B., Páramos, P. Santos, PRD 2009

Constraints from Galileo Navigation Satellite System

- **Relative frequency shift (Fig. 1) and Propagation time delay (Fig. 2)**

$$V_P = \frac{GM_E}{r}\left(\frac{R}{r}\right)^n$$

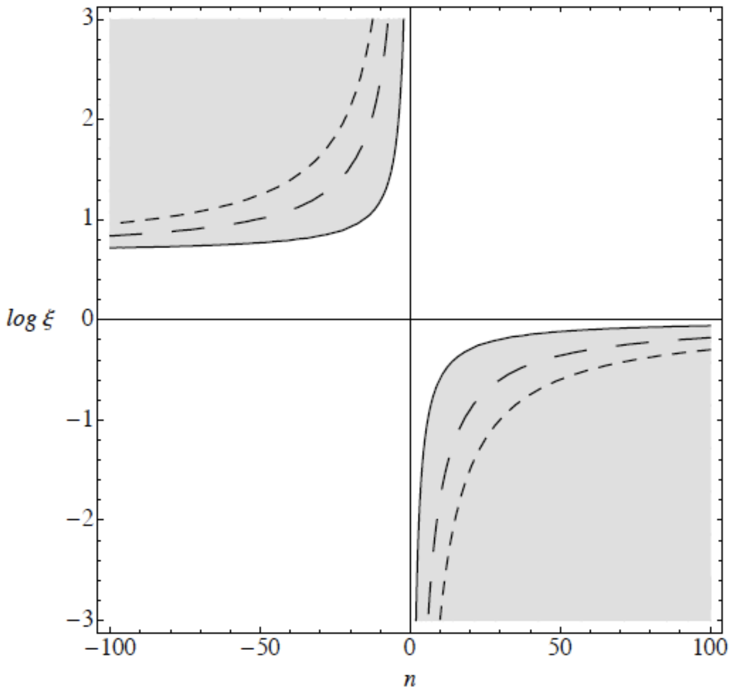


Figure 1. Contour plot for the relative frequency deviation ϵ as a function of $\xi = R/R_E$ and n, with contours for $\epsilon = 10^{-12}$ (solid line), 10^{-24} (long dash) and 10^{-36} (short dash), and allowed region grayed out.

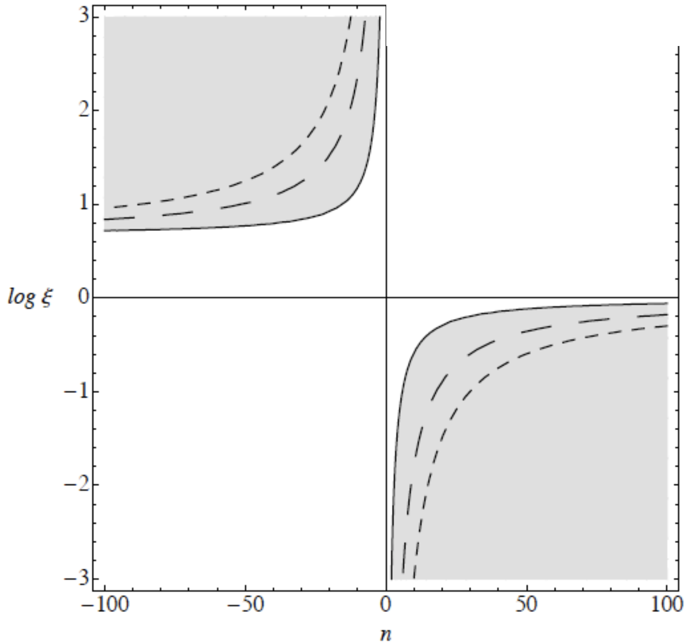


Figure 2. Contour plot for the propagation time delay Δt_P as a function of $\xi = R/R_E$ and n, with contours for $\Delta t_P = 10^{-9}$ s (solid line), 10^{-12} s (long dash) and 10^{-15} s (short dash), and allowed region grayed out.

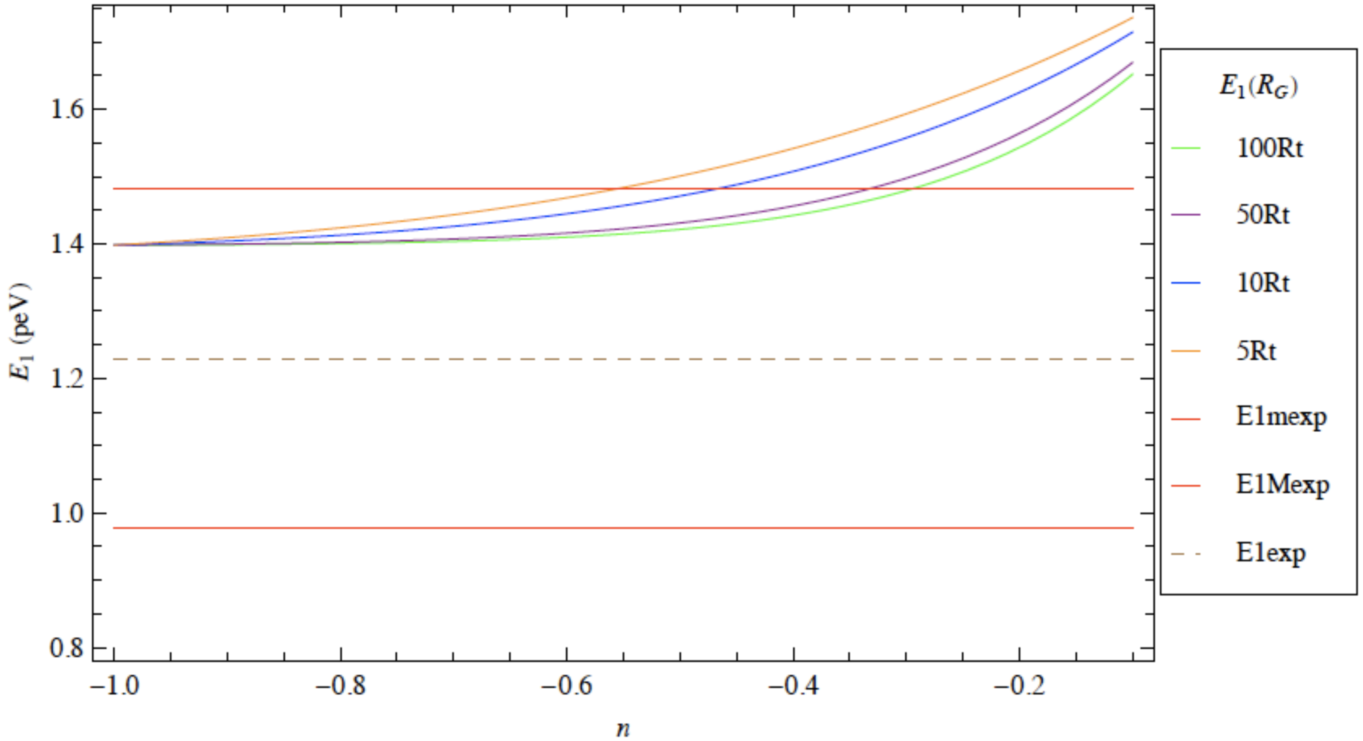
Páramos, O.B., 2009

Constraints from GRANIT I

- Effect on the first energy level of the GQW for tensor exchange (ungravity inspired):

$$V[r]=-\frac{G\,M}{2\,r}\,[1+[\frac{R_G}{r}]^n]$$

r=Rt



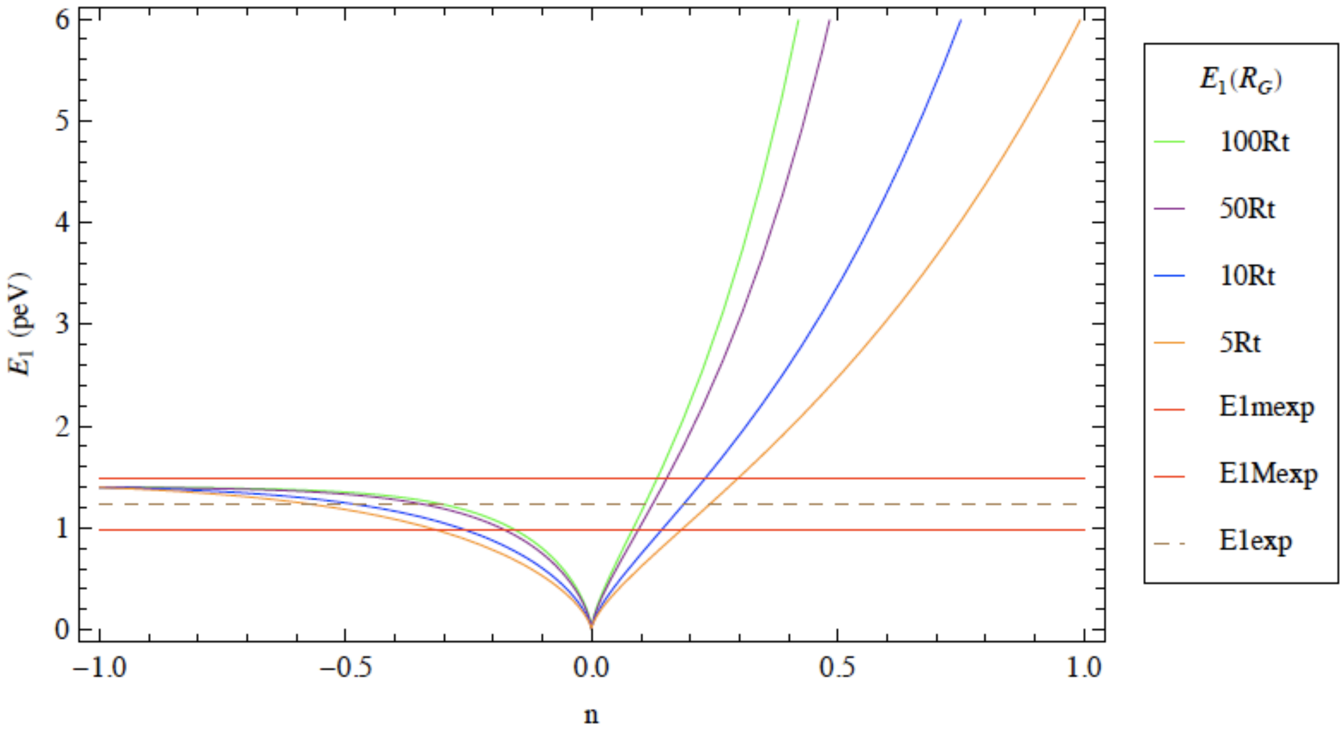
Alves, O.B., 2010

Constraints from GRANIT II

• Effect on the first energy level of the GQW for vector exchange (unparticle inspired):

$$V[r] = -\frac{GM}{r}\left[1 - \left[\frac{R_G}{r}\right]^n\right]$$

r=Rt



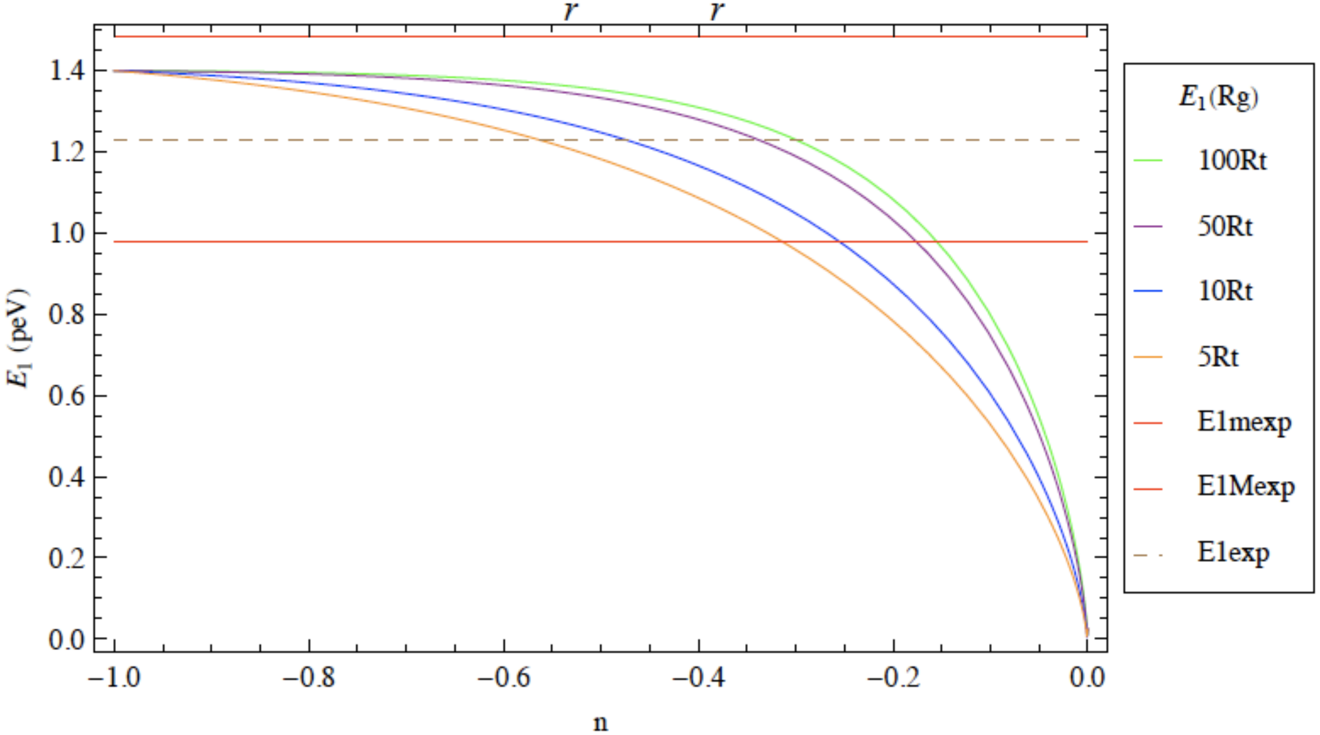
Alves, O.B., 2010

Constraints from GRANIT III

- **Effect on the first energy level of the GQW for vector exchange (unparticle inspired):**

$$V[r] = -\frac{GM}{r}\left[1 - \left[\frac{R_G}{r}\right]^n\right]$$

r=Rt



Alves, O.B., 2010

Other power-law corrections to ISL

- **Power-law corrections to ISL with integers exponents can be encountered in extra dimensional models. GRANIT bounds for these corrections have been previously studied and are complementary to ours (sub-mm range)**

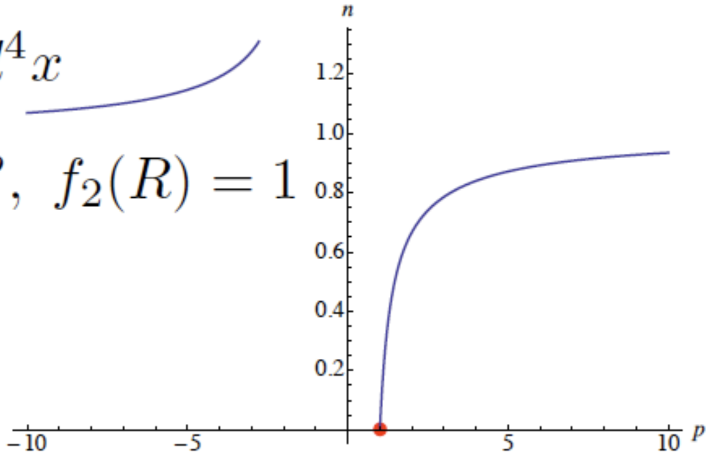
Buisseret, Silvestre-Brac, Mathieu, CQG 2007

- **Similar features can be found in generalized f(R) theories for galactic distances (kpc)**

O.B., Boehmer, Harko, Lobo, PRD 2007

$$S = \int [f_1(R) + f_2(R)\mathcal{L}] \sqrt{-g}\, d^4x$$

- purely gravitational: $f_1(R) \propto R^p, \;\; f_2(R) = 1$
- R_n integration constant, $n = n(p)$
- LSB best fit: $n = 0.817$ ($p = 3.5$)



Cappozzielo, Cardone, Troisi, MNRAS 2007

- Non-minimal matter-geometry coupling $f_2(R) = 1 + \lambda R^p$
  - Yields "dark" component $\rho_{dm} \propto \rho^{1/(1-p)}$

O.B., Páramos, JCAP 2010

Conclusions

- The GQW is a quite interesting tool for searching new physics and for testing the interplay between Quantum Mechanics (QM) and gravity

- NC geometry gives raise to interesting theoretical and phenomenological implications in field theory as well as in QM

- NCQGW is consistent with QM and it leads to a $0(10^{-24})$ correction to Planck´s constant that is universal and Lorentz invariant

- NCQM leads one to generalize NC features to Quantum Cosmology and to Black Holes. These approaches provide quite interesting insights on the initial conditions for the early universe and on the black hole singularity problem

- GRANIT may lead to interesting bounds for unparticle inspired corrections to Newtonian gravity. Improving the determination of the energy levels of GQW does directly constrain corrections to the gravitational coupling