Short-range forces, Casimir effect and geometry

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P. Andreucci & L. Durafourg (LETI),
C. Genet & T. Ebbesen (ISIS),
G. Ingold (Augsburg) …

European Science Foundation
Setting Science Agendas for Europe

Discussions with a number of other people, in particular within the ESF network "CASIMIR"

http://www.casimir-network.com

The many facets of the Casimir effect

- Casimir effect
  - an observable “mesoscopic” effect of quantum fluctuations

- A fascinating interface with other problems in fundamental physics
  - gravity : “vacuum energy” problem
  - non trivial effects of geometry :
    beyond the “Proximity Force Approximation”
  - “principle of relativity of motion” : dynamical Casimir-like effects
  - “new physics” : search for hypothetical new short-range forces expected to lie “beyond the standard model”

- A dominant force in the mesoscopic world : strong connections with
  - atomic and molecular physics, quantum optics
  - condensed matter physics, surface physics
  - chemical physics and biological physics
  - micro- and nano-technology : potential applications in new solutions for actuating or controlling micro/nanosystems …

Search for scale dependent modifications of the gravity force law ("fifth-force experiments")

The exclusion plot for deviations with a Yukawa form

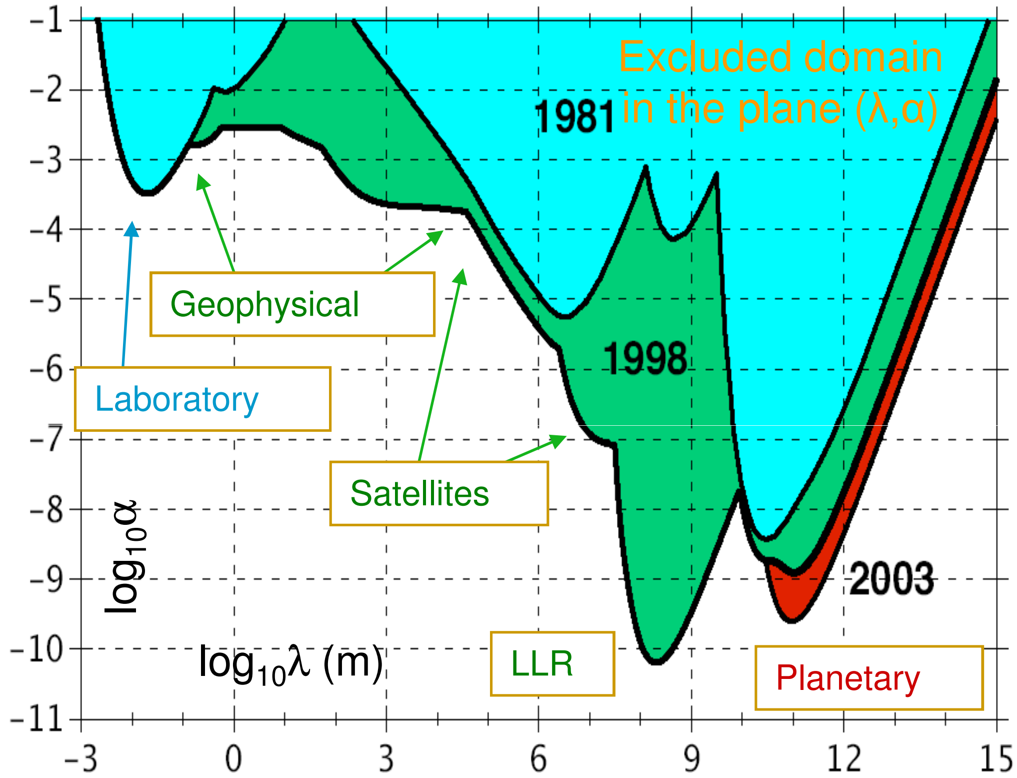
$$V(r) = -\frac{GMm}{r}\left(1+\alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations at short ranges

$\lambda < 1\,\mathrm{mm}$

or long ranges

$\lambda > 10^{16}\,\mathrm{m}$



Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge & E. M. Standish (2003) ; see M.T. Jaekel & S. Reynaud IJMP **A20** (2005)

The Search for Non-Newtonian Gravity, E. Fischbach & C. Talmadge (1998)

Constraints at sub-mm scales

- Best result to date : Eöt-wash group (U. Washington, Seattle)
- Cavendish-type experiments with torsion pendulum
- At 95% confidence, a Yukawa interaction with gravitational strength $\alpha > 1$ must have a range $\lambda < 56\mu m$

  - D.J. Kapner *et al* PRL **98** (2007) 021101
  - E.G. Adelberger et al PRL **98** (2007) 131104

➧ Talk S. Hoedl

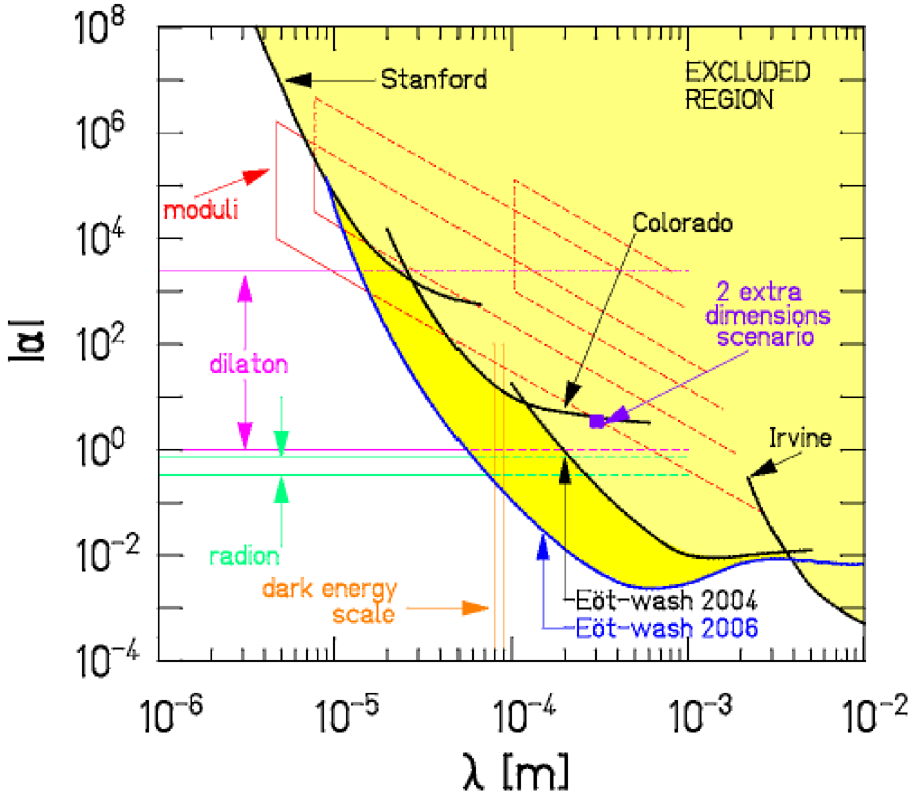


FIG. 6: Constraints on Yukawa violations of the gravitational $1/r^2$ law. The shaded region is excluded at the 95% confidence level. Heavy lines labeled Eöt-Wash 2006, Eöt-Wash 2004, Irvine, Colorado and Stanford show experimental constraints from this work, Refs. [11], [14], [15] and [16, 17], respectively. Lighter lines show various theoretical expectations summarized in Ref. [9].

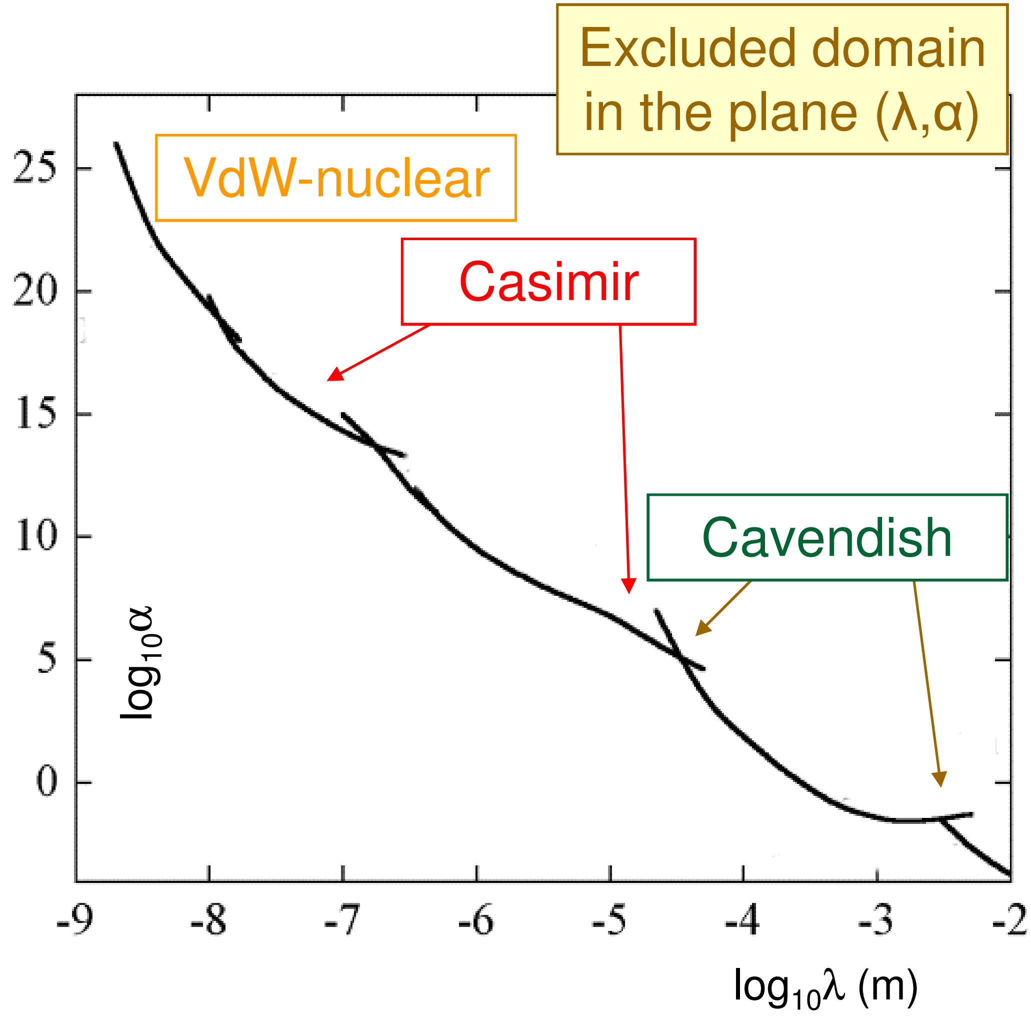
Constraints at shorter scales

- At (~) µm scales, comparison with theory of Casimir measurements
- Also worth with Van der Waals and nuclear forces at shorter scales

➧ Several talks on these topics today

- The hypothetical new force would be seen as a difference between experiment and theory

$$F_{\text{new}} \equiv F_{\text{exp}} - F_{\text{th}}$$



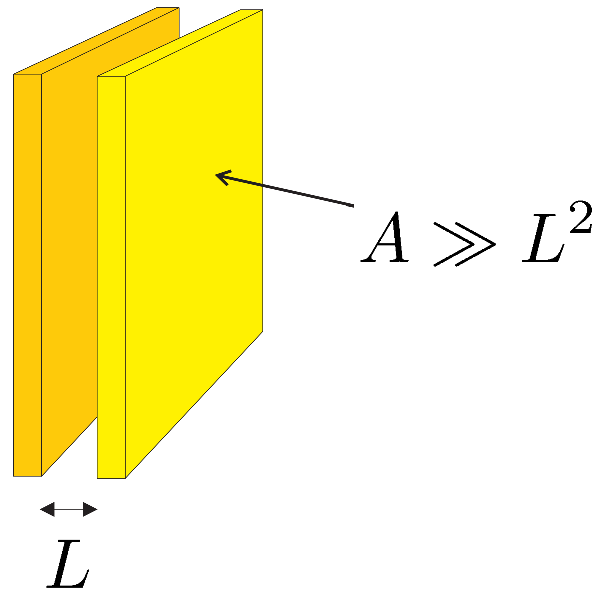
➧ The accuracy and reliability of theory and experiment have to be assessed <u>independently</u>

Casimir theory ..

- Between two perfect plane mirrors in vacuum

$$F_{\mathrm{Cas}} = P_{\mathrm{Cas}} A \ , \qquad P_{\mathrm{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$

$$E_{\mathrm{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3}$$



- Written for an ideal case
  - Parallel plane mirrors
  - Perfect reflection
  - Null temperature
  - Perfectly flat surfaces

- Attractive force

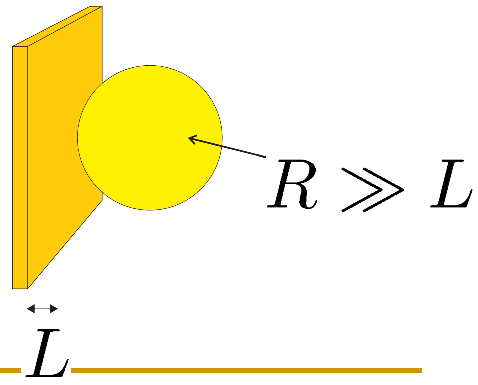
$$L = 100\mathrm{nm} \rightarrow |P| \sim 10\mathrm{Pa}$$

"Universality" : does not depend of anything else than *ħ, c, and geometry*

Hendrik Casimir 1948

Force between “real mirrors”

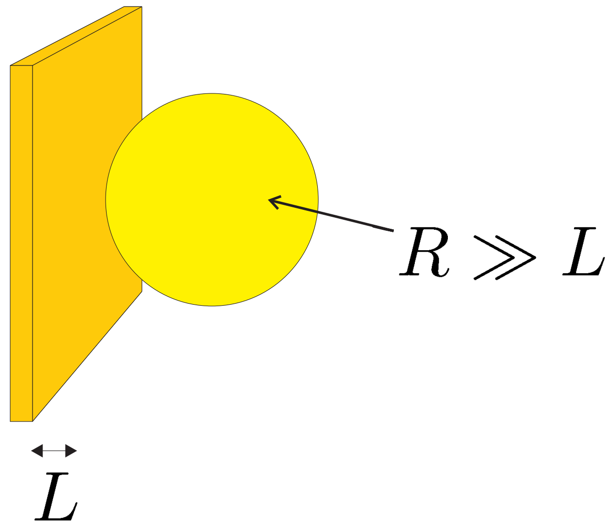
- Real mirrors are not perfectly reflecting
  - The Casimir force depends on non universal optical properties of the mirrors used in the experiments
- Experiments are performed at room temperature
  - The effect of thermal field fluctuations has to be added to that of vacuum fluctuations
- Experiments are not done in the ideal Casimir geometry
  - Plane-sphere geometry is used for most recent experiments
  - Surface state is not perfect (roughness)



A. Lambrecht, P. Maia Neto, S. Reynaud, New J. Physics **8** (2006) 243

The “Proximity Force Approximation” (PFA)

- Force between a plane and a sphere usually computed using the PFA
  - Contributions of surface elements corresponding to different inter-plate distances L' simply added as if they were independent



$$F_{\mathrm{PFA}} = \int_L^\infty \mathrm{d}A\, P\left(L'\right)$$

- For a plane and a sphere

$$\mathrm{d}A = 2\pi R\, \mathrm{d}L'$$

$$F_{\mathrm{PFA}} = 2\pi R \int_L^\infty \mathrm{d}L'\, P\left(L'\right)$$

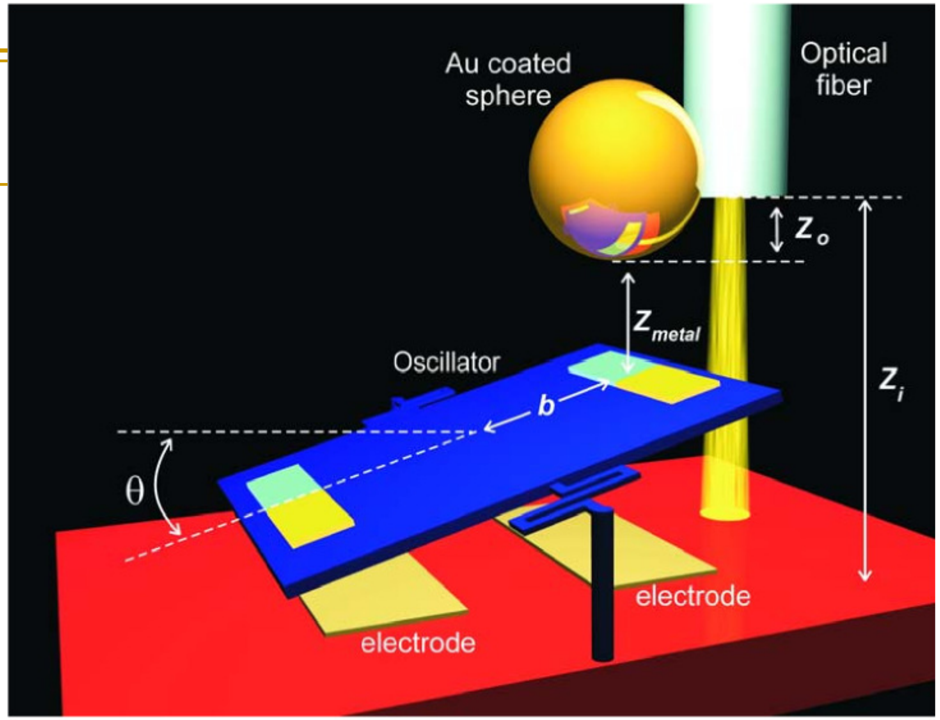
- Casimir forces are not additive
  - PFA is not a theorem !
  - It can only be an approximation valid for large spheres

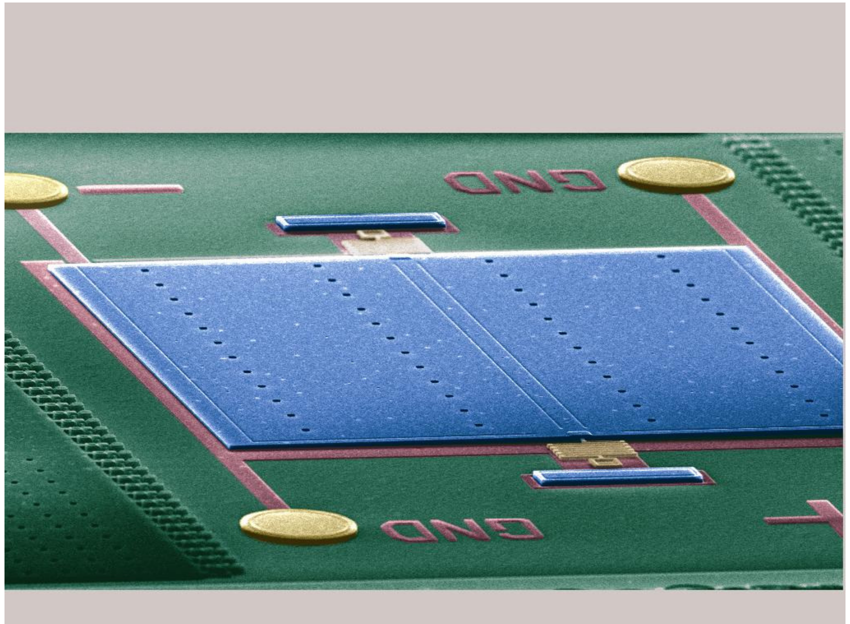
B. Deriagin *et al* (1968)

Casimir experiments ..

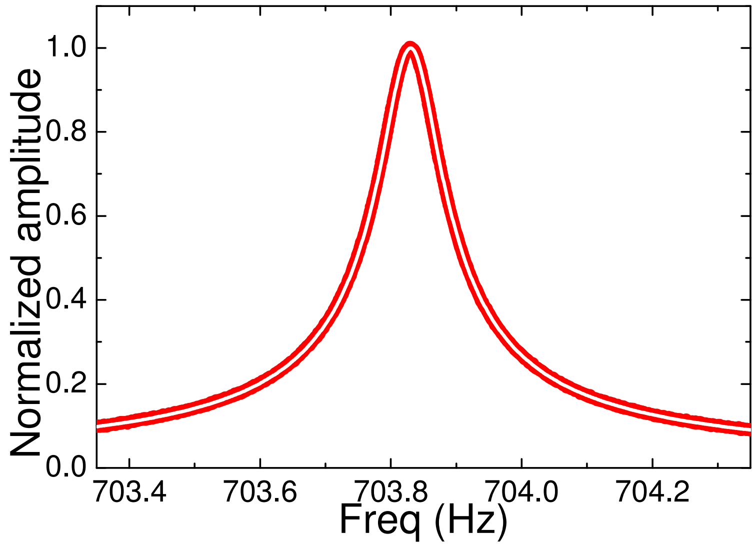
Most precise Casimir experiments :

dynamic measurements of the resonance frequency of a microelectromechanical resonator





Courtesy R.S. Decca
(Indiana U – Purdue U Indianapolis)



R.S. Decca, Talk at Sante Fe Workshop (09/2009) @ http://cnls.lanl.gov/casimir/

.. Casimir experiments ..

- Shift of the resonance determined by the gradient *G* of the Casimir force
- Plane-sphere force *F* calculated within the proximity force approximation

$$\omega_r^2 = \omega_0^2 - \frac{b^2}{I} G$$

$$G \equiv \frac{\partial F}{\partial L} = 2\pi R P(L)$$

- Pressure *P* between two planes calculated in the Lifshitz formulation of the Casimir force
  - Plasma model for the dielectric function
    - conduction electrons in the metals with relaxation accounted for (Drude model)
  - Gold has a finite conductivity
    - $\gamma \neq 0$ is certainly a better description than $\gamma = 0$

$$\boxed{\varepsilon(i\xi) = \bar{\varepsilon}(i\xi) + \frac{\sigma(i\xi)}{\xi}}$$

$$\boxed{\sigma(i\xi) = \frac{\omega_P^2}{\xi + \gamma}}$$

➧ **BUT** experiments agree better with $\gamma = 0$ than with the expected $\gamma \neq 0$

S. Reynaud et al, arXiv:1001.3375

.. Casimir experiments ..

BRIEF REPORTS PHYSICAL REVIEW D 75, 077101 (2007)

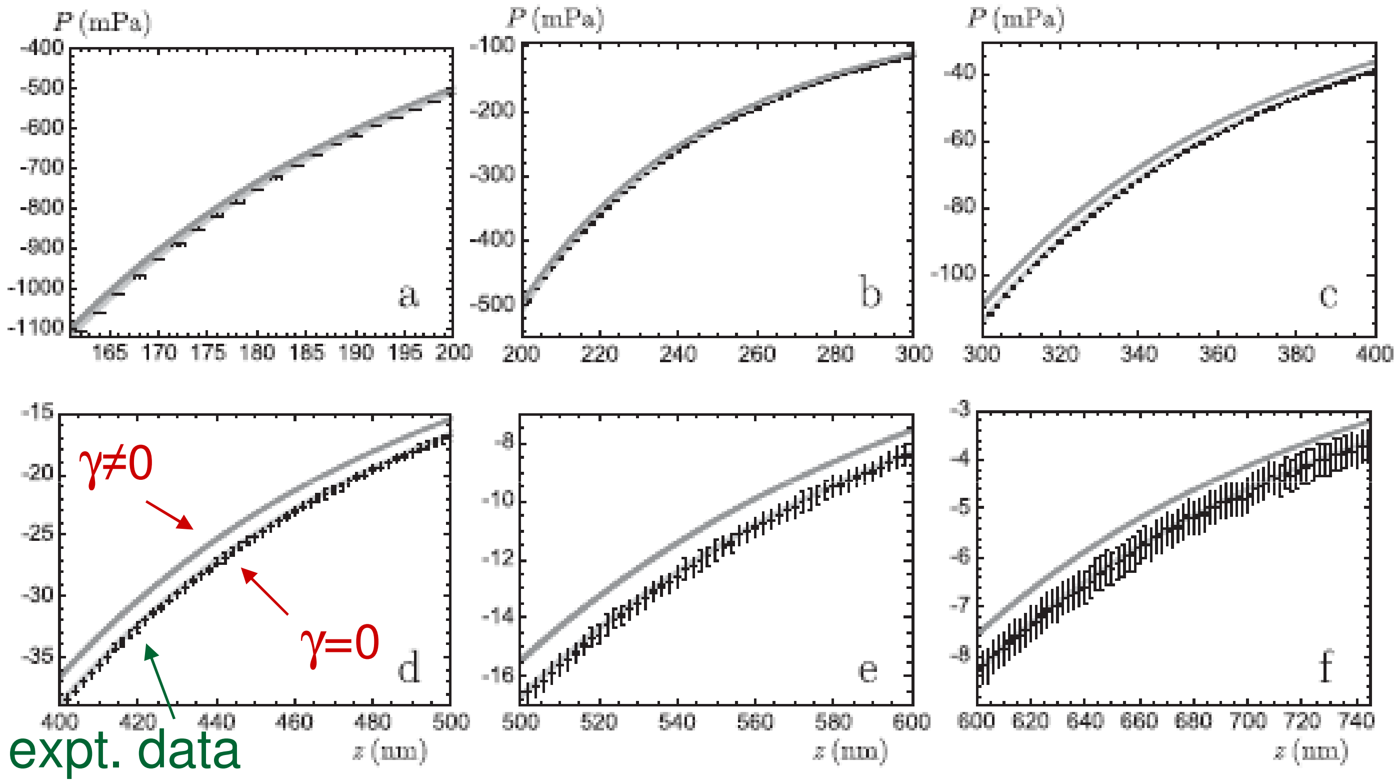


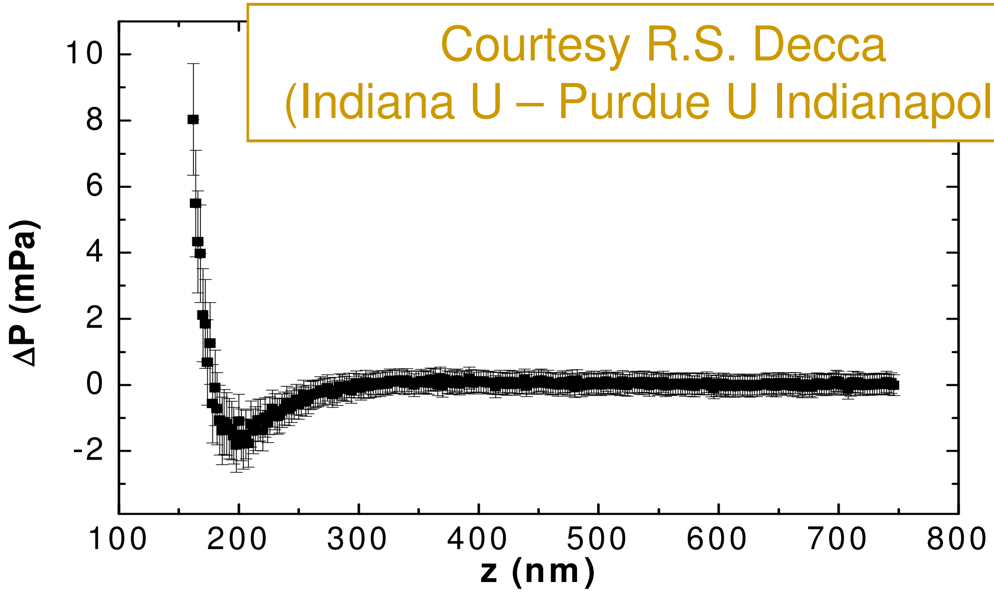
FIG. 1. Experimental data for the Casimir pressure as a function of separation z. Absolute errors are shown by black crosses in different separation regions (a–f). The light- and dark-gray bands represent the theoretical predictions of the impedance and Drude model approaches, respectively. The vertical width of the bands is equal to the theoretical error, and all crosses are shown in true scale.

Experimental results deviate from theoretical expectations

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

Casimir expt/theory comparison ..

The difference does not have a Yukawa or power-law form !



Courtesy R.S. Decca
(Indiana U – Purdue U Indianapolis)

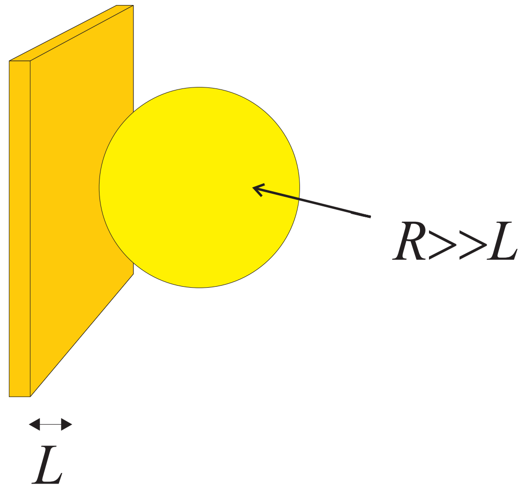
- What is the reason of the deviation between experiments and theory
  - Artifact in the experiments ?
  - Inaccuracy in the theoretical evaluations ?
  - Difference between the situation studied in theory and the experimental realization ?
  - Detection of a new weak force ???

Many talks at Oklahoma and Sante Fe Conferences (September 2009)

.. Casimir expt/theory comparison ..

- The geometry of the precise experiments is not the geometry of the precise calculations !

- The proximity force approximation (PFA) amounts to average the plane-plane result over the distance distribution
- It neglects diffraction effects
- It can only be expected to be accurate at the limit of large spheres



- But what is the accuracy for a given value of *L/R* ?
- How does this accuracy depend on the properties of the mirror, the distance, the temperature ?

A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud PRL **102** (2009) 230404

Going beyond the PFA :
The non-specular scattering formula

- In an arbitrary geometry with two scatterers in vacuum, the Casimir energy can be deduced from the scattering formula

$$E = \hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \mathrm{Tr} \ln \mathcal{D} \quad , \quad \mathcal{D} \equiv 1 - \mathcal{R}_1 e^{-\mathcal{K}L} \mathcal{R}_2 e^{-\mathcal{K}L}$$

  - $\mathcal{R}_1, \mathcal{R}_2$ are reflection matrices at imaginary frequencies $\omega \equiv i\xi$
    - They are defined in the absence of the other object (long-range)
    - They describe non-specular couplings between all modes with the same frequency but different wavevectors and polarizations
  - $e^{-\mathcal{K}L}$ are free propagators (diagonal in a plane wave basis)

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See also Balian & Duplantier - Emig, Kardar, Jaffe, et al - Kenneth, Klich et al, Milton et al - Bordag et al - Wirzba et al - Gies et al – Johnson et al …

The Lifshitz formula as a particular case

- With plane and parallel mirrors, the reflection matrices are diagonal in the plane wave basis
- This greatly simplifies the expression of the Casimir energy (specular scattering only)

- The Lifshitz formula is recovered
  - for semi-infinite bulk mirrors characterized by local dielectric response functions
  - with the reflection amplitudes given by Fresnel laws

- The scattering formula accommodates more general expressions for the reflection amplitudes
  - finite thickness, multilayer structure…
  - non local dielectric response, non isotropic response…
  - chiral materials…

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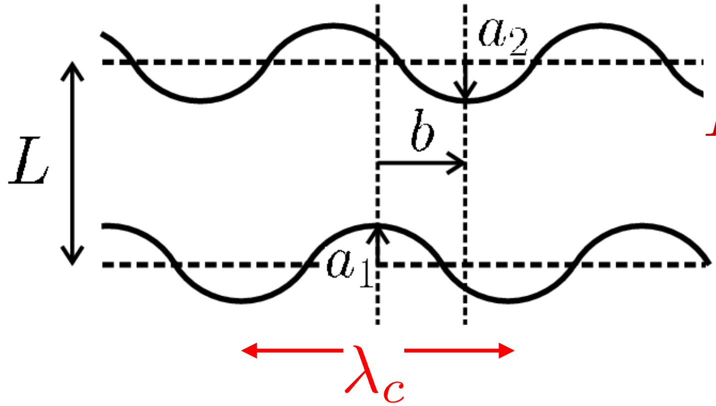
Lateral force between corrugated plates

Force



$$F^{\text{corrug}} = \frac{\partial \delta E^{\text{corrug}}}{\partial b}$$

Force

$$k = \frac{2\pi}{\lambda_c}$$

- Symmetry broken for transverse translations → Lateral forces
- The energy has to be obtained from the non-specular scattering formula except at the PFA limit $k \rightarrow 0$

- Perturbative expansion at lowest order in the corrugation amplitudes

$$\delta E^{\text{corrug}} = -\hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \mathrm{Tr} \left(\delta \mathcal{R}_1 \frac{e^{-\mathcal{K}L}}{\mathcal{D}_0} \delta \mathcal{R}_2 \frac{e^{-\mathcal{K}L}}{\mathcal{D}_0} \right)$$

R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRL **96** (2006) 100402

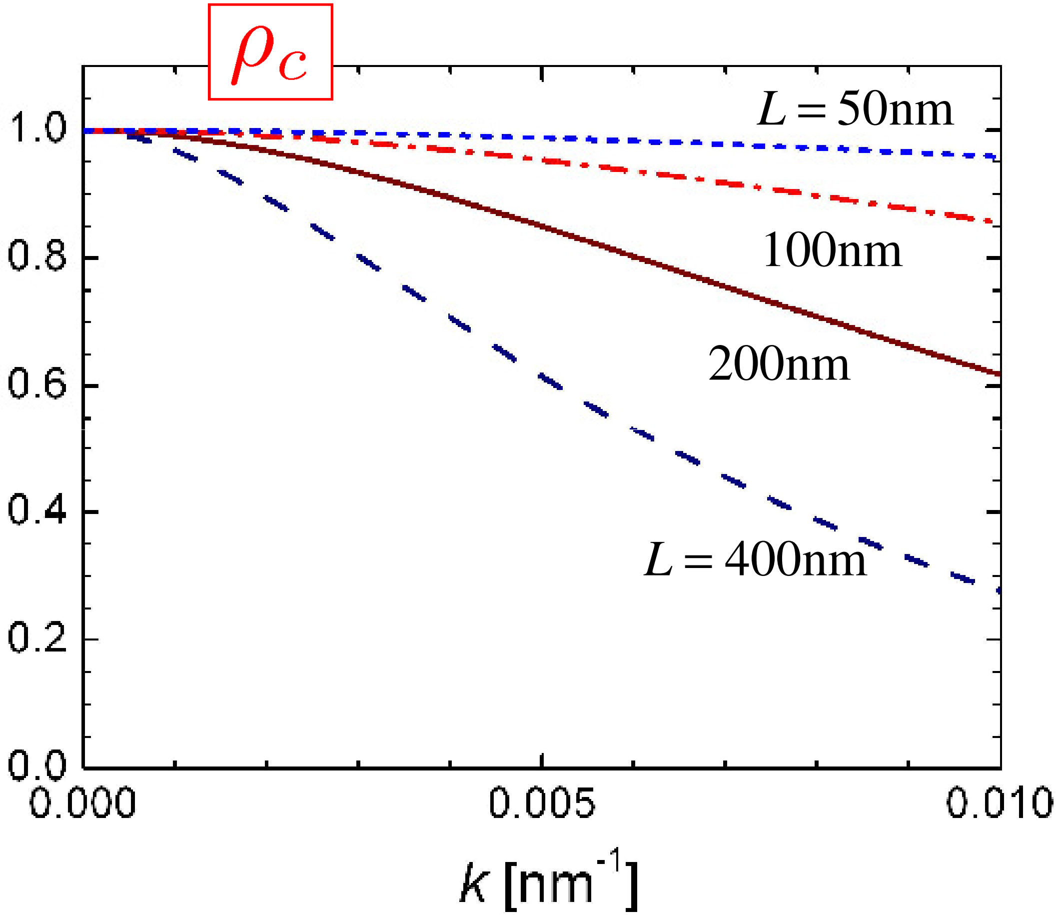
Non trivial effects of geometry

- Bulk metallic mirrors described by the plasma model

$$\lambda_{\mathrm{P}} = 136\mathrm{nm}$$

- The force is reduced with respect to PFA

$$\rho_c \equiv \frac{F^{\mathrm{corrug}}}{F^{\mathrm{corrug}}_{\mathrm{PFA}}} < 1$$

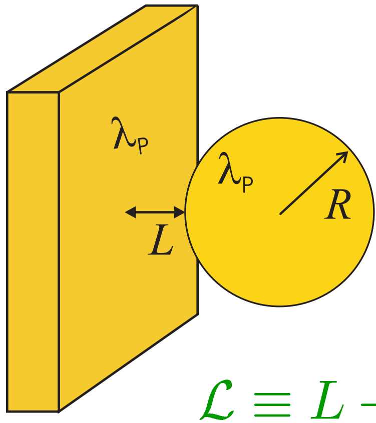


R. Rodriguez, P. Maia Neto, A. Lambrecht & S. Reynaud, PRL **98** (2007) 068902

The plane-sphere geometry beyond PFA

- We start from the non-specular scattering formula

$$E = \hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \mathrm{Tr} \ln \left(1 - \mathcal{R}_\mathrm{P} e^{-\mathcal{K}\mathcal{L}} \mathcal{R}_\mathrm{S} e^{-\mathcal{K}\mathcal{L}}\right)$$



$$\mathcal{L} \equiv L + R$$

  - We write the reflection matrices, $\mathcal{R}_\mathrm{P}$ for plane waves on the plane mirror, $\mathcal{R}_\mathrm{S}$ for spherical waves on the sphere (Mie amplitudes) …
  - … and the transformation from plane to spherical waves
  - All our calculations are performed for electromagnetic fields

- We obtain an “exact” multipolar expansion of the energy

$$x \equiv \frac{L}{R}$$

  - Spherical waves are labeled by ℓ and m with $|m| \leq \ell$
  - Sums are truncated at some ℓ_max for doing the numerics
  - The results are accurate for $x > x_\mathrm{min}$ with $x_\mathrm{min} \propto \dfrac{1}{\ell_\mathrm{max}}$

P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

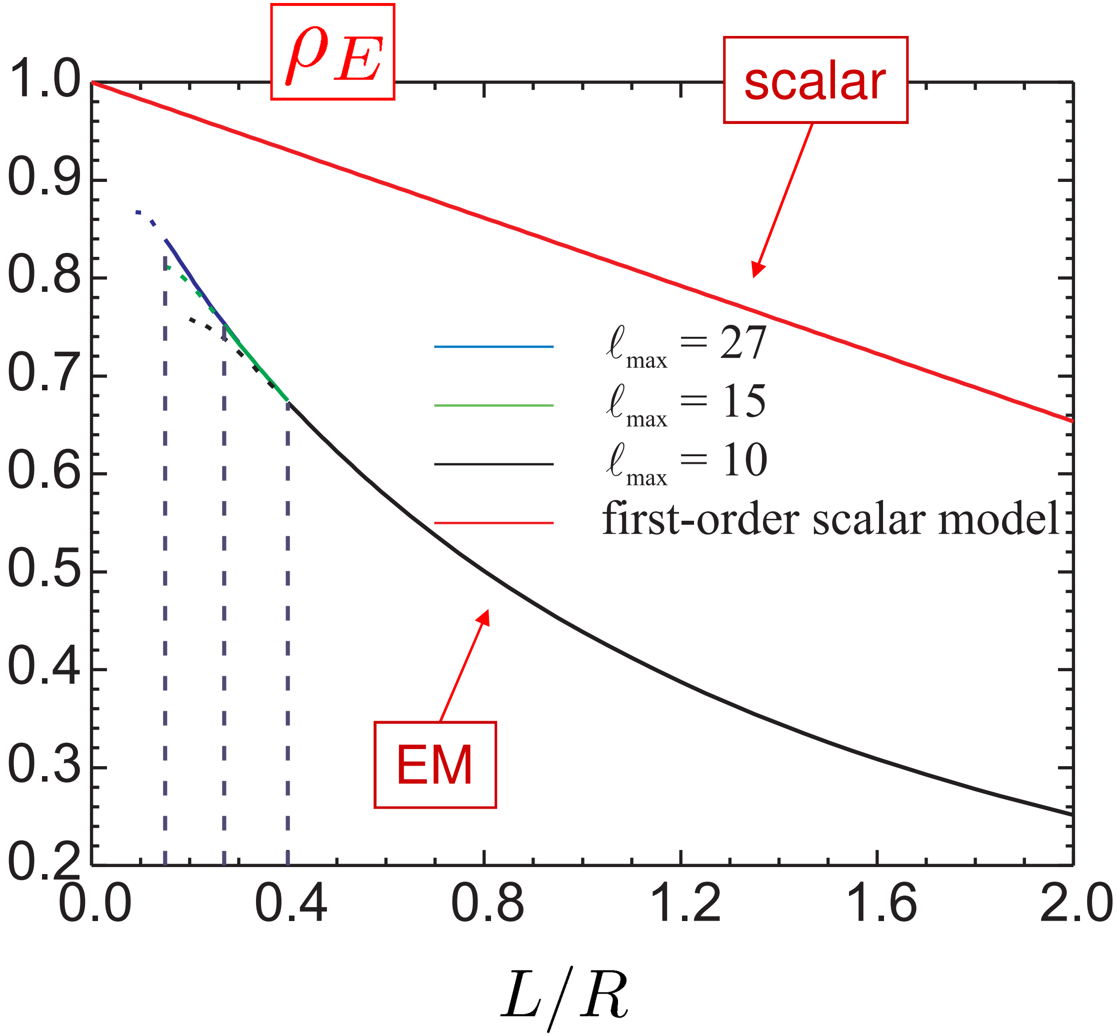
Calculation for perfect mirrors

➢ Plane and spherical perfect mirrors at *T=0*

- Energy is found to be reduced with respect to PFA

$$\rho_E \equiv \frac{E_{\mathrm{PS}}}{E_{\mathrm{PS}}^{\mathrm{PFA}}} < 1$$

- Electromagnetic result departs from PFA much more rapidly than scalar calculation



P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

Extrapolation at low values of $x \equiv \frac{L}{R}$

- Values at low values of x can be found through a polynomial extrapolation

$$\rho_E(x) \simeq 1 + \beta_E x + \dots$$

- Slope ~8 times larger than found in scalar calculations

$$(\beta_E)^{\mathrm{EM}} \simeq -1.4$$

$$(\beta_E)^{\mathrm{scalar}} \simeq -0.17$$

  - Agreement with results obtained independently by T. Emig, JSTAT (2008) P04007

- Same calculations done for the force *F* and gradient *G*

$$\rho_G \equiv \frac{G_{\mathrm{PS}}}{G_{\mathrm{PS}}^{\mathrm{PFA}}} \simeq 1 + \beta_G x + \dots$$

- The result of these calculations lies in conflict with the Purdue experiment

$$\beta_G \simeq -0.48$$

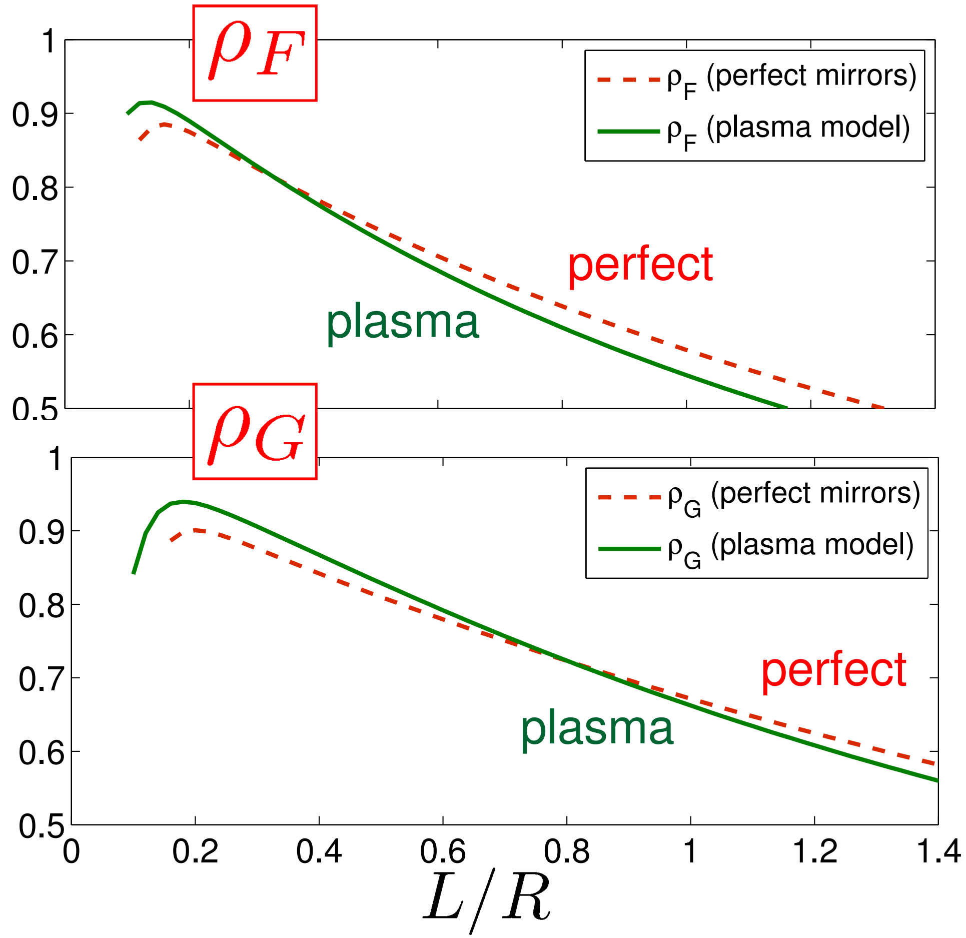
D. Krause et al, PRL (2007) $|\beta_G| \lesssim 0.4$

P. Maia Neto, A. Lambrecht, S. Reynaud, Phys. Rev. A **78** (2008) 012115

Interplay between geometry and material ..

Force *F* and gradient *G* between plane and spherical metallic plates

- Metallic mirrors described by the plasma model $\lambda_P = 136\text{nm}$
- Results close to the limit of perfect reflection for large spheres
- Differences seen for nanospheres : R=100nm on the plots



A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht, S. Reynaud PRL **102** (2009) 230404

.. Interplay between geometry and material

- Values for small x obtained through a polynomial extrapolation

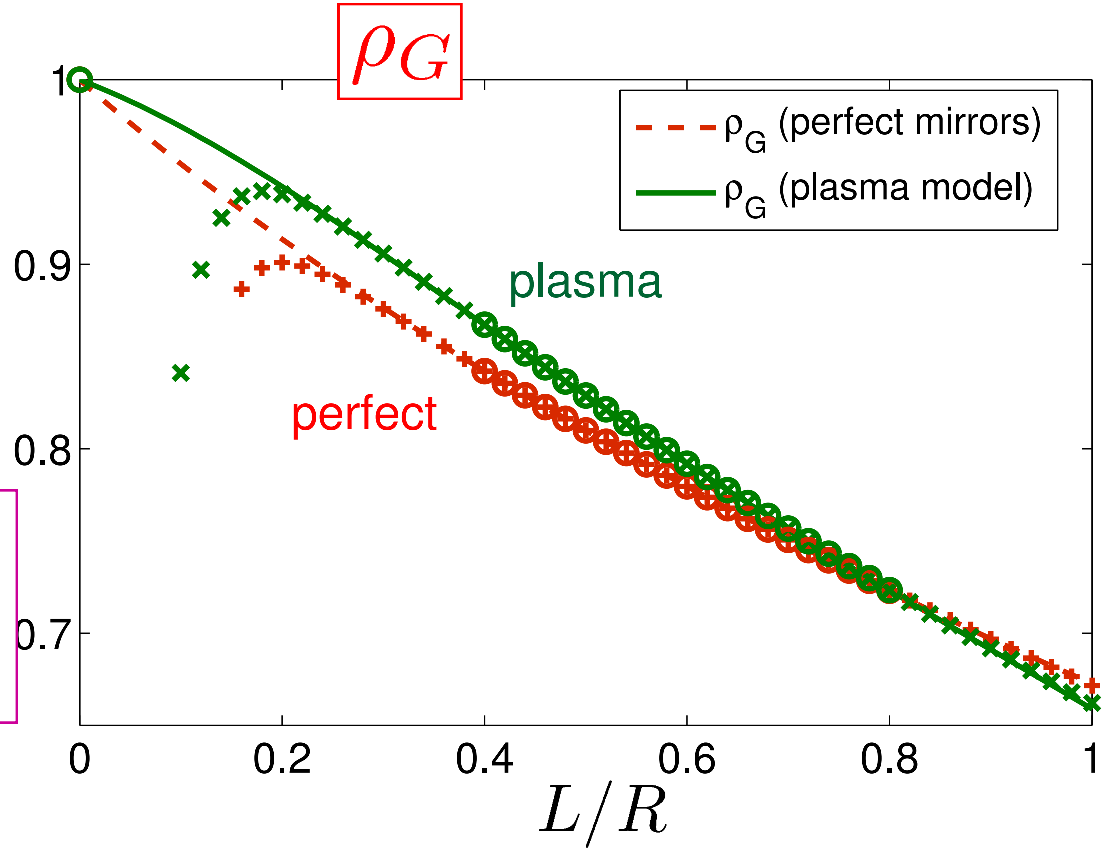
$$\rho_G \simeq 1 + \beta_G x + \dots$$

- Slope found for the plasma model is compatible with the Purdue experiment

$$\beta_G \sim -0.21$$

D. Krause et al, PRL (2007)

$$|\beta_G| \lesssim 0.4$$



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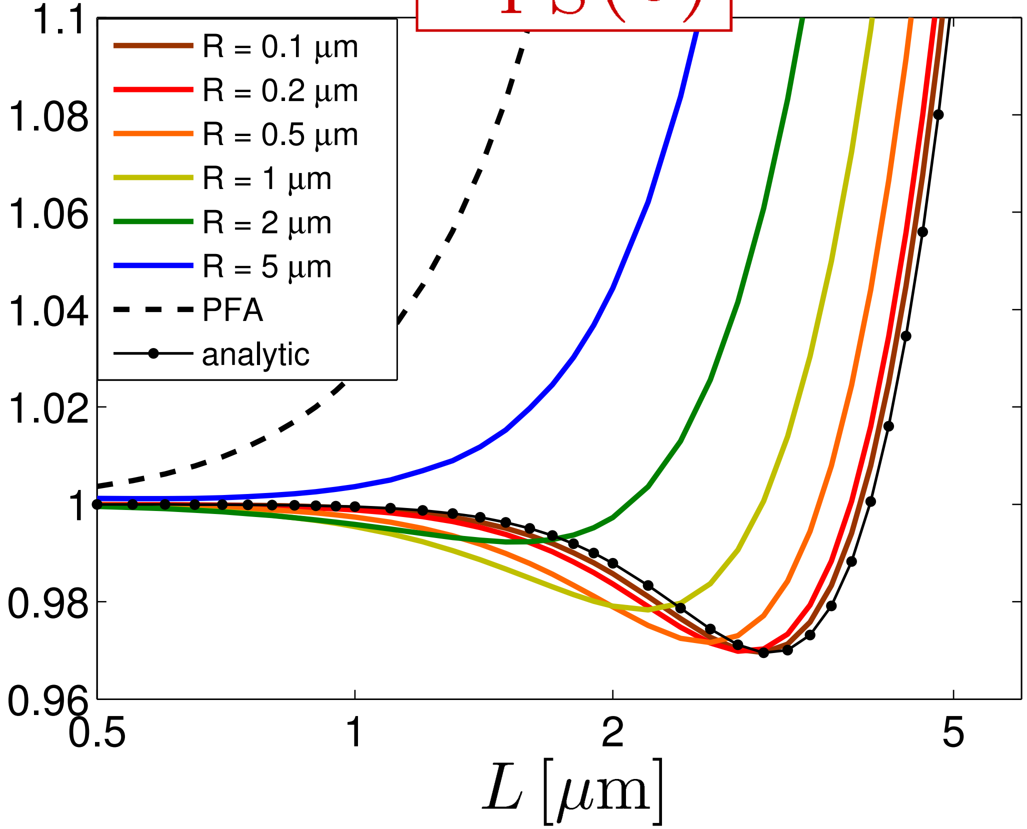
Interplay between geometry and temperature

➧ Force between plane and spherical perfect mirrors at ambient temperature

$$\frac{F_{\mathrm{PS}}(T)}{F_{\mathrm{PS}}(0)}$$

- Plane-sphere force at ambient temperature divided by plane-sphere force at *T=0*

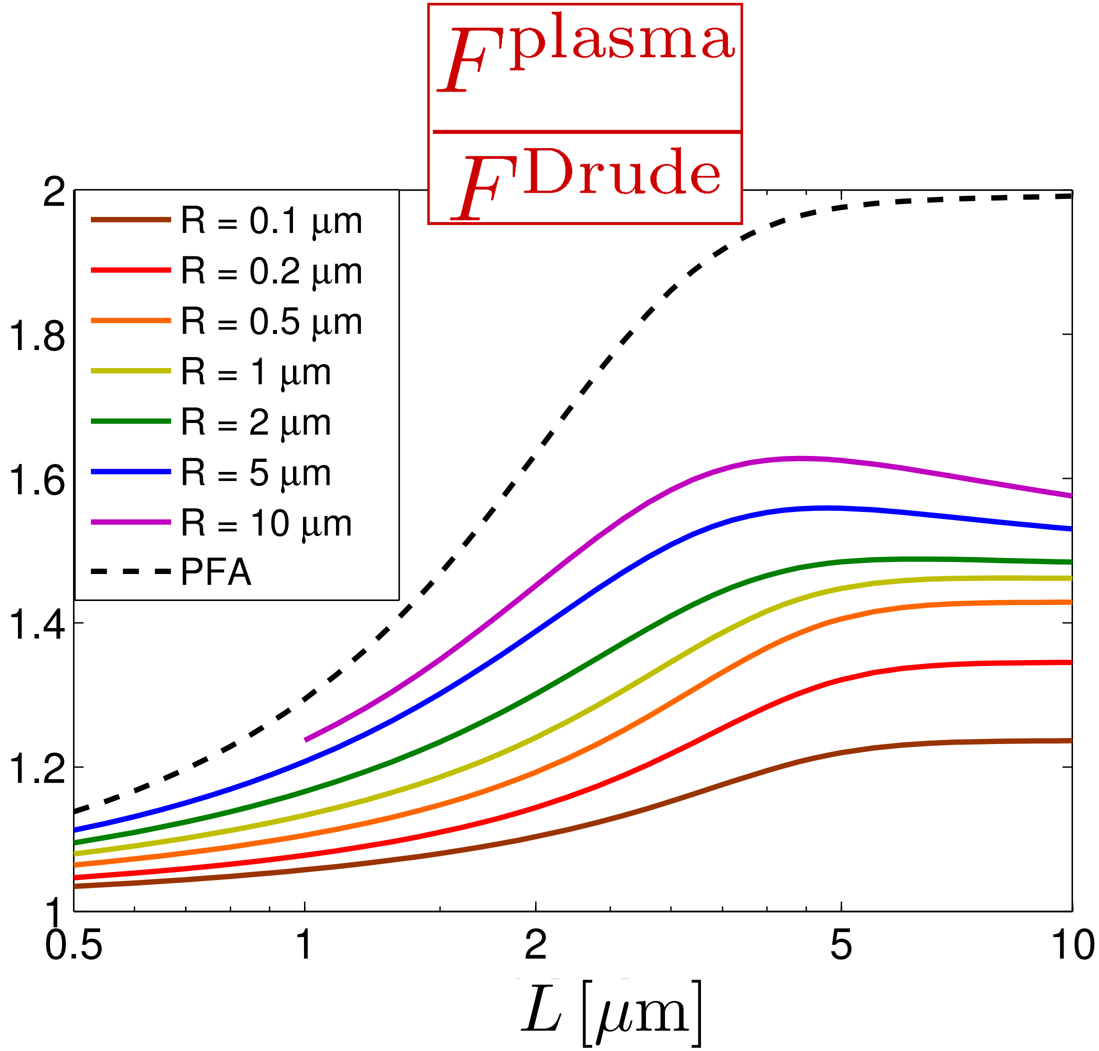


- Contribution of thermal photons repulsive at some distances !
- Entropy negative at some distances !

A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht, S. Reynaud
PRL **104** (2010) 040403

Interplay between geometry, temperature and dissipation

- Force obtained for the lossless plasma model (γ=0) divided by that for lossy Drude model (γ≠0)



- Plasma and Drude results always closer than expected from PFA
- Ratio of these results goes to 2 at large L with PFA, but to a value < $\frac{3}{2}$ with plane-sphere calculations

A. Canaguier-Durand, P.A. Maia Neto, A. Lambrecht, S. Reynaud
PRL **104** (2010) 040403