Standard Model contribution to paramagnetic EDM and beyond

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University of Florida

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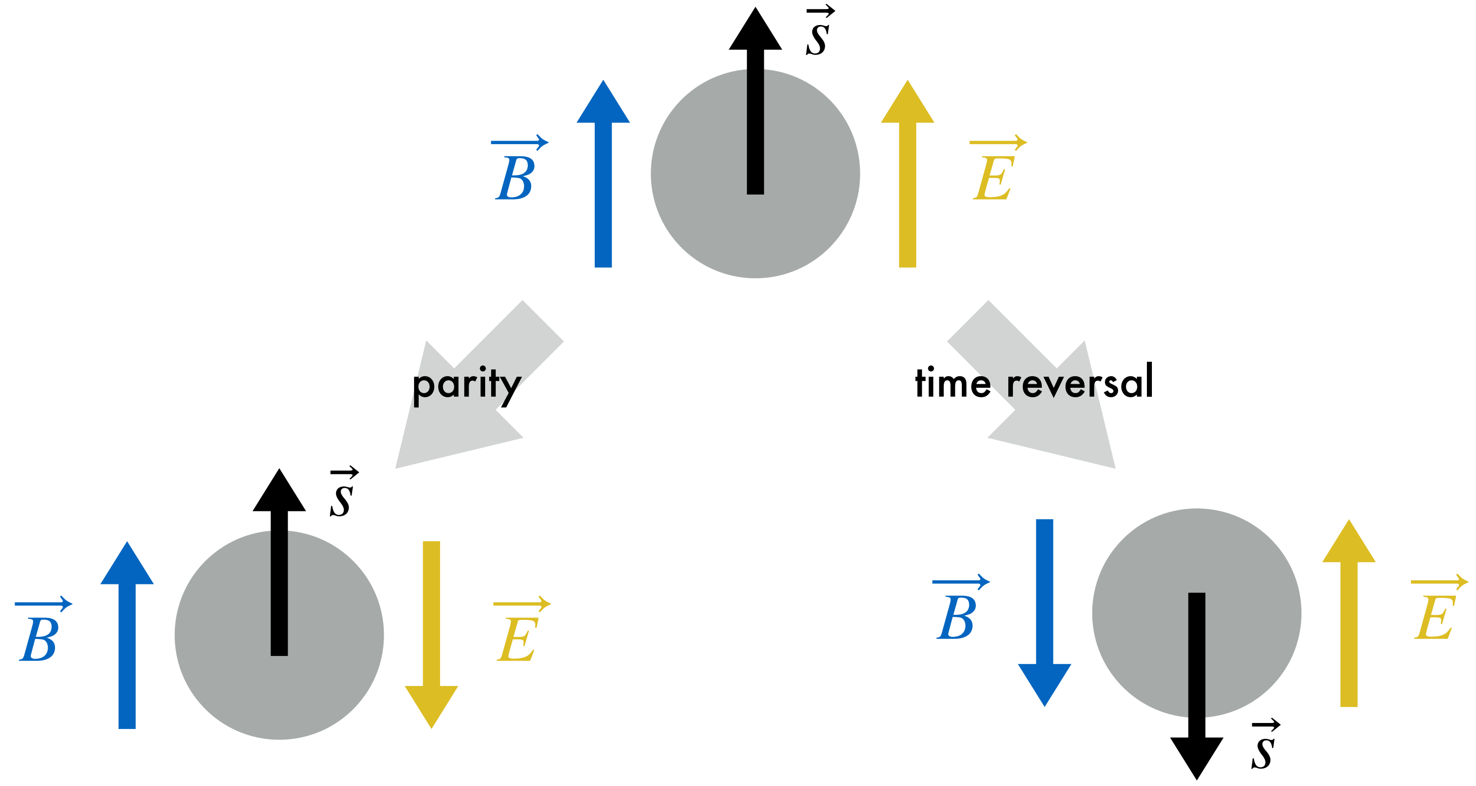
Based on works with Ting Gao, Maxim Pospelov, Adam Ritz

Electric dipole moment

- Electric dipole moment (EDM) violates P and T (= CP).

$$\mathscr{H} = -\vec{B}\cdot\vec{\mu} - \vec{E}\cdot\vec{d} = -2\vec{s}\cdot\left(\mu\vec{B}+d\vec{E}\right)$$

where μ : magnetic dipole moment, d : EDM.



	$\vec{B}$	$\vec{E}$	$\vec{s}$
P	$+$	$-$	$+$
T	$-$	$+$	$-$

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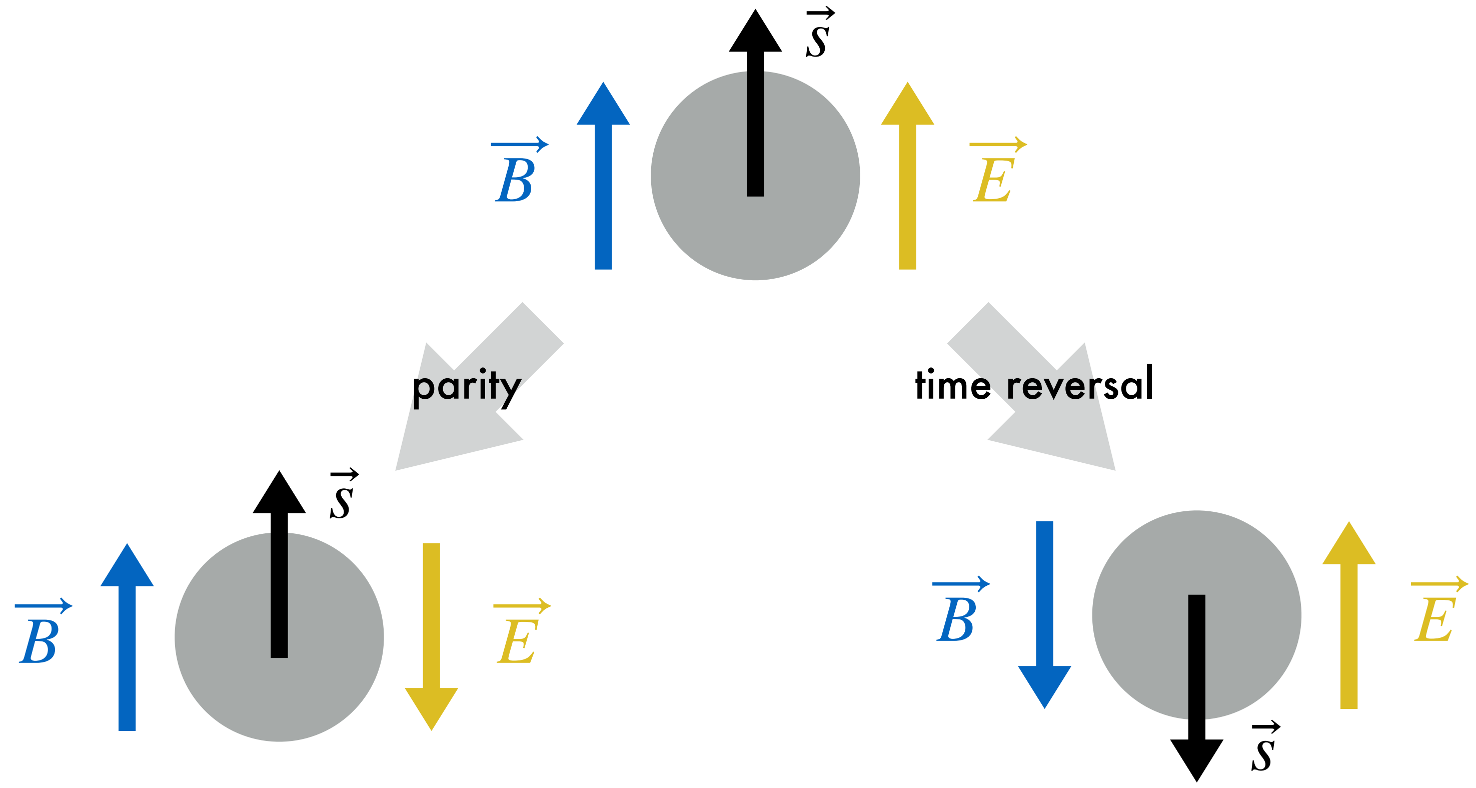


	$\vec{B}$	$\vec{E}$	$\vec{s}$
P	$+$	$-$	$+$
T	$-$	$+$	$-$

- Probe of CP violating physics well above LHC scale. [Roussay+ 22]

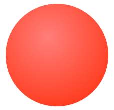
$$|d_e| < 4.1\times10^{-30}\,e\,\mathrm{cm} \;\Rightarrow\; \Lambda \gtrsim 80\,\mathrm{TeV} \;\text{ vs. }\; \Lambda_{\mathrm{LHC}} \gtrsim \mathcal{O}(1)\,\mathrm{TeV}$$

where $\frac{d_e}{e} \sim \frac{\alpha}{\pi}\frac{m_e}{\Lambda^2}$ assumed.

EDM experiments

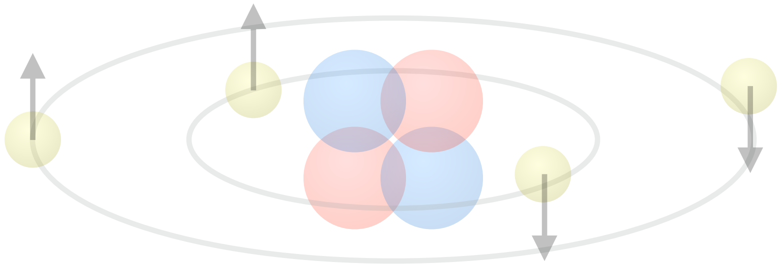
Impose electric field → charge-neutral system preferred.

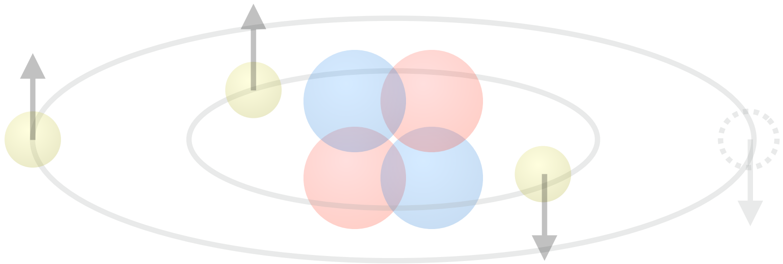
Neutron



- Sensitive to hadronic CP violation: $\theta_{\mathrm{QCD}},\ \ d_q,\ \ \cdots .$
- $|d_n| < 1.8 \times 10^{-26}\, e\,\mathrm{cm}\,.$ [PSI-nEDM 20] c.f. $e/m_n \sim 2 \times 10^{-14}\, e\,\mathrm{cm}\,.$

➡ $|\theta_{\mathrm{QCD}}| \lesssim 10^{-10}$: strong CP problem, can be solved by axion.





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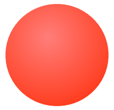

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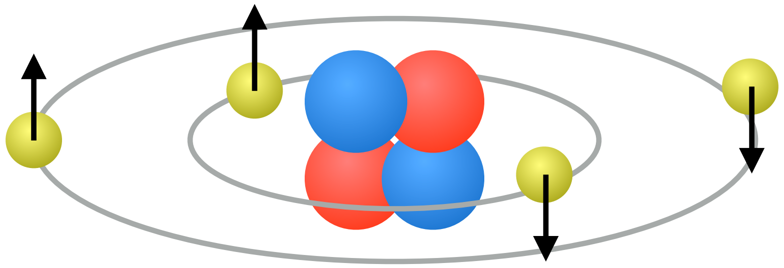
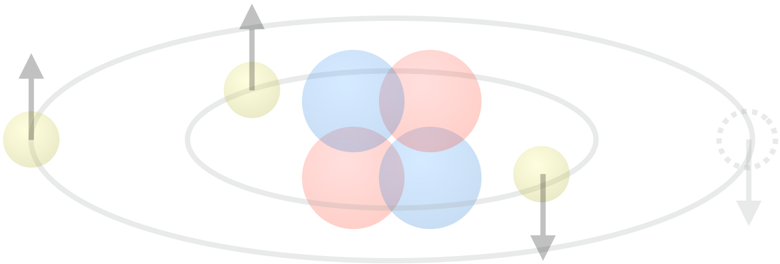
Diamagnetic atom (all electrons paired)



- Sensitive to hadronic CP violation.
- ${}^{199}{\rm Hg} : |d_{\rm Hg}| < 7.4 \times 10^{-30}\, e\,{\rm cm}\,.$ [Graner+ 16]



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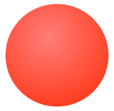

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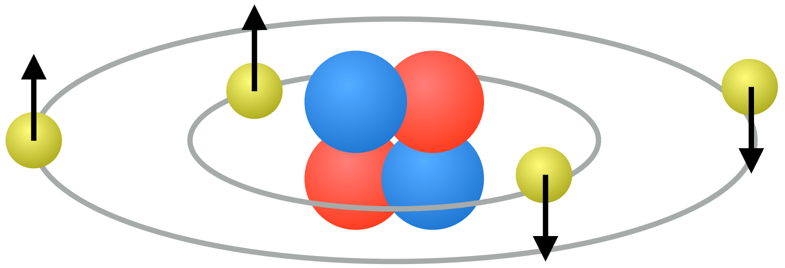

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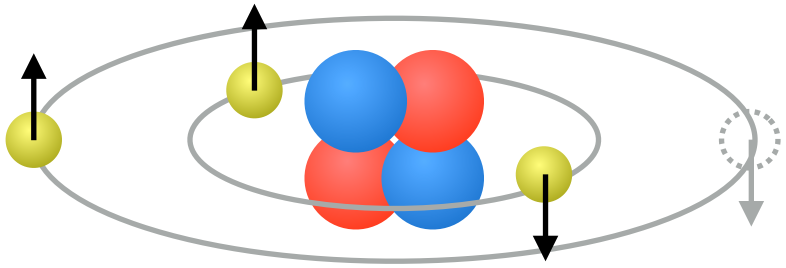
Paramagnetic atom/molecule (unpaired electron)



- Sensitive to leptonic CP violation.
- ${\rm ThO} : |d_e^{\rm (equiv)}| < 1.1 \times 10^{-29}\, e\, {\rm cm}\,.$ [ACME 18]
- ${\rm HfF}^+ : |d_e^{\rm (equiv)}| < 4.1 \times 10^{-30}\, e\, {\rm cm}\,.$ [Roussay+ 22]

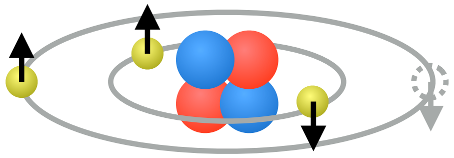
c.f. $e/m_e \sim 4 \times 10^{-11}\, e\, {\rm cm}\,.$

Today: CKM contribution to paramagnetic EDM

Atomic EDM

- Two types of contributions to atomic EDM: $\Delta\omega = \vec{E}_{\text{ext}} \cdot \vec{d}_A$ with

$$\vec{d}_A = \sum_k \left[\underbrace{\vec{d}_k}_{\text{direct}} + e_k \underbrace{\langle \Psi | \vec{r}_k | \Psi \rangle}_{\text{mixing}} \right].$$



Atomic wave function without P/CP violation:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left| \ominus \quad \oplus \right\rangle \pm \frac{1}{\sqrt{2}} \left| \oplus \quad \ominus \right\rangle \quad \Rightarrow \quad \langle \Psi | e\vec{E} \cdot \vec{r} | \Psi \rangle = 0.$$

Atomic wave function with P/CP violation:

$$|\Psi\rangle = \frac{1+\epsilon}{\sqrt{2}} \left| \ominus \quad \oplus \right\rangle \pm \frac{1-\epsilon}{\sqrt{2}} \left| \oplus \quad \ominus \right\rangle \quad \Rightarrow \quad \langle \Psi | e\vec{E} \cdot \vec{r} | \Psi \rangle \neq 0.$$

(screening theorem: cancellations of these two)

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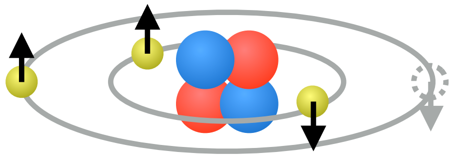

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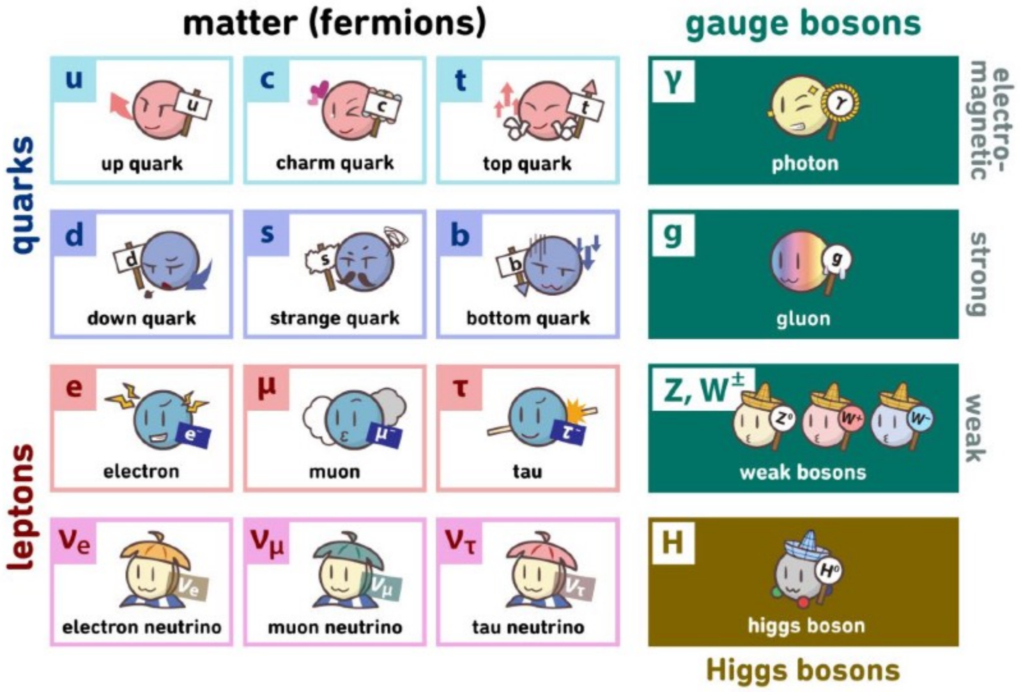
- Paramagnetic EDM sensitive to $\underbrace{\text{“}d_e \bar{e}\sigma \cdot \tilde{F} e\text{”}}_{\text{direct + mixing}}$ and $\underbrace{\text{“}C_S \bar{e} i\gamma_5 e \bar{N}N\text{”}}_{\text{mixing}}$.

$\Rightarrow$ “Equivalent” electron EDM $d_e^{(\text{equiv})} = d_e + C_S \times 1.5 \times 10^{-20}\, e\text{cm}$ e.g. for ThO.

SM contribution

CP violation in Standard Model

- CP violation $\simeq$ complex phase in the couplings.



[higgstan.com]

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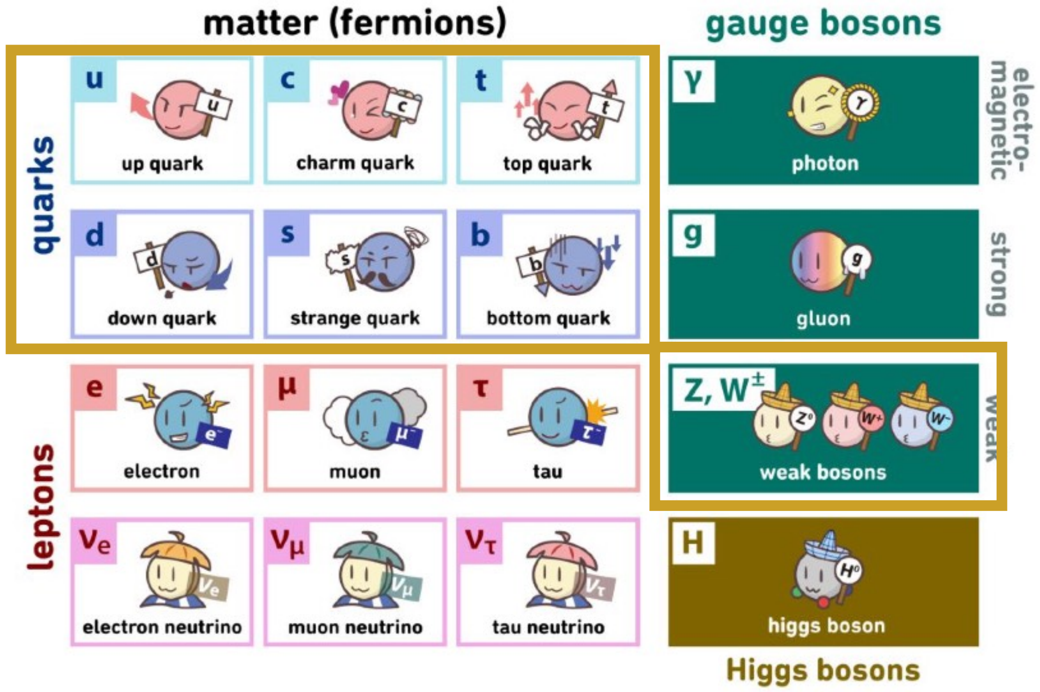
- Charged current in quark sector violates CP in SM:

$$\mathscr{L} = \frac{g}{2\sqrt{2}} V_{ij} W^{+}_{\mu} \bar{u}_i \gamma^{\mu} (1 - \gamma_5) d_j + (\mathrm{h.c.})$$

where V_{ij} : CKM matrix.



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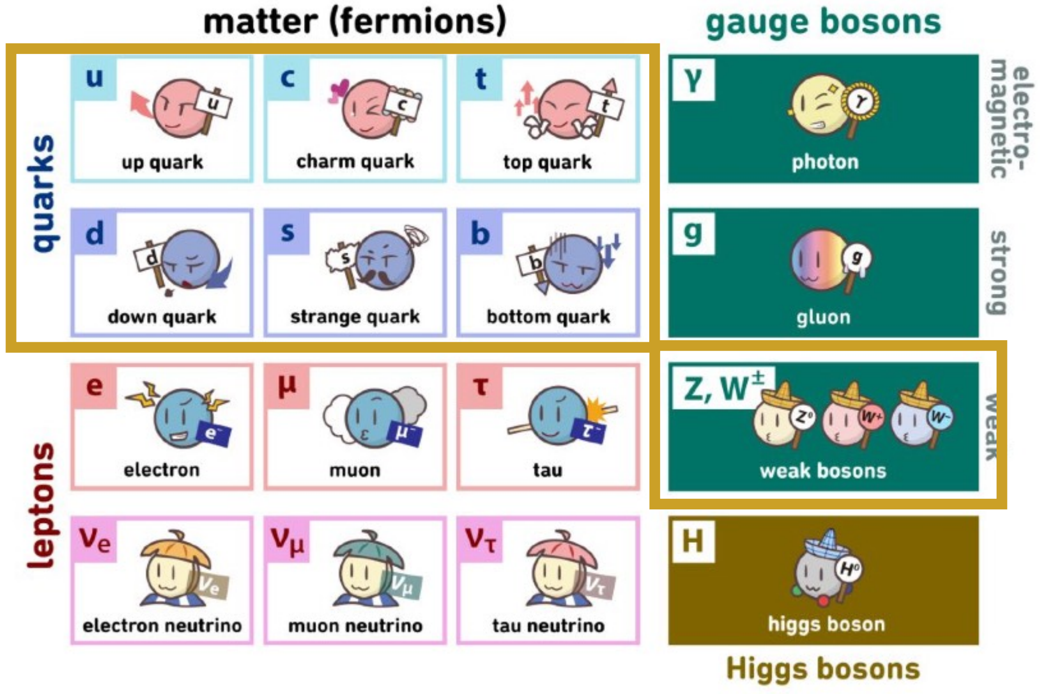

[higgstan.com]

- CP violation with $N = 3$ generations of quarks: CKM phase.

$$(\text{\# of physical phase}) = N^2 \;-\; \frac{1}{2}N(N-1) \;-\; (2N-1) = \frac{1}{2}(N-1)(N-2)\,.$$

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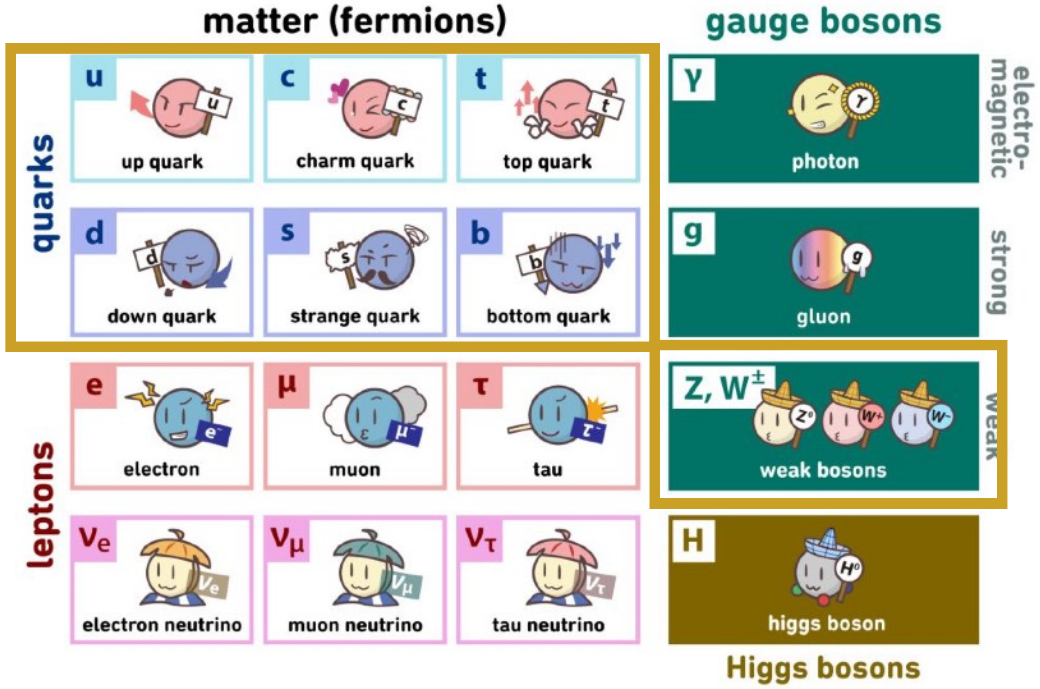

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$N \times N$ matrix

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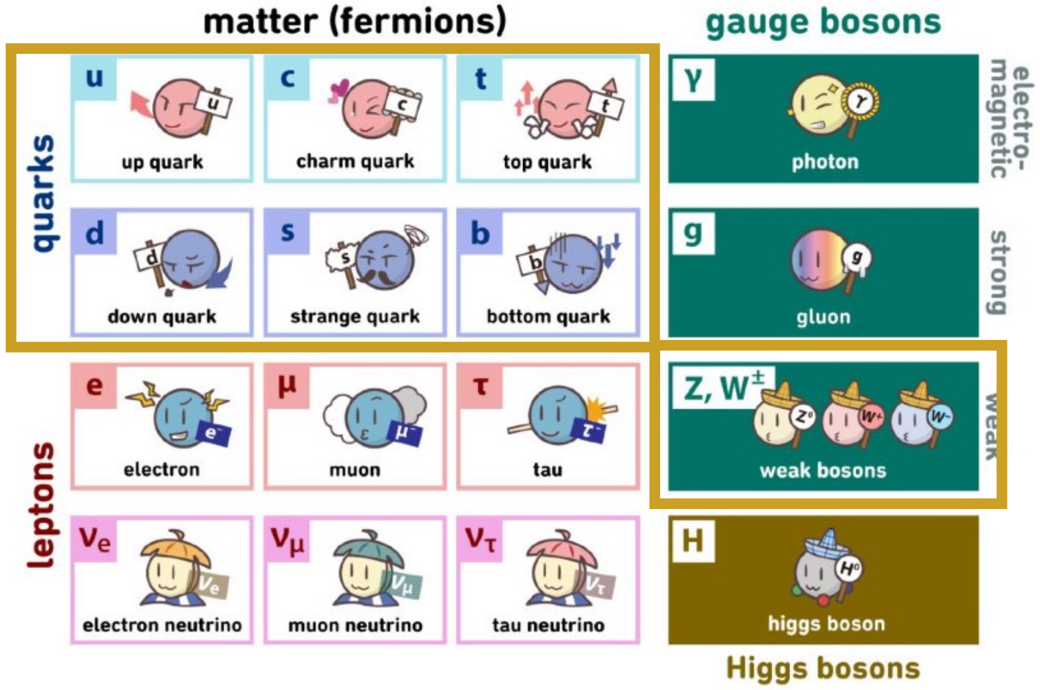

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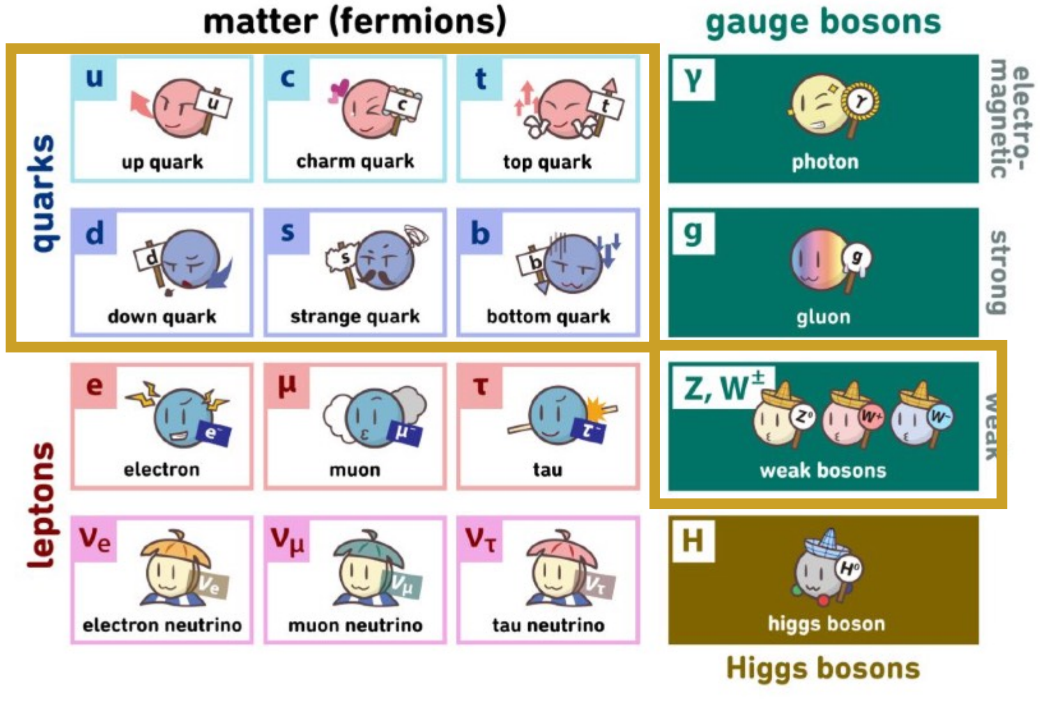

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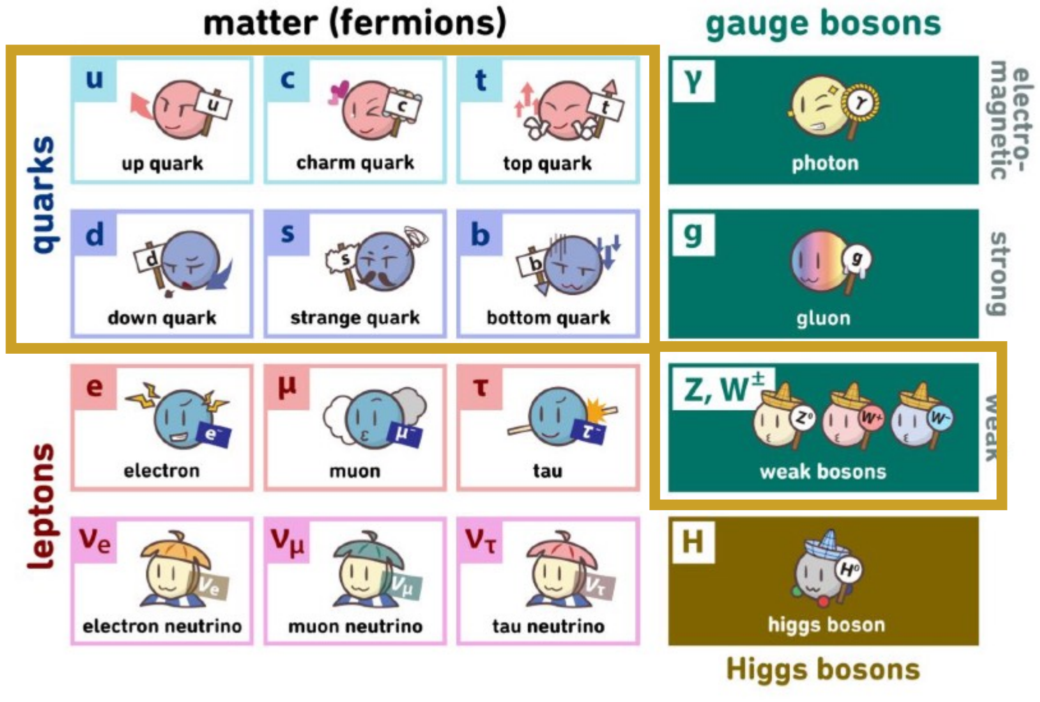

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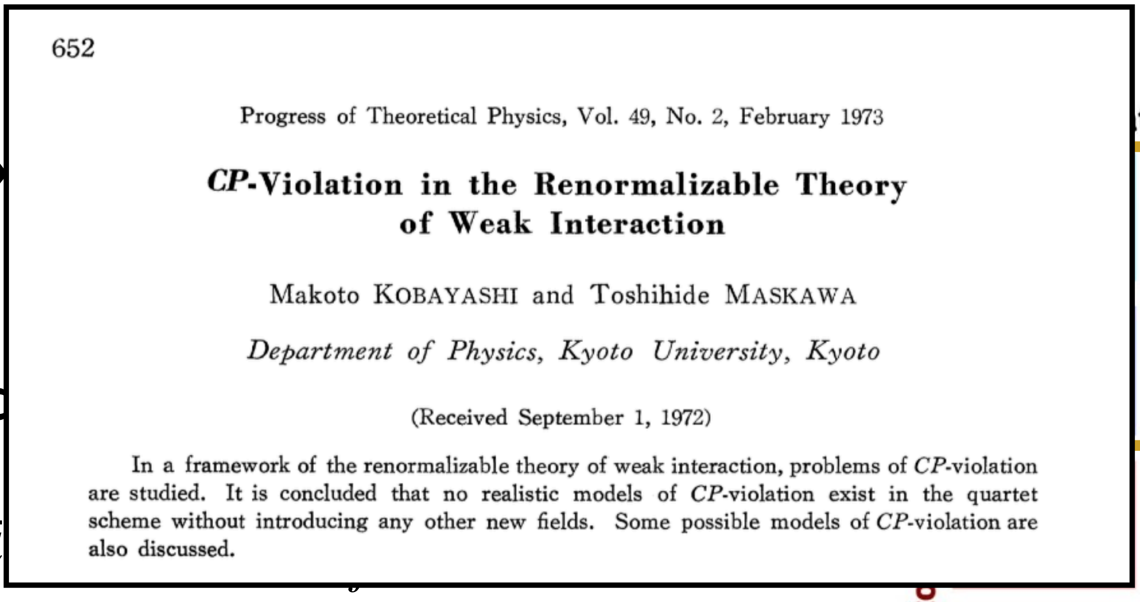

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Progress of Theoretical Physics, Vol. 49, No. 2, February 1973

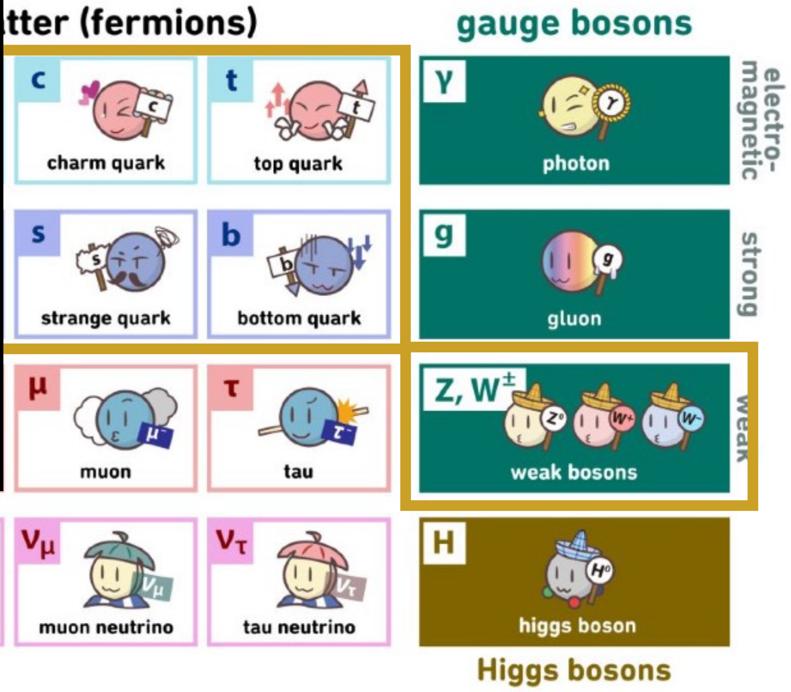
CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.



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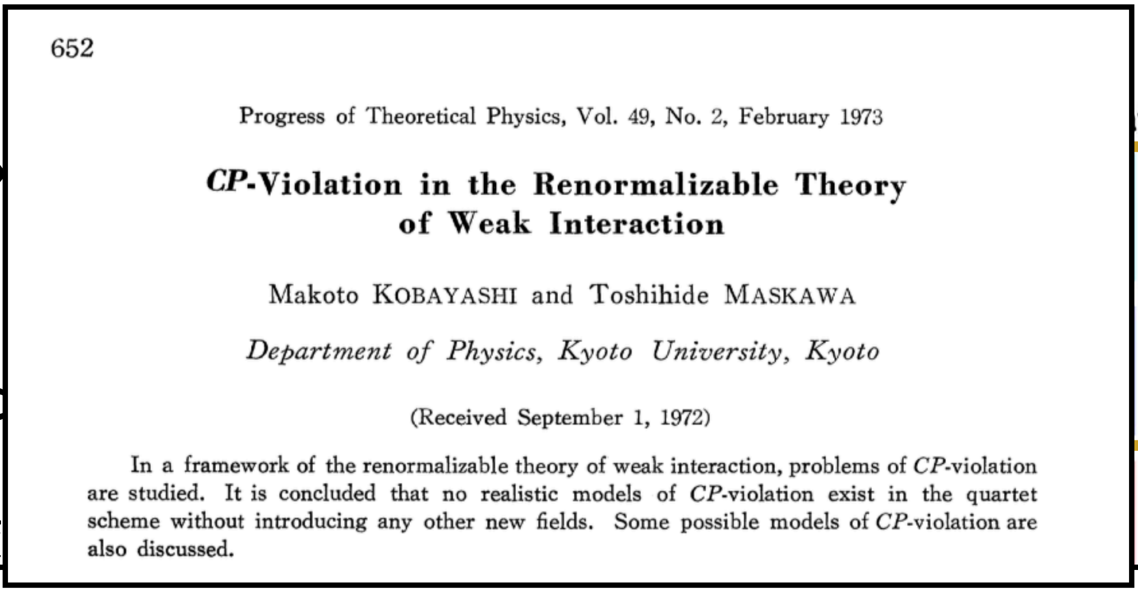
(# of physical pha $(N-1)(N-2)\,.$



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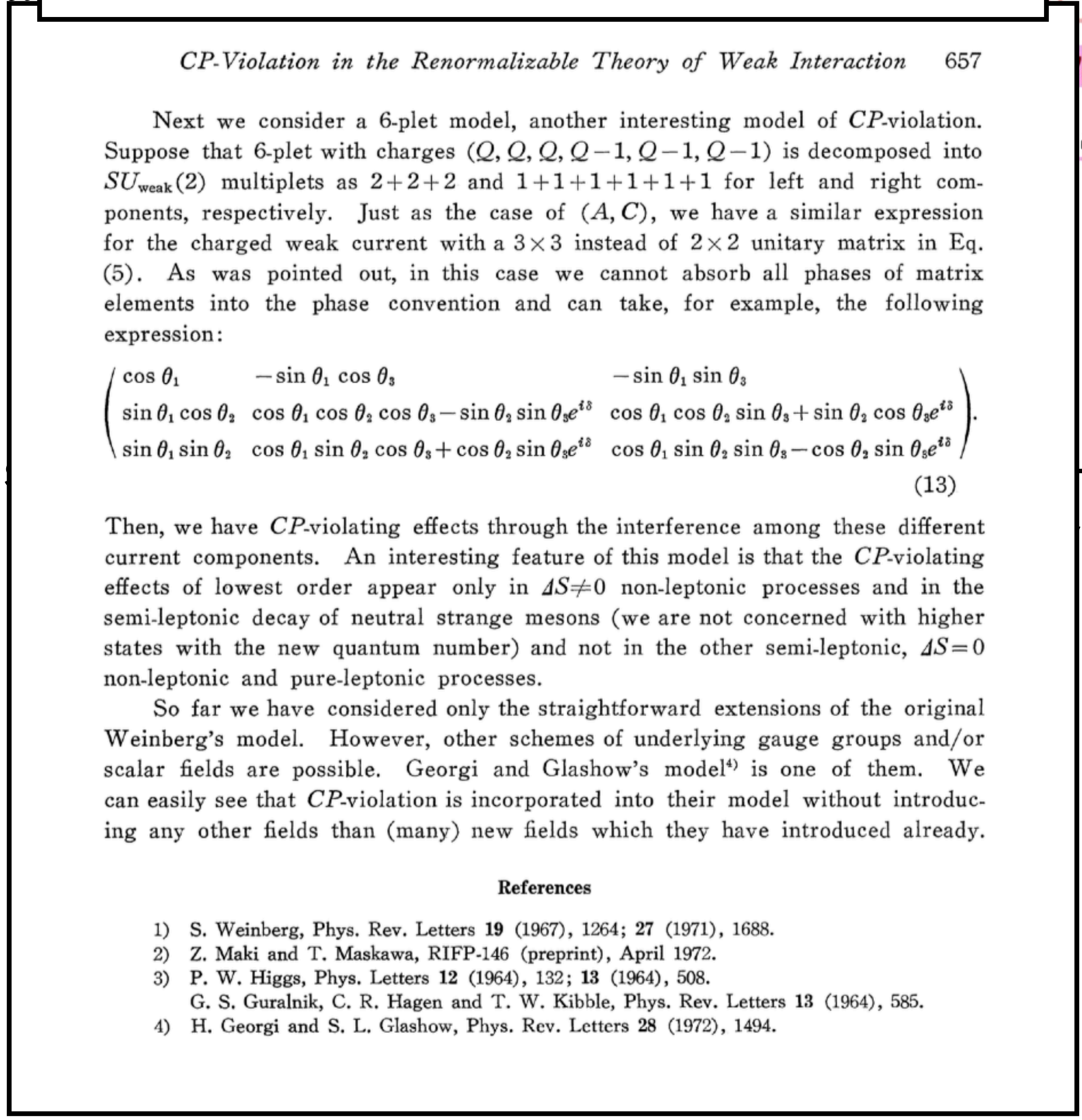


CP-Violation in the Renormalizable Theory of Weak Interaction 657

Next we consider a 6-plet model, another interesting model of *CP*-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

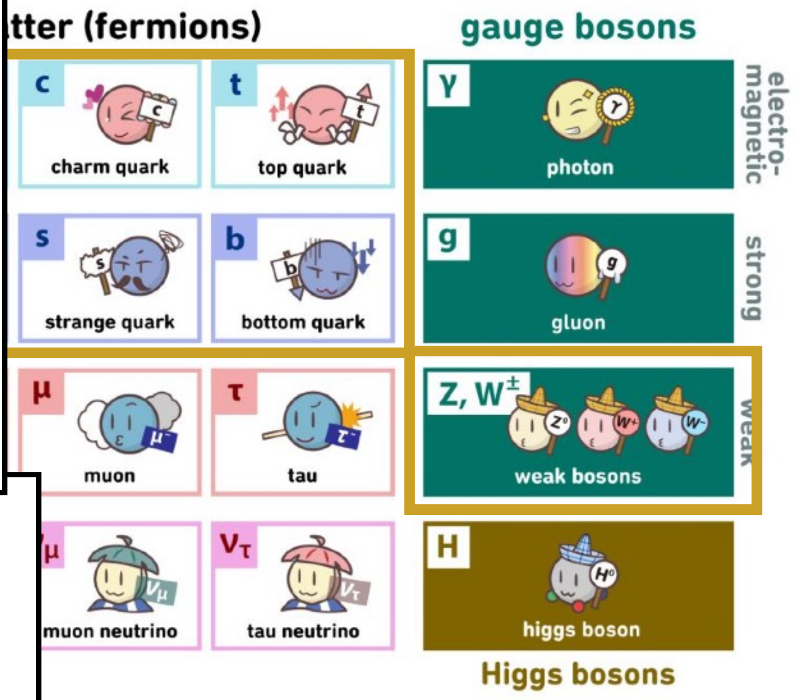
$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \tag{13}$$

Then, we have *CP*-violating effects through the interference among these different current components. An interesting feature of this model is that the *CP*-violating effects of lowest order appear only in $\Delta S\neq0$ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, $\Delta S=0$ non-leptonic and pure-leptonic processes.

So far we have considered only the straightforward extensions of the original Weinberg's model. However, other schemes of underlying gauge groups and/or scalar fields are possible. Georgi and Glashow's model[4] is one of them. We can easily see that *CP*-violation is incorporated into their model without introducing any other fields than (many) new fields which they have introduced already.

References

1) S. Weinberg, Phys. Rev. Letters **19** (1967), 1264; **27** (1971), 1688.
2) Z. Maki and T. Maskawa, RIFP-146 (preprint), April 1972.
3) P. W. Higgs, Phys. Letters **12** (1964), 132; **13** (1964), 508.
G. S. Guralnik, C. R. Hagen and T. W. Kibble, Phys. Rev. Letters **13** (1964), 585.
4) H. Georgi and S. L. Glashow, Phys. Rev. Letters **28** (1972), 1494.



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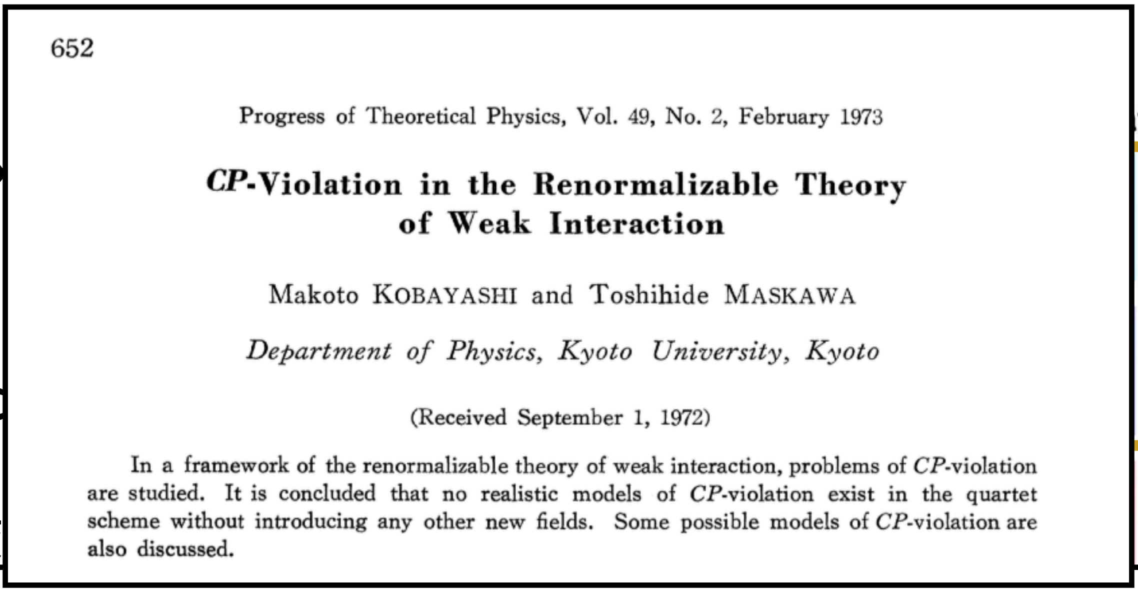

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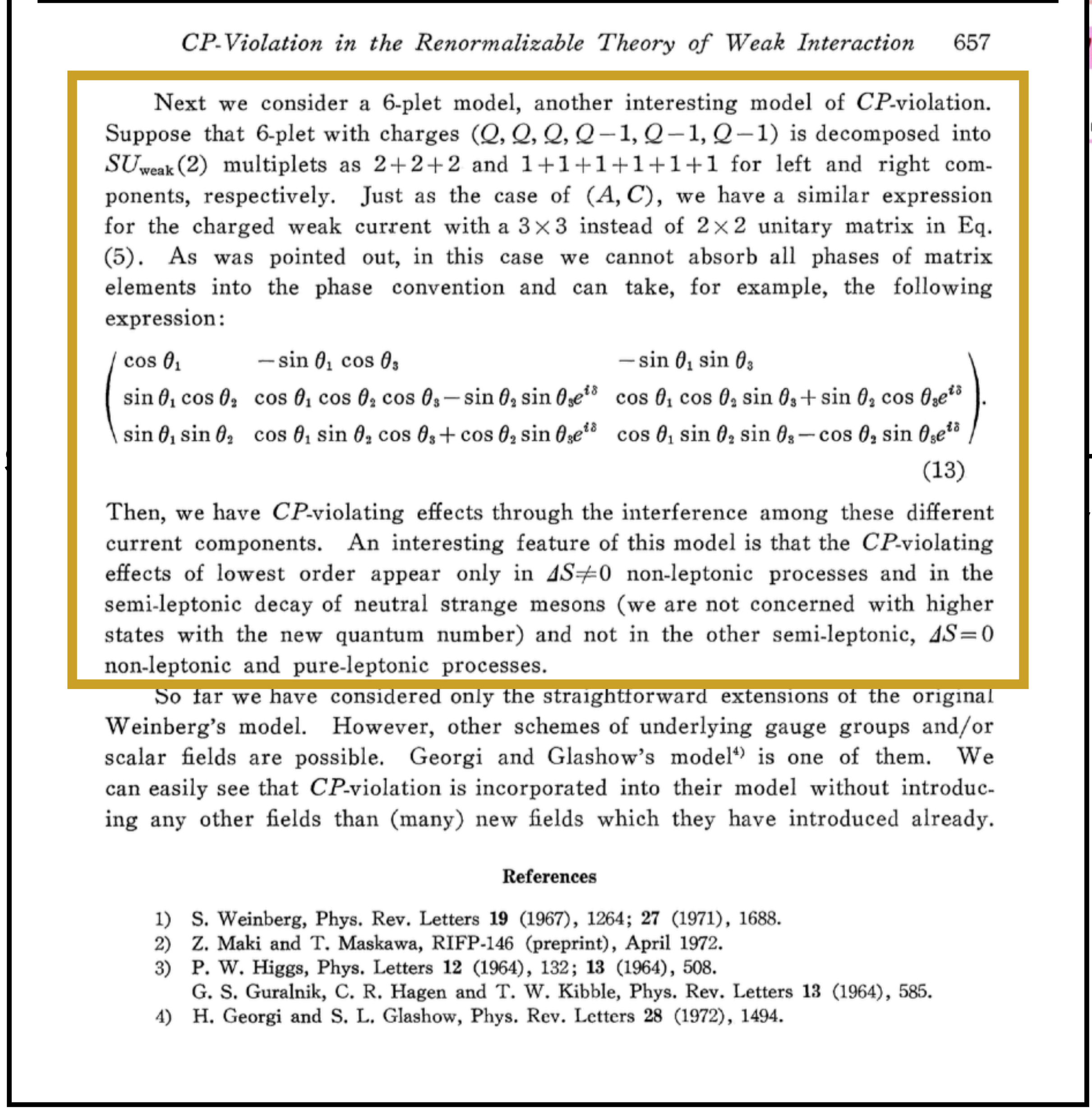

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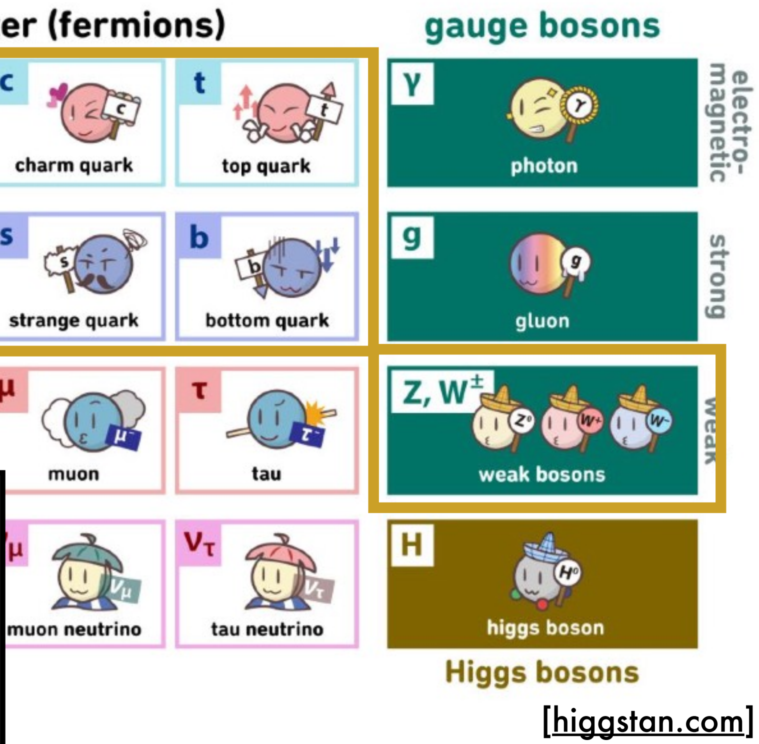
References

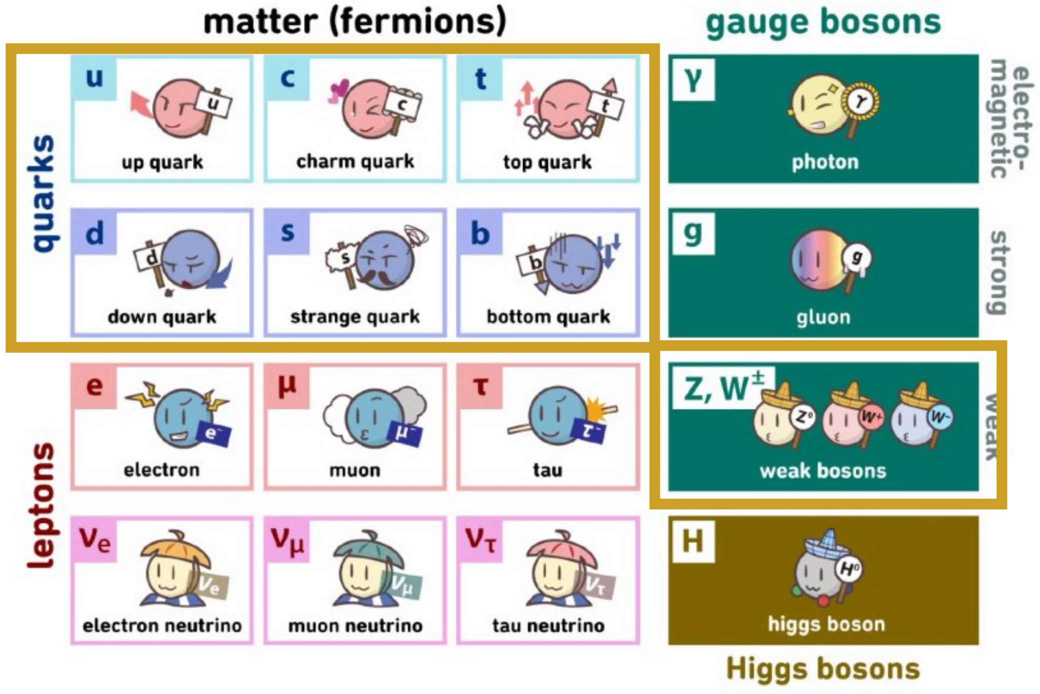
1) S. Weinberg, Phys. Rev. Letters **19** (1967), 1264; **27** (1971), 1688.
2) Z. Maki and T. Maskawa, RIFP-146 (preprint), April 1972.
3) P. W. Higgs, Phys. Letters **12** (1964), 132; **13** (1964), 508.
G. S. Guralnik, C. R. Hagen and T. W. Kibble, Phys. Rev. Letters **13** (1964), 585.
4) H. Georgi and S. L. Glashow, Phys. Rev. Letters **28** (1972), 1494.



[higgstan.com]

CP violation in Standard Model

- CP violation $\simeq$ complex phase in the couplings.



[higgstan.com]

- Charged current in quark sector violates CP in SM:

$$\mathscr{L} = \frac{g}{2\sqrt{2}} V_{ij} W_\mu^+ \bar{u}_i \gamma^\mu (1-\gamma_5) d_j + (\mathrm{h.c.})$$

where V_{ij} : CKM matrix.

- CP violation with $N=3$ generations of quarks: CKM phase.

$$(\text{\# of physical phase}) = N^2 - \frac{1}{2}N(N-1) - (2N-1) = \frac{1}{2}(N-1)(N-2)\,.$$

$N \times N$ matrix

$V^\dagger V = 1$

phase rotation

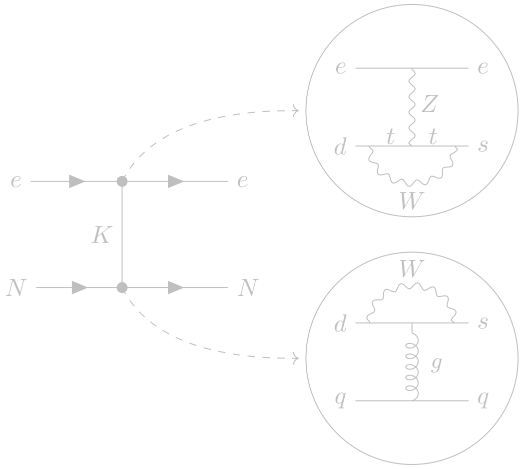
Need to pick up all three generations ("$m_W^{-4} \sim G_F^2$" at least)

"Jarlskog invariant" $J = \mathrm{Im}[V_{ts}^* V_{td} V_{ud}^* V_{us}] \simeq 3.08 \times 10^{-5}$.

SM value of paramagnetic EDM

$$\boxed{d_e^{(\mathrm{equiv})} = d_e + C_S \times 1.5 \times 10^{-20}\, e\,\mathrm{cm} \text{ for } \mathrm{ThO}}$$

- SM prediction of electron EDM: $d_e \sim 6 \times 10^{-40}\, e\,\mathrm{cm}\,.$ [Yamaguchi, Yamanaka 20]



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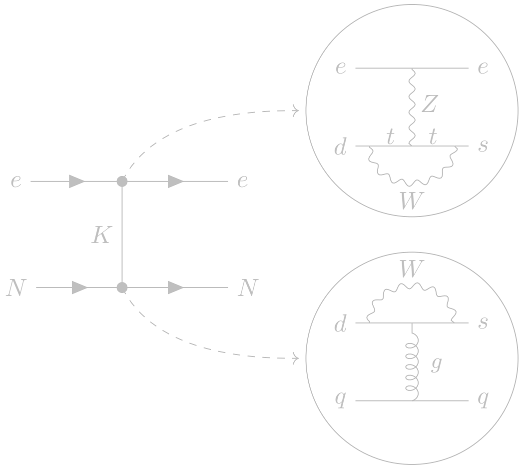
  coming from two photon exchange "$G_F^2 \times \alpha^2$." [Pospelov, Ritz 13]

Is there any contribution without "α^2"?



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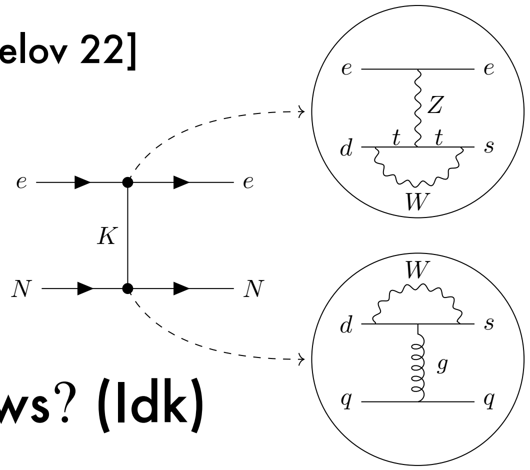
$$\boxed{\text{Is there any contribution without “}\alpha^2\text{”?}}$$

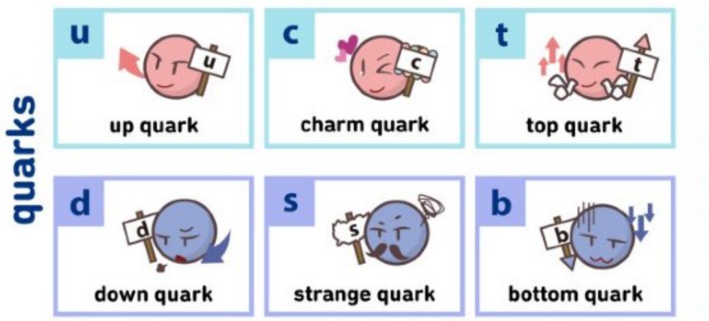
Yes, from Kaon exchange: $d_e^{(\mathrm{equiv})} = 1.0 \times 10^{-35}\, e\,\mathrm{cm}\,.$ [YE, Gao, Pospelov 22]

Formally “G_F^3” but numerically “$G_F^3 \times m_t^2 \sim G_F^2$”.



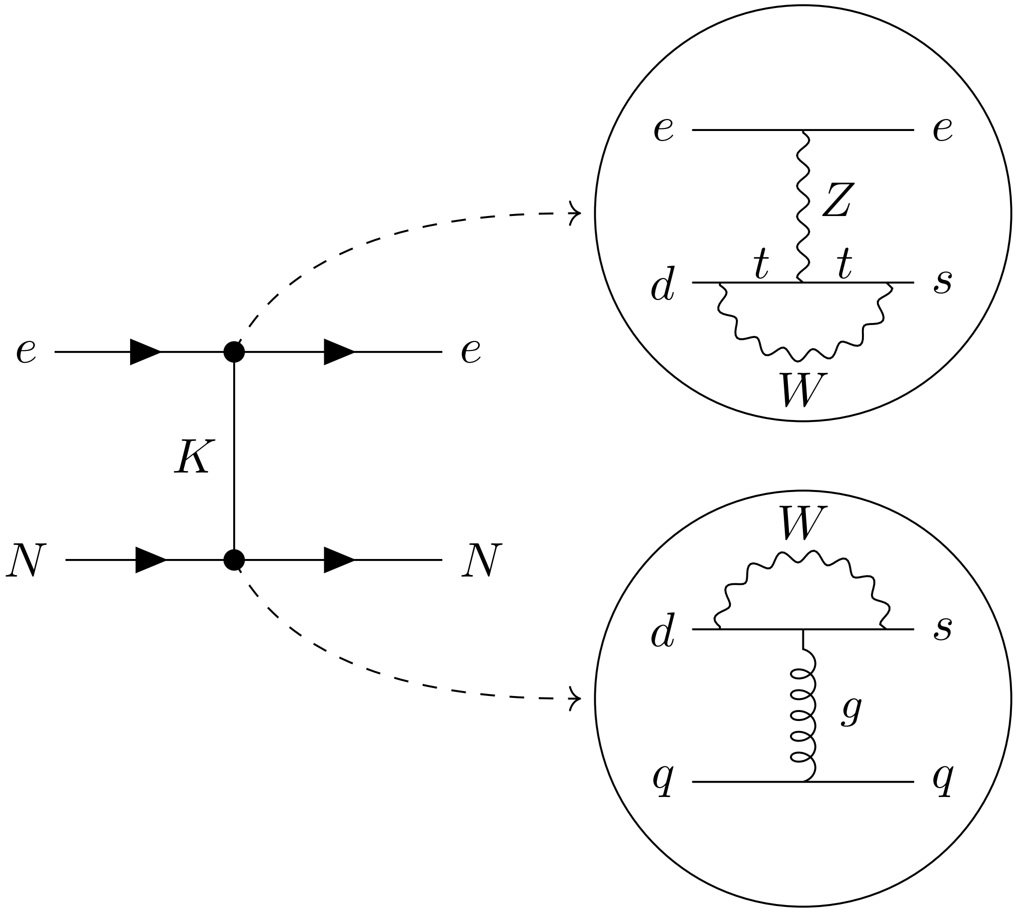
Still well below current experimental sensitivity $\gtrsim 10^{-30}\, e\,\mathrm{cm}$, but who knows? (Idk)

Kaon exchange



- Kaon exchange diagram picks up all three generations:

$$C_S \frac{G_F}{\sqrt{2}} \bar{e} i \gamma_5 e \times \bar{N} N =$$



Electroweak penguin "$G_F^2 \times m_t^2$".

$B_{s,d} \to \mu^+ \mu^-, \ldots$ [Inami, Lim 81; ...]

Strong penguin "G_F".

Weak hyperon decay $\Lambda \to N\pi, \cdots$.

[Bijnens+ 85; Jenkins 91; ...]

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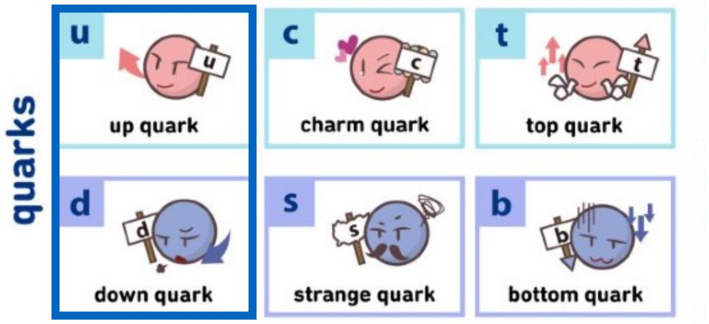

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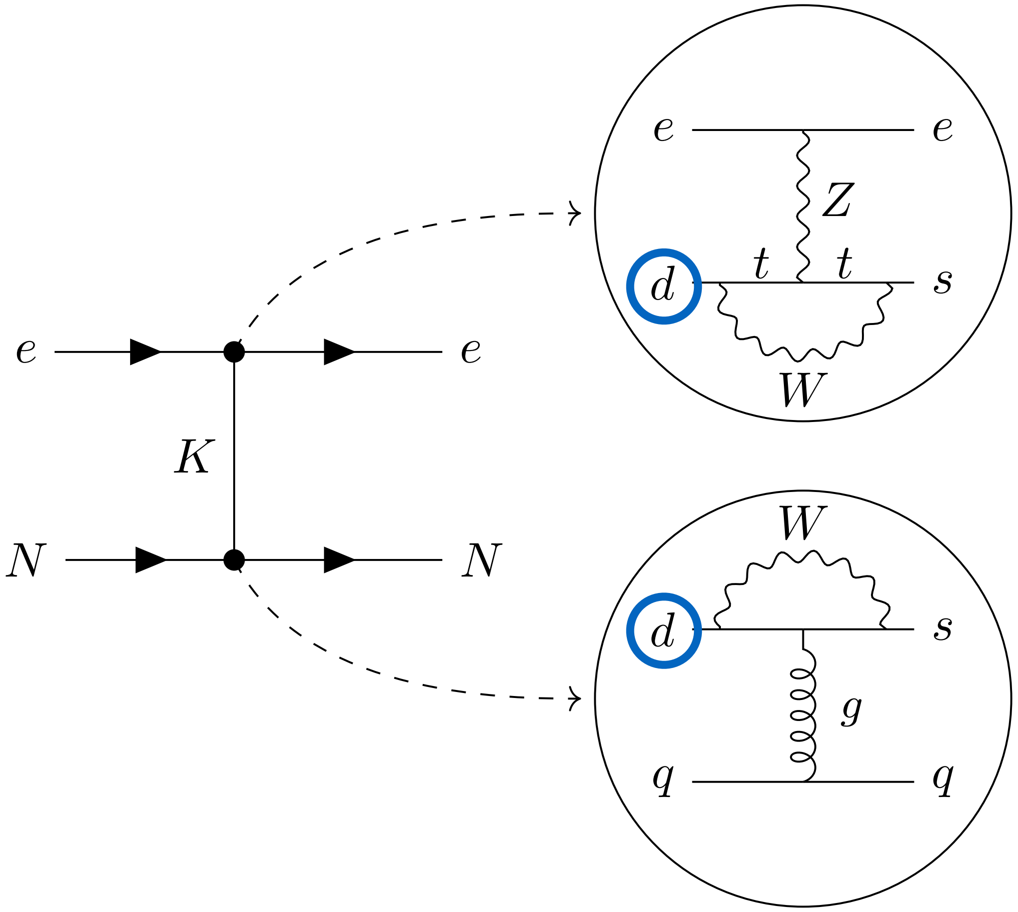

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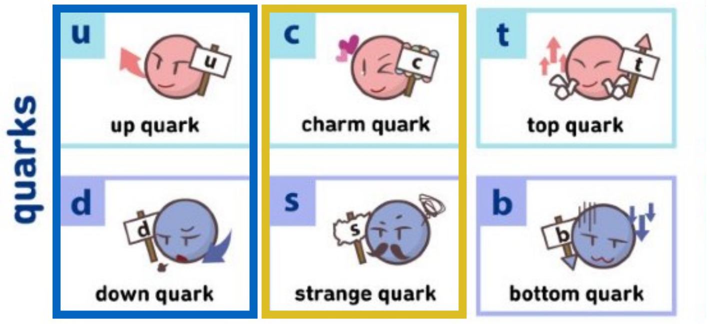

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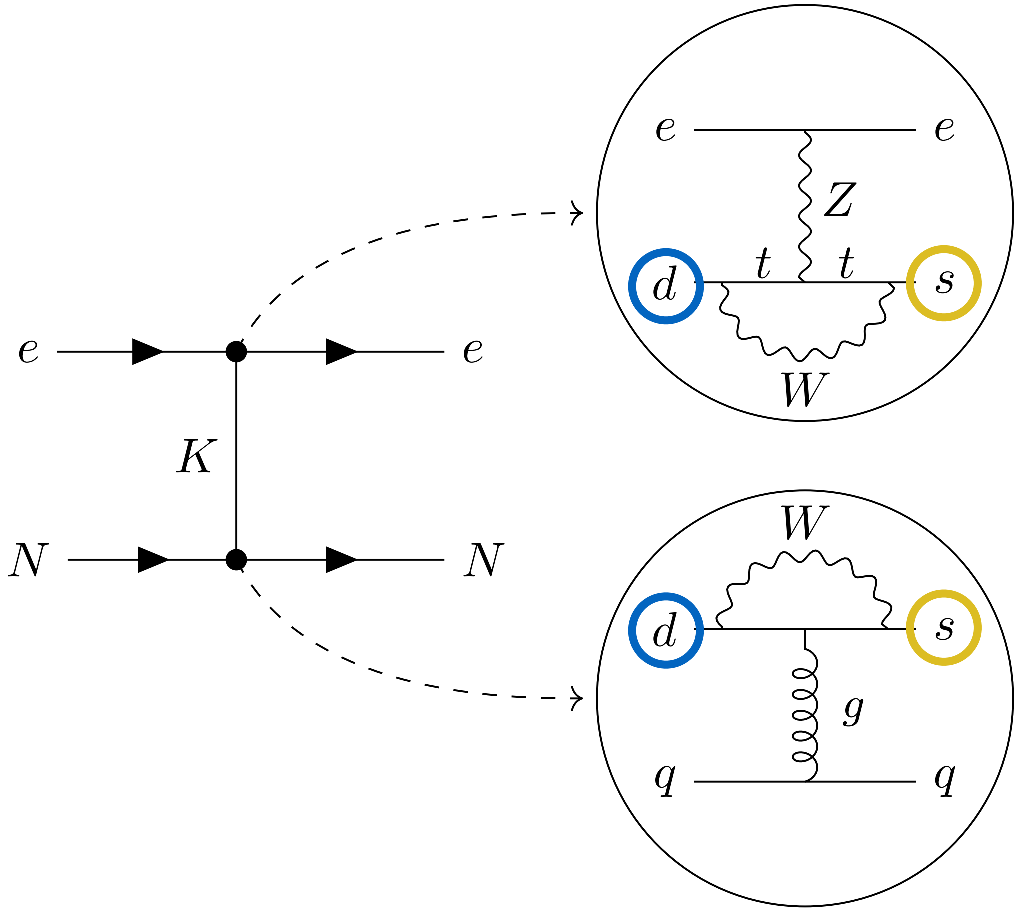

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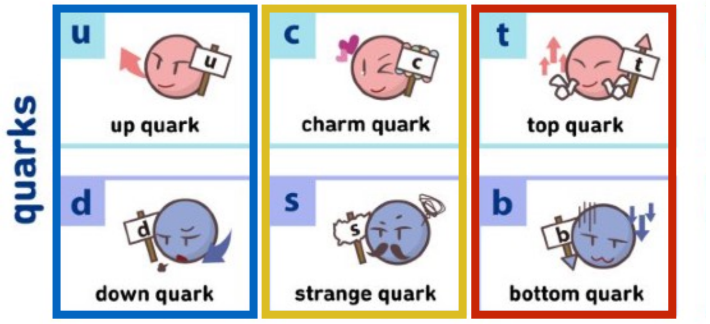

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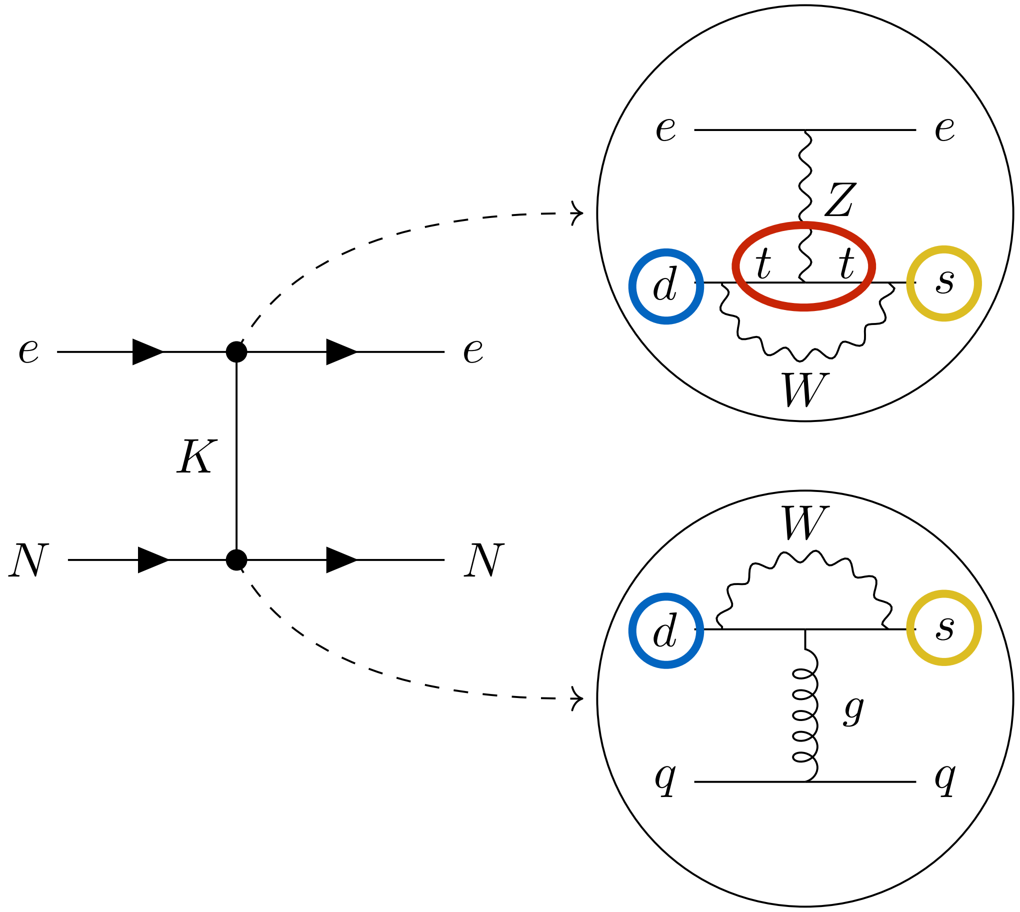

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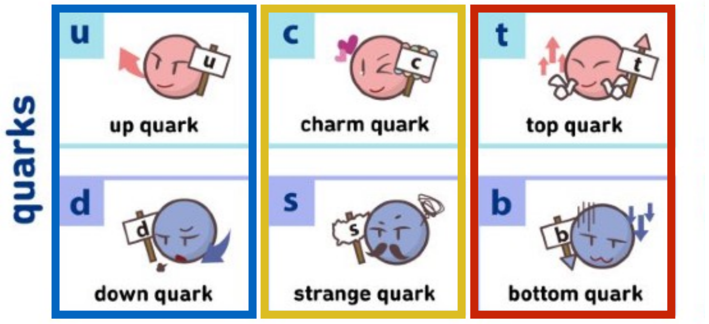

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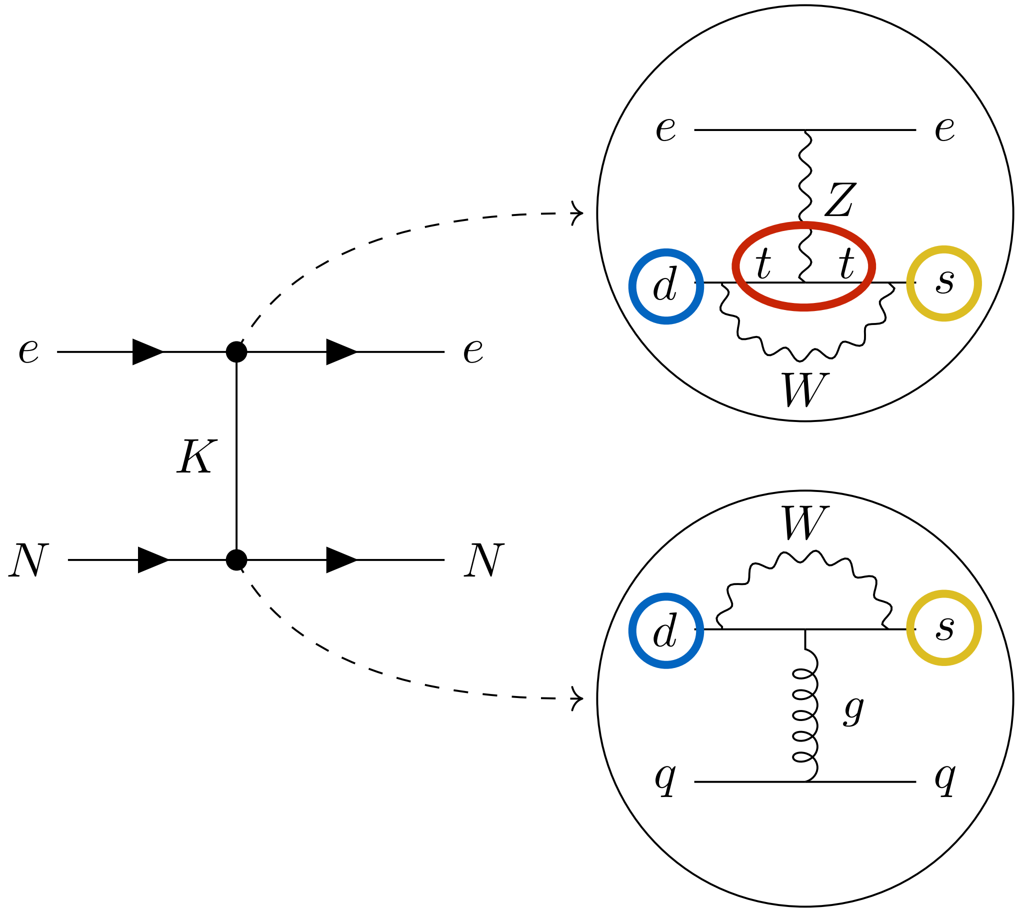

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- We obtain $G_F C_S = J \times \frac{N + 0.7Z}{A} \times \frac{13 m_\pi^2 f_\pi m_e G_F^2}{m_K^2} \times \frac{\alpha I(x_t)}{\pi s_W^2} \propto J \times G_F^3 m_t^2 \times m_e \times m_s^{-1} \times \Lambda_{\text{QCD}}^2$.

➡ $\boxed{d_e^{(\text{equiv})}(\text{ThO}) = 1.0 \times 10^{-35}\, e\,\text{cm}\,.}$ [YE, Gao, Pospelov 22]

Three orders of magnitude ($\sim \alpha^{-2}$) larger than previously believed.

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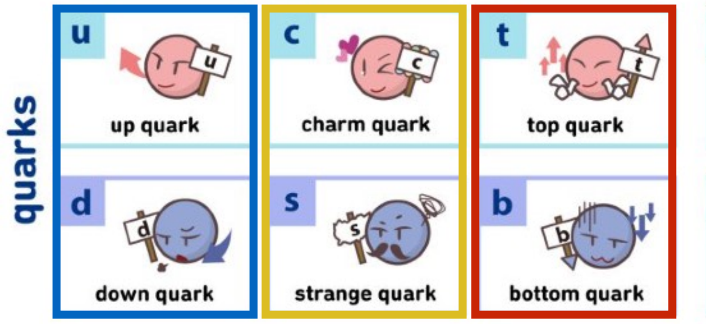

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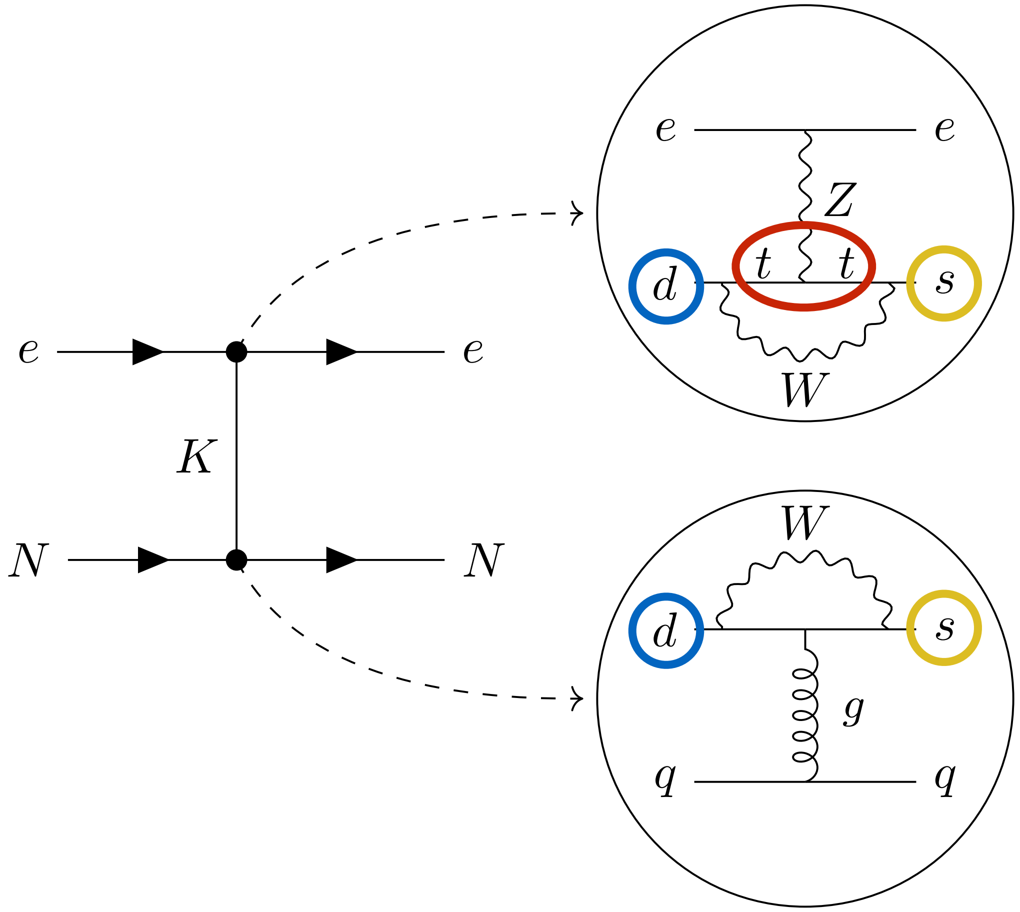

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Three orders of magnitude ($\sim \alpha^{-2}$) larger than previously believed.

- $1/m_K^2 \propto 1/m_s$ singularity in chiral limit → distinct from other contributions.

NLO correction

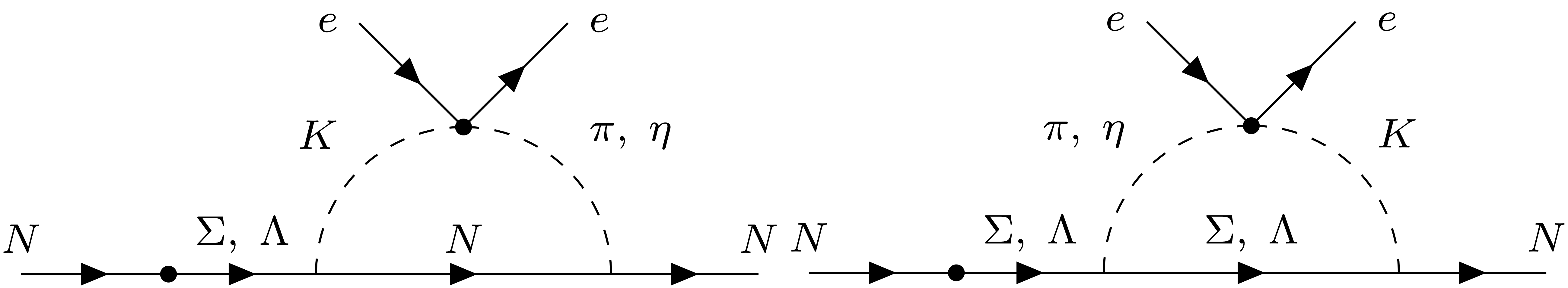
- Next-to-leading order (NLO) correction at $\frac{m_K}{\Delta m_B} \sim \sqrt{\frac{m_s}{\Lambda_{\rm had}}}$ from baryon pole diagrams.

- Calculable by heavy baryon chiral perturbation theory:

$$\frac{C_S^{\rm NLO}}{C_S^{\rm LO}}(p) \simeq 30\,\%\,, \quad \frac{C_S^{\rm NLO}}{C_S^{\rm LO}}(n) \simeq 40\,\%\,.$$

[YE, Gao, Pospelov 22]

➡ corrections calculable and under control.



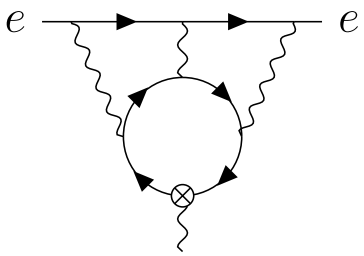
BSM contribution:

muon EDM

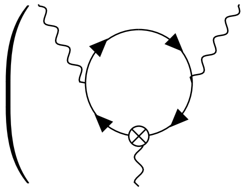
Beyond CKM: Muon EDM

- Muon EDM probed also by atomic/molecular EDM experiments.

chirality flipping

E.g. $d_e \sim$ 

$\sim d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 10^{-10} \times d_\mu.$

$\Bigg($  $\sim F_\mu{}^\nu F_\nu{}^\rho \tilde{F}_\rho{}^\mu = 0 \Bigg)$

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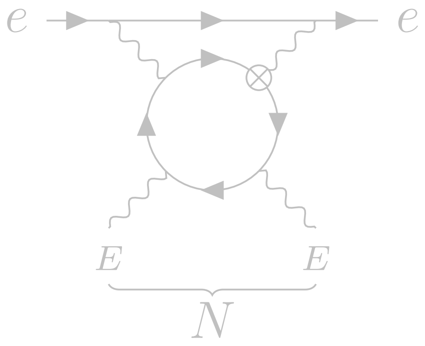

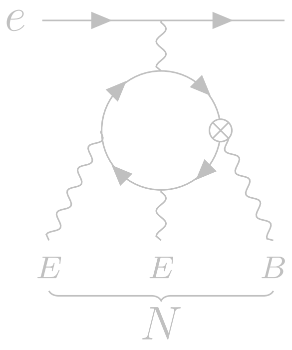

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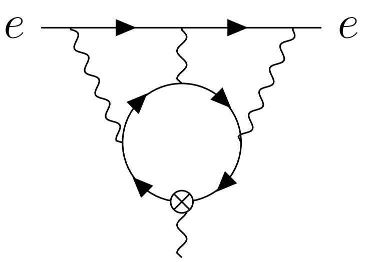
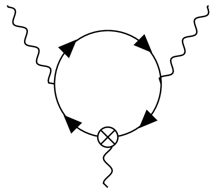
E.g. $d_e \sim$ 

$\sim d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 10^{-10} \times d_\mu .$ ( $\sim F_\mu{}^\nu F_\nu{}^\rho \tilde{F}_\rho{}^\mu = 0$)

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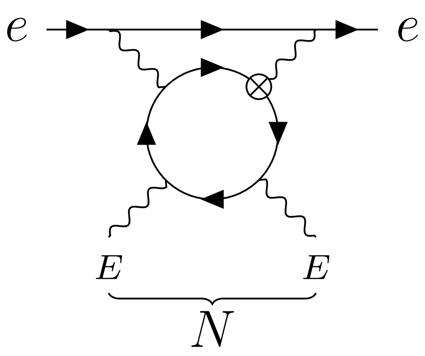
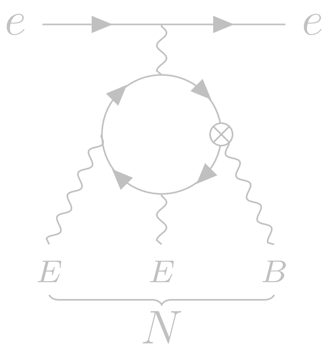


$$\boxed{|d_\mu(\mathrm{HfF})| < 8.9 \times 10^{-21}\, e\,\mathrm{cm}}$$

[YE, Gao, Pospelov 21, 22]

Tighter than BNL bound: $|d_\mu(\mathrm{BNL})| < 1.8 \times 10^{-19}\, e\,\mathrm{cm}\,.$



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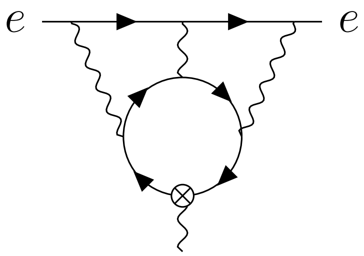
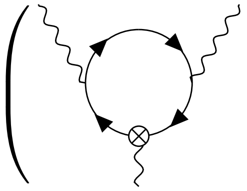
E.g. $d_e \sim$ 

$\sim d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \sim 10^{-10} \times d_\mu.$ ( $\sim F_\mu{}^\nu F_\nu{}^\rho \tilde{F}_\rho{}^\mu = 0$)

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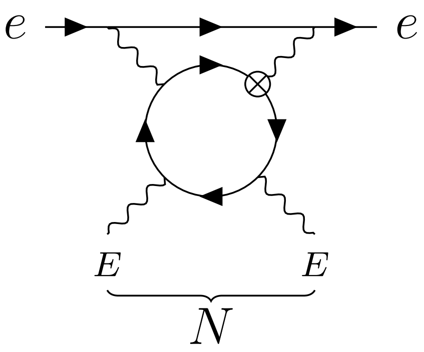

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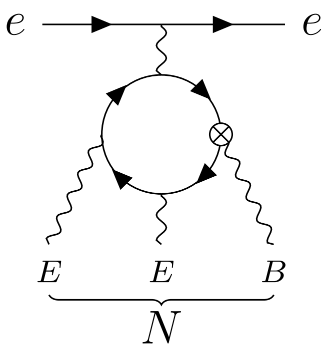
- Muon EDM also incudes Schiff moment, constrained by mercury experiment as



$$\boxed{|d_\mu(\text{Hg})| < 6.4 \times 10^{-20}\, e\text{cm}}$$

[YE, Gao, Pospelov 21]

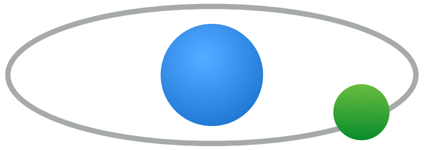
Topics for discussions

Muonic atoms?

Muonic atomic EDM

- Muonic atoms copiously produced in several facilities around the world.

➡ Could be useful for probing muonic CP violation?



- Atomic EDM is enhanced for $2s$ state due to $2s$ - $2p$ degeneracy.

[YE, Pospelov, in progress]

$$\begin{cases} d_{\mu p}(1S) \simeq -3.5 \times 10^{-5} d_\mu \text{ due to screening,} \\ d_{\mu p}(2S) = \dfrac{\alpha^4 m_\mu}{2(E_{2P} - E_{2S})} d_\mu \simeq 0.74\, d_\mu \text{ despite screening.} \end{cases}$$

Also sensitive to e.g. muonic "C_S" operator $\bar{\mu} i \gamma_5 \mu \bar{N} N$.

- Induce $2s - 2p$ mixing

➡ $E1$ transition of "$2s$" → $1s$ could be a signal?

Comment on

lattice calculation of $d_n(\theta)$

Neutron EDM

- SM has an additional source of CP violation: QCD θ term.
  - neutron EDM is a powerful probe: $|d_n| \lesssim 10^{-26}\, e\mathrm{cm} \to |\theta| \lesssim 10^{-10}\,.$

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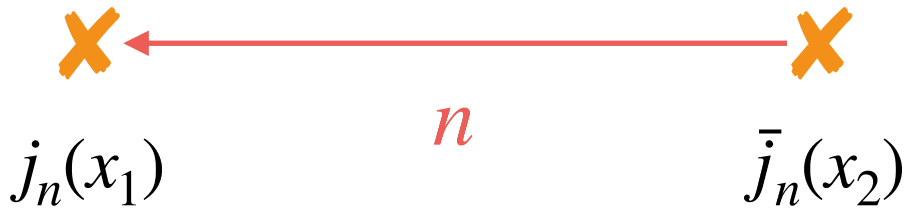

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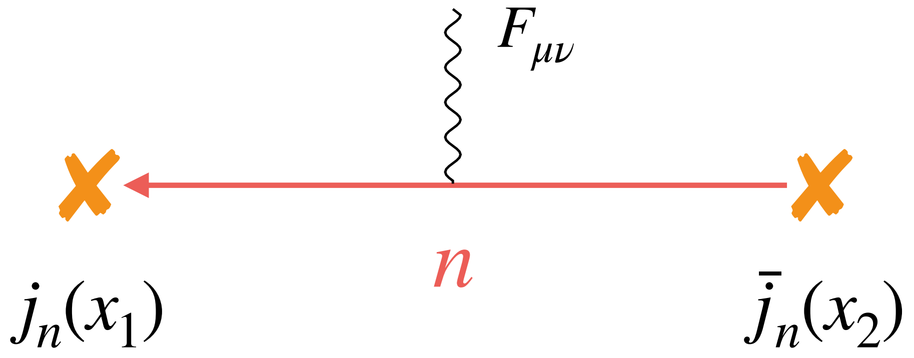

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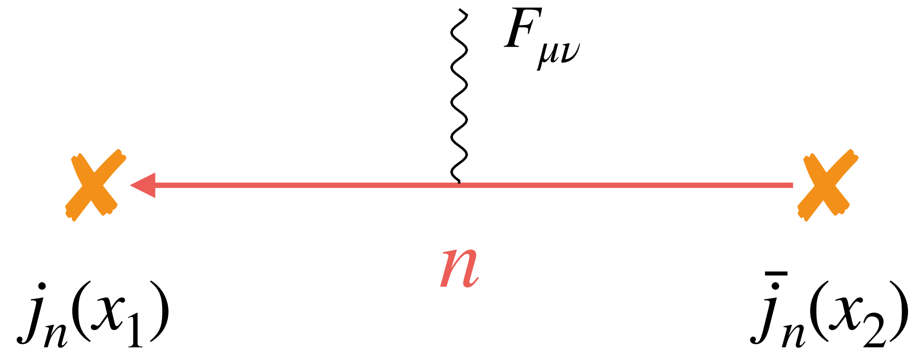

- Two operators having the same quantum number as neutron:

$$j_n^\beta = j_1 + \beta j_2, \quad \text{where } j_1 = 2(d^T C\gamma_5 u)d,\ j_2 = 2(d^T C u)\gamma_5 d\,.$$

The choice $\beta = 0$ is often made in lattice calculations.

Chiral covariance of j_n^β

- Ensuring chiral rotation invariance is crucial for evaluating $d_n(\theta)$.

Remember $d_n \propto m_u m_d \bar{\theta}$ due to the chiral invariance.

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- The choice $\beta = \pm 1$ is covariant under chiral rotation, while the others are not:

$$q \to e^{i\theta_A \gamma_5} q \, : \; j_n^\beta \to \frac{1+\beta}{2} e^{3i\theta_A \gamma_5} j_n^+ + \frac{1-\beta}{2} e^{-i\theta_A \gamma_5} j_n^- \, .$$

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- Lattice QCD subtracts chiral phase of 2pt fn from 3pt fn in the past studies.

[Abramczyk+ 17]

→ This procedure guarantees invariance for covariant choices $\beta = \pm 1$,

but NOT for other choices, including $\beta = 0$.

Chiral covariance of j_n^β

- Ensuring chiral rotation invariance is crucial for evaluating $d_n(\theta)$.

Remember $d_n \propto m_u m_d \bar{\theta}$ due to the chiral invariance.

- The choice $\beta = \pm 1$ is covariant under chiral rotation, while the others are not:

$$q \to e^{i\theta_A \gamma_5} q \,:\; j_n^\beta \to \frac{1+\beta}{2} e^{3i\theta_A \gamma_5} j_n^+ + \frac{1-\beta}{2} e^{-i\theta_A \gamma_5} j_n^- \,.$$

- Lattice QCD subtracts chiral phase of 2pt fn from 3pt fn in the past studies.

[Abramczyk+ 17]

➡ This procedure guarantees invariance for covariant choices $\beta = \pm 1$,

but NOT for other choices, including $\beta = 0$.

- In QCD sum rule, the correct dependence is obtained only for $\beta = \pm 1$.

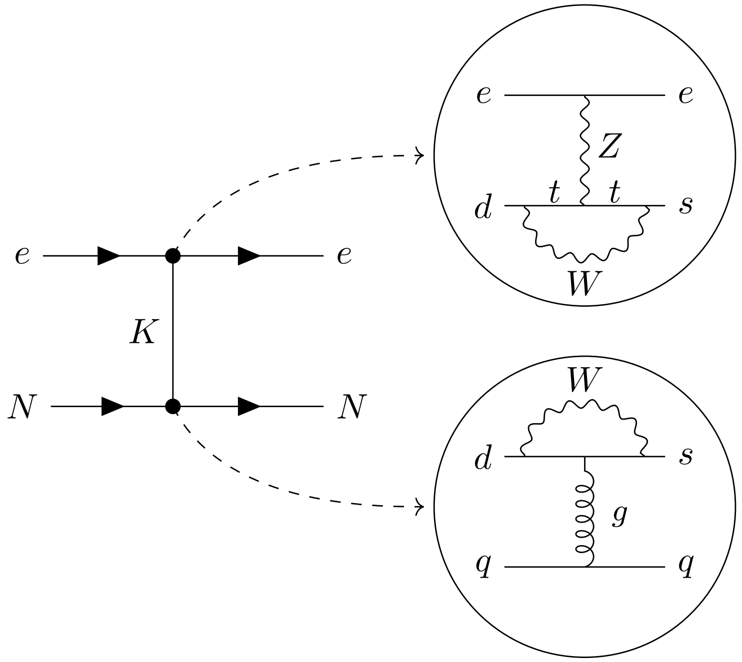
➡ Scrutiny needed, in our opinion. [YE, Gao, Pospelov, Ritz 24]

Summary

Summary

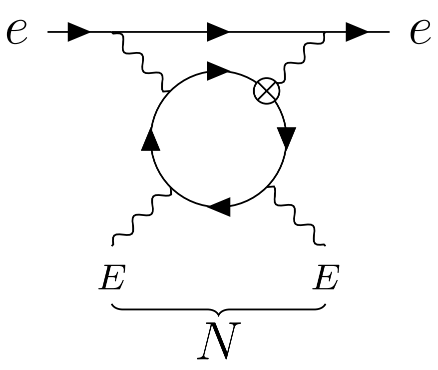
SM prediction of paramagnetic EDM:

- Paramagnetic EDM sensitive to both d_e and C_S.
- Kaon exchange induces $d_e^{(\text{equiv})} = 1.0 \times 10^{-35}\, e\,\text{cm}$.
- Distinct $1/m_s$ structure in the chiral limit.
- NLO correction calculable and under control.



Indirect constraints on muon EDM:

- Atomic/molecular EDMs also sensitive to e.g. muon EDM.
- JILA limits on $d_e^{(\text{equiv})}$ translates to $|d_\mu| < 8.9 \times 10^{-21}\, e\text{cm},$ surpassing BNL.
- Can muonic atoms be useful for probing muonic CP violation?



Comment on neutron EDM and lattice QCD:

- Chiral covariant interpolation function, $\beta = \pm 1$, could be a better choice for $d_n(\theta)$.

Back up

Shielding theorem

- (Non-relativistic) atomic Hamiltonian with external $\vec{E}$ and EDM:

$$\mathscr{H}_A = \mathscr{H}_N + \mathscr{H}_e + \Phi - \sum_k \left(e_k \vec{r}_k \cdot \vec{E}_{\text{ext}} + \vec{d}_k \cdot \vec{E}(\vec{r}_k) \right),$$

where Φ : coulomb potential btw particles and $\vec{E} = \vec{E}_{\text{int}} + \vec{E}_{\text{ext}}$.

$$\vec{E}_{\text{int}}(\vec{r}_k) = -\frac{\vec{\nabla}_k}{e_k}\Phi = -\frac{i}{e_k}\left[\vec{p}_k, \mathscr{H}_0\right] \text{ where } \mathscr{H}_0 = \mathscr{H}_N + \mathscr{H}_e + \Phi .$$

- EDM without $\vec{E}_{\text{ext}}$ induces mixing of (unperturbed) states as

$$|\Psi\rangle \simeq |0\rangle - \sum_{n\neq 0} \frac{\langle n | \sum_k \vec{d}_k \cdot \vec{E}_{\text{int}}(\vec{r}_k) | 0 \rangle}{E_0 - E_n} |n\rangle = \left(1 + \sum_k \frac{i}{e_k} \vec{d}_k \cdot \vec{p}_k \right) |0\rangle .$$

- This cancels the direct contribution to the atomic EDM:

$$\vec{d}_A = \langle \Psi | \sum_k \left(\vec{d}_k + e_k \vec{r}_k \right) | \Psi \rangle \simeq \sum_k \langle 0 | \left(\vec{d}_k - \sum_l \frac{i e_l}{e_k} \left[\vec{d}_k \cdot \vec{p}_k, \vec{r}_l \right] \right) | 0 \rangle = 0,$$

"Schiff shielding theorem"

[Purcell, Ramsey 50; Garwin, Lederman 59; Schiff 63]

Shielding theorem

- Two contributions to atomic EDM:

  (1) direct contribution from the constituent particle's EDM

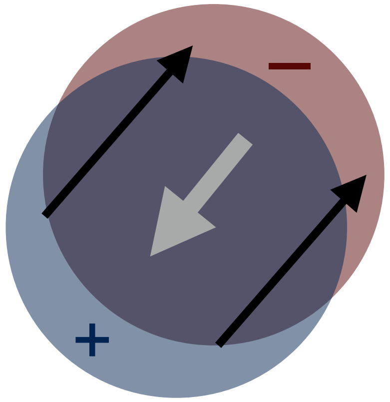
  (2) mixing of opposite parity wave functions through P,CP-odd interaction

  doesn't have to be EDM

$$\vec{d}_A \ni 2 \sum_{n \neq 0} \frac{\langle 0 | \sum_k e\vec{r}_k | n \rangle \langle n | \mathscr{H}_{\mathrm{int}} | 0 \rangle}{E_n - E_0} .$$

➡ these two cancel for non-relativistic neutral point particle's EDM.

- This is a rearrangement due to the constitutions.



- Two ways out:

  (a) relativistic correction → paramagnetic atom (an unpaired electron)

  (b) finite size correction → diamagnetic atom (all electrons paired)

Paramagnetic atom/molecule

- Electron actually relativistic $v \sim Z\alpha \;\; \to d_e$ can induce d_A:

$$\vec{d}_A = d_e \sum_{i=1}^{Z} \left[\langle 0_e | \underline{\left(\gamma_0^{(i)} - 1 \right)} \overrightarrow{\Sigma}^{(i)} | 0_e \rangle + 2 \sum_{n \neq 0} \frac{\langle 0_e | e\vec{r}_i | n_e \rangle}{E_0 - E_n} \langle n_e | \underline{\left(\gamma_0^{(i)} - 1 \right)} \overrightarrow{\Sigma}^{(i)} \cdot \overrightarrow{E}_{\text{int}} | 0_e \rangle \right]$$

relativistic correction to shielding

where $\overrightarrow{\Sigma} = \gamma_5 \gamma^0 \vec{\gamma}$.

$(\gamma_0 - 1)\overrightarrow{\Sigma} = -2 \begin{pmatrix} 0 & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$ in Dirac rep. → need an unpaired electron = paramagnetic atom.

- The latter (mixing of states) dominant,

  and this is actually an enhancement: $d_A / d_e \sim Z^3 \alpha^2 \sim \mathcal{O}(10^2)$. [Sandars 65; ...]

- CP-odd operator $C_S (G_F / \sqrt{2}) \, \bar{e} i \gamma_5 e \times \bar{N} N$ also induces mixing of states.

➡ d_e and C_S degenerate.

$$d_e^{\text{equiv}} = d_e + C_S \times 1.5 \times 10^{-20} \, e\text{cm} \;\; \text{for} \;\; \text{ThO.}$$

Diamagnetic atom

- Nucleus is not point-like → d_N can induce d_A through mixing of states.

$$\vec{d}_A = \sum_{n\neq 0} \frac{1}{E_0 - E_n} \left[\langle 0_e | e \sum_{i=1}^{Z} \vec{r}_i | n_e \rangle \langle n_e | \mathscr{H}_{\rm int} | 0_e \rangle + \mathrm{h.c.} \right]$$

where $\mathscr{H}_{\rm int} = \int d^3r \underbrace{\left(\frac{\vec{d}_N(\vec{r})}{e} - \rho_q(\vec{r}) \frac{\langle \vec{d}_N \rangle}{e} \right)}_{\text{(partial) shielding}} \cdot \vec{\nabla}_e \frac{\alpha}{|\vec{r} - \vec{r}_e|}$: difference btw charge and EDM distribution.

avoid complete suppression: $d_A / d_N \sim 10 Z^2 (R_N / R_A)^2 \sim \mathscr{O}(10^{-3})$.

- Expanding this w.r.t. $r \sim r_N \ll r_e$:

$$\mathscr{H}_{\rm int} = \int d^3r \left(\frac{\vec{d}_N}{e} - \rho_q \frac{\langle \vec{d}_N \rangle}{e} \right) \cdot \vec{\nabla}_e \frac{\alpha}{|\vec{r} - \vec{r}_e|} = -4\pi\alpha \frac{\vec{S}}{e} \cdot \vec{\nabla}_e \delta^{(3)}(\vec{r}_e) + \cdots$$

[Schiff 63]

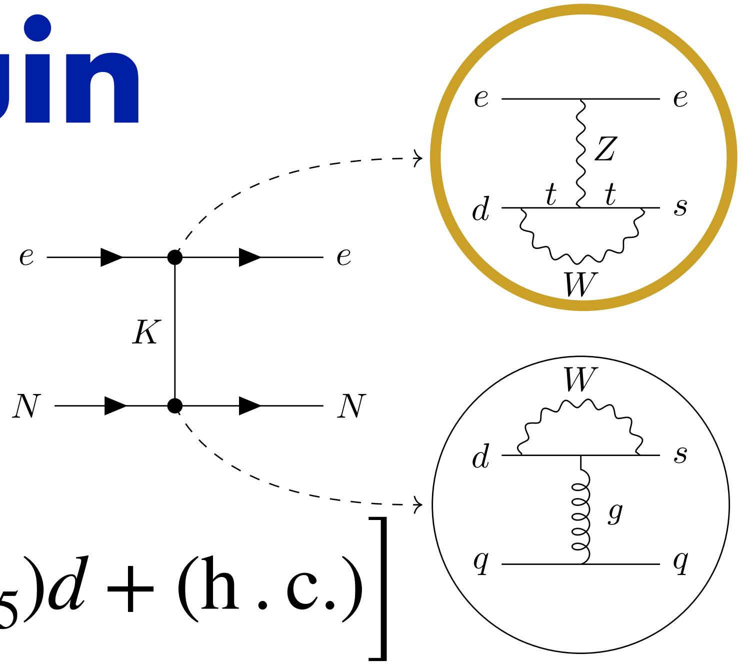
where $\vec{S}$: Schiff moment.

- Diamagnetic atom EDM exp. puts constraints on S .

e.g. ^{199}Hg constraint: $|S_{^{199}\mathrm{Hg}}| < 3.1 \times 10^{-13}\, e\mathrm{fm}^3$.

[Graner et.a. 16]

Electroweak penguin



- EW penguin (+ W box) induces

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi s_W^2} \left[\bar{e}\gamma^\mu(1-\gamma_5)e\right] \sum_{i=c,t} I(x_i) \Big[V_{is}^* V_{id} \bar{s}\gamma_\mu(1-\gamma_5)d + ({\rm h.c.})\Big]$$

where $x_i = m_i^2/m_W^2$ and $I(x) = \frac{3}{4}\left(\frac{x}{x-1}\right)^2 \log x + \frac{x}{4} - \frac{3}{4}\frac{x}{x-1}$. [Inami, Lim 81]

* Essentially no G_F suppression for top, $m_t^2/m_W^2 \sim \mathcal{O}(1)$.

- Below QCD scale:

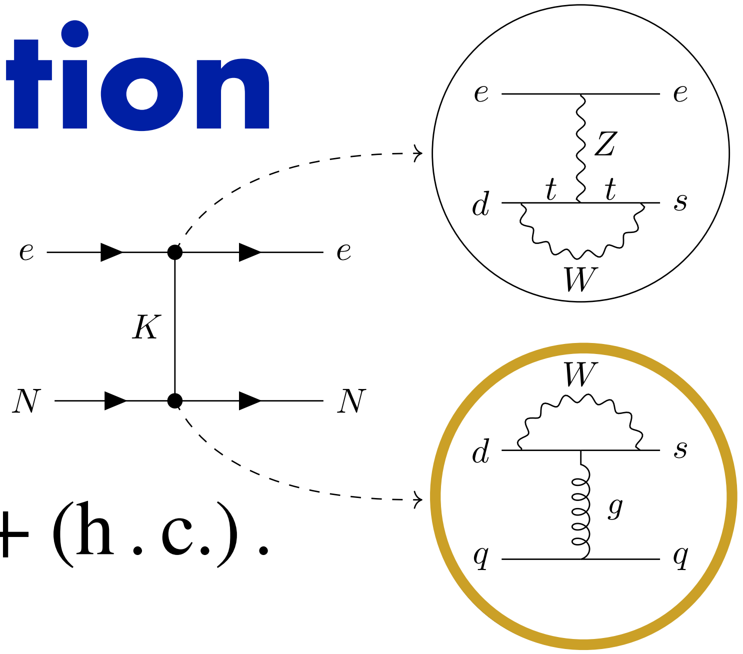
$$V_{is}^* V_{id} \bar{s}\gamma_\mu(1-\gamma_5)d + ({\rm h.c.}) \to \sqrt{2} f_K \partial_\mu \Big({\rm Re}[V_{is}^* V_{id}]K_L + {\rm Im}[V_{is}^* V_{id}]K_S\Big).$$

➡ $\mathcal{L}_{\rm eff} = \frac{\alpha}{2\pi s_W^2} G_F f_K m_e I(x_t) {\rm Im}[V_{ts}^* V_{td}] \times K_S \bar{e} i\gamma_5 e + \cdots,$ with electron EoM used.

- This induces e.g. (short distance contribution to) $K_S \to \mu^+\mu^-$. [Isidori, Unterdorfer 03]

Not observed yet, ${\rm Br}(K_S \to \mu^+\mu^-)_{\rm short} \sim 10^{-13}$ vs ${\rm Br}(K_S \to \mu^+\mu^-)_{\rm exp} < 2.1 \times 10^{-10}$.

$\Delta I = 1/2$ weak transition



- Tree-level W exchange:

$$\mathscr{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{us}^{*} V_{ud} \left[\bar{s}\gamma^{\mu}(1-\gamma_5)u\right] \left[\bar{u}\gamma_{\mu}(1-\gamma_5)d\right] + (\mathrm{h\,.\,c.})\,.$$

- This contains **27** and **8** but **8** is enhanced below QCD scale: $\Delta I = 1/2$ rule.

➡ $$\mathscr{L}_{\rm eff} = -\,a\mathrm{Tr}_F\left[\bar{B}\{\xi^{\dagger}h\xi, B\}\right] - b\mathrm{Tr}_F\left[\bar{B}[\xi^{\dagger}h\xi, B]\right],$$

where B : baryon octet, $\xi = \exp(i\pi/f)$ with π: meson octet, $h_{ij} = \delta_i^2\delta_j^3$: spurion.

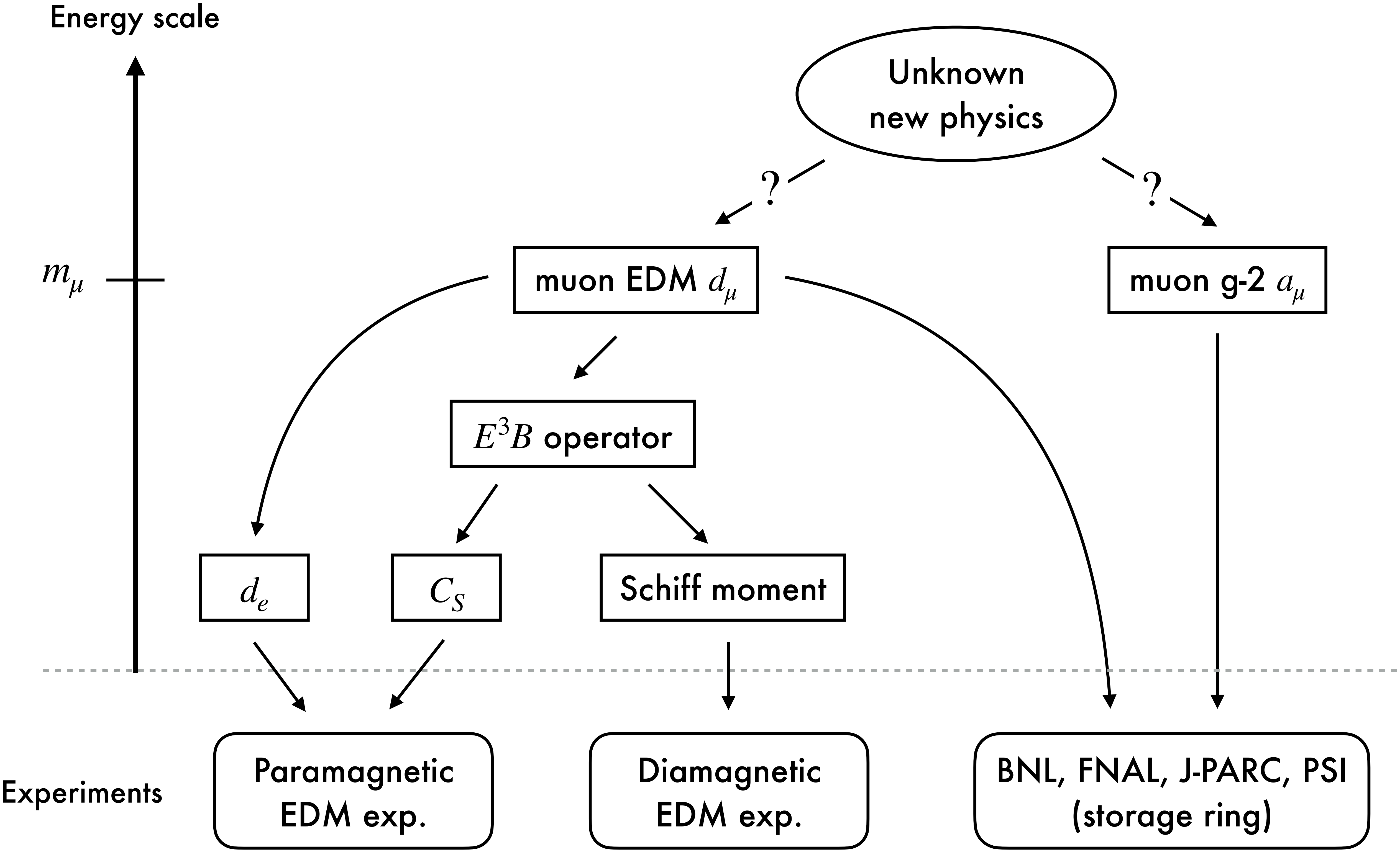
- This induces weak hyperon decay $\Sigma \to N\pi, \;\; \Lambda \to N\pi, \;\; \Xi \to \Lambda\pi\,.$

➡ $a = 0.56G_F[m_\pi^2]f_\pi\,, \;\; b = -\,1.42G_F[m_\pi^2]f_\pi\,,$ from experiments [Bijnens+ 03].

- Focusing on Kaon part:

$$\mathscr{L}_{\rm eff} = -\,\frac{\sqrt{2}K_S}{f_\pi}\left[(b-a)\bar{p}p + 2b\bar{n}n\right] + \cdots\,.$$

Towards observables



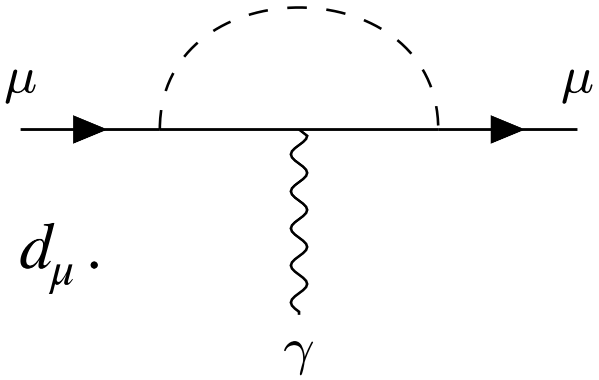
Muon EDM

- Recently FNAL confirms BNL muon g-2 result:

$$a_\mu(\text{exp}) - a_\mu(\text{SM}) = \left(251 \pm 59\right) \times 10^{-11} \quad (4.2\sigma)$$

[FNAL muon g-2 21]

- Muon g-2 and EDM can be closely related:



$$\mathscr{L} = -\frac{c}{2}\bar{\psi}_R\,\sigma\cdot F\,\psi_L + \text{h.c.} \;\; \Rightarrow \;\; \text{Re}[c] = \frac{e\Delta a_\mu}{2m_\mu}, \;\; \text{Im}[c] = d_\mu\,.$$

⇒ $d_\mu \simeq 2 \times 10^{-22}\, e\,\text{cm} \times \tan\phi \times \left(\frac{\Delta a_\mu}{2.5 \times 10^{-9}}\right)$ with $c = |c|\,e^{i\phi}$.

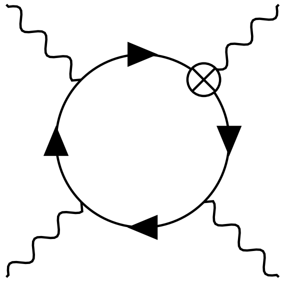
- Current/future direct measurements of muon EDM:

$$\begin{cases} \text{BNL (existing limit): } |d_\mu| < 1.8 \times 10^{-19}\, e\,\text{cm}\,. \\ \text{FNAL, J-PARC: } |d_\mu| \lesssim 10^{-21}\, e\,\text{cm}\,. \\ \text{PSI (“frozen spin”): } |d_\mu| < 6 \times 10^{-23}\, e\,\text{cm}\,. \end{cases}$$

⇒ understand indirect limits from atomic/molecular EDM experiments.

CP-odd photon operator

- Muon EDM induces CP-odd photon operator at one-loop:



$$= -\frac{e^3 d_\mu}{96\pi^2 m_\mu^3} \tilde{F}_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$

where cross-dot: muon EDM $d_\mu \bar{\mu}\sigma \cdot \tilde{F}\mu$ insertion.

- Atomic EDM exp. has large Z → strong nuclear electric field.

➡ $\tilde{F}_{\mu\nu} F^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \ \ni \ E^3 B$ can induce sizable CP-odd effects.

- In particular this operator induces

$$\begin{cases} \text{Schiff moment} & \rightarrow \text{diamagnetic EDM (Hg)} \\ \text{semi-leptonic CP-odd operator} & \rightarrow \text{paramagnetic EDM (ThO)} \end{cases}$$

Nuclear electric field

- Nuclear electric field given by the charge distribution inside nuclei:

$$e\overrightarrow{E}_N(\vec{r}) = \frac{Ze^2}{4\pi}\int d^3r_N \rho_q(\vec{r}_N)\,\overrightarrow{\nabla}\frac{1}{|\vec{r}-\overrightarrow{r_N}|}\,.$$

- We simply take the charge distribution as

$$\rho_q(r_N) = \frac{3}{4\pi R_N^3}\Theta\left(R_N - r_N\right), \quad R_N = \sqrt{\frac{5}{3}}r_c, \quad r_c = 5.45\,\mathrm{fm}\ \text{ for } {}^{199}\mathrm{Hg}.$$

- The Woods-Saxon shape different only within $10\,\%$ in the final result.

Nuclear magnetic field

- $^{199}\mathrm{Hg}$ has an unpaired outermost neutron with $2p_{1/2}$ $(n = 2, l = 1, j = 1/2)$.

➡ $\overrightarrow{B}_N$ dominantly provided by this neutron.

- As a result $\overrightarrow{B}_N$ is given by

$$e\overrightarrow{B}_N(\vec{r}) = \frac{2e\mu_n}{3}\psi_n^\dagger(\vec{r})\vec{\sigma}\psi_n(\vec{r}) + \frac{e\mu_n}{4\pi}\left[\vec{\nabla}\left(\vec{\nabla}\cdot\right) - \frac{\vec{\nabla}^2}{3}\right]\int d^3r_n \frac{\psi_n^\dagger(\vec{r}_n)\vec{\sigma}\psi_n(\vec{r}_n)}{|\vec{r}_n - \vec{r}|}$$

$$= e\mu_n\frac{\left|R(r)\right|^2}{4\pi}\chi^\dagger\left[\left(\vec{n}\cdot\vec{\sigma}\right)\vec{n} - \vec{\sigma}\right]\chi + \frac{e\mu_n}{4\pi}\int_0^\infty dr_n\, r_n^2\left|R(r_n)\right|^2\chi^\dagger\vec{g}(\vec{r}, r_n)\chi,$$

where $\mu_n \simeq -1.91\frac{e}{2m_p}$: neutron magnetic moment, ψ_n : neutron wave function,

R_n: neutron radial wave function, χ neutron spinor, $\vec{n} = \vec{r}/r$.

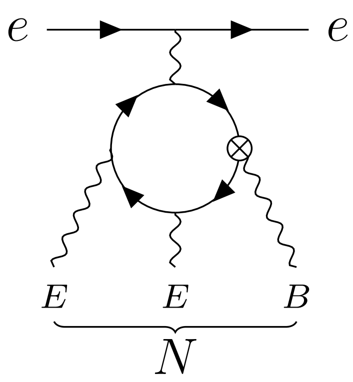
- We use the nuclear shell model to obtain ψ_n.

Schiff moment

- Schiff moment: $\mathscr{H}_{\rm int} = -4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)\,.$

$$\vec{d}_A = \sum_{i=1}^{Z}\langle\Psi\,|\,e\vec{r}_i\,|\,\Psi\rangle = -\sum_{n\neq 0}\frac{2}{E_0-E_n}\sum_{i=1}^{Z}\langle 0_e|e\vec{r}_i|n_e\rangle\langle n_e|4\pi\alpha(\vec{S}/e)\cdot\vec{\nabla}_i\delta^{(3)}(\vec{r}_i)|0_e\rangle\,.$$

- E^3B with two E_N and one B_N induces effective EDM distribution:



$$= \int d^3r\left(\vec{\nabla}_e\frac{\alpha}{|\,\vec{r}-\vec{r}_e\,|}\right)\cdot\frac{\vec{d}_N(\vec{r})}{e}\,,\quad \vec{d}_N \propto d_\mu\left(2\vec{E}_N(\vec{E}_N\cdot\vec{B}_N)+\vec{B}_N E_N^2\right).$$

- Difference between EDM and charge distribution gives Schiff moment:

$$\mathscr{H}_{\rm eff} = \int d^3r\left(\frac{\vec{d}_N}{e}-\rho_q\frac{\langle\vec{d}_N\rangle}{e}\right)\cdot\vec{\nabla}_e\frac{\alpha}{|\,\vec{r}-\vec{r}_e\,|} = -4\pi\alpha\frac{\vec{S}}{e}\cdot\vec{\nabla}_e\delta^{(3)}(\vec{r}_e)+\cdots\,.$$

- ^{199}Hg constraint: $|\,S_{^{199}\rm Hg}\,| < 3.1\times 10^{-13}\,e\mathrm{fm}^3\,.$ [Graner et.a. 16]

$$\boxed{|\,d_\mu(\mathrm{Hg})\,| < 6.4\times 10^{-20}\,e\mathrm{cm}}$$

[**YE**, Gao, Pospelov 21]

Magnetic quadrupole moment

- Magnetic quadrupole moment (MQM) also violates P and CP:

$$\mathscr{H}_{\rm eff} = -\frac{M}{6}\nabla_j B_i I_{ij}, \quad I_{ij} \equiv \frac{3}{2I(I-1)}\left(I_iI_j + I_jI_i - \frac{2}{3}\delta_{ij}I(I+1)\right).$$

- The E^3B operator converts EQM Q to MQM as

$$\frac{M}{e} \simeq -\frac{Z^2\alpha^3 d_\mu/e}{5\pi m_\mu^2 R_N^3}\frac{Q}{e} \simeq 1.1\times 10^{-4}\,\mathrm{fm}\left(\frac{Q/e}{300\,\mathrm{fm}^2}\right)\times d_\mu/e\,.$$

- Q can be large in nuclei with $I \geq 1$ and large deformation.

➡ can be an interesting observable in future.

Fudge factor from E_N^2 to $\bar{N}N$

- E_N^2 and $\bar{N}N$ are both localized around nuclei, but not exactly the same.

- Relevant electron transition between $s_{1/2}$ and $p_{1/2}$ states.

➡ matrix element:

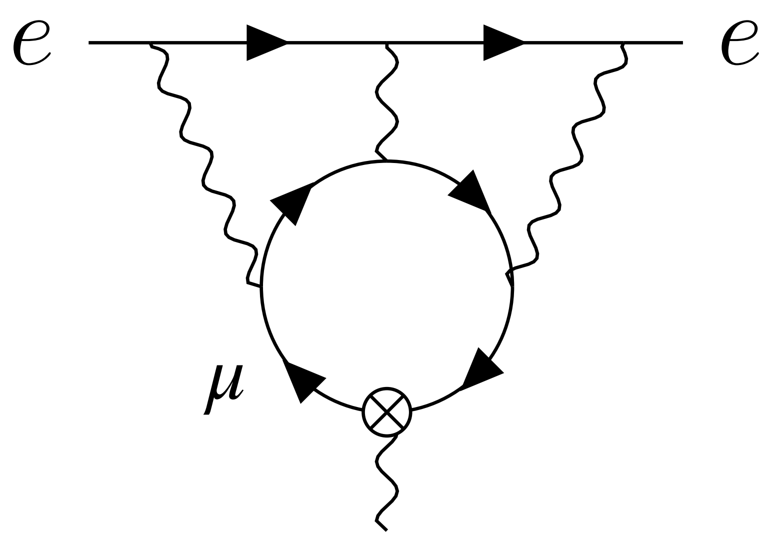
$$\begin{cases} \int d^3r \rho_N \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right) & \text{for} \quad \bar{N}N\bar{e}i\gamma_5 e, \\ \int d^3r \,|\, \overrightarrow{E}_N |^2 \psi_p^\dagger \gamma^0 \gamma_5 \psi_s \propto \int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right) & \text{for} \quad E_N^2 \bar{e}i\gamma_5 e\,. \end{cases}$$

- We compute the fudge factor κ by solving the Dirac equation and get

$$\kappa = \frac{\int dr\, r^2 \bar{\rho}_{E^2} \left(f_p g_s + f_s g_p \right)}{\int dr\, r^2 \bar{\rho}_N \left(f_p g_s + f_s g_p \right)} \simeq 0.66.$$

Electron EDM

- Muon EDM induces electron EDM at three-loop:



+ permutations (⊗: EDM operator)

- Two types of contributions:

$$i\mathcal{M} = i\tilde{F}^{\mu\nu}\,\bar{e}(p)\left[S^{(1)}m_e\sigma_{\mu\nu} + S^{(2)}\left\{\sigma_{\mu\nu}, \not{p}\right\}\right]e(p)\,.$$

[Grozin, Khriplovich, Rudenko 08] overlooked $S^{(2)}$.

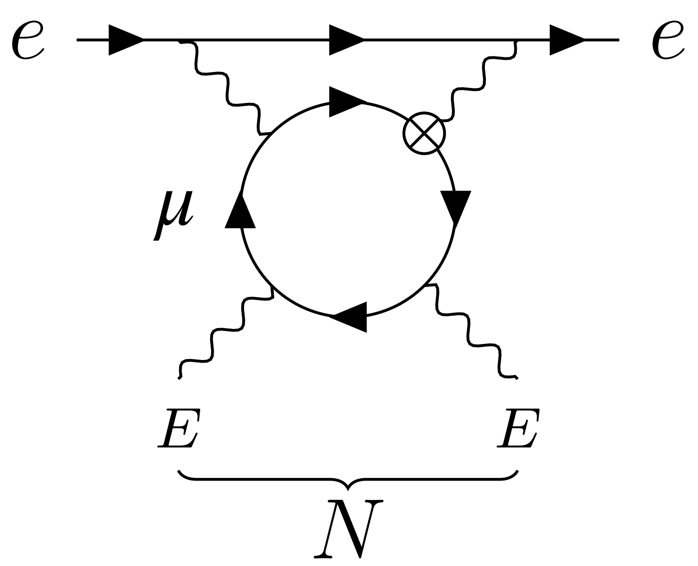
- Combining $S^{(1)}$ and $S^{(2)}$, the result is $40\,\%$ larger:

$$d_e = 2.75 \times d_\mu \left(\frac{\alpha}{\pi}\right)^3 \frac{m_e}{m_\mu} \simeq 1.7 \times 10^{-10} \times d_\mu \text{ (UV finite).}$$

[**YE**, Gao, Pospelov 22]

Semi-leptonic CP-odd operator

- Muon EDM induces



$$\sim d_\mu \times \bar{e} i \gamma_5 e \times E_N^2 .$$

- Nuclear electric field E_N^2 localized around nucleus.

$$\bar{e} i \gamma_5 e \times E_N^2 \;\sim\; \bar{e} i \gamma_5 e \times n_N \sim\; \bar{e} i \gamma_5 e \times \bar{N} N$$: equivalent to C_S.

- Combining C_S and d_e, ACME translated as

$$\boxed{|d_\mu(\mathrm{ThO})| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}}$$

[**YE**, Gao, Pospelov 21, 22]

Better than BNL bound: $|d_\mu(\mathrm{BNL})| < 1.8 \times 10^{-19}\, e\,\mathrm{cm}$.

- Recent Colorado result [Roussay+ 22] even stronger: $|d_\mu(\mathrm{HfF})| < 8.9 \times 10^{-21}\, e\,\mathrm{cm}$.

Tau/charm/bottom EDMs

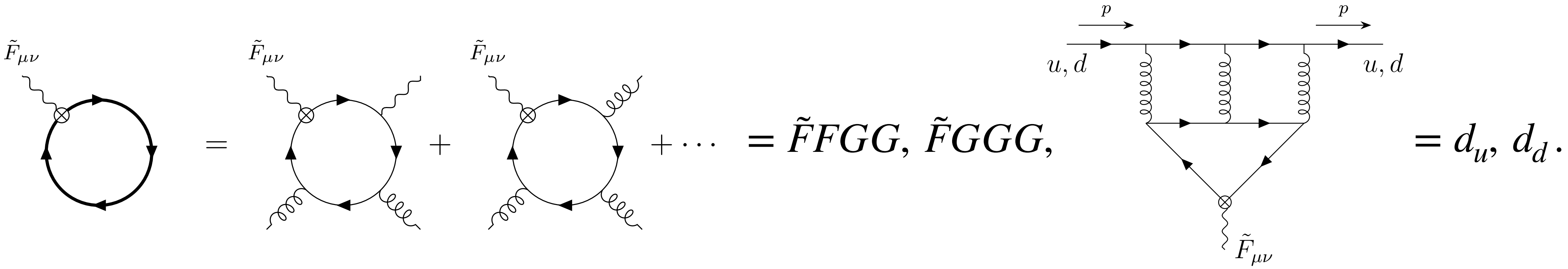
- Tau EDM constraint obtained by $m_\mu \to m_\tau$:

$$|d_\tau(\text{ThO})| < 1.1 \times 10^{-18}\, e\,\text{cm}.$$

[**YE**, Gao, Pospelov 22b]

$$\begin{cases} \text{Belle} : -2.2 \times 10^{-17} < \text{Re}(d_\tau)/e\,\text{cm} < 4.5 \times 10^{-17},\ -2.5 \times 10^{-17} < \text{Im}(d_\tau)/e\,\text{cm} < 8.0 \times 10^{-19}, \\ \text{Belle-II (future)} : |\text{Re}(d_\tau)|, |\text{Im}(d_\tau)| \lesssim \mathcal{O}(10^{-18} - 10^{-19})\, e\,\text{cm}\,. \end{cases}$$

- Similar constraints on charm/bottom quark EDMs.



$$= \cdots + \cdots + \cdots = \tilde F F G G,\ \tilde F G G G, \qquad = d_u,\ d_d\,.$$

$$\begin{cases} |d_c| < 1.3 \times 10^{-20}\, e\,\text{cm}, \quad |d_b| < 7.6 \times 10^{-19}\, e\,\text{cm} \quad \text{from paramagnetic EDM}, \\ |d_c| < 6 \times 10^{-22}\, e\,\text{cm}, \quad |d_b| < 2 \times 10^{-20}\, e\,\text{cm} \quad \text{from neutron EDM}. \end{cases}$$

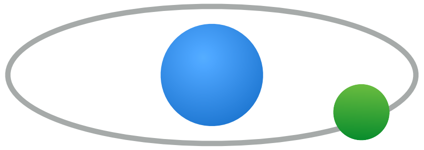
[**YE**, Gao, Pospelov 22c]

Constraints from d_n stronger but with more hadronic uncertainties.

Muonic atomic EDM

- Muonic atoms copiously produced in several facilities around the world.

➡ Could be useful for probing muonic CP violation?



- Atomic EDM is enhanced for $2s$ state due to $2s$ - $2p$ degeneracy.

[YE, Pospelov, in progress]

$$\begin{cases} d_{\mu p}(1S) \simeq -3.5 \times 10^{-5} d_\mu \text{ due to screening,} \\ d_{\mu p}(2S) = \dfrac{\alpha^4 m_\mu}{2(E_{2P} - E_{2S})} d_\mu \simeq 0.74\, d_\mu \text{ despite screening.} \end{cases}$$

Also sensitive to e.g. muonic "C_S" operator $\bar{\mu} i \gamma_5 \mu \bar{N} N$.

- Induce $2s - 2p$ mixing

➡ $E1$ transition of "$2s$" → $1s$ could be a signal?