Implications of FI-Terms in Orbifold Compactifications

Kai Schmidt-Hoberg

In Collaboration with W. Buchmüller, R. Catena, P. Hosteins, R. Kappl and M. Ratz

`arXiv:0803.4501`, `arXiv:0902.4512`

`arXiv:0905.3323`

Grenoble
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Introduction: SUSY GUTs in 4 Dimensions

- Doublet-triplet splitting problem
  Standard Model Higgs comes together with color triplet that leads to proton decay $\Rightarrow$ must be very heavy
- Dimension-5 proton decay operators
  Decay too fast even if triplet mass is $\mathcal{O}(M_{\text{GUT}})$
- Gauge symmetry breaking needs large Higgs representations
- μ problem
  μ parameter must be small to get correct EWSB
- SUSY flavour problem
  Squark and slepton mass matrices must be almost diagonal to avoid FCNCs

$\Rightarrow$ Supersymmetric Orbifold GUTs

Introduction: Orbifold Compactification

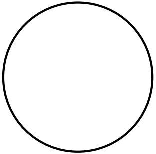
- Starting point: higher-dimensional setup

Simplest example: one extra dimension, compactified on circle

- Compactification scale: $M_c \equiv 1/R \sim M_{\mathrm{GUT}}$
- Kaluza-Klein mode expansion:

$$\Phi(x,y) = \sum_{n=0}^{\infty} \Phi_+^{(n)}(x)\,\cos\left(\frac{ny}{R}\right) + \sum_{n=1}^{\infty} \Phi_-^{(n)}(x)\,\sin\left(\frac{ny}{R}\right)$$

$\rightsquigarrow$ In 4D effective theory: Tower of states with masses n/R



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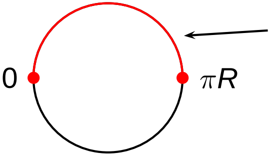
- Impose symmetry $\mathbb{Z}_2$: $y \to -y$



Orbifold $S^1/\mathbb{Z}_2$

Fixed points $=$ branes at 0, πR
Fields can be localized there

- Fields either even or odd: $\Phi(x,y) \xrightarrow{\mathbb{Z}_2} \pm\Phi(x,-y)$

Introduction: Virtues of Orbifold GUTs

- Only even fields have zero modes ($n = 0 \Rightarrow$ massless)
- All odd fields are heavy (mass $\sim M_c$)

$\Rightarrow$ Unwanted fields can be removed from low-energy spectrum

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- Only even fields have zero modes ($n = 0 \Rightarrow$ massless)
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$\Rightarrow$ Unwanted fields can be removed from low-energy spectrum

- Higgs doublets even, triplets odd $\Rightarrow$ doublet-triplet splitting
- Only SM gauge bosons even $\Rightarrow$ gauge symmetry breaking without large Higgs representations
- No dimension-5 proton decay Hall, Nomura, Phys Rev **D 64** (2001)
- Higher-dimensional supersymmetry broken to $N = 1$ SUSY $\Rightarrow$ chiral fermions

Overview

- Challenge: Size and Shape of the extra dimensions undetermined
  $\Rightarrow$ Moduli problem
- Casimir energy induces a nontrivial potential
- Radiative corrections generically induce Fayet-Iliopoulos terms at the fixed points.
  Lee, Nilles, Zucker, Nucl.Phys.B **680** (2004)
  Buchmüller, Lüdeling, Schmidt, JHEP **0709** (2007)
- Combination of Casimir energy with FI-terms can lead to small extra dimensions
  $\Rightarrow$ Part 1
- Fayet-Iliopoulos-terms also have an important impact on couplings
  $\Rightarrow$ Part 2

Outline

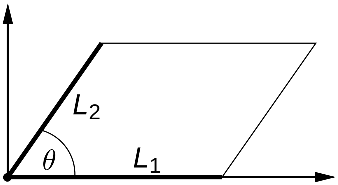
1. Stabilisation of Extra Dimensions
   - Example: An Orbifold GUT Model
   - Casimir Energy
   - Stabilisation
2. Gauge-Top Unification
   - GTU in GUTs
   - String theory input
   - Phenomenological implications

Orbifold Compactification

- Starting point: higher-dimensional setup

Here: two extra dimensions, compactified on a torus

- Torus specified by the volume $\mathcal{A}$ and shape τ



$$\mathcal{A} = L_1 L_2 \sin\theta$$
$$\tau = L_2/L_1 e^{i\theta}$$

Orbifold Compactification

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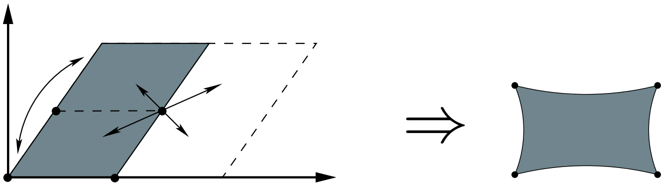
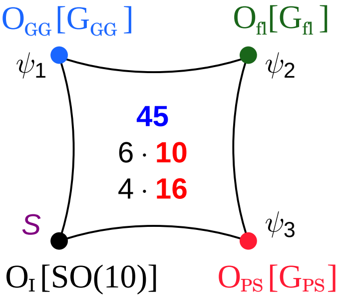
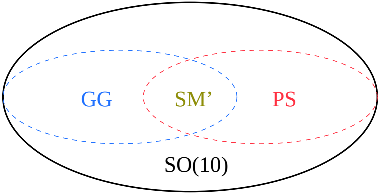


- Impose symmetry $\mathbb{Z}_2 : \; y \to -y$
- Values for τ, $\mathcal{A}$?
- Casimir energy of bulk fields induces nontrivial potential
- Supersymmetry $\Rightarrow$ vanishing Casimir energy $\Rightarrow$ SUSY breaking

Gaugino Mediation in a 6D Orbifold GUT Model

Asaka, Buchmüller, Covi, Phys. Lett. **B563** (2003)





Gaugino Mediation

Kaplan, Kribs, Schmaltz, Phys. Rev. **D62** (2000)
Chacko, Luty, Nelson, Ponton, JHEP **01** (2000)

In general: soft masses for all bulk fields

Gaugino masses: $m_g = \frac{\lambda \mu}{\Lambda^2 \mathcal{A}}$ Scalar masses: $m_H^2 = -\frac{\lambda' \mu^2}{\Lambda^2 \mathcal{A}}$

Casimir Energy

- Consider one-loop Casimir energy of a real scalar field
- Geometry: $T^2/\mathbb{Z}_2^3$
- Fields can be either even or odd wrt a $\mathbb{Z}_2$ symmetry
- Only fields which couple to SUSY breaking brane contribute
- Boundary conditions encoded in $\alpha, \beta \in \{0, 1/2\}$

$\Rightarrow$ Four different contributions,

$$V_M^{\alpha,\beta} = \frac{1}{2} \left[\sum\right]_{m,n}^{(\alpha,\beta)} \int \frac{d^4 k_E}{(2\pi)^4} \log\left(k_E^2 + \mathcal{M}_{m,n}^2 + M^2\right)$$

$$\mathcal{M}_{m,n}^2 = \frac{4(2\pi)^2}{\mathcal{A}\tau_2} |n + \beta - \tau(m+\alpha)|^2$$

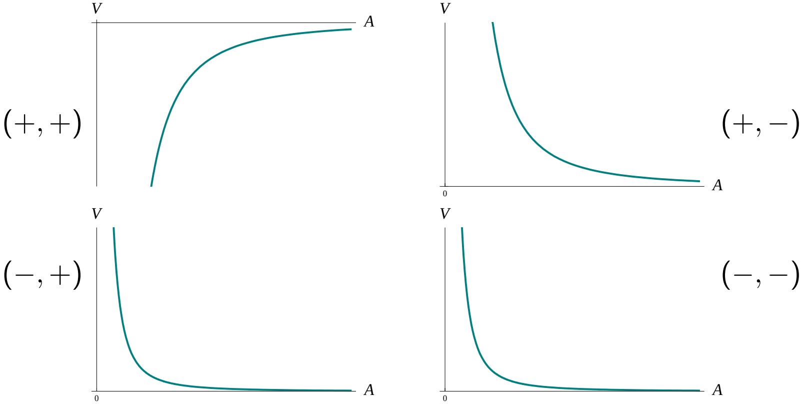
- Zeta function regularisation

Casimir Energy

$$
\begin{aligned}
V_M^{\alpha,\beta} = &+ \frac{M^6 \mathcal{A}}{3072\pi^3}\left[\frac{11}{12} - \log\left(\frac{M}{\mu_r}\right)\right] \\
&- \frac{M^4}{64\pi^2}\left[\frac{3}{4} - \log\left(\frac{M}{\mu_r}\right)\right]\delta_{\alpha 0}\delta_{\beta 0} \\
&- \frac{M^3 \tau_2^{3/2}}{4\pi^3 \mathcal{A}^{1/2}} \sum_{p=1}^{\infty} \frac{\cos(2\pi p \alpha)}{p^3} K_3\left(p \frac{\sqrt{\mathcal{A}} M}{2\sqrt{\tau_2}}\right) \\
&- \frac{32}{\mathcal{A}^2 \tau_2^2} \sum_{p=1}^{\infty}\sum_{m=0}^{\infty} \frac{1}{2^{\delta_{\alpha 0}\delta_{m0}}} \frac{\cos(2\pi p(\beta - (m+\alpha)\tau_1))}{p^{5/2}} \left(\tau_2^2 (m+\alpha)^2 + \frac{\mathcal{A}\tau_2 M^2}{(4\pi)^2}\right)^{\frac{5}{4}} \\
&\quad K_{5/2}\left(2\pi p \sqrt{\tau_2^2(m+\alpha)^2 + \frac{\mathcal{A}\tau_2 M^2}{(4\pi)^2}}\right)
\end{aligned}
$$

- Dependence on regularization scale μ_r remnant of divergent bulk and brane cosmological terms

Casimir Energy - Volume



- Sign and strength of Casimir force depends on boundary conditions
- General potential: $a\,V^{(+,+)} + b\,V^{(+,-)} + c\,V^{(-,+)} + d\,V^{(-,-)}$

Casimir Energy

- Analytical behaviour for small volume with τ_1 and τ_2 in the minimum:

$$V_M^{(0,0)}(\tau_1 = \tfrac{1}{2}, \tau_2 = \tfrac{1}{2}, \mathcal{A}) \simeq -\frac{4\pi^3}{945\mathcal{A}^2} + \frac{\pi M^2}{360\mathcal{A}} + \mathcal{O}(M^4)$$

- Contributions for bosons and fermions come with opposite sign
$\Rightarrow$ Leading term cancels within supermultiplet
- $M^2 = M^2_{\text{SUSY}} + m^2_{\text{soft}}$
- Leading term in supermultiplet $\propto m^2_{\text{soft}}$

Casimir Energy in the Orbifold GUT Model

- Can neglect contribution from vector multiplet $\rightarrow$ Hypermultiplets
- Example: H_3 and H_4

SM$'$	$(\mathbf{1},\mathbf{2};-\frac{1}{2},-2)$		$(\mathbf{1},\mathbf{2};\frac{1}{2},2)$		$(\overline{\mathbf{3}},\mathbf{1};\frac{1}{3},-2)$		$(\mathbf{3},\mathbf{1};-\frac{1}{3},2)$	
	$\mathbb{Z}_2^{ps}$	$\mathbb{Z}_2^{GG}$	$\mathbb{Z}_2^{ps}$	$\mathbb{Z}_2^{GG}$	$\mathbb{Z}_2^{ps}$	$\mathbb{Z}_2^{GG}$	$\mathbb{Z}_2^{ps}$	$\mathbb{Z}_2^{GG}$
H_3	$-$	$+$	$-$	$-$	$+$	$+$	$+$	$-$
H_4	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$+$

$$
\begin{aligned}
V_H &= 12\left(V_{m_H}^{(0,0)} - V^{(0,0)}\right) + 12\left(V_{m_H}^{(0,1/2)} - V^{(0,1/2)}\right) \\
&\quad +8\left(V_{m_H}^{(1/2,0)} - V^{(1/2,0)}\right) + 8\cdot\left(V_{m_H}^{(1/2,1/2)} - V^{(1/2,1/2)}\right) \\
&\simeq -\frac{\pi}{36}\frac{\mu^2\lambda'}{\Lambda^2\mathcal{A}^2}
\end{aligned}
$$

Casimir Energy in the Orbifold GUT Model

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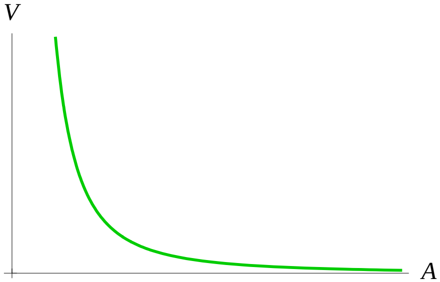


$\Rightarrow$ Can achieve repulsive force at short distances
But: Need additional ingredient for stabilisation

Breaking of $U(1)_X$

- 4D gauge symmetry: $G_{SM'} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_X$
- Vev $\langle\Phi\rangle$ breaks the additional $U(1)_X$
  $\Rightarrow$ Bulk mass $M \sim g_6\langle\Phi\rangle$
- Quantum corrections generically induce Fayet-Iliopoulos terms at the fixed points
  Lee, Nilles, Zucker, Nucl.Phys.B **680** (2004)
  Buchmüller, Lüdeling, Schmidt, JHEP **0709** (2007)
- Localised FI terms can induce vev for bulk fields in turn
- D-flatness implies $\mathcal{A}\langle\Phi\rangle^2 \sim C\,\Lambda^2$, $C \ll 1$

Volume Stabilisation

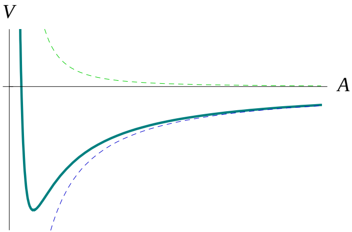
- Classical contribution to the vacuum energy density

$$V^{(0)} = -\frac{\lambda''}{\Lambda^4}\int d^4\theta \langle S^\dagger S \Phi^\dagger \Phi\rangle$$
$$\simeq -\lambda'' \frac{\mu^2 C}{\mathcal{A}}$$

- attractive for $\lambda'' > 0$
- Combine with the repulsive Casimir energy

$$V_{\text{tot}} = V^{(0)} + V^{(1)} = -\frac{\pi}{36}\frac{\mu^2\lambda'}{\Lambda^2\mathcal{A}^2} - \frac{\lambda''\mu^2 C}{\mathcal{A}}$$

Volume Stabilisation

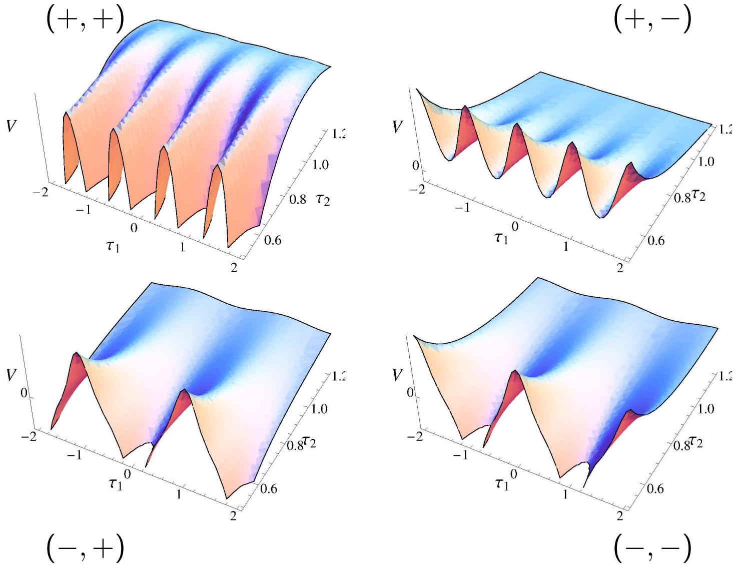


- Stable minimum at

$$\mathcal{A}_{\text{min}} = -\frac{\pi\lambda'}{36\lambda''}\frac{1}{M^2} \lesssim \frac{1}{M^2}$$

- Independent of supersymmetry breaking scale μ^2
- Cosmological constant has to be tuned to zero by a brane cosmological term

Casimir Energy - Shape

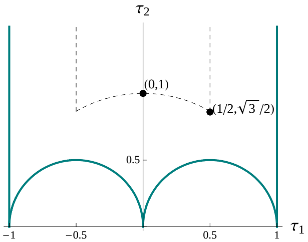


Casimir Energy - Shape

- Casimir energy invariant under modular transformations

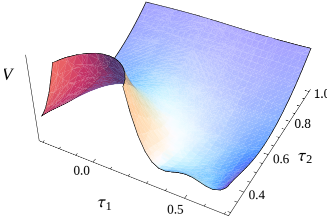
$$\tau \to \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1$$

- For boundary conditions $(+,+)$ $a, b, c, d \in \mathbb{Z} \Rightarrow SL(2, \mathbb{Z})$
- For other boundary conditions: subgroups of $SL(2, \mathbb{Z})$
- For a general potential: $a, c = 1 \bmod 2, b, d = 0 \bmod 2 \Rightarrow \Gamma(2)$
- Fundamental domain



Casimir Energy - Shape

- Modular transformation can have fixed points which correspond to extrema in the effective potential
- In our case we have $c = 0 \bmod 2$ and $d = 1 \bmod 2$
- These transformations have a fixed point at $\tau_1 = \tau_2 = 1/2$ which corresponds to a minimum in the effective potential



- Equivalent to $R_1 = \sqrt{2} R_2$ and $\theta = \pi/4 \Rightarrow$ Root lattice of SO(5)
  $\Rightarrow$ Shape moduli stabilised at symmetry enhanced point

Summary Part 1

- Extra dimensions can be stabilised by interplay of Casimir energy and Fayet-Iliopoulos term
- Compactification scale naturally of $\mathcal{O}(M_{\mathrm{GUT}})$ independently of supersymmetry breaking scale μ^2
- Leads to consistent picture of Orbifold GUTs
- Shape moduli stabilised at symmetry enhanced points

... Part 2

(shorter!)

Motivation GTU

- Couplings in nature seem to come in two different classes: g, y_t are $\mathcal{O}(1)$, other Yukawas are suppressed
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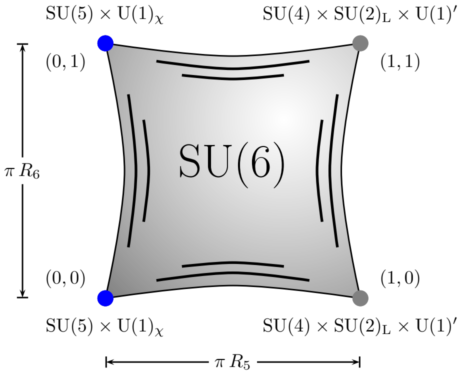
- Possible solution: Gauge-top unification

GUTs in extra dimensions

- Higher dimensional GUT with $\mathcal{M}_4 \times \mathbb{T}^2/\mathbb{Z}_2$ geometry



- First two generations $\rightarrow$ brane fields
- Third SM family lives in the bulk (split)
- Higgs doublet comes from the 6D gauge multiplet (V, Φ)

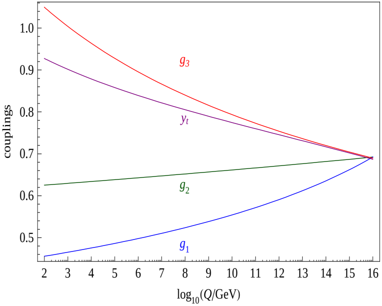
Gauge-top unification

- Setup results in

$$g\overline{u}_3 q_3 h_u$$

Buchmüller, Lüdeling, Schmidt, JHEP **0709** (2007)

- All other Yukawas are suppressed



We obtain the tree level relation $y_t = g$

Corrections to $y_t = g$

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- Diagonalization effects

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mathcal{O}(g) \end{pmatrix} + \begin{pmatrix} s^{n_{11}} & s^{n_{12}} & s^{n_{13}} \\ s^{n_{21}} & s^{n_{22}} & s^{n_{23}} \\ s^{n_{31}} & s^{n_{32}} & s^{n_{33}} \end{pmatrix}$$

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Leading effect

Main topic of Part 2!

Localization effects

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Localization effects

- Third family corresponds to zero modes in the bulk
  Usual assumption: flat profiles!
- Consider additional U(1) symmetry with $\mathrm{Tr}(q_I) \neq 0$ at different fixed points $\Rightarrow$ local FI term
- Bulk fields charged under this U(1) obtain non-trivial profile through the local FI term Lee, Nilles, Zucker, Nucl.Phys.B **680** (2004)
- Effect even occurs when the effective FI term in 4D vanishes $\Rightarrow$ Local effect!

Zero mode profile

- Zero mode profile:

$$\psi \simeq f \prod_I \left| \vartheta_1 \left(\frac{z - z_I}{2\pi} \middle| \tau \right) \right|^{\frac{1}{2\pi} g_6 q_\psi \xi_I}$$
$$\exp \left(-\frac{1}{8\pi^2 \tau_2} g_6 q_\psi \xi_I (\mathrm{Im}(z - z_I))^2 \right)$$

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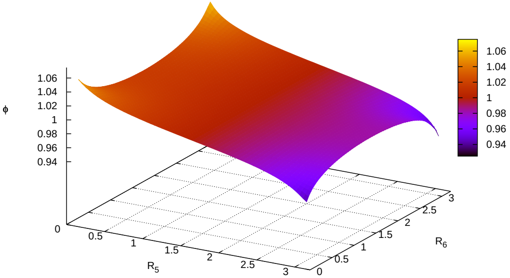
- q_ψ is the charge of the field under the considered $U(1)$
- ξ_I is the FI term:

$$\xi_I = \frac{1}{16\pi^2} g_6 \Lambda^2 \,\mathrm{Tr}(q_I), \quad \Lambda = \text{UV cutoff}$$

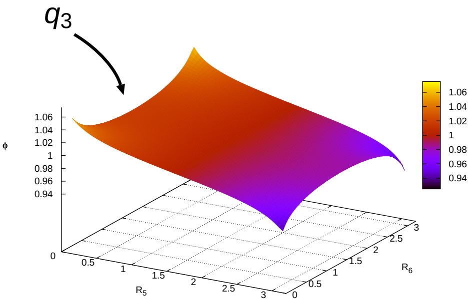
Zero mode profile



- Localization becomes more pronounced for larger q_ψ, q_l

Effect on $y_t = g$

- y_t and g are proportional to overlap integrals in the extra dimensions
- $y_t \sim \int d^2z \, h_u q_3 \overline{u}_3$



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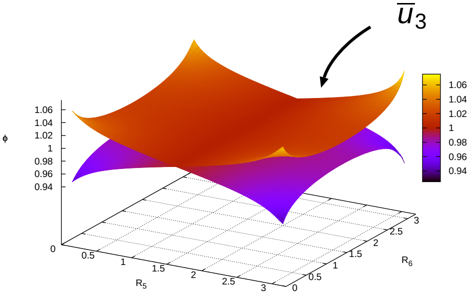

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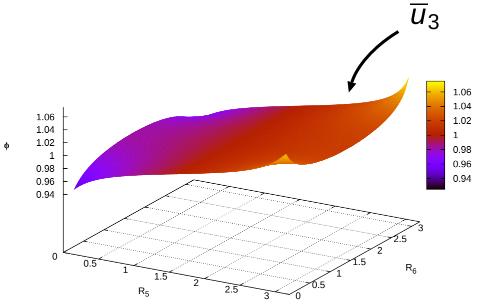
- $g \sim \int d^2z\, A \overline{u}_3 \overline{u}_3^\dagger$



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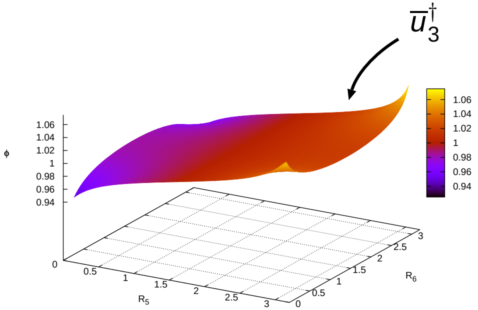

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Overlap integrals differ, $y_t < g$

Suppression of top Yukawa coupling

- The ratio y_t/g depends mainly on two features:
  - Charges under the U(1) $\Rightarrow$ model dependent
  - $R_5/R_6 \Rightarrow$ anisotropy of the extra dimensions
- We fix R_5 to be the inverse GUT scale

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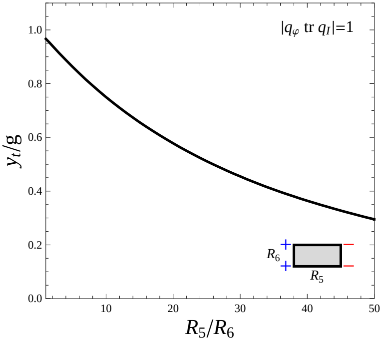
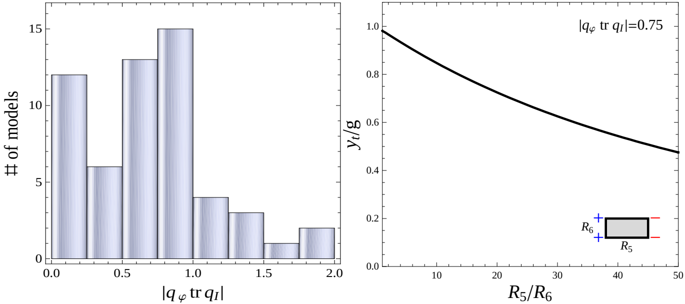


The heterotic MiniLandscape

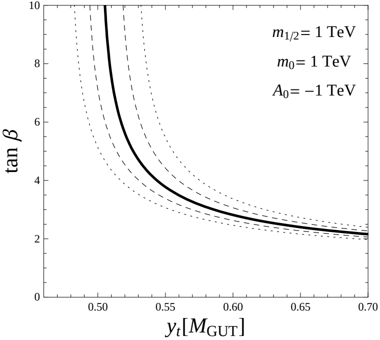
- Consider the heterotic MiniLandscape
- A large subset has gauge-top unification



- What does this imply?

Phenomenological implications: tan β

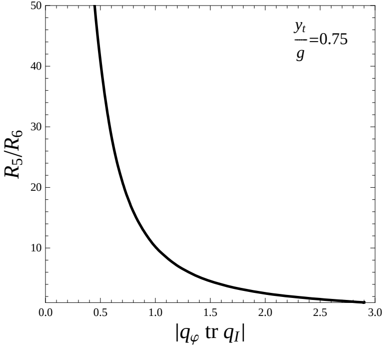
- y_t at the GUT scale $\Rightarrow$ related to tan β



$\Rightarrow$ Allowed values for tan β result in narrow range for y_t/g

Anisotropy of extra dimensions

- Given the charges the anisotropy is fixed by $y_t/g \sim 0.75$
- MiniLandscape: Anisotropic compactifications seem to be favored



Summary Part 2

- Gauge-top unification can explain why the top Yukawa coupling is large
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- Localization effects change the tree level relation to $y_t \lesssim g$
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Thank you for your attention!

Zeta function regularisation

$$V = -\frac{\mathrm{d}\zeta(s)}{\mathrm{d}s}\bigg|_{s=0}$$

where

$$\zeta(s) = \frac{1}{2}\left[\sum\right]_{m,n} \mu_r^{2s} \int \frac{\mathrm{d}^4 k_E}{(2\pi)^4} \left(k_E^2 + \frac{4}{R_z^2}\left[e^2(m+\alpha)^2 + (n+\beta)^2\right)\right] + M^2\right)^{-s}$$