Magnetic field-induced transitions between quantum states

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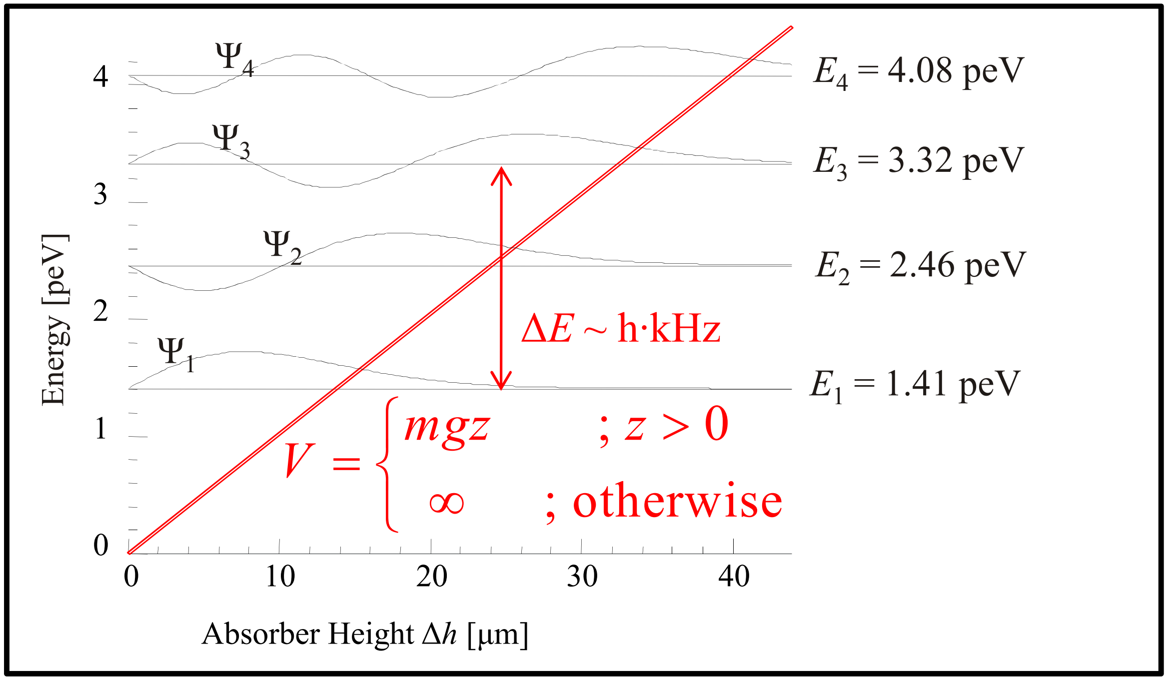
Advantage of spectroscopy



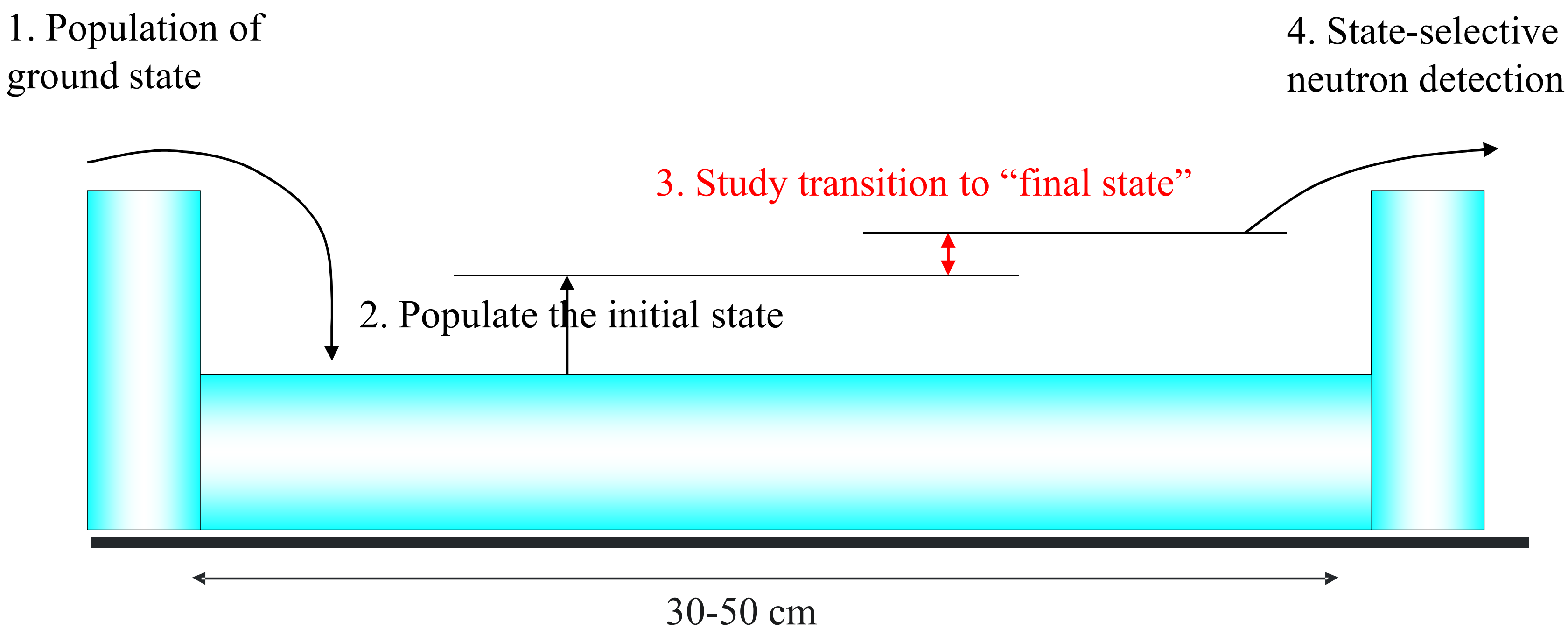
Accuracy of position-type observables: ~ 10%

Improvement: do **spectroscopy**

Induce state transitions through:

- **Oscillating magnetic field gradients**
- Vibrations → Konstantin Protassov's talk
- Oscillating Masses

The GRANIT spectrometer in storage mode



Challenge: Tolerances to get a high neutron lifetime in a given state:

$$\Delta E \cdot t_{\text{transition}} \approx \hbar$$

→ Guillaume Pignol’s talk

Resonance transitions = Rabi oscillations

Schroedinger equation:

$$H = \underbrace{\frac{\hbar^2}{2m_{\mathrm{n}}}\frac{\partial^2}{\partial z^2} + mgz}_{H_0,\ \text{defines } \left|N\right\rangle} + V(z)\cdot\cos\omega t$$

Ansatz:

$$\left|\psi(t)\right\rangle = \sum_{N=1} a_N(t)\mathrm{e}^{-i\omega_N t}\left|N\right\rangle \quad \text{with } H_0\left|N\right\rangle = \hbar\omega_N\left|N\right\rangle$$

Insert in Schroedinger equation, restrict to 2 states “1” and “3”:

$$\hbar\Omega_{13} = \left\langle 1\right|V(z)\left|3\right\rangle \qquad \omega_{13} = \omega_3 - \omega_1 \qquad \hbar\Omega_{11} = \left\langle 1\right|V(z)\left|1\right\rangle$$

$$i\hbar\frac{da_3(t)}{dt} = \Omega_{13}a_1(t)\mathrm{e}^{-i\omega_{13}t}\cos\omega t + \Omega_{33}a_3(t)\cos\omega t$$

$$\hbar\Omega_{33} = \left\langle 3\right|V(z)\left|3\right\rangle$$

$$i\hbar\frac{da_1(t)}{dt} = \Omega_{13}a_3(t)\mathrm{e}^{-i\omega_{13}t}\cos\omega t + \Omega_{11}a_1(t)\cos\omega t$$

Neglect fast self coupling and fast oscillating terms:

Rabi formula:

$$\boxed{P_{3\to 1}\left(\tau_{\text{passage}}\right) = \Omega_{13}^{\ 2}\frac{\sin^2\left(\sqrt{\left(\omega-\omega_{13}\right)^2 + \Omega_{13}^{\ 2}}\,\frac{\tau_{\text{passage}}}{2}\right)}{\left(\omega-\omega_{13}\right)^2 + \Omega_{13}^{\ 2}}}$$

Transition probability at maximum for $\Omega_{13}\cdot\tau_{\text{passage}} = \pi$

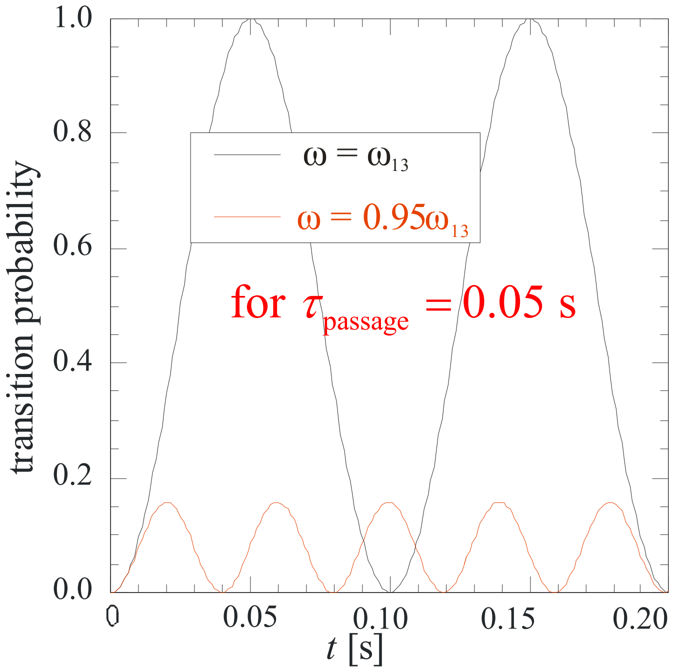
Resonance transitions = Rabi oscillations

Rabi formula:

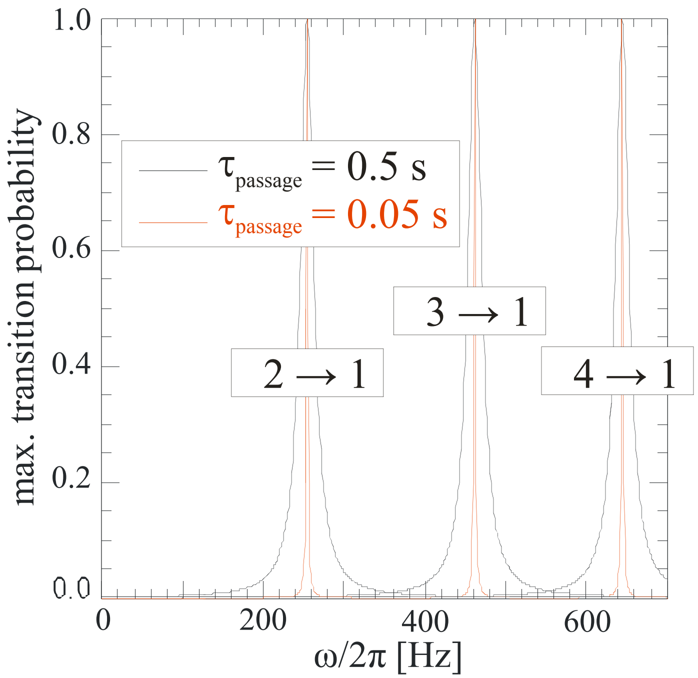
$$\boxed{P_{3\to 1}\left(\tau_{\text{passage}}\right) = \Omega_{13}^{\ 2}\,\frac{\sin^2\left(\sqrt{\left(\omega-\omega_{13}\right)^2+\Omega_{13}^{\ 2}}\,\dfrac{\tau_{\text{passage}}}{2}\right)}{\left(\omega-\omega_{13}\right)^2+\Omega_{13}^{\ 2}}}$$

Transition probability at maximum for $\Omega_{13}\cdot\tau_{\text{passage}} = \pi$

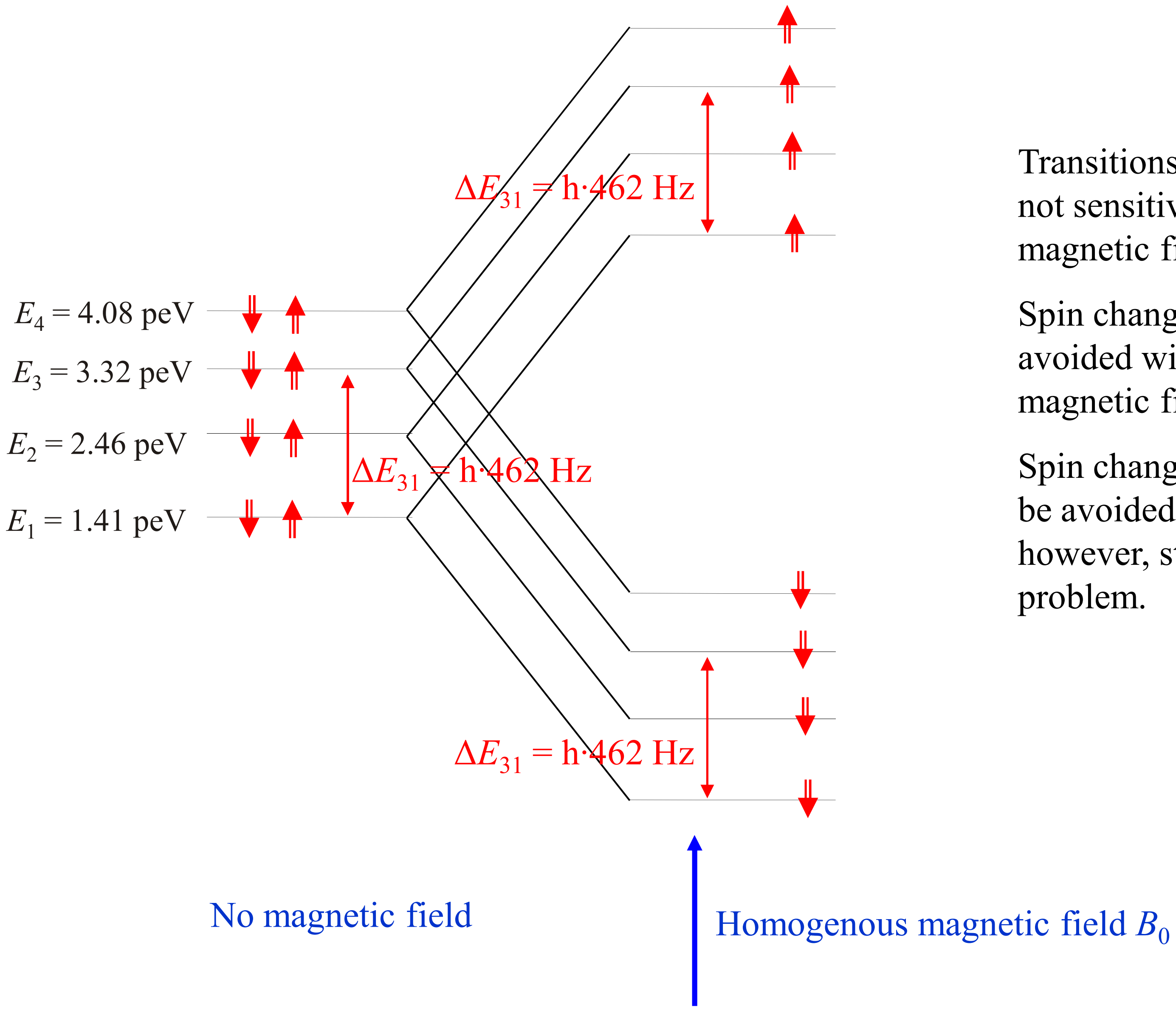
“Rabi oscillations”



Resonance behavior



Selection of transitions



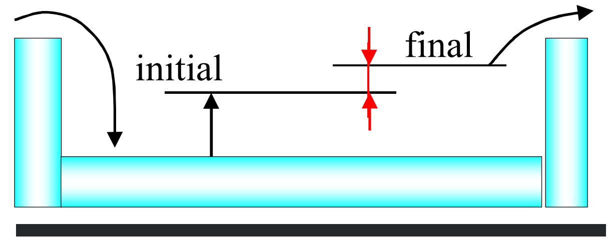
Transitions w/o spin change are not sensitive to (constant) magnetic field.

Spin change-transitions can be avoided with a homogeneous magnetic field.

Spin change-transitions can also be avoided w/o magnetic field, however, stray fields are a problem.

Properties of the magnetic field

$$V(z)\cdot\cos\omega t = \vec{\mu}_{\mathrm{n}}\cdot\vec{B}(x,y,z)\cdot\cos\omega t$$



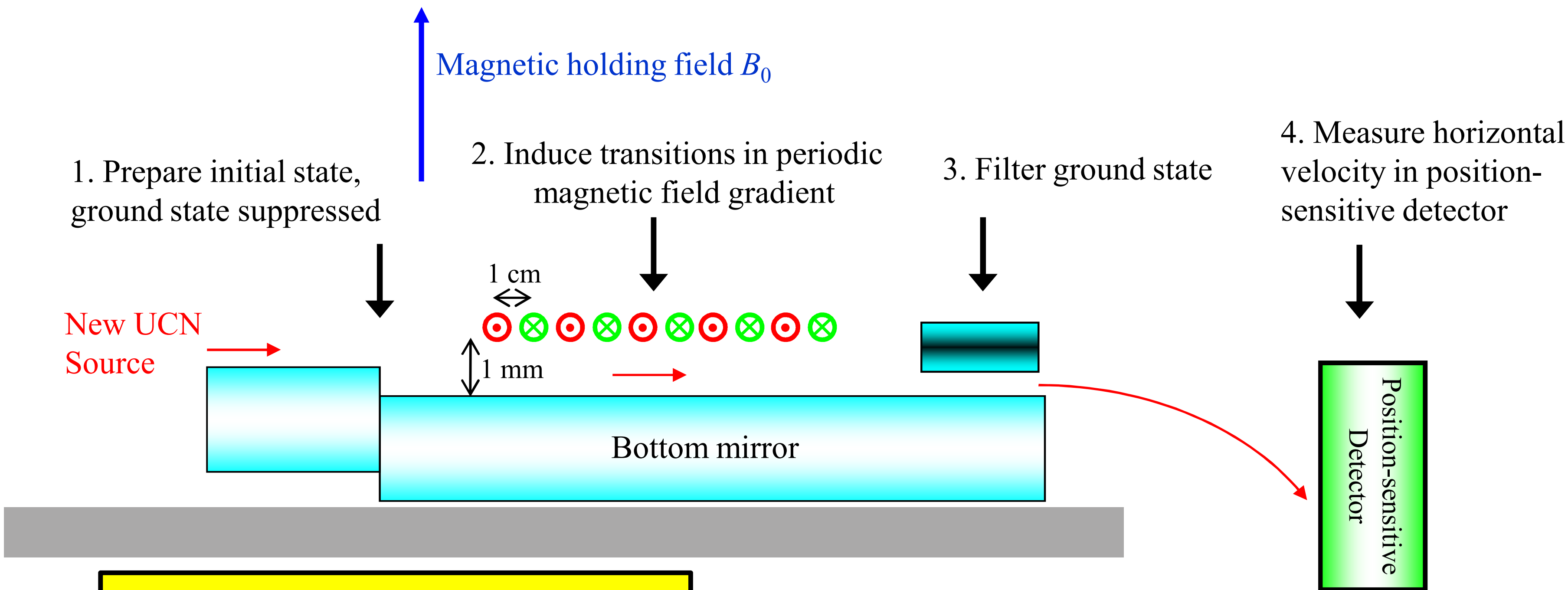
Assuming adiabaticity, the magnetic moment will stay aligned with the magnetic field

$$\vec{\mu}_{\mathrm{n}}\cdot\vec{B}(x,y,z) = \cancel{\vec{\mu}_{\mathrm{n}}\cdot\vec{B}(x_0,y_0,z_0)} + \begin{pmatrix} \cancel{\mu_{\mathrm{n,x}}\cdot\frac{\partial B_{\mathrm{x}}}{\partial x}} & \cancel{\mu_{\mathrm{n,x}}\cdot\frac{\partial B_{\mathrm{x}}}{\partial y}} & \cancel{\mu_{\mathrm{n,x}}\cdot\frac{\partial B_{\mathrm{x}}}{\partial z}} \\ \cancel{\mu_{\mathrm{n,y}}\cdot\frac{\partial B_{\mathrm{y}}}{\partial x}} & \cancel{\mu_{\mathrm{n,y}}\cdot\frac{\partial B_{\mathrm{y}}}{\partial y}} & \cancel{\mu_{\mathrm{n,y}}\cdot\frac{\partial B_{\mathrm{y}}}{\partial z}} \\ \cancel{\mu_{\mathrm{n,z}}\cdot\frac{\partial B_{\mathrm{z}}}{\partial x}} & \cancel{\mu_{\mathrm{n,z}}\cdot\frac{\partial B_{\mathrm{z}}}{\partial y}} & \mu_{\mathrm{n,z}}\cdot\frac{\partial B_{\mathrm{z}}}{\partial z} \end{pmatrix}\cdot\begin{pmatrix} x-x_0 \\ y-y_0 \\ z-z_0 \end{pmatrix}$$

1. $\vec{B}(x_0,y_0,z_0)$ doesn't contribute, as $\langle n|\vec{\mu}_{\mathrm{n}}\cdot\vec{B}(x_0,y_0,z_0)|m\rangle = 0$ for $n \neq m$
2. No magnetic field or gradient in y direction
3. $\frac{\partial B_{\mathrm{x}}}{\partial x}(x-x_0)$, $\frac{\partial B_{\mathrm{z}}}{\partial x}(x-x_0)$ doesn't contribute, as $\langle n|\mu_{\mathrm{n,x}}\frac{\partial B_{\mathrm{x}}}{\partial x}(x-x_0)|m\rangle = 0$, $\langle n|\mu_{\mathrm{n,z}}\frac{\partial B_{\mathrm{z}}}{\partial x}(x-x_0)|m\rangle = 0$ for $n \neq m$
4. $\langle n|\mu_{\mathrm{n,x}}\frac{\partial B_{\mathrm{x}}}{\partial z}(z-z_0)|m\rangle$ couples states with opposite spin. Thanks to the holding field B_0, the energy change is big, and the resonance frequency is high

$\mu_{\mathrm{n,z}}\cdot\frac{\partial B_{\mathrm{z}}}{\partial z}(x_0,y_0,z_0)$ is the only first order transition operator

Projected first measurement of resonance transitions



Apparatus → F. Vezzu's talk

1. Prepare initial state, ground state suppressed
2. Induce Transitions in time-dependent magnetic field gradient
3. Filter ground state
4. Detect neutrons in dependence of free fall height

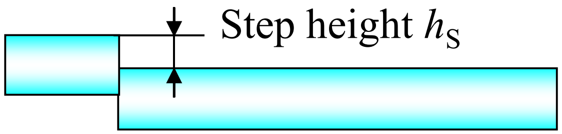
   (corresponding to horizontal velocity, corresponding to oscillation frequency)

G. Pignol et al., Thesis UJF Grenoble, 2009

Initial state preparation

Before step: $\psi(z-h_s)=\sum_n c_n\psi_n(z-h_s)$ for $z>h_s$

After step: $\psi(z)=\sum_N \underbrace{\sum_n c_n \int_{h_s} \psi_N(z')\psi_n(z'-h_s)dz'}_{b_N} \cdot \psi_N(z)$

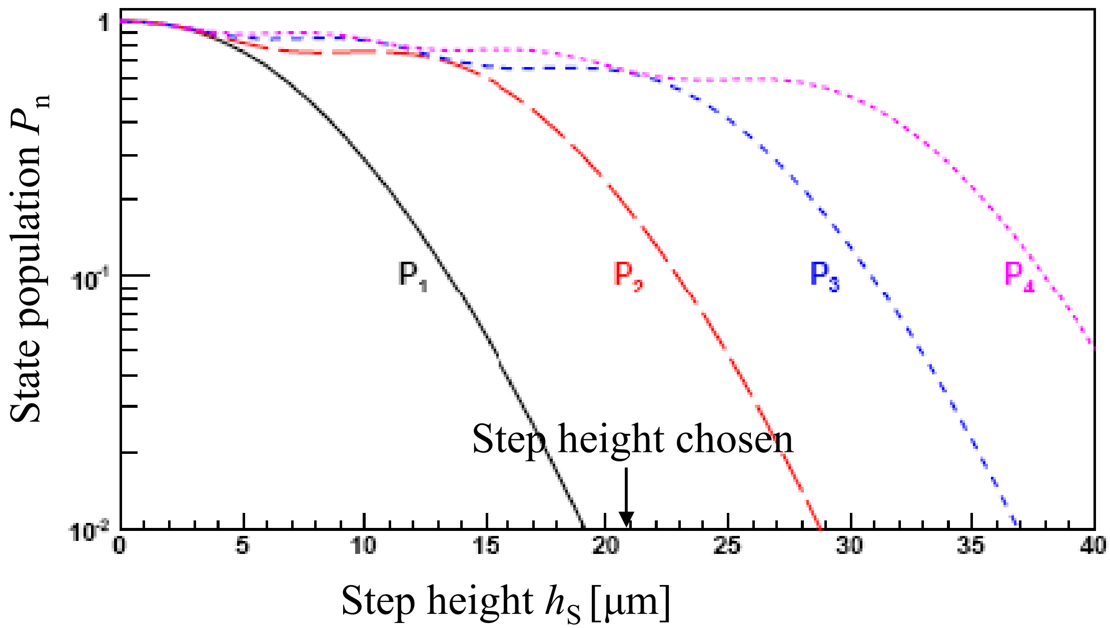


Population of state N: $|b_N|^2$

Here, the initial state (before the step) is described by the random (but normalized) variables c_n

Population of state N is proportional to:

$$P_N=\sum_{n=1}^{10}\left|\int_{h_s}\psi_N(z')\psi_n(z'-h_s)dz'\right|^2$$



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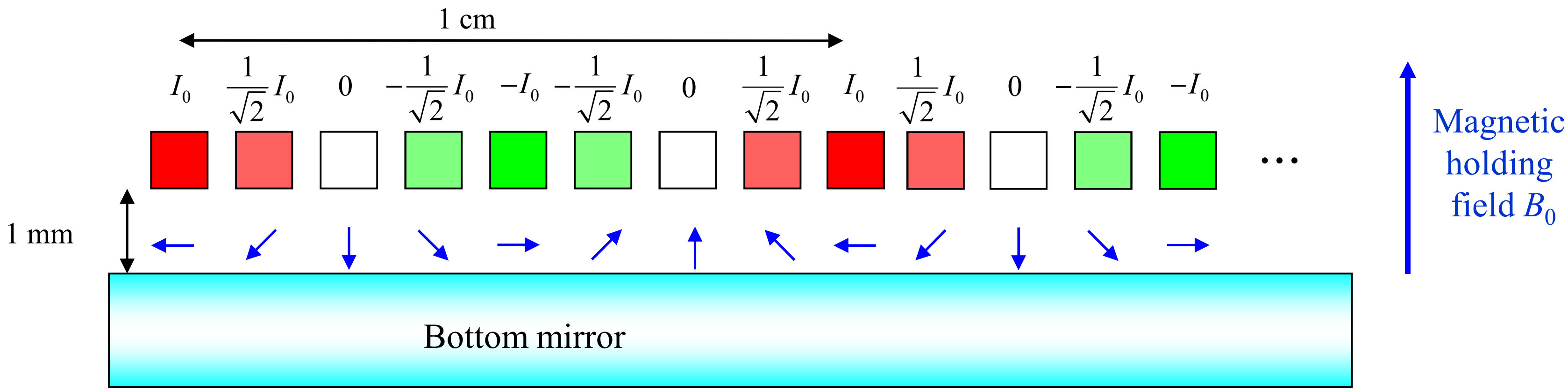
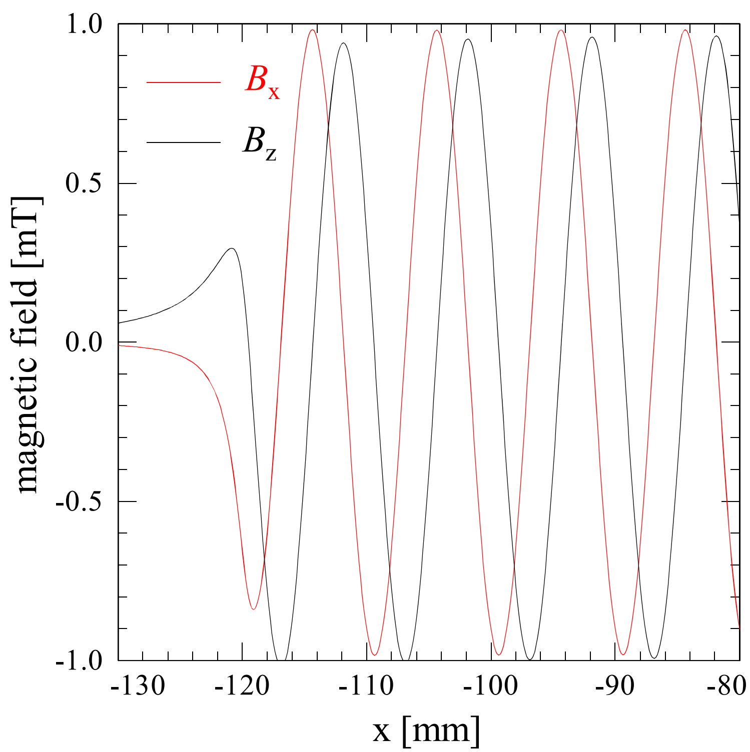
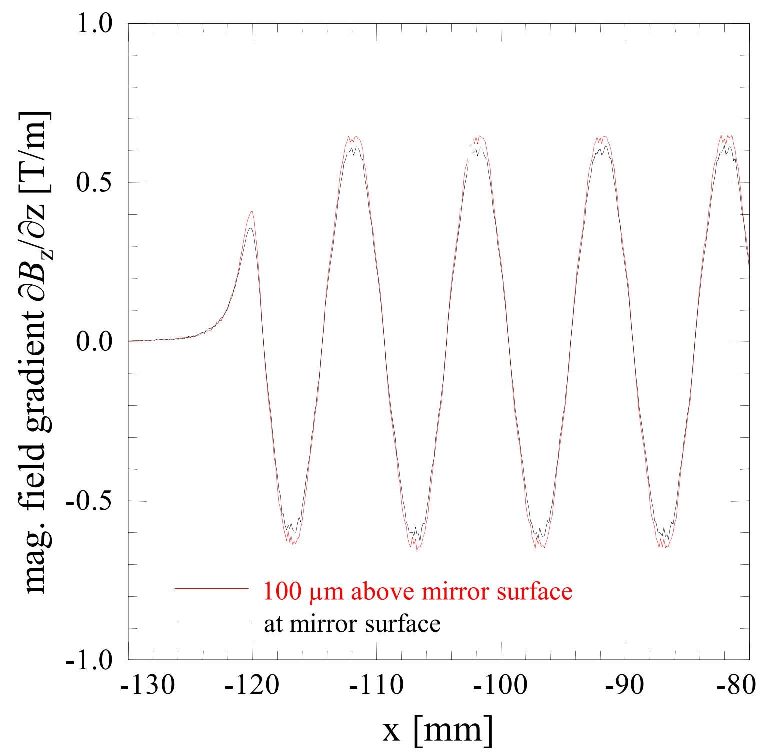
Fluctuating magnetic field







High frequency terms and self coupling

$$i\hbar \frac{da_3(t)}{dt} = \Omega_{13} a_1(t) \mathrm{e}^{-i\omega_{13}t} \cos \omega t + \Omega_{33} a_3(t) \cos \omega t$$

$$i\hbar \frac{da_1(t)}{dt} = \Omega_{13} a_3(t) \mathrm{e}^{-i\omega_{13}t} \cos \omega t + \Omega_{11} a_1(t) \cos \omega t$$

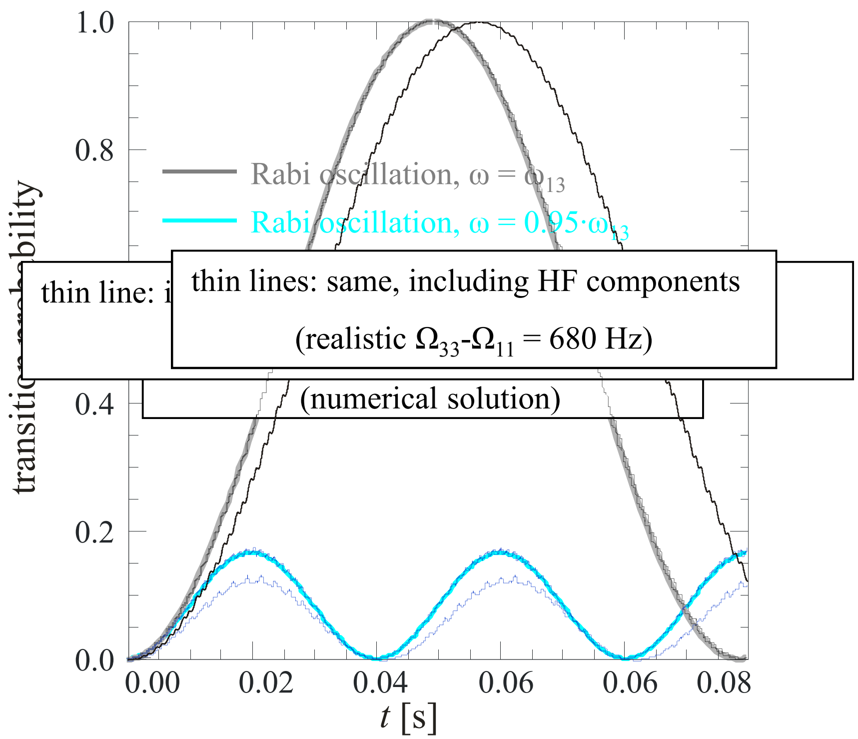
Notes:

1. We have $\omega_{13} \gg \Omega_{13}$, but the ratio
 is 1 order of magnitude,
 and not 9 as in atomic physics

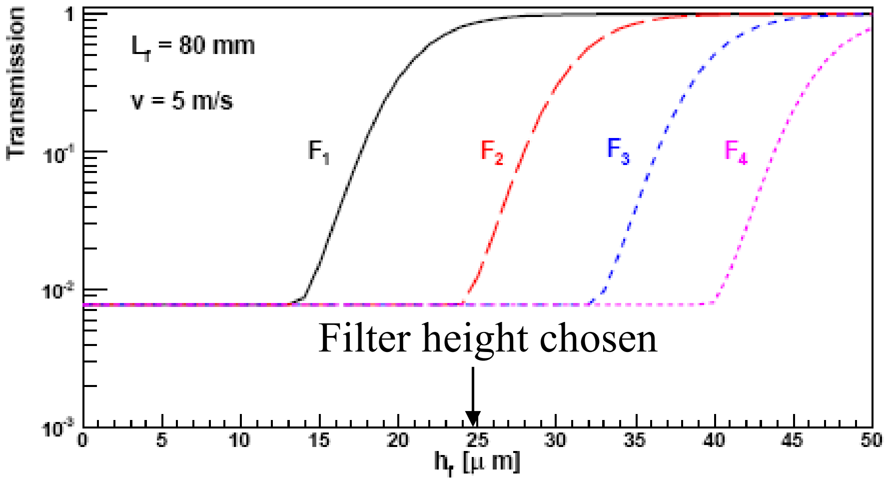
2. Self-coupling exists, is non-trivial if

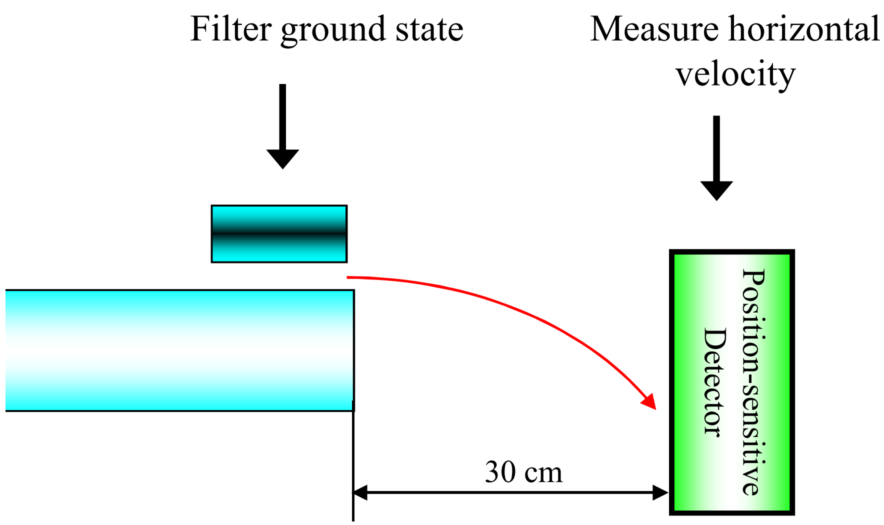
$$\Omega_{33} - \Omega_{11} \neq 0$$

$$\langle 3 | \mu_{\mathrm{n,z}} \frac{\partial B_z}{\partial z} z | 3 \rangle - \langle 1 | \mu_{\mathrm{n,z}} \frac{\partial B_z}{\partial z} z | 1 \rangle \neq 0$$



Velocity-selective detection





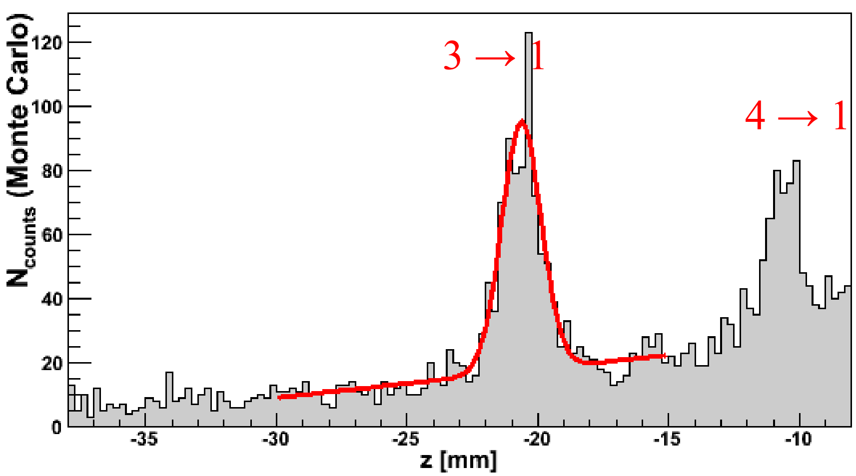
Detector → A. Muzychka's talk

Monte-Carlo-Simulation, takes into account:

- 10000 incoming neutrons per incoming state
- Background: 10% in ground state
- Filter: 80 mm long, slit height: 25 μm
- Wave function evolves in free fall
- Detector resolution: 0.2 mm

Other systematic uncertainties:

Geometry, Stern-Gerlach effect, Adiabaticity, Interference effects



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Summary

• The detection of magnetic field-induced transitions allows for spectroscopy of gravitationally bound quantum states

• A first, “proof of principle” experiment is done in flow-through mode. To be done: Size of B_0, 3D picture, interferences

• The accuracy of the flow-through experiment is planned to be several percent, compared to about 10% for previous transmission experiments.

• Ultimately, in storage mode, $\Delta E/E \sim 10^{-6}$ @ $E \sim peV$ might be reached.

Thank you for your interest!