NOISE-INDUCED REACTIVITY SHIFTS IN THE POINT KINETICS EQUATIONS

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The point-kinetics modelling of neutron noise

- Power noise affects the cross-sections of the problem $\longrightarrow$ $\rho(t) = \rho + \sigma\Lambda\delta\rho(t)$

$$\frac{dN}{dt} = \frac{\rho-\beta}{\Lambda}N + \lambda C + \sigma N\,\delta\rho(t)$$
$$\frac{dC}{dt} = \frac{\beta}{\Lambda}N - \lambda C$$

Common simplifications

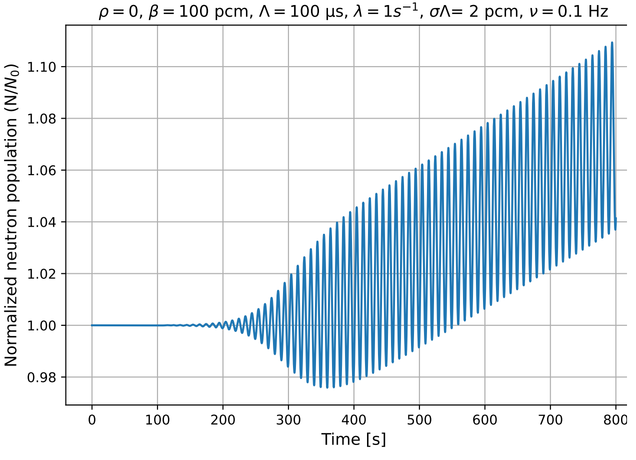
- Study the linear response of the system to a sinusoidal excitation at a frequency ν
  - Linearization $\longrightarrow$ $N(t) = N_{ref}(t) + \delta N(t)$, and neglect terms of the form $\delta N \times \delta\rho$
  - Sinusoidal excitations $\delta\rho(t) = \sin(\omega t)$ $\longrightarrow$ What about "true" noise ?

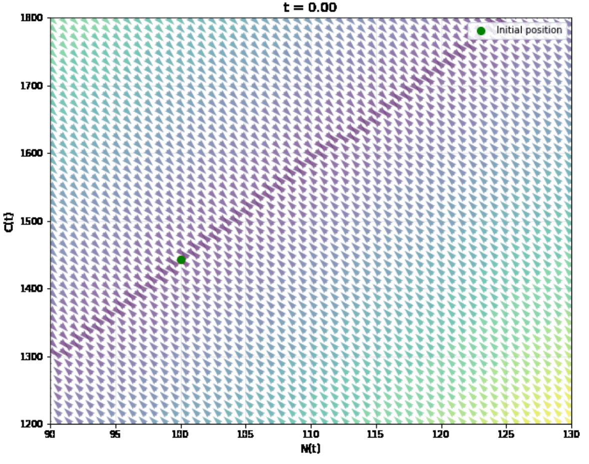
Study the impact of the hypothesis

- Do not linearize the equation $\longrightarrow$ this work
- Use stochastic and temporally correlated noise $\longrightarrow$ upcoming paper
  - "An enquiry into noise-induced reactivity shifts in the point-kinetics equations" (Submitted to ANE)

A noise-induced drift of the neutron population

- The precursor population has a destabilizing role ! *(Lewins 1995, Ravetto 1997)*
- The precise mechanism at play remains elusive ... (to my knowledge at least !)





Setting up a perturbative approach

- Start from stationary initial conditions $\longrightarrow$ set $\rho = 0$ and $(N(0), N'(0)) = (N_0, 0)$

$$N'(t) = -(\beta/\Lambda)N + \lambda C + \sigma N \sin(\omega t)$$
$$C'(t) = (\beta/\Lambda)N - \lambda C$$

- $C(t)$ can be eliminated by algebraic manipulations of the system of equations (set $\kappa = \frac{\beta}{\Lambda} + \lambda$)

$$N''(t) + \kappa N'(t) = \sigma \left\{ [\lambda \sin(\omega t) + \omega \cos(\omega t)] \, N(t) + N'(t) \sin(\omega t) \right\}$$

- Seek a solution of the form $N(t) = N_0 + \sigma N_1(t) + \sigma^2 N_2(t) + \ldots$ $\longrightarrow$ One has

$$\boxed{N_n''(t) + \kappa N_n'(t) = g_n(t)}$$

with

$$\boxed{g_n(t) = N_{n-1}(t) \, [\lambda \sin(\omega t) + \omega \cos(\omega t)] + N_{n-1}'(t) \sin(\omega t)}$$

Solution at order n

- The equation $N_n''(t) + \kappa N_n'(t) = g_n(t)$ admits a "simple" solution
  - Recall $g_n(t) = N_{n-1}(t)\,[\lambda \sin(\omega t) + \omega \cos(\omega t)] + N_{n-1}'(t) \sin(\omega t)$

$$\boxed{N_n(t) = \frac{1}{\kappa} \int_0^t g_n(\tau) - \frac{1}{\kappa} \int_0^t e^{-\kappa(t-\tau)} g_n(\tau)}$$

N(t) is obtained by successive integrations of $g_n(t)$

- Start with N_0 =const.$= N(0)$
- $N_1(t) = K_1 + E_1\, e^{-\kappa t} + S_1 \sin(\omega t) + C_1 \cos(\omega t)$
- $g_2(t)$ thus contains a term of the form $\sin^2(\omega t)$
- At order $o(\sigma^2)$, one can show

$$N(t) = N_0 \frac{\lambda\beta/\Lambda}{2\kappa(\kappa^2+\omega^2)} \times \sigma^2 t + \left\{ \begin{array}{c} \text{constant} \\ \text{oscillating} \\ \text{decaying} \end{array} \right. \text{terms} + o(\sigma^3)$$

Neutron noise without linearization

Emergence of the drift term

A simple iterative process, that generates a plethora of terms...

Entrée [9]:

```
g1 = N0*(l*sin(w*t)+w*cos(w*t))
N1 =  (1/g)*integral( g1, t,0,t) - (exp(-g*t)/g)*integral( expand(g1*exp(g*t)),t,0,t  ); N1.full_simplify()
```

Out[9]:

$$\frac{\left(\left(N_0\gamma^2 - N_0\gamma\lambda_d\right)\omega e^{(\gamma t)}\sin(\omega t) + \left(N_0\gamma - N_0\lambda_d\right)\omega^2 - \left(N_0\gamma^2\lambda_d + N_0\gamma\omega^2\right)\cos(\omega t)e^{(\gamma t)} + \left(N_0\gamma^2\lambda_d + N_0\lambda_d\omega^2\right)e^{(\gamma t)}\right)e^{(-\gamma t)}}{\gamma^3\omega + \gamma\omega^3}$$

Entrée [11]:

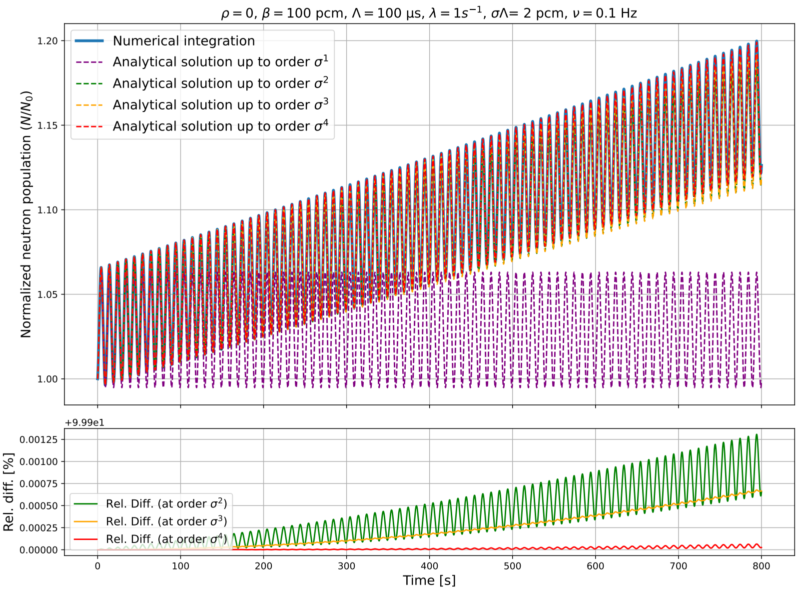
```
g2 = N1*(l*sin(w*t)+w*cos(w*t)) + diff(N1,t)*sin(w*t)
N2 =  (1/g)*integral( g2, t,0,t) - (exp(-g*t)/g)*integral( expand(g2*exp(g*t)),t,0,t  ); N2.full_simplify().expand(
```

Out[11]:

$$\frac{N_0\gamma^6\lambda_d\omega^2 t}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} - \frac{N_0\gamma^5\lambda_d^2\omega^2 t}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} + \frac{5N_0\gamma^4\lambda_d\omega^4 t}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)}$$
$$- \frac{5N_0\gamma^3\lambda_d^2\omega^4 t}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} + \frac{2N_0\gamma^2\lambda_d\omega^6 t}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{2N_0\gamma\lambda_d^2\omega^6 t}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$- \frac{N_0\gamma^6\omega^2\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{2N_0\gamma^5\lambda_d\omega^2\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{N_0\gamma^4\lambda_d^2\omega^2\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$- \frac{5N_0\gamma^4\omega^4\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{9N_0\gamma^3\lambda_d\omega^4\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{4N_0\gamma^2\lambda_d^2\omega^4\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$- \frac{4N_0\gamma^2\omega^6\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{4N_0\gamma\lambda_d\omega^6\cos(\omega t)e^{(-\gamma t)}}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{3N_0\gamma^6\lambda_d\omega\cos(\omega t)\sin(\omega t)}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)}$$
$$+ \frac{3N_0\gamma^5\lambda_d^2\omega\cos(\omega t)\sin(\omega t)}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} - \frac{3N_0\gamma^5\omega^3\cos(\omega t)\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{3N_0\gamma^4\lambda_d\omega^3\cos(\omega t)\sin(\omega t)}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)}$$
$$+ \frac{3N_0\gamma^3\lambda_d^2\omega^3\cos(\omega t)\sin(\omega t)}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} - \frac{3N_0\gamma^3\omega^5\cos(\omega t)\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{3N_0\gamma^2\lambda_d\omega^5\cos(\omega t)\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$- \frac{N_0\gamma^4\lambda_d\omega^3 e^{(-\gamma t)}\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} + \frac{N_0\gamma^3\lambda_d^2\omega^3 e^{(-\gamma t)}\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{4N_0\gamma^2\lambda_d\omega^5 e^{(-\gamma t)}\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$+ \frac{4N_0\gamma\lambda_d^2\omega^5 e^{(-\gamma t)}\sin(\omega t)}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8} - \frac{N_0\gamma^6\lambda_d^2\sin(\omega t)^2}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} + \frac{N_0\gamma^6\omega^2\sin(\omega t)^2}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$
$$- \frac{9N_0\gamma^5\lambda_d\omega^2\sin(\omega t)^2}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} + \frac{N_0\gamma^4\lambda_d^2\omega^2\sin(\omega t)^2}{2\left(\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8\right)} - \frac{N_0\gamma^4\omega^4\sin(\omega t)^2}{\gamma^8\omega^2 + 6\gamma^6\omega^4 + 9\gamma^4\omega^6 + 4\gamma^2\omega^8}$$

Comparison to a numerical integration

- 4th order term has a strong impact on the relative differences



Drift term at 4th order

- $N_2(t) = \sigma^2 \frac{\lambda\beta/\Lambda}{2\gamma(\gamma^2+\omega^2)} \times t + \cdots$
- $N_4(t) = \frac{\sigma^4}{2} \left(\frac{\lambda\beta/\Lambda}{2\gamma(\gamma^2+\omega^2)}\right)^2 \times t^2 + \cdots$

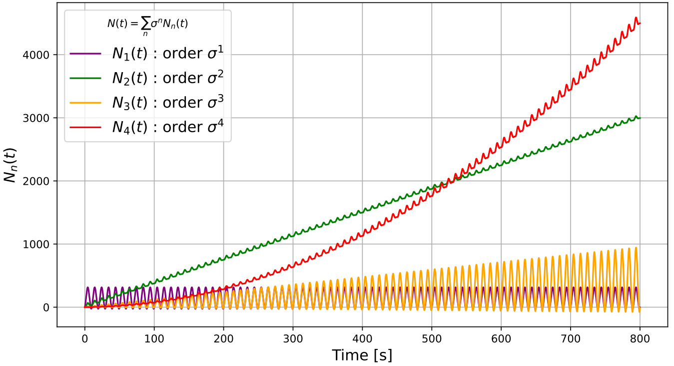
Hypothesis

Drift term of the form $N_0 \exp\left(\frac{\lambda\beta/\Lambda}{2\gamma(\gamma^2+\omega^2)} \times \sigma^2 t\right)$

What about the odd orders ?

- A careful analysis up to order 5 points to a contribution

$$N_0 \exp\left(\frac{\lambda\sigma}{\kappa\omega} \frac{\lambda\beta/\Lambda}{2\gamma(\gamma^2+\omega^2)} \times \sigma^2 t\right)$$



Characterization of the drift term

Muss es sein ?

A complete characterization of the drift term ?

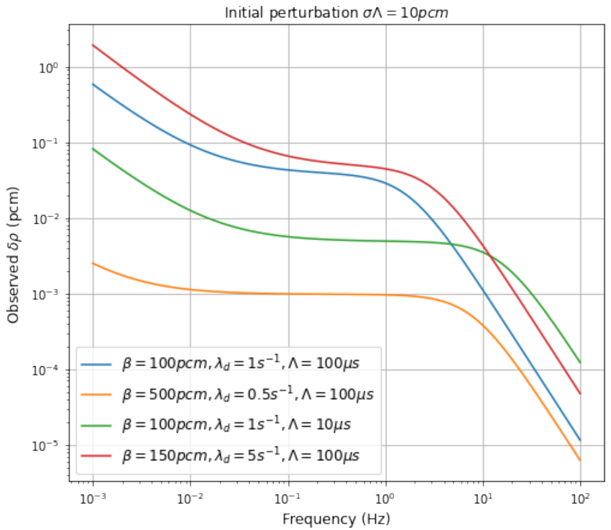
- A detailed analysis suggests (recall $\kappa = \frac{\beta}{\Lambda} + \lambda$)
  - Numerically verified over different sets of kinetic parameters

$$\boxed{N(t) = F(t) \times N_0 \exp\left\{ \frac{\lambda\beta/\Lambda}{2\kappa(\kappa^2 + \omega^2)} \left(1 + \frac{\lambda\sigma}{\kappa\omega}\right) \times \sigma^2 t \right\}}$$

- The unknown function $F(t)$ encapsulate all of the constant + oscillating terms

Response to the initial perturbation

- Write the drift as $N_0 \exp\left(\frac{\delta\rho t}{\Lambda}\right)$
- The observed response $\delta\rho$ is found to be consistently lower than the input perturbation $\sigma\Lambda$



Conclusion

Towards a better understanding

- Set up an original method to extract the drift term associated to sinusoidal excitations
  - Is this truly original ? Has any other approach been tried before ?
  - Some more elaborate treatment might reveal more adapted (e.g. multiple scale method ?)
- The reactivity shift seems to be always quite low ! Observable ?
- Does this give a better understanding of the mechanism at play ?
  - Meh ...

Perspective

- Add spatial dimension and decompose into local/global components ?
- Try to use symbolic regression ? PYSR, PYSINDY
- Results for stochastic *temporally correlated* noise in upcoming paper (Submitted to ANE)
  - *"An enquiry into noise-induced reactivity shifts in the point-kinetics equations"*