A new concept to extract and collimate UCN without loosing phase-space density

A. Mietke[1,2], S. Baessler[2], V. Nesvizhevsky[3]

[1] Technical University Dresden, [2] University of Virginia, [3] Institut Laue-Langevin

February 16, 2010

Outline

1. Introduction of two collimating systems
2. Simulationdetails and -data
3. Alternative application: Neutron reflectometry

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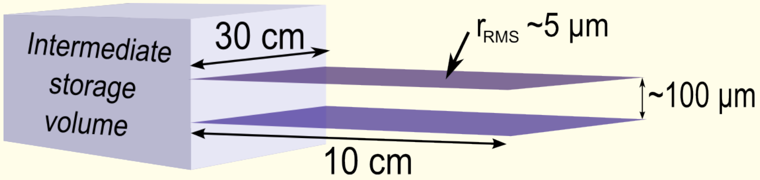

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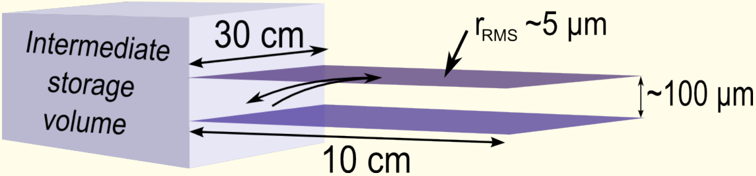
Neutrons:

- propagate on classical trajectories
- can be diffusely reflected, specularly reflected or absorbed
- are accepted at the slit if: $E_{\perp} < m_n g h_{slit} \approx 10\ peV$

1. The present collimation system - Setup



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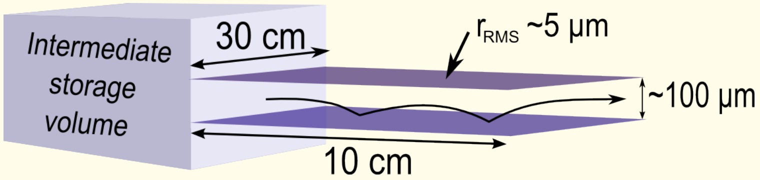

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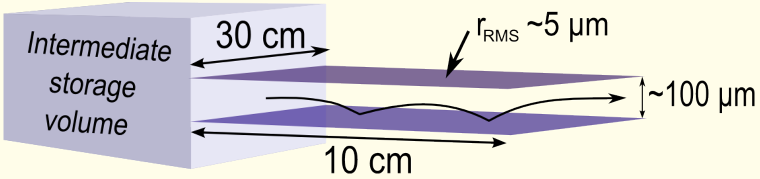

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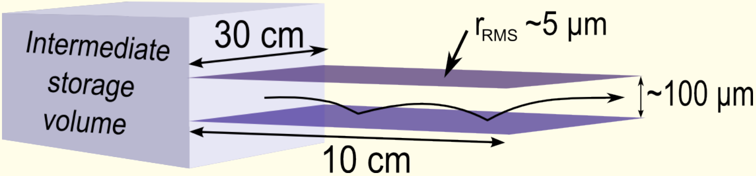

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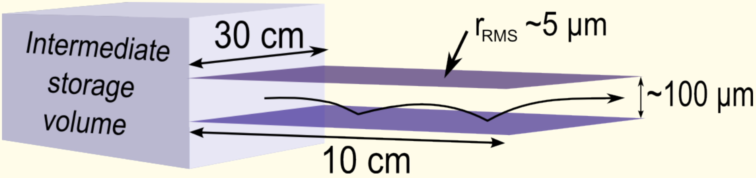

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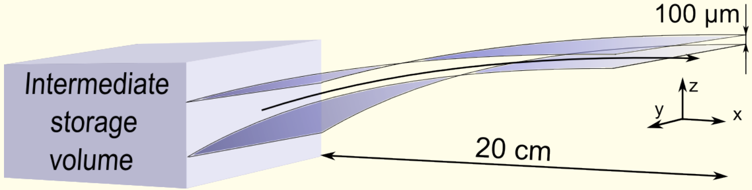

Collimator with parabola-shaped edges - the slope corresponds to $v_n \sim 7\ \frac{m}{s}$

Now we adapt the collimating system for the neutron trajectory and expect the following improvements:

- Higher rate of reflections back (P_R) into the storage volume
  $\Rightarrow$ A larger aperture, that still provides a proper phase space density of the UCN, becomes possible
- $P_{diff} \propto v_\perp^2$ is essential for the effectiveness of the system
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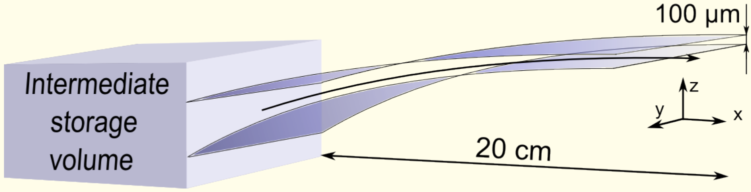

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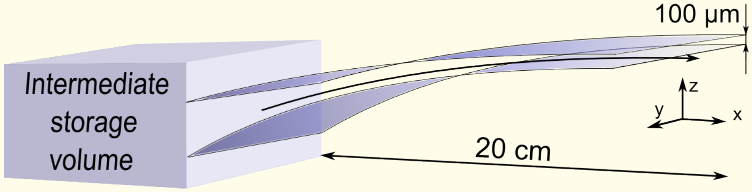

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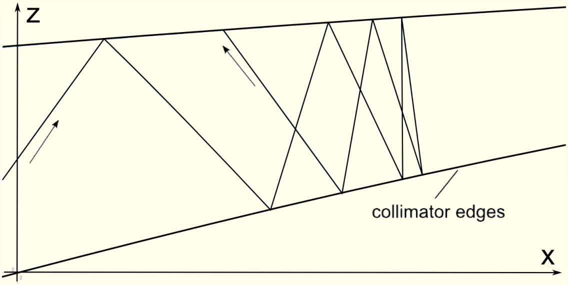


Top:

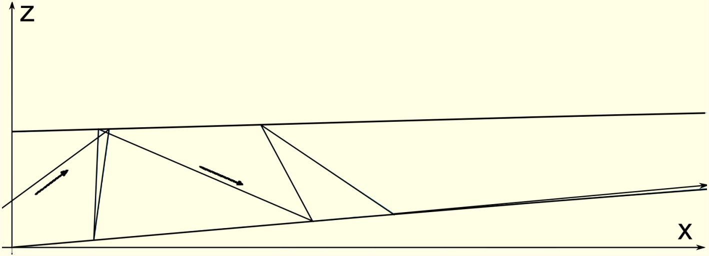
Trajectory at

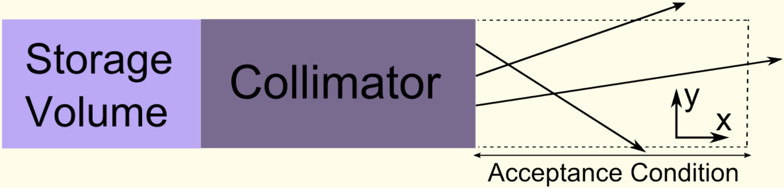
$r_{RMS} = 0$

Bottom:

Trajectory at

$r_{RMS} = 5\ \mu m$





Collimationsystem top view - Visualization Acceptance Condition

- Neutrons are counted with respect to their alignment
  $\Rightarrow$ What intensity can be expected in a certain distance from the exit slit

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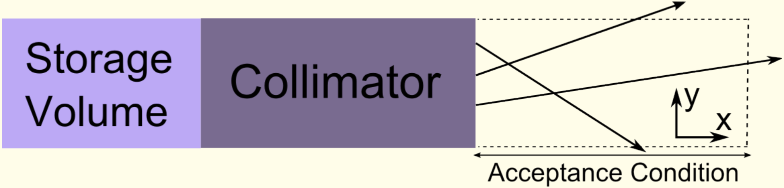

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1. An **absorbtion** of a neutron takes place with a probability[1,2] of $P_{abs} = 3 \cdot 10^{-5}$
2. The probability for **diffuse reflection** depends on the velocity component perpendicular to the surface[1]:
   $P_{diff} = \left(\frac{k_\perp r_{RMS}}{2\pi}\right)^2 \quad \overset{k_\perp = \frac{2\pi p_\perp}{h}}{\Longrightarrow} \quad P_{diff} = \frac{r^2 m_n^2}{h^2} v_\perp^2$
3. The neutrons are then scatterd with an angle distribution, that is based on the scattering law[1] and yields:
   $\mathrm{d}\Omega_{out} = \cos\theta_{out} \mathrm{d}\cos\theta_{out} \mathrm{d}\phi_{out} = \frac{1}{2} \mathrm{d}\cos^2\theta_{out} \mathrm{d}\phi_{out}$
4. The probability for **specular reflection** is then given by $P_{spec} = 1 - P_{abs} - P_{diff}$

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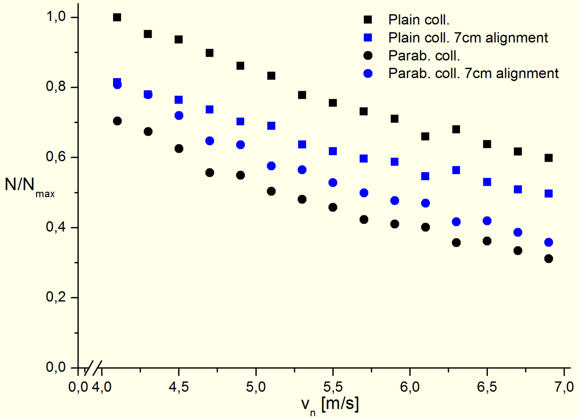
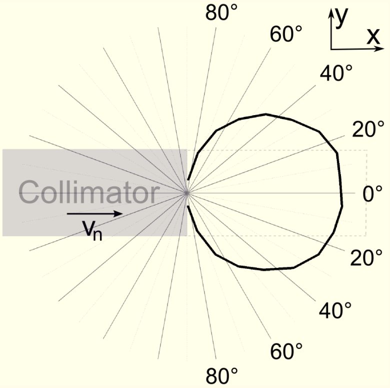
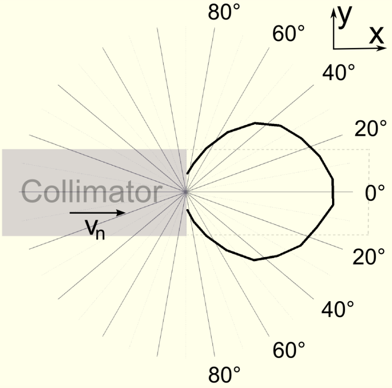


Velocity distributions for parabola-shaped and plain collimator



Angle distribution plain collimator



Angle distribution parabola- shaped collimator

5. Approaching P_R - Passing and reflection rates

A_0: Aperture area, P_p: Passing probability, P_b: Back reflection probability

Flux relation:

$$\frac{f_{parabola}}{f_{plain}} = \frac{A_{0,par} P_{p,par}}{A_{0,plain} P_{p,plain}}$$

Effective aperture:

$$A_{eff} = A_0(1 - P_R)$$

Main goal:

$$A_{eff} < 1.5\,mm^2$$

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Plain collimator:

Width	$A_{0,plain}$ $[mm^2]$	A_{eff} $[mm^2]$	P_b	$P_{p,plain}$
30 cm	30	1.05	96.5 %	0.7/0.45 %

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Parabola-shaped collimator:

Width	$A_{0,par}$ $[mm^2]$	A_{eff} $[mm^2]$	P_b	$P_{p,par}$	$\frac{f_{parabola}}{f_{plain}}$
26 cm	56.42	1.2	97.8 %	0.43/0.27 %	1.3
28 cm	60.76	1.3	97.8 %	0.46/0.28 %	1.4
30 cm	65.1	1.4	97.8 %	0.48/0.3 %	1.5

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- Inserting a mirror at the bottom decreases P_p rapidly
- Inserting mirrors as sidewalls increases P_p significantly

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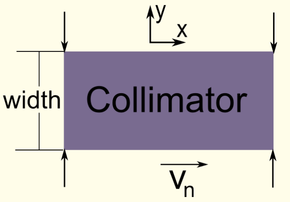
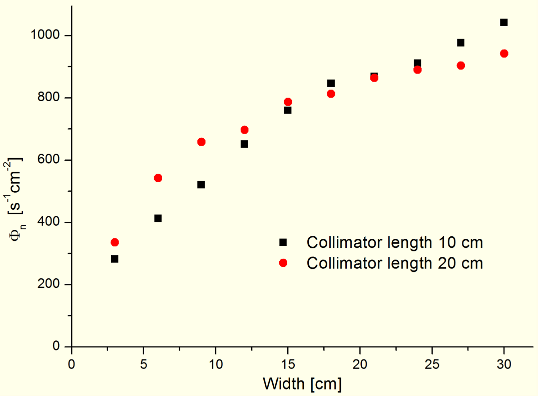
- Only a *width* $< 5\,cm$ is useful for this measurement technique



$$3\text{cm} \Rightarrow \Phi = 350 s^{-1} cm^{-2}$$
$$5\text{cm} \Rightarrow \Phi = 550 s^{-1} cm^{-2}$$



Conclusions

- A Monte-Carlo simulation was successfully developed, to investigate and compare two collimation systems
- While taking a limit for the effective aperture into account, the parabola-shaped collimator was optimized

Results

- The optimized collimator is capable of a flux increase of a factor of more than 2 compared to the plain arrangement
- The setup is sensitive to the roughness and mirror combination that is choosen
- It is found, that a setup that might be applicable for neutron reflectometry could produce a neutron flux of about 600 $s^{-1}\,cm^{-2}$

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Outlook

Investigations will be carried out on the following questions:

- Can we manipulate the velocity distribution via tilting the collimator?
- Can similar shapes reproduce the improvements that were found for the parabola-shaped collimator?
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The condition of detailed balance states, that the number of neutrons reflected from Ω_0 to Ω must be equal to the number of neutrons reflected from Ω to Ω_0 and can be expressed as:

$$\begin{aligned} \cos\theta_0 d\Omega_0 W_{ref}(\Omega_0, \Omega) d\Omega = \cos\theta d\Omega W_{ref}(\Omega, \Omega_0) d\Omega_0 \\ \Rightarrow \cos\theta_0 W_{ref}(\Omega_0, \Omega) = \cos\theta W_{ref}(\Omega, \Omega_0) \end{aligned} \tag{1}$$

Where θ und θ_0 are the angles between normal of the surface and direction of passing neutrons. Because of the required equilibrium, W_{ref} must be symmetric in Ω and Ω_0 and with respect to (1) can be satisfied by functions as: $W_{ref}(\Omega_i, \Omega_j) = f(\Omega_i, \Omega_j) \cos\theta_j$, where f is an arbitrary function, symmetric in its arguments. Assuming $f = const.$ allows us to normalize W_{ref} to satisfy $\int W_{ref} d\Omega = 1$ and leads to:

$$W_{ref} d\Omega = \frac{1}{\pi} \cos\theta d\Omega = \frac{1}{2\pi} d\cos^2\theta d\phi$$