Spatial effects in Cohn's method for neutron noise analysis

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1.Context

Application of neutron noise ?

- Qualification of delayed nuclear data
- Measure of the reactivity of a subcritical reactor
- Sizing of startup neutron source
- Application on ADS

Main interest : Kinetic parameters are used as a reference for differential measure

Measure of β_{eff} using neutron noise

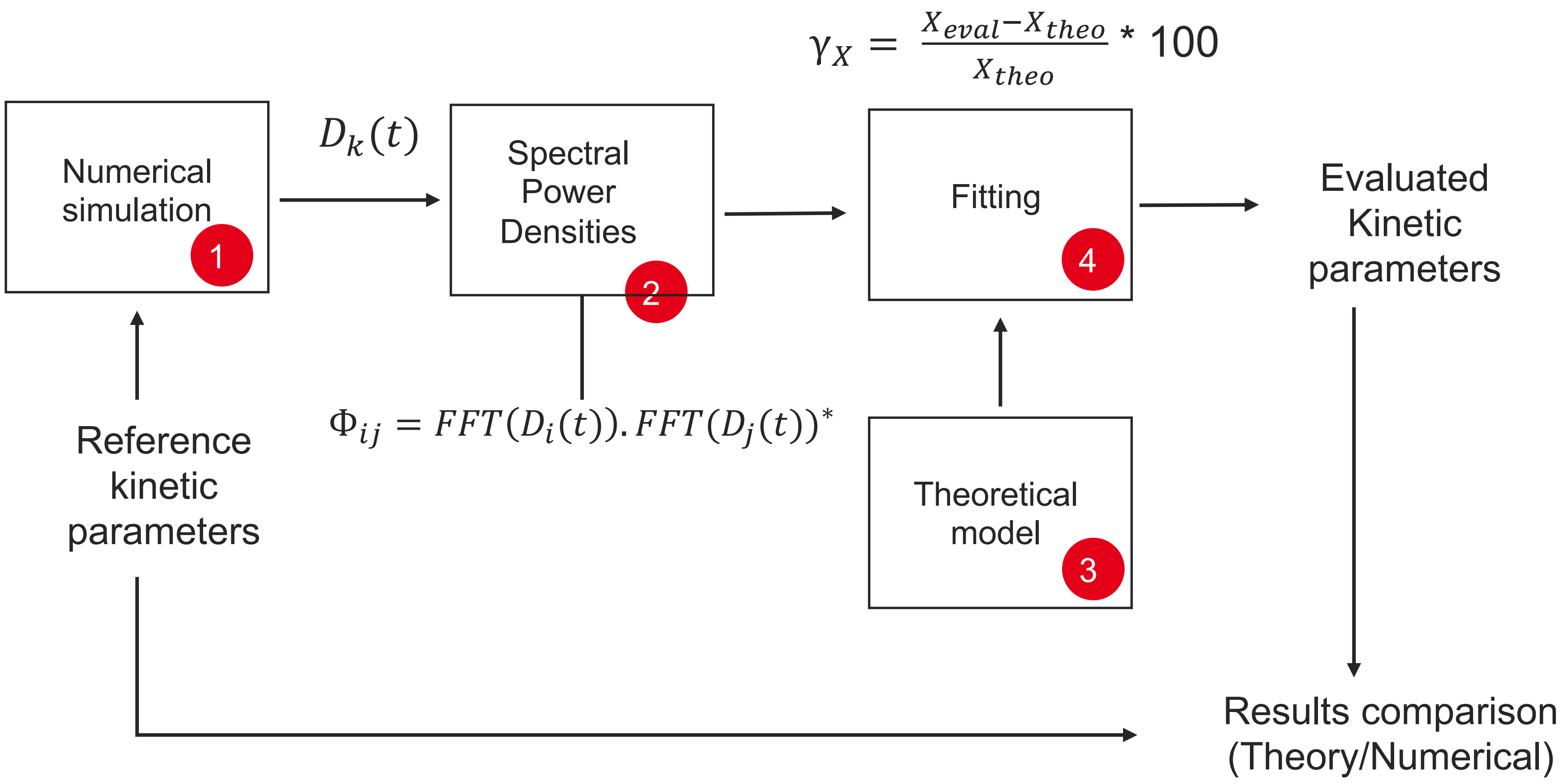
Discrepencies observed between theory and measures

What's missing: Interpretation? Data?

Study of spatial effects

2.Methodology and model

Goal : Validate the interpretation of the signal that lead to the measured kinetic parameters



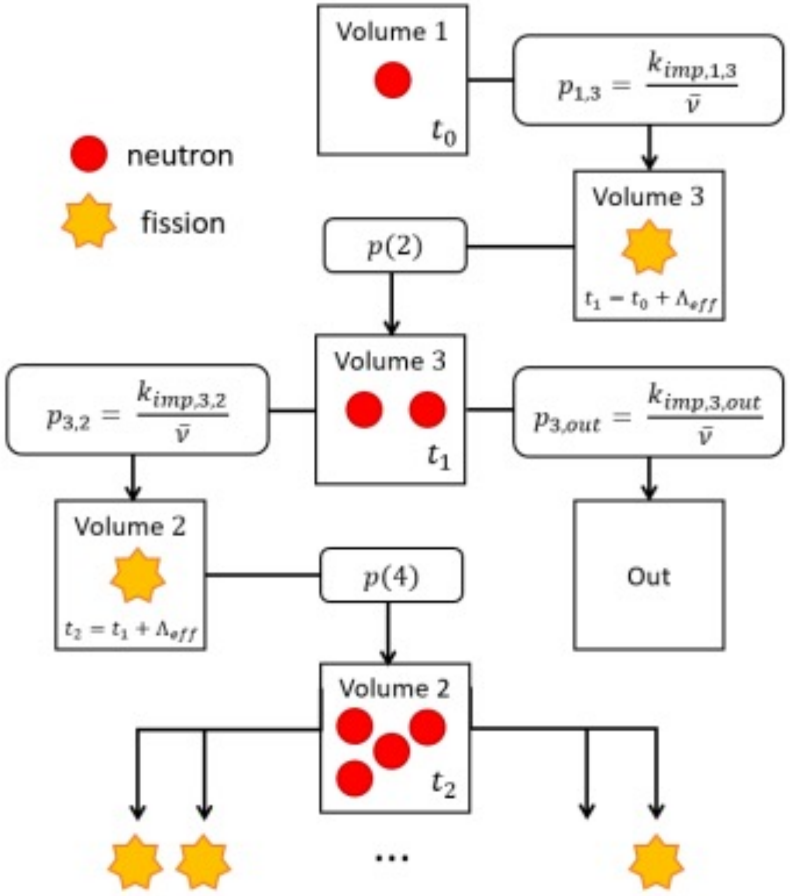
1. Simulation
2. Signal treatment
3. Theory
4. Fitting

2.1. Numerical simulation

Algorithm of the dedicated Monte-Carlo code :

1. A neutron is sampled in volume j at time t_0 1
2. The neutron is either transported to volume i, with the time updated accordingly ($t^{(g+1)} = \Lambda + t^{(g)}$), or leaks out of the system
3. Production of υ new neutrons transported at step 2
4. The delayed neutrons are simulated by introducing a time delay δt after neutron production.
5. The events are tallied as a function of volume and time.

A history ends when a fission event produces zero neutrons or when the neutron leaves the reactor.



2.2. Analytical model (classical expression)

Classical cohn-α expression (based on point kinetics)

The Cohn-α method is based on the analysis of the Power Spectral Density (PSD) of neutron noise signals. Unlike time-domain techniques, it characterizes neutron correlations in the frequency domain.

Its main advantages are as follows:

- The spectral representation reduces the influence of uncorrelated random coincidences, thereby improving the signal-to-noise ratio.
- The low-frequency region is particularly sensitive to delayed neutron effects, facilitating the estimation of the delayed neutron fraction. The frequency-domain representation also enhances the visibility of long-time correlations in the PSD.

- Point kinetics : $|S_{ij}(f)| = \left(\frac{\left(D_\upsilon - \frac{\rho}{\upsilon}\right)R_p}{1-\rho} \frac{C_i C_j}{2\Lambda^2 \alpha^2 F} \frac{1}{1+(\frac{\omega}{\alpha})^2} + \frac{C_i C_j}{\sqrt{C_i C_j}} \delta_{ij}\right)$

Where , $\frac{C_i C_j}{\sqrt{C_i C_j}}$ can be called P_0 and $C_i C_j . \frac{D_\upsilon R_P}{2\Lambda^2 F}$ is P_c, the red part is the reactor transient function H(ω)

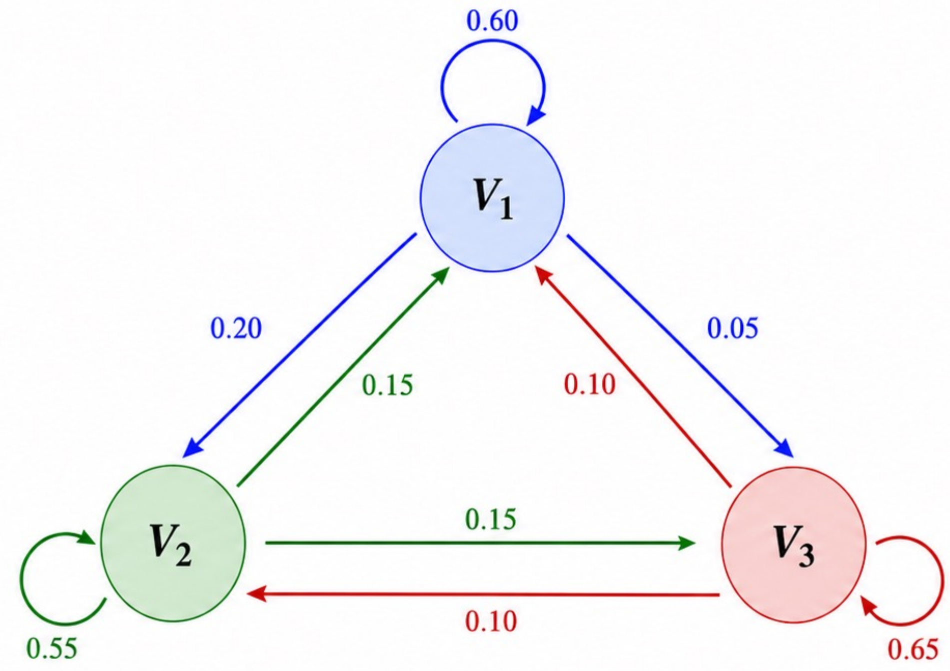
$$\rightarrow |S_{ij}(f)| = \left(\frac{P_c}{1+(\frac{\omega}{\alpha})^2} + P_0\ \delta_{ij}\ \right)$$

2.2. Analytical model (multi-modal expression)

1/ Spatial expansion :

Fission matrix K

K_{ij} : average number of neutrons produced in volume i at generation $g+1$
by one neutron present in volume j at generation g



Fission matrix K

$$K = \begin{array}{c} \\ \text{to } V_1 \\ \text{to } V_2 \\ \text{to } V_3 \end{array} \begin{array}{c} \begin{array}{ccc} \text{from } V_1 & \text{from } V_2 & \text{from } V_3 \end{array} \\ \begin{pmatrix} 0.60 & 0.15 & 0.10 \\ 0.20 & 0.55 & 0.15 \\ 0.05 & 0.10 & 0.65 \end{pmatrix} \end{array}$$

- The diagonal terms (K_{ii}) represent neutrons produced in the same volume (self-loops).
- The off-diagonal terms ($K_{ij},\ i \neq j$) represent neutrons produced in other volumes.
- In general, $K_{ij} \neq K_{ji}$.

The model becomes :

$$n^{(g)} = (n_1, \ldots, n_M)$$
$$\mathrm{E}[n^{(g+1)}] = K\mathrm{E}[n^{(g)}]$$

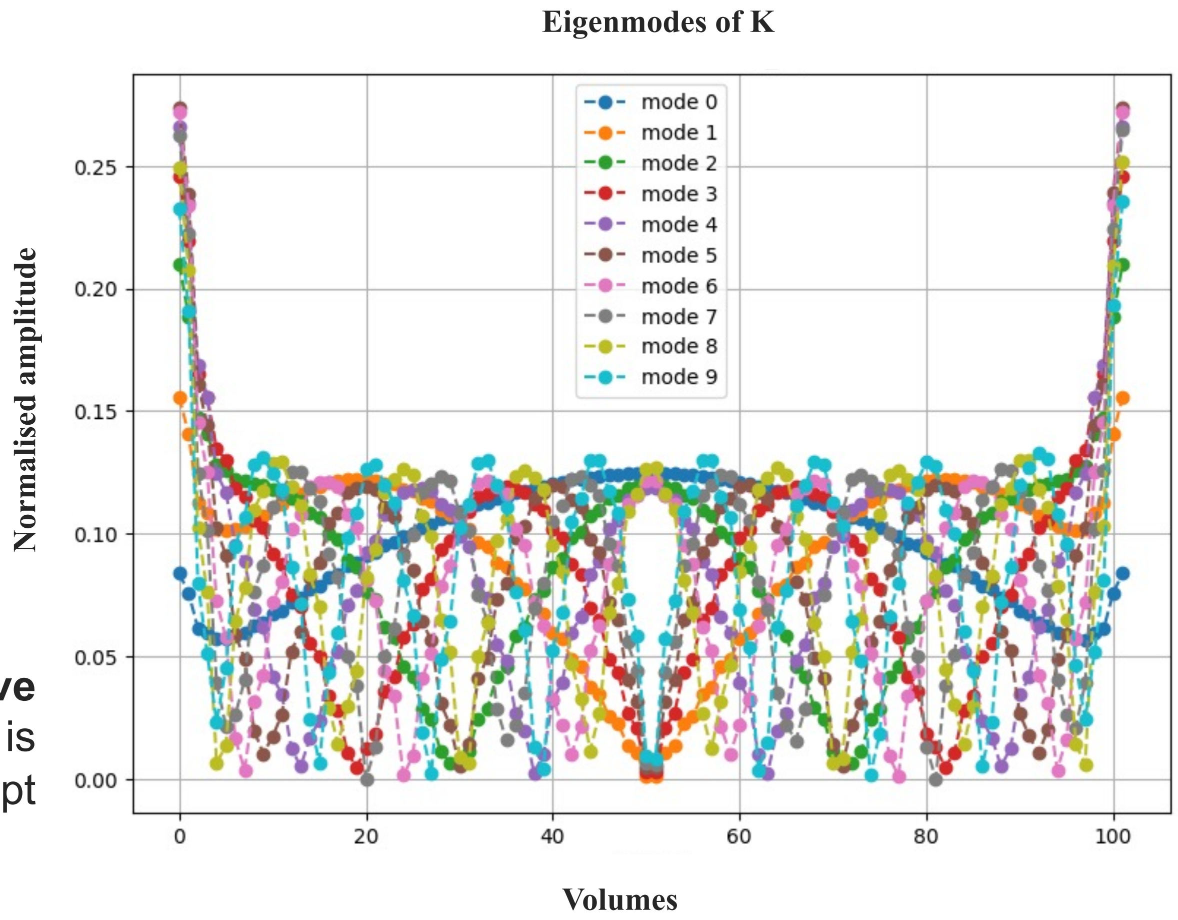
Thus, a dicrete Langevin equation can be written as :

$$\delta n^{(g+1)} = K\delta n^{(g)} + \eta^{(g)}$$
$$< \eta^{(g)}\eta^{(g')} > = Q\delta_{gg'}$$

2.2. Analytical model (multi-modal expression)

2/ Modal decomposition of K :

- Projection onto the eigenmodes of K
- Each modes are independant
  ➔ model order reduction



Let ϕ_k and χ_k be the right and left eigenvectors, respectively. :

$$K\phi_k = \rho_k\phi_k$$
$$a_k = \chi_k^T\delta n$$
$$a_k^{(g+1)} = \rho_k a_k^{(g)} + \eta_k$$

This latter equation corresponds to a **first-order autoregressive (AR(1)) process**. The coefficient is given by $\rho_k = e^{-\alpha_k\Lambda}$, where α_k is the decay constant associated with mode k, and Λ is the prompt neutron generation time of the reactor.

2.2. Analytical model

3/ Frequency domain and CPSD:

Lets now look at the frequency domain, The CPSD then is :

$$S_k(\omega) = \sum_m C_k(m) e^{-i\omega m\Lambda}$$

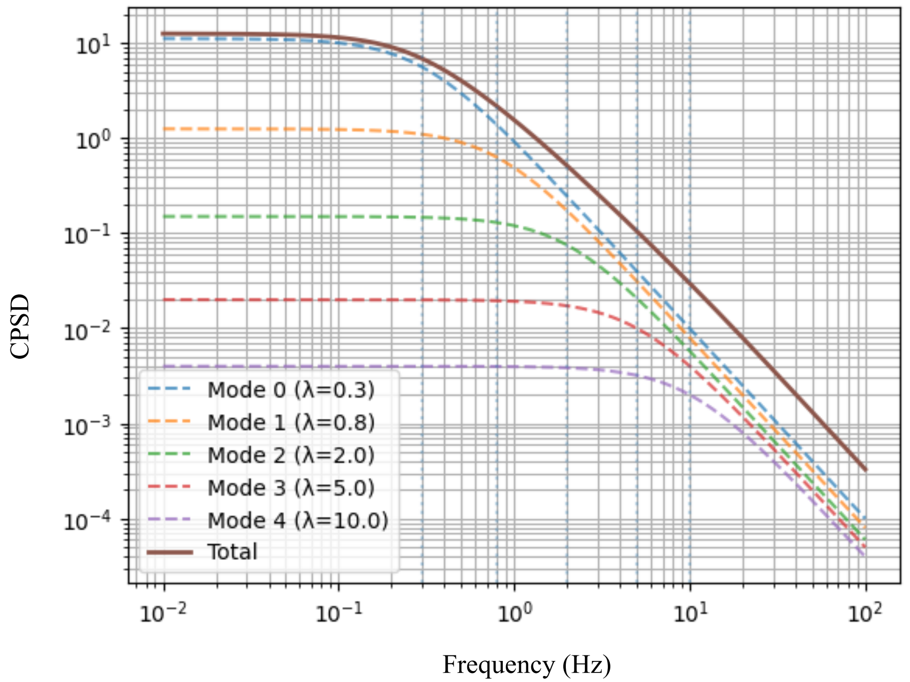
With : $C_k(m) = < a_k^{(g)} a_k^{(g+m)} >$

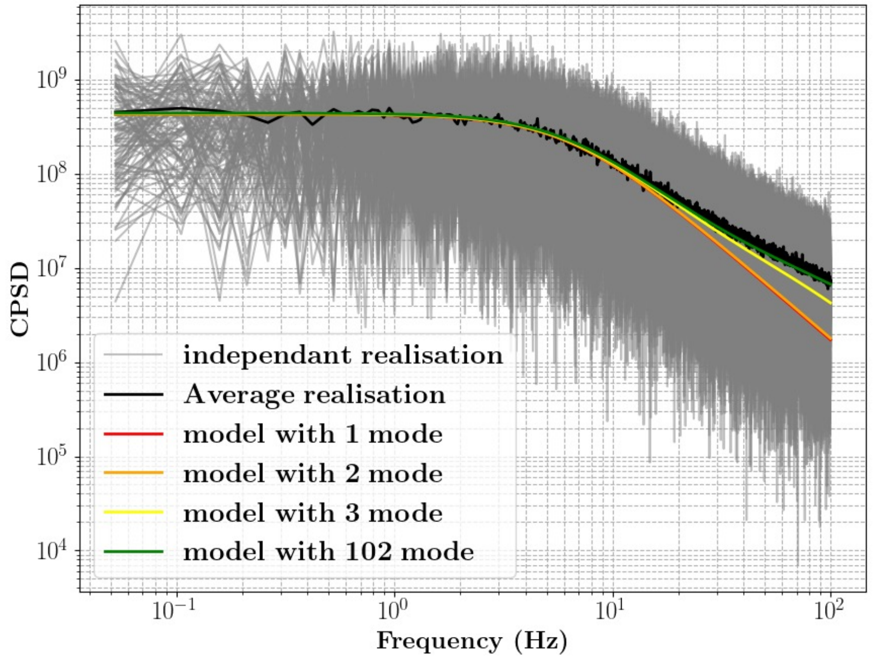
By taking the continuous limit, it becomes :

$$S_k(\omega) = \frac{\mu_k}{\lambda_k^2 + \omega^2}$$

Finally, $\boldsymbol{S\,(\omega) = \sum_k S_k(\omega)}$

The values of λ_k enlightened a dominance of the fundamental mode ➔ Point kinetic model is a valid approximation as long as $\lambda_0 \ll \lambda_{k>0}$





2.2. Analytical model

Two expressions have been studied :

- Point kinetic model : $|S_{ij}(f)| = \left(\frac{\left(D_\upsilon - \frac{\rho}{\bar{\upsilon}}\right)R_p}{1-\rho}\,\frac{C_iC_j}{2\Lambda^2\alpha^2 F}\,\frac{1}{1+\left(\frac{\omega}{\alpha}\right)^2} + \frac{C_iC_j}{\sqrt{C_iC_j}}\,\delta_{ij}\right)$

- Multi-modal model : $|S_{ij}(f)| = \left(\sum_{k=0}^{N}\frac{\frac{\mu_k}{\Lambda^2\alpha_k^2}}{1+\frac{\omega^2}{\alpha_k^2}} + \frac{C_iC_j}{\sqrt{C_iC_j}}\,\delta_{ij}\right),$

μ_k is defined as : $\mu_k = <\chi_k^*\,(Q_{branch})\chi_k.\,Q_{detec}>$ with Q_{branch} the spatial covariance of the branching process and $Q_{detec} = [\Phi_k^*\,D_1.\,D_2\Phi_k]$.

The point kinetic based expression is the mono-modal limits of the second expression.
Each mode contribute to the CPSD with :

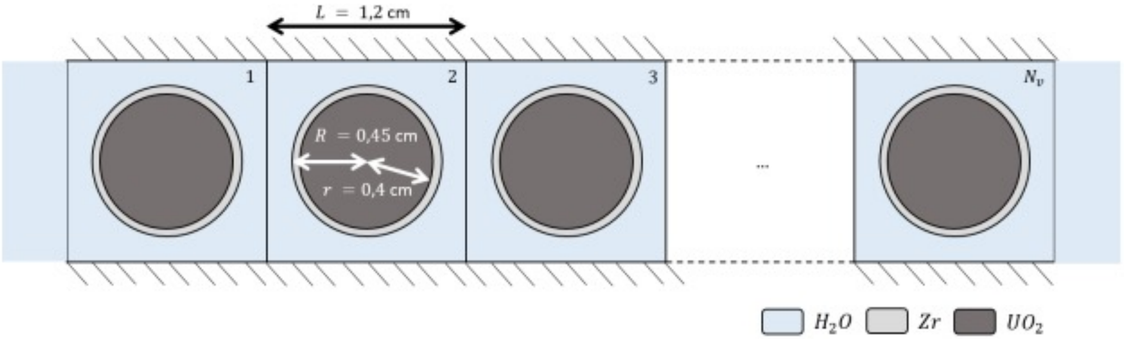
- Its own decay constant α_k,
- Its own spatial structure,
- Its own weigth depending on the detector positions.

2.3. Reactor models

Several 1D reactor are studied to show the impact of spatial effects in neutron noise measurement, the length of the reactor is modified : 61.2, 122.4, 183.6, 244.8 and 306 cm.

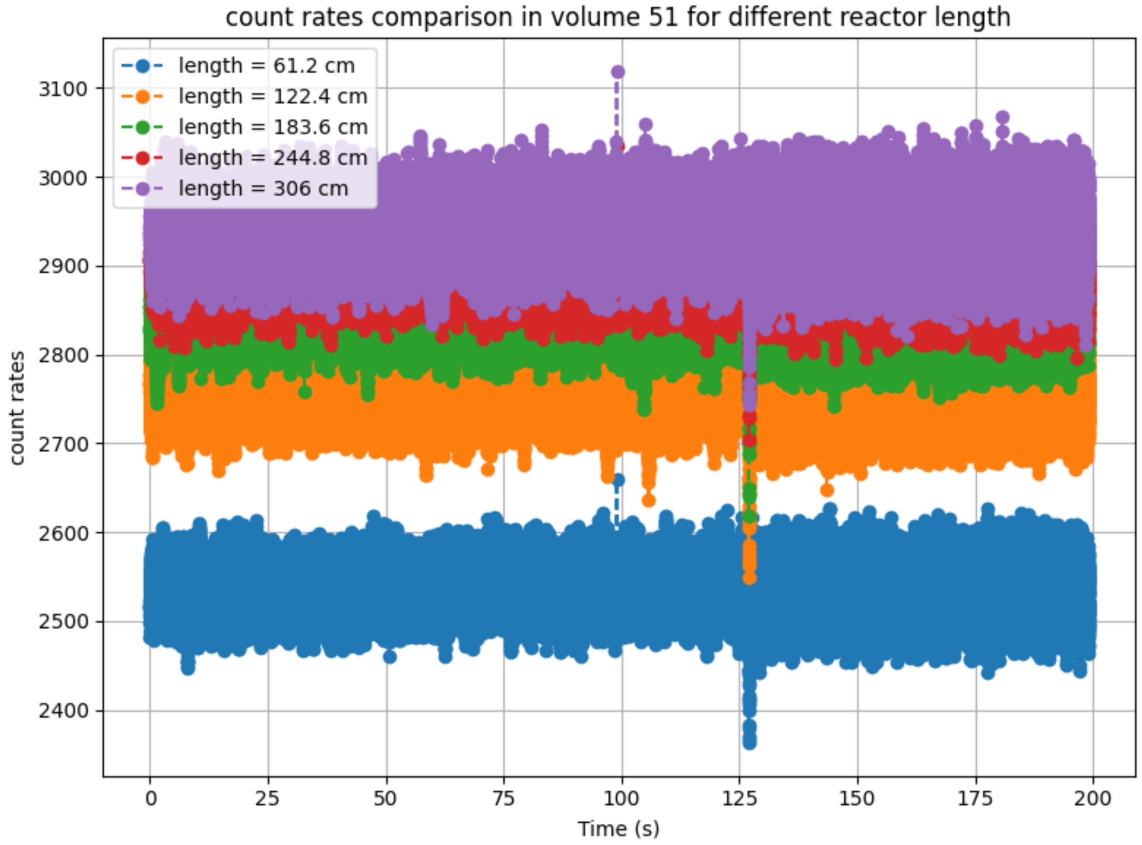
The Kij matrices are obtained using TRIPOLI4 with 102 volumes (mesh) for each lengths.

➔ Kinetic parameters are obtained by fitting for each combination of detector positions.

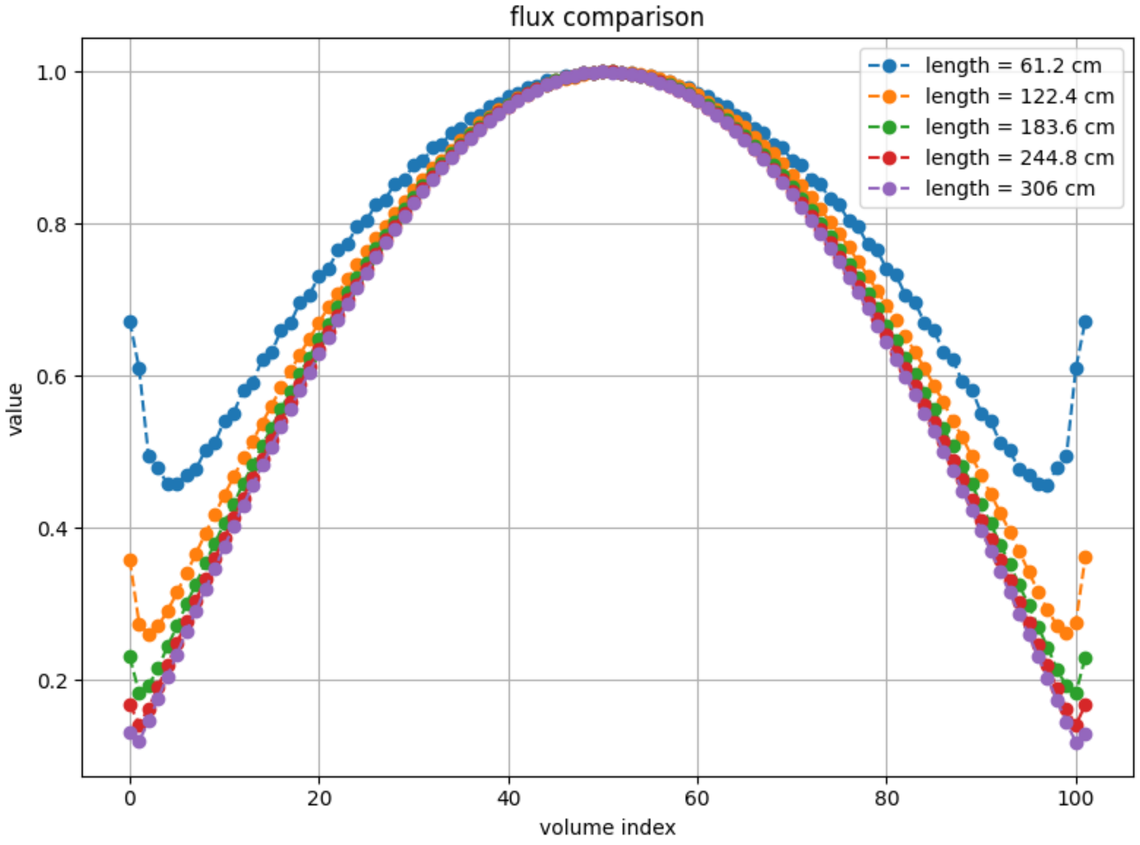


Parameters	Values
Reactivity (pcm)	-400
Neutron multiplicity probabilities per fission p(v = 0, 1, 2, 3, 4, 5)	0.0, 0.2, 0.4, 0.2, 0.1, 0.1
Source intensity (neutron/s)	10^6
Sampling time T_e (µs)	100
Mean generation time Λ_{eff} (µs)	100
Simulation time Tsimu (s)	200
Number of simulation	100

2.3. Reactor models



$D_{51}(t)$ after 1 for different reactor lengths



Normalised flux

3. Results

Small reactor (L=61,2 cm) :

- detector 1 ➔ volume 51 (centre)
- detector 2 ➔ volume 90 (boundaries)

Point kinetics

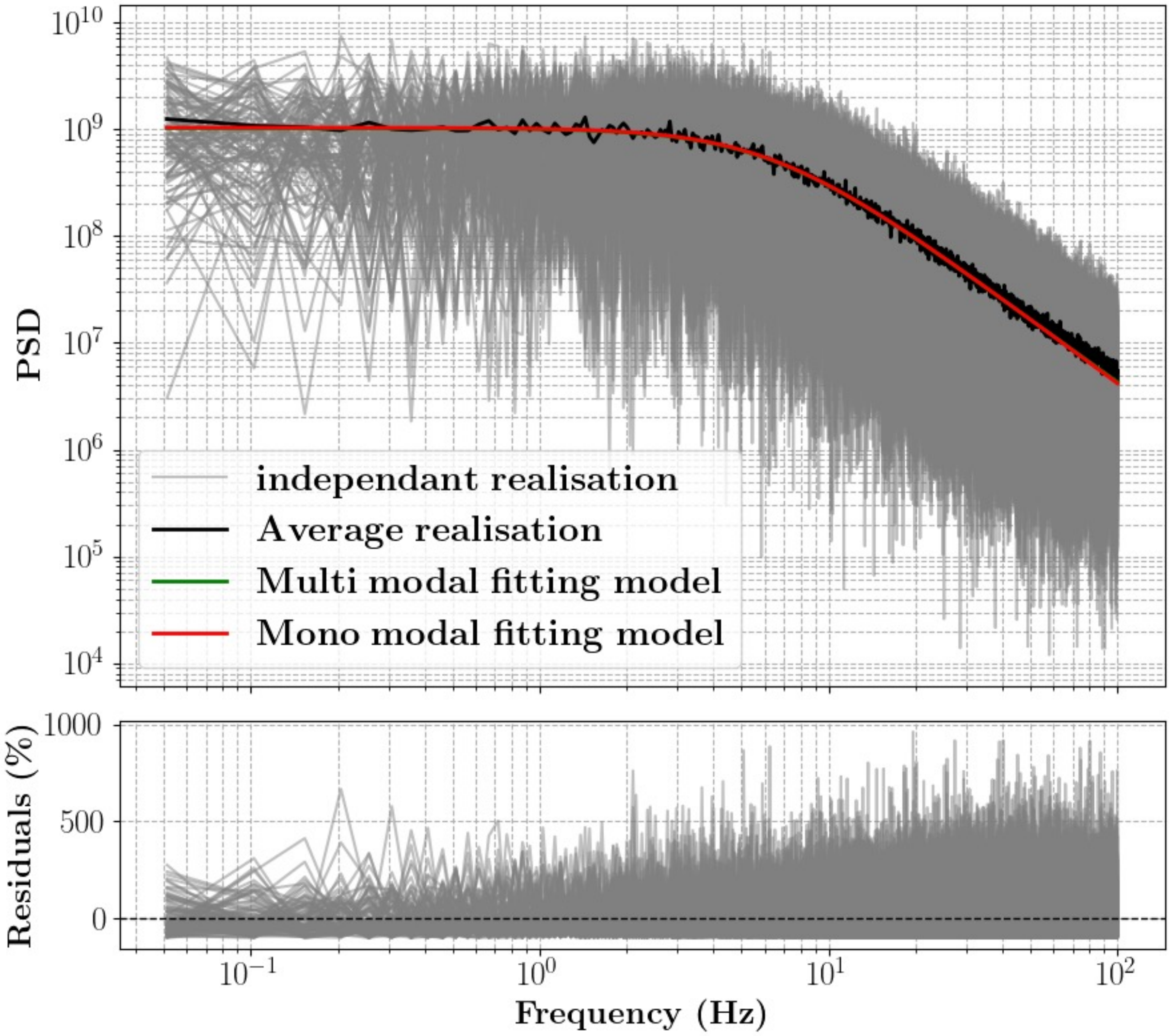
$$\gamma_{Pc} = 0{,}639 \pm 1{,}339\%$$
$$\gamma_{\alpha} = -0{,}522 \pm 1{,}088\ \%$$

Multi modal model

$$\gamma_{Pc} = 0{,}156 \pm 1{,}53\%$$
$$\gamma_{\alpha} = -1{,}337 \pm 0{,}526\ \%$$

In small reactor :
good agreement between the classical model and the simulations
good agreement between multi-modal model and the simulations

The fundamental mode dominates, while higher-order modes are strongly damped ➔ the classical model remains valid



3. Results

Large reactor (L=306 cm) :

- detector 1 ➔ volume 51 (centre)
- detector 2 ➔ volume 90 (boundaries)

Point kinetics

$\gamma_{Pc} = 36{,}717 \pm 2{,}155\%$

$\gamma_{\alpha} = 16{,}778 \pm 1{,}581\ \%$

Multi-modal model

$\gamma_{Pc} = -2{,}713 \pm 1{,}454\%$

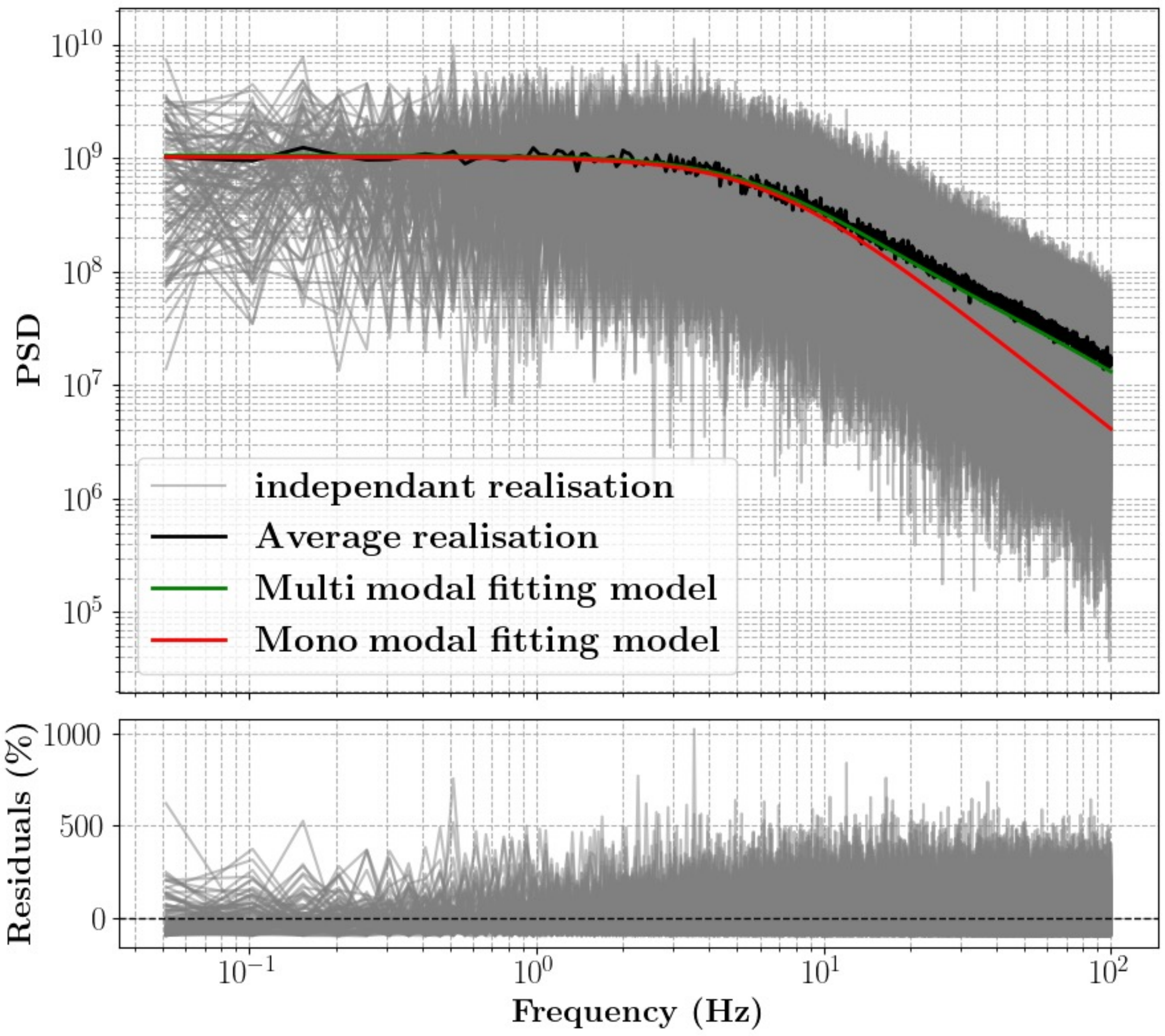
$\gamma_{\alpha} = -0{,}735 \pm 0{,}583\ \%$

For large reactor :

discrepencies at high frequencies between classical expression and the simulations

good agreement between the multi-modal model and the simulations

Higher-order modes contaminate the CPSD.



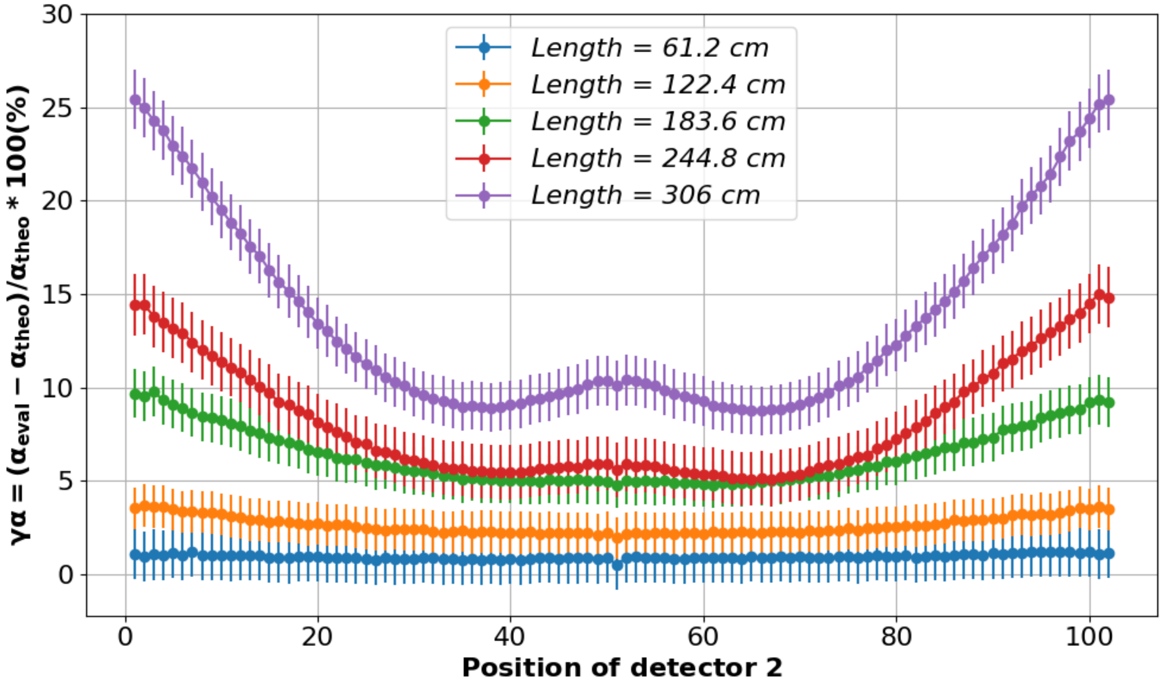
3. Results

Conditions :

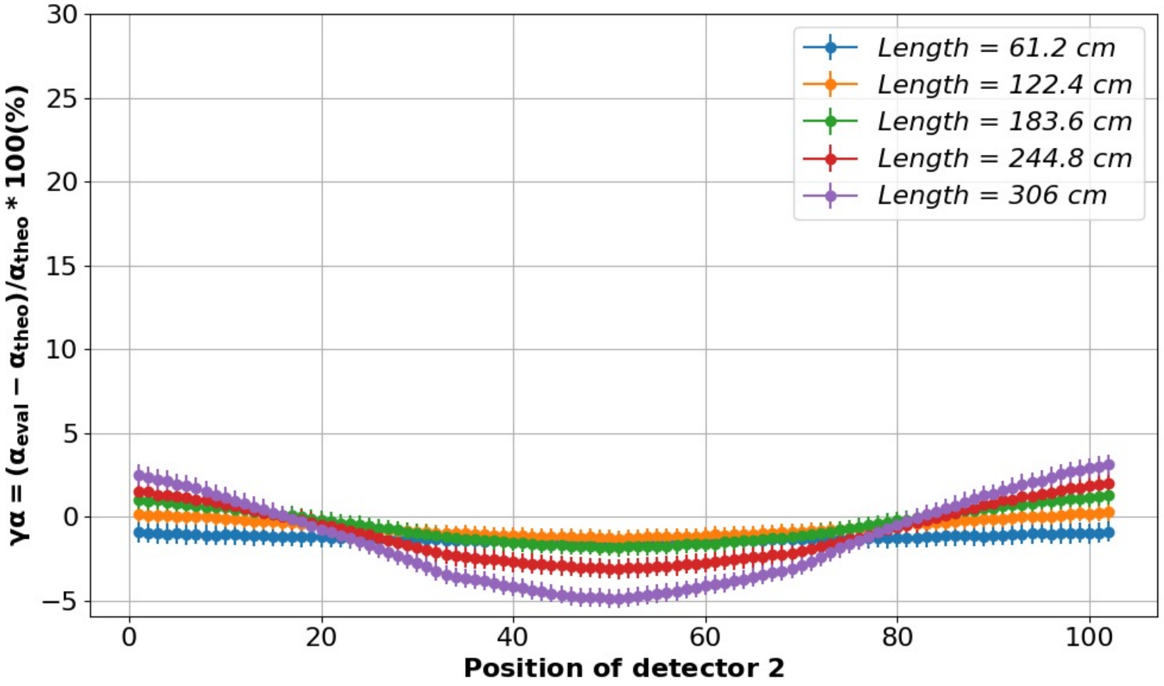
- Detector 1 : fixed at the centre
- Detector 2 : Every possible positions

For each positions of Detector 2, α is evaluated and compared with the reference value (the fundamental eigenvalue of the fission matrix)

Point kinetics



Multi-modal model



4. Conclusion

Result 1 :

The single-mode assumption is no longer valid when multiple neutron modes contribute simultaneously to the system dynamics.

Result 2 :

The experimentally observed biases mainly arise from :

- higher-order spatial modes
- detector-mode coupling.

Result 3 :

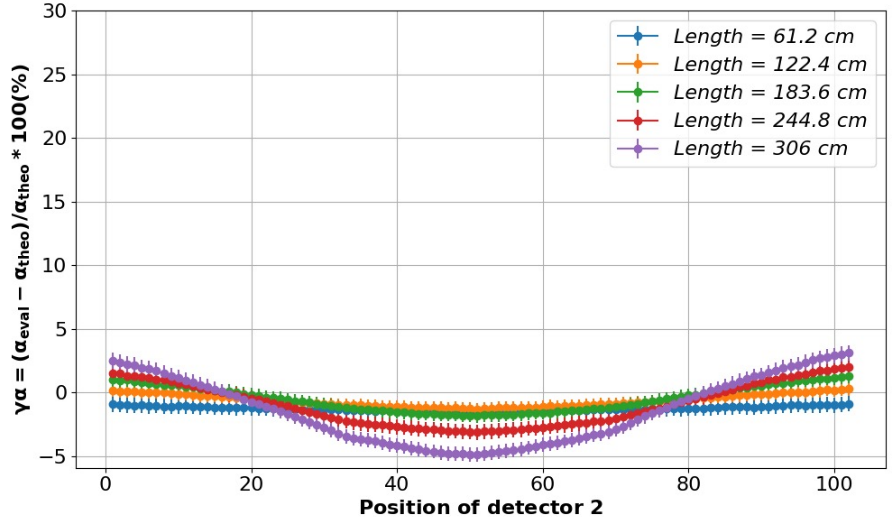
The multi-modal model correctly reproduces

- the shape of the CPSD,
- the detector dependence,
- the observed discrepancies in the fitted decay constants.

The Reactivity play a non negligeable role on the evaluation obtained for the classical cohn-α method but almost no role in the evaluation of the multi-modal approach

Outlook :

- Taking into account the delayed neutron
- Study real experiment
- Publish a Paper on ANE



Despite these improvements, a residual spatial dependance remains.

This suggest that not all physical mechanisms are fully captured, or that certain couplings are still neglected in the model.

Also, that method requires the prior knowledge of the fission matrix which complicates its application.

The multi-modal nature constitutes the dominant mechanism behind the observed biases, but it likely does not capture the full physics of neutron noise.

Thank you