Two-group theory of noise in fast Molten Salt Reactors

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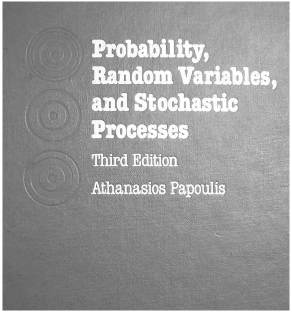
Motto

"**Scientific theories deal with concepts, never with reality**"

Athanasios Paopoulis, Foreword to the first edition of his book

Probability, Random Variables and Stochastic Processes



Reactor noise diagnostics for fast reactors

- Noise methods were traditionally developed for the currently used thermal reactors with measuring only the thermal flux/thermal noise
- The simplest manageable but realistic model is based on two-group theory
- Most of next generation reactors will use a fast spectrum
- The 2-group methodology/theory has to be suited for fast systems
- The methodology was extended to fast systems for reactors with **solid fuel**. A comprehensive edited book was written recently on these extensions
- In this talk the extensions to fast **Molten Salt Reactors** will be presented.

Surveillance and Diagnostics of Next Generation Nuclear Reactors

Next generation nuclear reactors are expected to contribute to the future energy mix, forming part of the transition to an emission-free energy system. Diagnostics and monitoring are a necessary part of the design, development and operation of the new generation of nuclear power plants.

Reactor diagnostics, particularly methods based on neutron noise analysis, have been used for traditional light and heavy water reactors, and offer benefits for early detection, identification and quantification of incipient failures. Diagnostic techniques will be useful for the next generation of power plants, including small, medium and small modular reactors (SMR), which use alternative coolant types.

To expedite the planning of effective diagnostics, one needs knowledge of what problems might occur, and what methods are suitable for detecting and quantifying those problems. This book conveys a thorough study of the dynamic transfer properties of the main Gen-IV and SMR types; it also serves as a handbook for two-group power reactor noise theory for both traditional and fast systems.

Chapters cover principles of reactor diagnostics and noise analysis, the structure of the core response in basic planned Gen-IV types and SMRs to some basic perturbations, a survey of dynamic core transfer calculational codes, zero power noise methods, instrumentation and control systems, and worldwide activities in design and planned diagnostics of SMRs.

Surveillance and Diagnostics of Next Generation Nuclear Reactors is a concise and logically-structured reference for nuclear scientists, researchers and engineers involved in exploring new reactor concepts and developing designs for the coming generation of nuclear power plants.

About the Editors

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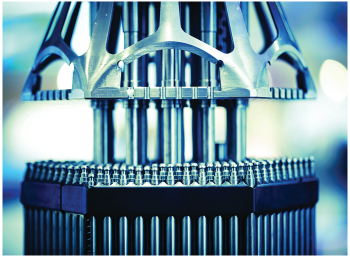

Surveillance and Diagnostics of Next Generation Nuclear Reactors

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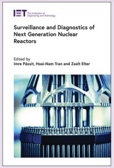

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Description

Next generation nuclear reactors are expected to contribute to the future energy mix, forming part of the transition to an emission-free energy system. Diagnostics and monitoring are a necessary part of the design, development and operation of the new generation of nuclear power plants.

Reactor diagnostics, particularly methods based on neutron noise analysis, have been used for traditional light and heavy water reactors, and offer benefits for early detection, identification and quantification of incipient failures. Diagnostic techniques will be useful for the next generation of power plants, including small, medium and small modular reactors (SMR), which use alternative coolant types.

To expedite the planning of effective diagnostics, one needs knowledge of what problems might occur, and what methods are suitable for detecting and quantifying those problems. This book conveys a thorough study of the dynamic transfer properties of the main Gen-IV and SMR types; it also serves as a handbook for two-group power reactor noise theory for both traditional and fast systems.

Chapters cover principles of reactor diagnostics and noise analysis for both thermal and fast systems, the structure of the core

Noise equations for thermal systems

The equations for the space-frequency dependent neutron noise, $\delta\phi(x,\omega)$, which are obtained from the time-dependent equations for the flux and the precursors, after subtracting the static equations and neglecting second order terms, read as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) & \nu\Sigma_{f2}(\omega) \\ \Sigma_R & D_2\nabla^2 - \Sigma_2(\omega) \end{bmatrix} \begin{bmatrix} \delta\phi_1(x,\omega) \\ \delta\phi_2(x,\omega) \end{bmatrix} = \begin{bmatrix} S_1(x,\omega) \\ S_2(x,\omega) \end{bmatrix} \tag{1}$$

Here, the following standard notations and definitions were used:

$$\Sigma_{f,i}(\omega) = \Sigma_{f,i}\left(1 - \frac{i\omega\beta}{\lambda + i\omega}\right); \qquad i = 1,2 \tag{2}$$

$$\Sigma_1(\omega) = \Sigma_{a1} + \Sigma_R + \frac{i\,\omega}{v_1} - \nu\Sigma_{f1}(\omega) \tag{3}$$

and

$$\Sigma_2(\omega) = \Sigma_{a2} + \frac{i\omega}{v_2} \tag{4}$$

Thermal systems: the noise sources

The noise sources $S_1(x,\omega)$ and $S_2(x,\omega)$ are defined in terms of the cross section fluctuations and the static fluxes as follows:

$$S_1(x,\omega) = \left[\delta\Sigma_R(x,\omega) + \delta\Sigma_{a_1}(x,\omega) - \delta\nu\Sigma_{f_1}(x,\omega)\left(1 - \frac{i\omega\beta}{i\omega+\lambda}\right)\right]\phi_1(x) - \delta\Sigma_{f_2}(x,\omega)\left(1 - \frac{i\omega\beta}{i\omega+\lambda}\right)\phi_2(x) \tag{5}$$

and

$$S_2(x,\omega) = -\delta\Sigma_R(x,\omega)\phi_1(x) + \delta\Sigma_{a_2}(x,\omega)\phi_2(x). \tag{6}$$

Thermal systems: the Green's function

The equation for the Green's matrix that connects the noise in the fast and thermal groups with the fast and thermal noise sources can then be written as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) & \nu\Sigma_{f2}(\omega) \\ \Sigma_R & D_2\nabla^2 - \Sigma_2(\omega) \end{bmatrix} \begin{bmatrix} G_{11}(x,x',\omega) & G_{12}(x,x',\omega) \\ G_{21}(x,x',\omega) & G_{22}(x,x',\omega) \end{bmatrix} = \begin{bmatrix} \delta(x-x') & 0 \\ 0 & \delta(x-x') \end{bmatrix}. \tag{7}$$

From Eq. (7) the noise can be obtained as:

$$\begin{bmatrix} \delta\phi_1(x,\omega) \\ \delta\phi_2(x,\omega) \end{bmatrix} = \int_0^H \begin{bmatrix} G_{11}(x,x',\omega) & G_{12}(x,x',\omega) \\ G_{21}(x,x',\omega) & G_{22}(x,x',\omega) \end{bmatrix} \begin{bmatrix} S_1(x',\omega) \\ S_2(x',\omega) \end{bmatrix} \mathrm{d}x' \tag{8}$$

Fast systems - energy separation between groups

- Thermal systems: energy separation between groups 1 and 2 at 0.625 eV (the minimum of the flux between the fission spectrum and the thermal peak)
- Fast systems: at the threshold for fast fission - 1.35 MeV (around the top of the fast spectrum.)

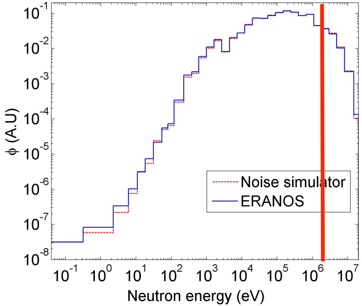

Figure 1: Energy spectrum of the flux in a fast reactor

Fast systems - fission neutron source

The second modification concerns the fact that with the energy separation of 1.35 MeV, both the prompt and the delayed neutrons will appear both in group 1 and 2 with the corresponding spectral weight. Correspondingly, we define

$\chi_{p,1}$ - fraction of prompt fission neutrons appearing in group 1

$\chi_{p,2}$ - fraction of prompt fission neutrons appearing in group 2

$\chi_{d,1}$ - fraction of delayed neutrons appearing in group 1

$\chi_{d,2}$ - fraction of delayed neutrons appearing in group 2.

For thermal systems, $\chi_{p,1} = \chi_{d,1} = 1$ and $\chi_{p,2} = \chi_{d,2} = 0$.

Noise equations in fast systems (solid fuel)

The equations for the neutron noise in the two groups will read as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) & \nu\Sigma_{f2}(\omega) \\ \Sigma_{R2}(\omega) & D_2\nabla^2 - \Sigma_2(\omega) \end{bmatrix} \begin{bmatrix} \delta\phi_1(x,\omega) \\ \delta\phi_2(x,\omega) \end{bmatrix} = \begin{bmatrix} S_1(x,\omega) \\ S_2(x,\omega) \end{bmatrix} \tag{9}$$

with

$$\chi_{1,2}(\omega) = \chi_{p1,2}(1-\beta) + \chi_{d1,2}\,\frac{\lambda\,\beta}{i\,\omega + \lambda} \tag{10}$$

$$\Sigma_1(\omega) = \Sigma_{a1} + \frac{i\,\omega}{v_1} + \Sigma_R - \chi_1(\omega)\,\nu\Sigma_{f1} \tag{11}$$

$$\Sigma_2(\omega) = \Sigma_{a2} + \frac{i\,\omega}{v_2} - \chi_2(\omega)\,\nu\Sigma_{f2} \tag{12}$$

$$\Sigma_{f2}(\omega) = \chi_1(\omega)\,\nu\Sigma_{f2}. \tag{13}$$

and

$$\Sigma_{R2}(\omega) = \Sigma_R + \chi_2(\omega)\,\nu\Sigma_{f1} \tag{14}$$

The noise sources in fast systems

The components of the noise source are given as

$$S_1(x,\omega) = \Big(\delta\Sigma_{a_1}(x,\omega) + \delta\Sigma_R(x,\omega) - \chi_1(\omega)\,\delta\nu\Sigma_{f1}(x,\omega)\Big)\phi_1(x) - \chi_1(\omega)\,\delta\nu\Sigma_{f_2}(x,\omega)\,\phi_2(x) \tag{15}$$

and

$$S_2(x,\omega) = -\left(\delta\Sigma_R(x,\omega) + \chi_2(\omega)\,\delta\nu\Sigma_{f1}(\omega)\right)\phi_1(x) + \left(\delta\Sigma_{a_2}(x,\omega) - \chi_2(\omega)\,\delta\nu\Sigma_{f2}\right)\,\phi_2(x). \tag{16}$$

If $\chi_{p,1} = \chi_{d,1} = 1$ and $\chi_{p,2} = \chi_{d,2} = 0$ (thermal systems), then one has

$$\chi_1(\omega) = (1-\beta) + \frac{\lambda\,\beta}{i\,\omega + \lambda} = \left(1 - \frac{i\omega\beta}{\lambda + i\omega}\right) \quad \text{and} \quad \chi_2(\omega) = 0, \tag{17}$$

i.e. all expressions revert to the former ones for thermal system. The above is therefore a generalisation of the formalism which is valid for both thermal and fast systems.

Solution for the Green's matrix

With these definitions, the roots of the characteristic equation of (9) read as

$$\mu^2(\omega) = -\frac{1}{2}\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right) + \frac{1}{2}\sqrt{\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right)^2 + 4\frac{\Sigma_{R2}(\omega)\,\nu\Sigma_{f2}(\omega) - \Sigma_1(\omega)\,\Sigma_2(\omega)}{D_1\,D_2}}, \tag{18}$$

and

$$\nu^2(\omega) = \frac{1}{2}\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right) + \frac{1}{2}\sqrt{\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right)^2 + 4\frac{\Sigma_{R2}(\omega)\,\nu\Sigma_{f2}(\omega) - \Sigma_1(\omega)\,\Sigma_2(\omega)}{D_1\,D_2}}, \tag{19}$$

Solution

The equation can be solved columnwise for the Green's matrix, and the solution reads as follows. Define the basic solutions which fulfil the boundary conditions as

$$g_\mu(x,x',\omega) = \begin{cases} \sin[\mu(\omega)\,(a+x)]\;\sin[\mu(\omega)\,(a-x')]; & x < x'; \\ \\ \sin[\mu(\omega)\,(a-x)]\;\sin[\mu(\omega)\,(a+x')]; & x > x'; \end{cases} \tag{20}$$

and

$$g_\nu(x,x',\omega) = \begin{cases} \sinh[\nu(\omega)\,(a+x)]\;\sinh[\nu(\omega)\,(a-x')]; & x < x'; \\ \\ \sinh[\nu(\omega)\,(a-x)]\;\sinh[\nu(\omega)\,(a+x')]; & x > x', \end{cases} \tag{21}$$

The solution for the Green's matrix

Then, one has for $j = 1,2$:

$$\begin{bmatrix} G_{1,j}(x,x',\omega) \\ G_{2,j}(x,x',\omega) \end{bmatrix} = \begin{bmatrix} 1 \\ c_\mu(\omega) \end{bmatrix} A_{\mu,j}(\omega)\, g_\mu(x,x',\omega) + \begin{bmatrix} 1 \\ c_\nu(\omega) \end{bmatrix} A_{\nu j}(\omega)\, g_\nu(x,x',\omega) \tag{22}$$

with

$$c_\mu(\omega) = \frac{\Sigma_1(\omega) + D_1\, \mu^2(\omega)}{\nu\Sigma_{f2}(\omega)} \tag{23}$$

and

$$c_\nu(\omega) = \frac{\Sigma_1(\omega) - D_1\, \nu^2(\omega)}{\nu\Sigma_{f2}(\omega)} \tag{24}$$

The four coefficients $A_{\mu j}$ and $A_{\nu j}$, $j = 1,2$ are obtained from the continuity of the Green's function components and the discontinuity of the derivatives at $x = x'$.

Examples of Green's functions for thermal and fast systems

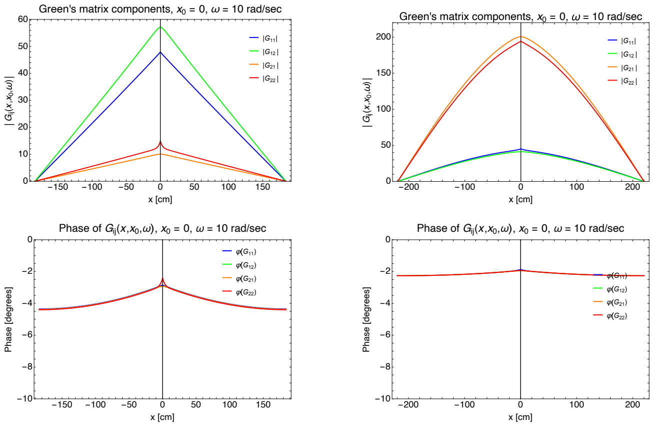

Figure 2: Space dependence of the amplitude and the phase of the components of the Green's function for $x_0 = 0$ and $\omega = 10$ rad/s for a commercial BWR (left column) and for a large fluoride salt cooled fast core

Neutron noise in fast MSRs

The following notations are introduced:

u: the speed of the fuel flow

H : height of the core

L : length of the external loop

$T = H + L$: the total length of the loop

$\tau_c = H/u$: the transit time of the fuel in the core

$\tau_L = L/u$: the transit time of the fuel in the external loop

$\tau = \tau_c + \tau_L = T/u$: the total recirculation time.

Static equations for fast MSRs

The static two-group equations for an MSR read as follows in the x coordinate system, with the system boundaries at $x = \pm a$:

$$D_1 \nabla^2 \phi_1(x) - (\Sigma_{a1} + \Sigma_R)\phi_1(x) + \chi_{p1} \left[\nu\Sigma_{f1}(1-\beta)\phi_1(x) + \nu\Sigma_{f2}(1-\beta)\,\phi_2(x)\right] + \chi_{d1}\lambda\, C_0(x) = 0$$

$$D_2 \nabla^2 \phi_2(x) - \Sigma_{a2}\phi_2(x) + \Sigma_R\phi_1(x) + \chi_{p2} \left[\nu\Sigma_{f1}(1-\beta)\phi_1(x) + \nu\Sigma_{f2}(1-\beta)\,\phi_2(x)\right] + \chi_{d2}\lambda\, C_0(x) = 0$$

$$u\frac{dC_0(x)}{dx} = \beta\left[\nu\Sigma_{f1}\phi_1(x) + \nu\Sigma_{f2}\,\phi_2(x)\right] - \lambda C_0(x) \tag{25}$$

Boundary conditions :

- vanishing of the fluxes at the extrapolated boundaries,
- for the precursors one has

$$C_0(-x) = C_0(x)\, e^{-\lambda\tau_L} \tag{26}$$

Static equations: delayed neutron precursors

The equation for the delayed neutron precursors can be integrated, to express the precursors as functions of the fluxes:

$$C_0(x) = e^{-\frac{x\lambda}{u}} \frac{\beta}{u} \left(\frac{1}{e^{\lambda\tau} - 1} \int_{-a}^{a} e^{\frac{x'\lambda}{u}} \left[\nu\Sigma_{f1}\phi_1(x') + \nu\Sigma_{f2}\, \phi_2(x') \right] \mathrm{d}x' + \int_{-a}^{x} e^{\frac{\lambda x'}{u}} \left[\nu\Sigma_{f1}\phi_1(x') + \nu\Sigma_{f2}\, \phi_2(x') \right] \mathrm{d}x' \right). \tag{27}$$

Analytical solution can only be obtained in the prompt recirculation (infinite fuel velocity) approximation, in which case the above will simplify to

$$C_0(x) = \frac{\beta}{\lambda\, T} \int_{-a}^{a} \left[\nu\Sigma_{f1}\phi_1(x') + \nu\Sigma_{f2}\, \phi_2(x') \right] \mathrm{d}x'. \tag{28}$$

Static equations for the fluxes

Introducing the notations

$$\Sigma_1 = \Sigma_{a1} + \Sigma_R - \chi_{p1}\,\nu\Sigma_{f1}(1-\beta), \tag{29}$$

$$\Sigma_2 = \Sigma_{a2} - \chi_{p2}\,\nu\Sigma_{f2}(1-\beta), \tag{30}$$

$$\Sigma_{R2} = \Sigma_R + \chi_{p2}\,\nu\Sigma_{f1}(1-\beta), \tag{31}$$

one obtains the static equations for the fast and slow flux in a matrix form as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1 & \chi_{p1}\,\nu\Sigma_{f2}(1-\beta) \\ \Sigma_{R2} & D_2\nabla^2 - \Sigma_2 \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix} = - \begin{bmatrix} \chi_{d1} \\ \chi_{d2} \end{bmatrix} \frac{\beta}{T} \int_{-a}^{a} \left[\nu\Sigma_{f1}\phi_1(x') + \nu\Sigma_{f2}\,\phi_2(x')\right] \mathrm{d}x'. \tag{32}$$

The character of the static equation

The definite integrals on the r.h.s. of (32) are constants, hence the equation is formally an inhomogeneous equation. Its solution can be sought as such (general solution of the homogeneous plus a particular solution of the inhomogeneous.

On the other hand, even the constant term is proportional to the solution itself (it is its integral), hence the equation is still homogeneous, whose solution is undetermined to a constant factor, and has only a solution with a specific combination of the parameters (=criticality condition).

The solution for the static flux

The particular solution of the inhomogeneous equation is a constant, thus a general solution which satisfies the boundary conditions at $x = \pm a$ has the form

$$\begin{bmatrix} \phi_1(x) \\ \phi_2(x) \end{bmatrix} = A \begin{bmatrix} 1 \\ c_\mu \end{bmatrix} \left[\cos(\mu x) - \cos(\mu a)\right] + B \begin{bmatrix} 1 \\ c_\nu \end{bmatrix} \left[\cosh(\nu x) - \cosh(\nu a)\right] \tag{33}$$

where μ^2 and $-\nu^2$ are the two roots of the characteristic equation,

$$c_\mu = \frac{\Sigma_{R2}}{\Sigma_2 + D_2\mu^2} = \frac{\Sigma_1 + D_1\mu^2}{\chi_{p1}\nu\Sigma_{f2}\,(1-\beta)} \tag{34}$$

and

$$c_\nu = \frac{\Sigma_{R2}}{\Sigma_2 - D_2\nu^2} = \frac{\Sigma_1 - D_1\nu^2}{\chi_{p1}\nu\Sigma_{f2}\,(1-\beta)} \tag{35}$$

Substituting the above into (32) yields a relationship between A and B, as well as the criticality equation which has to be fulfilled.

The dynamic equations

The equation for the space- and frequency dependent Green's function matrix is given as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) + \dfrac{\chi_{d1}\,\nu\Sigma_{f1}\,\beta(\omega)}{T}\displaystyle\int_{-a}^{a}\mathrm{d}x' & \chi_{p1}\,\nu\Sigma_{f2}(1-\beta) + \dfrac{\chi_{d1}\,\nu\Sigma_{f2}\,\beta(\omega)}{T}\displaystyle\int_{-a}^{a}\mathrm{d}x' \\ \Sigma_{R2} + \dfrac{\chi_{d2}\,\nu\Sigma_{f1}\,\beta(\omega)}{T}\displaystyle\int_{-a}^{a}\mathrm{d}x' & D_2\nabla^2 - \Sigma_2(\omega) + \dfrac{\chi_{d2}\nu\Sigma_{f2}\,\beta(\omega)}{T}\displaystyle\int_{-a}^{a}\mathrm{d}x' \end{bmatrix} \times$$

$$\begin{bmatrix} G_{11}(x,x_0,\omega) & G_{12}(x,x_0,\omega) \\ G_{21}(x,x_0,\omega) & G_{22}(x,x_0,\omega) \end{bmatrix} = \begin{bmatrix} \delta(x-x_0) & 0 \\ 0 & \delta(x-x_0) \end{bmatrix}. \tag{36}$$

The integrals on the l.h.s. of Eq. (36) have to be taken with respect to the first argument of the Green's function.

The dynamic equations

Here

$$\beta(\omega) = \beta\,\frac{\lambda}{\lambda + i\omega} \tag{37}$$

$$\Sigma_1(\omega) = \Sigma_{a1} + \Sigma_R - \chi_{p1}\nu\Sigma_{f1}(1-\beta) + \frac{i\,\omega}{v_1}, \tag{38}$$

$$\Sigma_2(\omega) = \Sigma_{a2} - \chi_{p2}\nu\Sigma_{f2}(1-\beta) + \frac{i\,\omega}{v_2}, \tag{39}$$

and

$$\Sigma_{R2} = \Sigma_R + \chi_{p2}\,\nu\Sigma_{f1}(1-\beta). \tag{40}$$

Solution of the dynamic equations

Eq. (36) can be solved with a method similar to the solution of the transport equation as a sum of the uncollided and collided flux.

The equation for the uncollided part has a Dirac delta source but no scattering (no integral term), whereas the collided part has no Dirac delta singularity, but its source contains the integral of the uncollided solution.

In an analogous manner, the full solution for the Greens function $\hat{\mathbf{G}}$ of the MSR will be sought as the sum of the unrecirculated ($\hat{\mathbf{G}^{\mathbf{u}}}$) and the recirculated ($\hat{\mathbf{G}^{\mathbf{c}}}$) terms:

$$\hat{\mathbf{G}} = \hat{\mathbf{G}^{\mathbf{u}}} + \hat{\mathbf{G}^{\mathbf{c}}} \tag{41}$$

The equation for the unrecirculated part

The equation for the unrecirculated compoment $\hat{\mathbf{G}^{\mathbf{u}}}$ of the Green's function reads as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) & \chi_{p1}\,\nu\Sigma_{f2}(1-\beta) \\ \\ \Sigma_{R2} & D_2\nabla^2 - \Sigma_2(\omega) \end{bmatrix} \begin{bmatrix} G^u_{11}(x,x_0,\omega) & G^u_{12}(x,x_0,\omega) \\ \\ G^u_{21}(x,x_0,\omega) & G^u_{22}(x,x_0,\omega) \end{bmatrix} = \begin{bmatrix} \delta(x-x_0) & 0 \\ \\ 0 & \delta(x-x_0) \end{bmatrix}. \tag{42}$$

This equation has formally the same form as for the traditional reactors, only the parameters are different. Hence the solution of this component can be written down immediately and has the same form as (22) (see next page).

The solution for the unrecirculated Green's matrix

The roots of the characteristic equation are given as

$$\mu^2(\omega) = -\frac{1}{2}\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right) + \frac{1}{2}\sqrt{\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right)^2 - 4\frac{\Sigma_1(\omega)\,\Sigma_2(\omega) - \Sigma_{R2}\,\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1\,D_2}}, \tag{43}$$

and

$$\nu^2(\omega) = \frac{1}{2}\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right) + \frac{1}{2}\sqrt{\left(\frac{\Sigma_1(\omega)}{D_1} + \frac{\Sigma_2(\omega)}{D_2}\right)^2 - 4\frac{\Sigma_1(\omega)\,\Sigma_2(\omega) - \Sigma_{R2}\,\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1\,D_2}}. \tag{44}$$

The solution for the unrecirculated Green's matrix

The coupling constants $c_\mu(\omega)$ and $c_\nu(\omega)$ are given by

$$c_\mu = \frac{\Sigma_{R2}}{\Sigma_2(\omega) + D_2\mu^2(\omega)} = \frac{\Sigma_1(\omega) + D_1\mu^2(\omega)}{\chi_{p1}\,\nu\Sigma_{f2}\,(1-\beta)} \tag{45}$$

and

$$c_\nu = \frac{\Sigma_{R2}}{\Sigma_2(\omega) - D_2\nu^2(\omega)} = \frac{\Sigma_1(\omega) - D_1\nu^2(\omega)}{\chi_{p1}\,\nu\Sigma_{f2}\,(1-\beta)}. \tag{46}$$

Then, one has for $j = 1{,}2$:

$$\begin{bmatrix} G^u_{1,j}(x,x',\omega) \\ \\ G^u_{2,j}(x,x',\omega) \end{bmatrix} = \begin{bmatrix} 1 \\ \\ c_\mu(\omega) \end{bmatrix} A_{\mu,j}(\omega)\, g_\mu(x,x',\omega) + \begin{bmatrix} 1 \\ \\ c_\nu(\omega) \end{bmatrix} A_{\nu j}(\omega)\, g_\nu(x,x',\omega) \tag{47}$$

The solution for the unrecirculated component

The coefficients $A_{\mu j}(\omega)$ and $A_{\nu j}(\omega)$, $j = 1{,}2$ are given by

$$A_{\mu 1}(\omega) = \frac{c_\nu(\omega)\,\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1^2\,\mu(\omega)\,\sin[2\,\mu(\omega)\,a]\,(\mu^2(\omega)+\nu^2(\omega))}, \tag{48}$$

$$A_{\nu 1}(\omega) = -\frac{c_\mu(\omega)\,\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1^2\,\nu(\omega)\,\sinh[2\,\nu(\omega)\,a]\,(\mu^2(\omega)+\nu^2(\omega))}, \tag{49}$$

$$A_{\mu 2}(\omega) = -\frac{\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1\,D_2\,\mu(\omega)\,\sin[2\,\mu(\omega)\,a]\,(\mu^2(\omega)+\nu^2(\omega))} \tag{50}$$

and

$$A_{\nu 2}(\omega) = \frac{\chi_{p1}\,\nu\Sigma_{f2}(1-\beta)}{D_1\,D_2\,\nu(\omega)\,\sin[2\,\nu(\omega)\,a]\,(\mu^2(\omega)+\nu^2(\omega))} \tag{51}$$

The equation for the recirculated part

The equation for the recirculated compoment $\hat{\mathbf{G}}^{\mathbf{c}}$ of the Green's function reads as

$$\begin{bmatrix} D_1\nabla^2 - \Sigma_1(\omega) & \chi_{p1}\,\nu\Sigma_{f2}(1-\beta) \\ \\ \Sigma_{R2} & D_2\nabla^2 - \Sigma_2(\omega) \end{bmatrix} \begin{bmatrix} G^c_{11}(x,x_0,\omega) & G^c_{12}(x,x_0,\omega) \\ \\ G^c_{21}(x,x_0,\omega) & G^c_{22}(x,x_0,\omega) \end{bmatrix} =$$

$$-\frac{\beta(\omega)}{T}\int_{-a}^{a}\begin{bmatrix} \chi_{d1}(\nu\Sigma_{f1}\,G_{11} + \nu\Sigma_{f2}\,G_{21}) & \chi_{d1}(\nu\Sigma_{f1}\,G_{12} + \nu\Sigma_{f2}\,G_{22}) \\ \chi_{d2}(\nu\Sigma_{f1}\,G_{11} + \nu\Sigma_{f2}\,G_{21}) & \chi_{d2}(\nu\Sigma_{f1}\,G_{12} + \nu\Sigma_{f2}\,G_{22}) \end{bmatrix} \mathrm{d}x' \tag{52}$$

On the right hand side of (52), one has the full Green's function under the integral sign, i.e.

$$G_{ij} = G^u_{ij} + G^c_{ij} \tag{53}$$

and the integrals need to be performed w.r.t the first argument of the Green's function

The solution for the recirculated part

The equation has the same structure as the one for the static flux, and hence the solution can be sought with the same method. Define the functions

$$\varphi_\mu(x,\omega) = \cos[\mu(\omega)\, x] - \cos[\mu(\omega)\, a] \tag{54}$$

and

$$\varphi_\nu(x,\omega) = \cosh[\nu(\omega)\, x] - \cosh[\nu(\omega)\, a] \tag{55}$$

The two columns ($j = 1,2$) of the recirculated Green's function matrix are then written as

$$\begin{bmatrix} G^c_{1j}(x,x_0,\omega) \\ G^c_{2j}(x,x_0,\omega) \end{bmatrix} = \begin{bmatrix} 1 \\ c_\mu(\omega) \end{bmatrix} A_j(x_0,\omega)\, \varphi_\mu(x,\omega) + \begin{bmatrix} 1 \\ c_\nu(\omega) \end{bmatrix} B_j(x_0,\omega)\, \varphi_\nu(x,\omega) \tag{56}$$

The solution for the recirculated part

To calculate the coefficients $A_j(x_0, \omega)$ and $B_j(x_0, \omega)$, we need to calculate the integrals on the right hand side of (52). We will use the shorthand notations

$$\tilde{g}_\mu(x_0,\omega) \equiv \int_{-a}^{a} g_\mu(x,x_0,\omega)\mathrm{d}x \equiv \frac{2\,\sin[\mu(\omega)\,a]\,\varphi_\mu(x_0,\omega)}{\mu(\omega)}, \tag{57}$$

$$\tilde{g}_\nu(x_0,\omega) = \int_{-a}^{a} g_\nu(x,x_0,\omega)\mathrm{d}x = -\frac{2\,\sinh[\nu(\omega)\,a]\,\varphi_\nu(x_0,\omega)}{\nu(\omega)}, \tag{58}$$

$$\tilde{\varphi}_\mu(\omega) \equiv \int_{-a}^{a} \varphi_\mu(x,\omega)\mathrm{d}x = 2\left(\frac{\sin[\mu(\omega)\,a]}{\mu(\omega)} - a\,\cos[\mu(\omega)]\right) \tag{59}$$

and

$$\tilde{\varphi}_\nu(\omega) \equiv \int_{-a}^{a} \varphi_\nu(x,\omega)\mathrm{d}x = 2\left(\frac{\sinh[\nu(\omega)\,a]}{\nu(\omega)} - a\,\cosh[\nu(\omega)]\right). \tag{60}$$

The solution for the recirculated part

This procedure will lead to a 2 × 2 matrix equation for the coefficient matrix $\hat{\mathbf{A}}(x_0,\omega)$

$$\hat{\mathbf{A}}(x_0,\omega) = \begin{bmatrix} A_1(x_0,\omega) & A_2(x_0,\omega) \\ \\ B_1(x_0,\omega) & B_2(x_0,\omega) \end{bmatrix} \tag{61}$$

in the form

$$\hat{\mathbf{M}}(\omega)\,\hat{\mathbf{A}}(x_0,\omega) = \hat{\mathbf{L}}(x_0,\omega) \tag{62}$$

From this the searched coefficients are obtained by inverting the equation,

$$\hat{\mathbf{A}}(x_0,\omega) = \hat{\mathbf{M}}^{-1}(\omega)\,\hat{\mathbf{L}}(x_0,\omega) \tag{63}$$

The solution for the recirculated part

The elements of the matrix $\hat{\mathbf{M}}(x_0,\omega)$ are given as follows:

$$M_{11}(\omega) = -D_1\,\mu^2(\omega)\cos[\mu(\omega)\,a] + \frac{\chi_{d,1}\,\beta(\omega)}{T}\left(\nu\Sigma_{f1} + c_\mu\,\nu\Sigma_{f2}\right)\tilde{\varphi}_\mu(\omega) \tag{64}$$

$$M_{12}(\omega) = D_1\,\nu^2(\omega)\cosh[\nu(\omega)\,a] + \frac{\chi_{d,1}\,\beta(\omega)}{T}\left(\nu\Sigma_{f1} + c_\nu\,\nu\Sigma_{f2}\right)\tilde{\varphi}_\nu(\omega) \tag{65}$$

$$M_{21}(\omega) = -D_2\,c_\mu\,\mu^2(\omega)\cos[\mu(\omega)\,a] + \frac{\chi_{d,2}\,\beta(\omega)}{T}\left(\nu\Sigma_{f1} + c_\mu\,\nu\Sigma_{f2}\right)\tilde{\varphi}_\mu(\omega) \tag{66}$$

and

$$M_{22}(\omega) = D_2\,c_\nu\,\nu^2(\omega)\cosh[\nu(\omega)\,a] + \frac{\chi_{d,1}\,\beta(\omega)}{T}\left(\nu\Sigma_{f1} + c_\nu\,\nu\Sigma_{f2}\right)\tilde{\varphi}_\nu(\omega) \tag{67}$$

The solution for the recirculated part

The elements of the matrix $\hat{\mathbf{L}}(x_0,\omega)$ are given as

$$L_{i,j}(x_0,\omega) = -\frac{\chi_{d,i}\,\beta(\omega)}{T} \quad \Big\{\nu\Sigma_{f1}\left[A_{\mu j}(x_0,\omega)\,\tilde{g}_\mu(x_0,\omega) + A_{\nu j}(x_0,\omega)\,\tilde{g}_\nu(x_0,\omega)\right] + \nu\Sigma_{f2}\left[c_\mu A_{\mu j}(x_0,\omega)\,\tilde{g}_\mu(x_0,\omega) + c_\nu A_{\nu j}(x_0,\omega)\,\tilde{g}_\nu(x_0,\omega)\right]\Big\} \tag{68}$$

The solution for the recirculated part

With the inverse $\hat{\mathbf{M}}^{-1}(\omega)$ written out, the solution for the coefficients $A_j(x_0,\omega)$ and $B_j(x_0,\omega)$ are given as follows:

$$A_1(x_0,\omega) = |M|^{-1}[M_{22}(\omega)\, L_{11}(x_0,\omega) - M_{12}(\omega)\, L_{21}(x_0,\omega)], \tag{69}$$

$$A_2(x_0,\omega) = |M|^{-1}[M_{22}(\omega)\, L_{12}(x_0,\omega) - M_{12}(\omega)\, L_{22}(x_0,\omega)], \tag{70}$$

$$B_1(x_0,\omega) = |M|^{-1}[-M_{21}(\omega)\, L_{11}(x_0,\omega) + M_{11}(\omega)\, L_{21}(x_0,\omega)] \tag{71}$$

and

$$B_2(x_0,\omega) = |M|^{-1}[-M_{21}(\omega)\, L_{12}(x_0,\omega) + M_{11}(\omega)\, L_{22}(x_0,\omega)]. \tag{72}$$

Here, $|M|$ stands for the (signed, not absolute value) determinant of $\hat{\mathbf{M}}(\omega)$.

The full solution

With the above, the full analytical solution of the dynamic Green's matrix of the MSR in two group theory, applicable for both thermal and fast MSRs in the prompt recirculation approximation is available. The space-frequency dependent coefficients $A_{\mu j}$, A_{nuj}, A_j, and B_j, $j = 1{,}2$ can all be calculated from the formulae above. The full solution in a columnwise representation reads as

$$\begin{bmatrix} G_{1,j}(x,\ x_0,\ \omega) \\ G_{2,j}(x,\ x_0,\ \omega) \end{bmatrix} = \begin{bmatrix} 1 \\ c_\mu(\omega) \end{bmatrix} A_{\mu,j}(\omega)\, g_\mu(x,\ x_0,\ \omega) + \begin{bmatrix} 1 \\ c_\nu(\omega) \end{bmatrix} A_{\nu,j}(\omega)\, g_\nu(x, x_0,\ \omega) + \begin{bmatrix} 1 \\ c_\mu(\omega) \end{bmatrix} A_j(x_0,\ \omega)\, \varphi_\mu(x,\ \omega) + \begin{bmatrix} 1 \\ c_\nu(\omega) \end{bmatrix} B_j(x_0,\ \omega)\, \varphi_\nu(x,\ \omega) \tag{73}$$

Conclusion, future work

- Full solution of the 2-group MSR equations for fast systems was obtained in the prompt recirculation approximation
- Quantitative work is underway and will be reported in a journal publication soon
- The main goal is to confirm that the suggestion put forward in the recent book that a thermal MSR can be modelled by the equations of a traditional (solid fuel) reactor, but with a reduced delayed neutron fraction, is also valid for fast systems.