Stochastic Subspace Identification for the Modal Analysis of Neutron Noise

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1.Context

Application of neutron noise ?

- Qualification of delayed nuclear data
- Measure of the reactivity of a subcritical reactor
- Sizing of startup neutron source
- Application on ADS

Measure of β_{eff} using neutron noise

Discrepencies observed between theory and measures

Who’s wrong : Interpretation? Data?

Study of spatial effects

1.Context (recall of yesterday)

Result 1 :

The single-mode assumption is no longer valid when multiple neutron modes contribute simultaneously to the system dynamics.

Result 2 :

The experimentally observed biases mainly arise from :

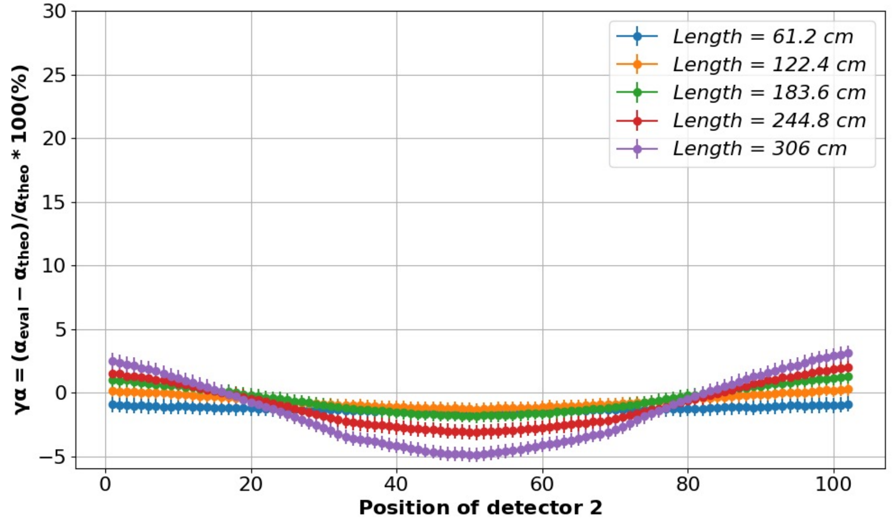
- higher-order spatial modes
- detector-mode coupling.

Result 3 :

The multi-modal model correctly reproduces

- the shape of the CPSD,
- the detector dependence,
- the observed discrepancies in the fitted decay constants.

- Multi-modal model : $|S_{ij}(f)| = \left(\sum_{k=0}^{N} \frac{\frac{\mu_k}{\Lambda^2 \alpha_k^2}}{1+\frac{\omega^2}{\alpha_k^2}} + \frac{C_i C_j}{\sqrt{C_i C_j}} \delta_{ij}\right),$



Despite these improvements, a residual spatial dependance remains.

This suggest that not all physical mechanisms are fully captured, or that certain couplings are still neglected in the model.

That method requires the prior knowledge of the fission matrix which complicates its application.

The multi-modal nature constitutes the dominant mechanism behind the observed biases, but it likely does not capture the full physics of neutron noise.

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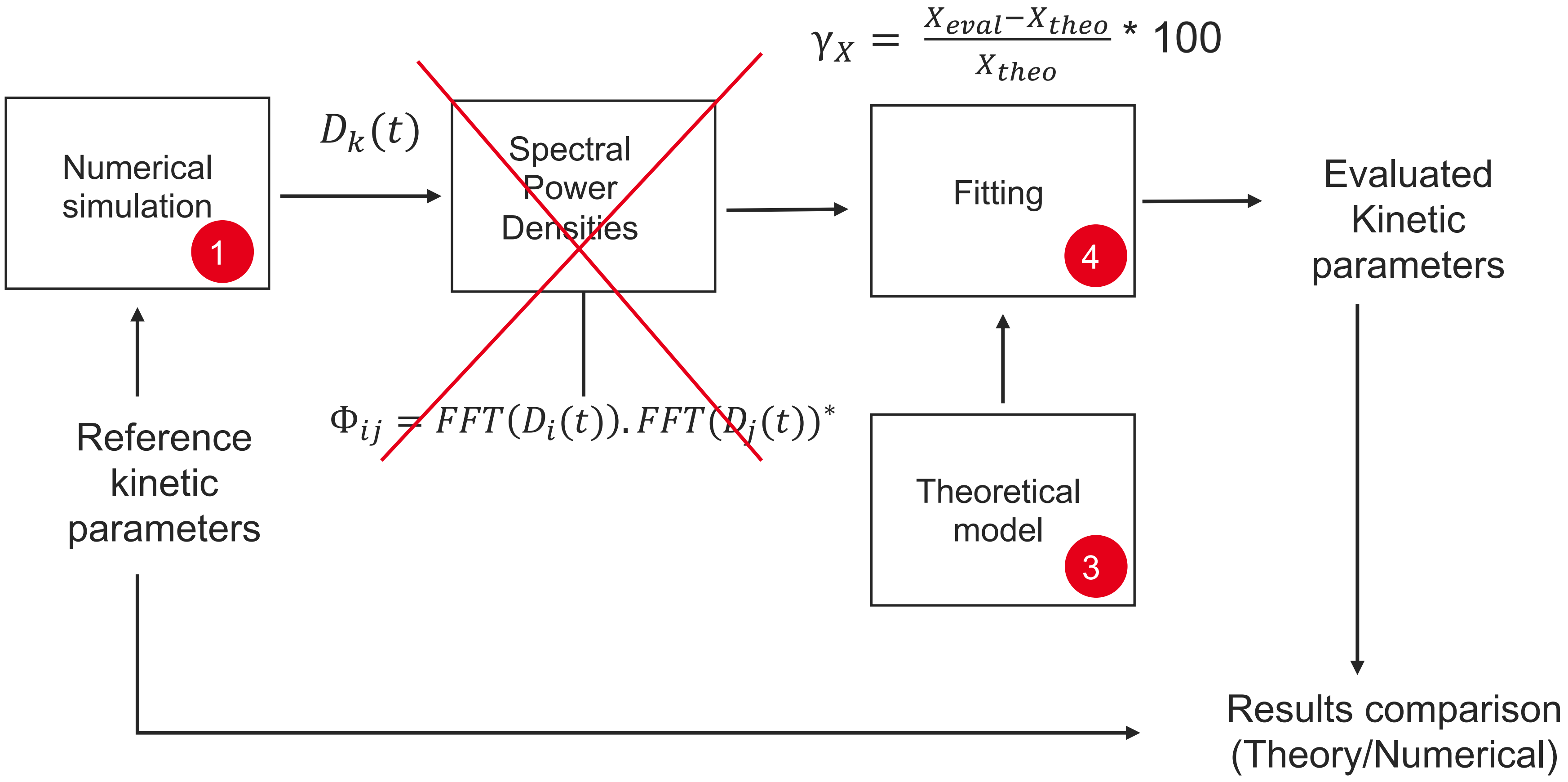
1.Context

Goal : avoid the use of CPSD and the need for rior knowledge of the fission matrix



1. Simulation
2. Signal treatment
3. Theory
4. Fitting

1.Context

SSI: reconstructing hidden dynamics from output correlations

Originally developed in structural dynamics, SSI identifies state-space models directly from output measurements.

Structural dynamics / vibration analysis

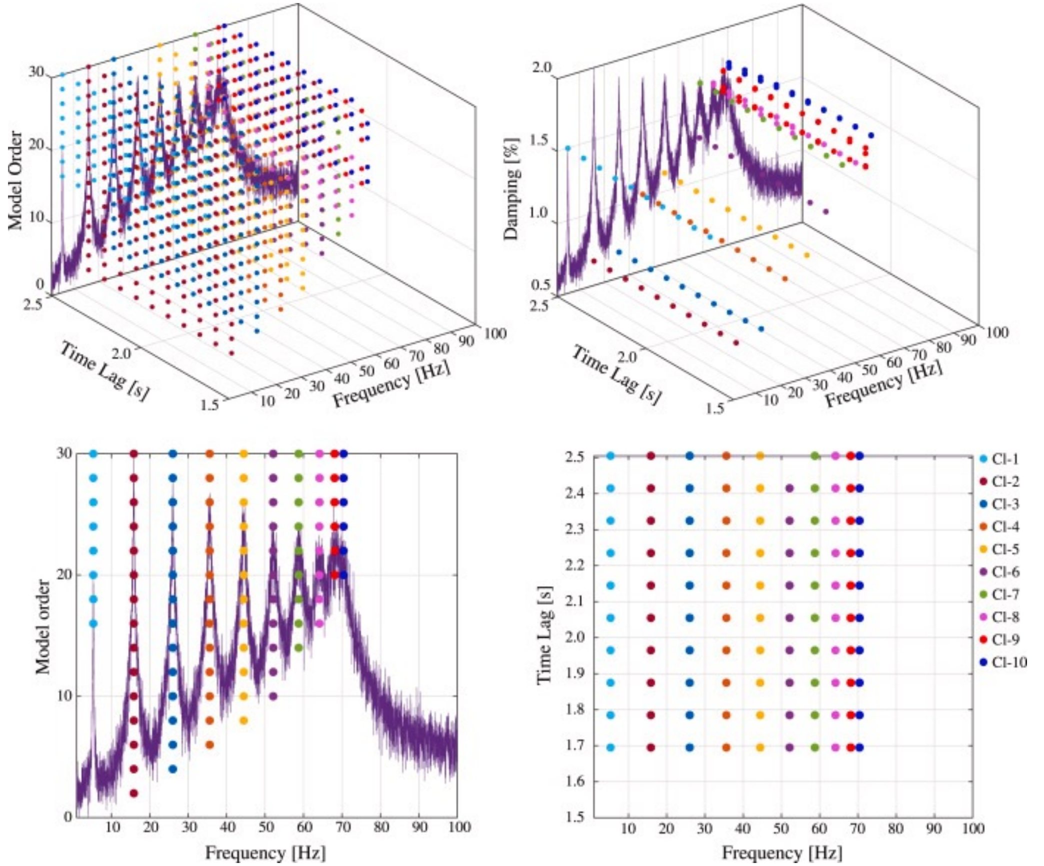
- Modal analysis of buildings
- Aeroelastic systems
- Control engineering

Why is that interesting here :

- Neutron noise follows an AR(1) process

$$a_k^{(g+1)} = \rho_k a_k^{(g)} + \eta_k$$

- Detectors are noisy outputs of hidden state
- Correlations contain full dynamical structure
- No need for parametric model and prior knowledge



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SSI extracts the system operator directly from measured correlations, enabling a non-parametric reconstruction of reactor dynamics.

2.1. SSI methodology

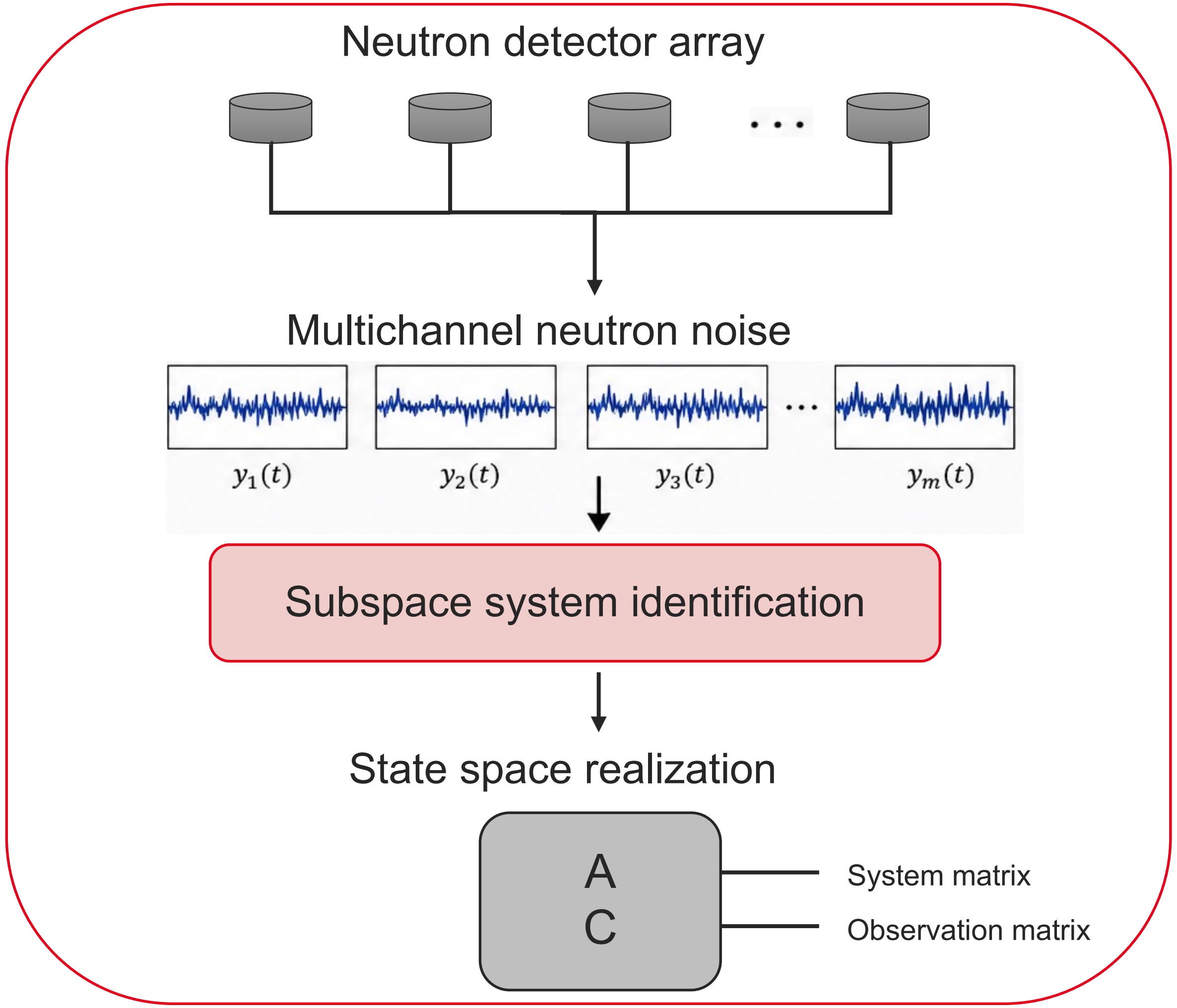
Subspace System Identification Principle :

Neutron noise measurements are modeled as the output of a linear stochastic dynamical system :

$$x_{k+1} = Ax_k + w_k$$
$$y_k = Cx_k + v_k$$

Where :

- x_k : hidden reactor state
- y_k : detector measurements
- A : unknown system matrix (here the Fission matrix)
- C : observation matrix
- w_k, v_k : process and measurements noise



Objective : reconstruct the unknown system matrix A directly from neutron noise measurements.

2.1. SSI methodology

Correlation structure (Hankel matrices)

Output covariance at lag i :

$$R_i = E\left[x_{k+i} x_k^T\right]$$

$$H_0 = \begin{bmatrix} R_0 & R_1 & \cdots & R_{s-1} \\ R_1 & R_2 & \cdots & R_s \\ \vdots & \vdots & \ddots & \vdots \\ R_{s-1} & R_s & \cdots & R_{2s-2} \end{bmatrix}$$

Shift by 1 lag

$$H_1 = \begin{bmatrix} R_1 & R_2 & \cdots \\ R_2 & R_3 & \cdots \\ \vdots & \vdots & \end{bmatrix}$$

The system follows a first order autoregressive process, thus :

$$H_1 = AH_0$$

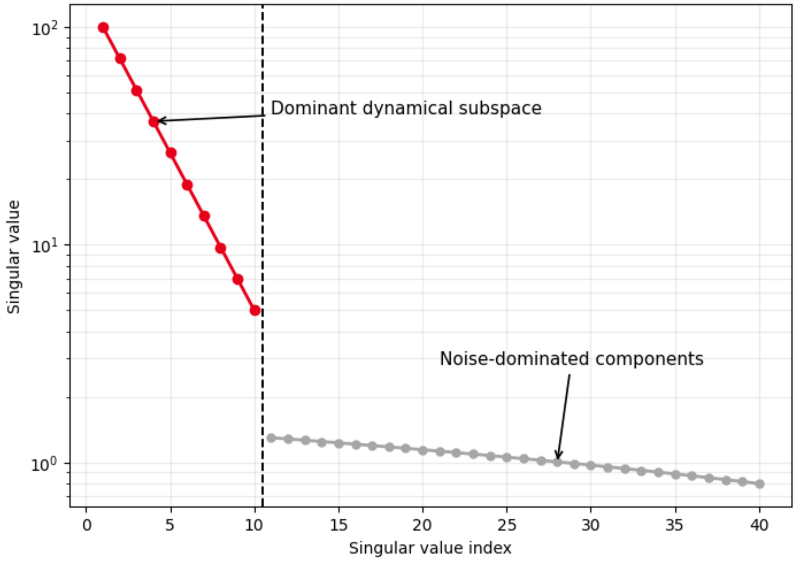
Similar reactor dynamics appear repeatedly along shifted rows, creating a low-rank structure that can be exploited.

Dominant dynamical subspace

$$H_0 = U\Sigma V^T$$

$$H_0 \approx U_r \Sigma_r V_r^T$$

Signal energy is concentrated in a few singular values, while noise spreads over the remaining ones.



Reconstruction of hidden state-space

Lets introduce :

- $O = U_r \Sigma_r^{1/2}$
- $Z_0 = \Sigma_r^{1/2} V_r^T$

With : O, an empirical observability matrix capturing how latent dynamics are observed by the detectors. and Z_0, a latent state sequence representing the reconstructed hidden system evolution.

Thus :

$$H_0 = OZ_0$$

And

$$H_1 = OAZ_0$$

Isolating A :

$$A_{SSI} = \Sigma_r^{-1/2} U_r^T H_1 V_r \Sigma_r^{-1/2}$$

If λ_k are the eigenvalue of A_{SSI},

$$\lambda_k = e^{\alpha_k dt}$$

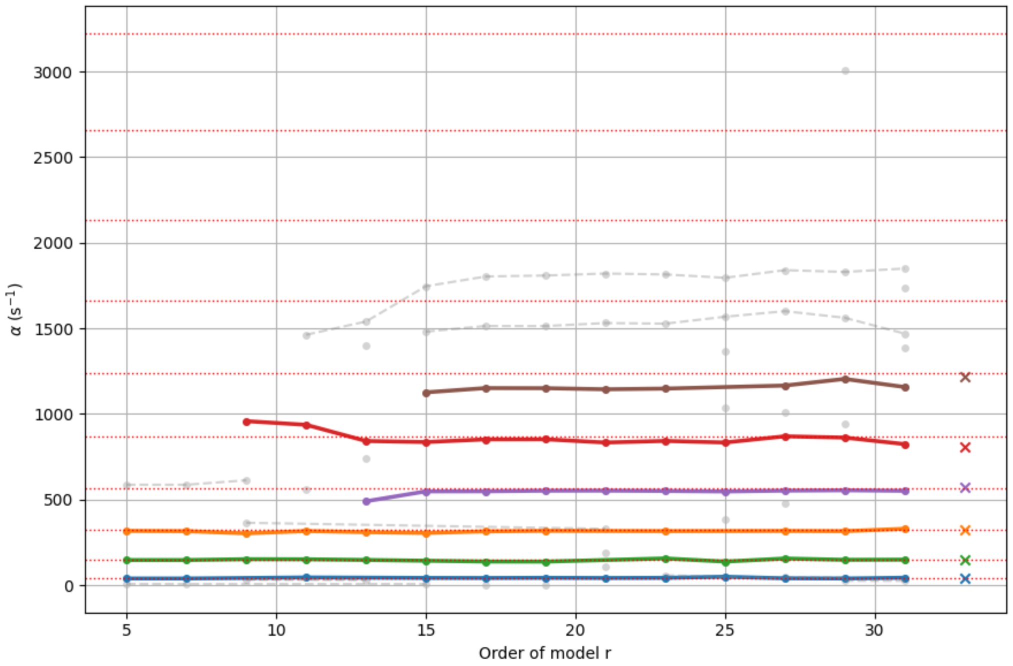
Eigenvalues of the identified operator correspond to the reactor α-modes.

SSI transforms temporal correlations into a low-dimensional dynamical model whose eigenvalues are the reactor α-modes.

2.1. SSI methodology

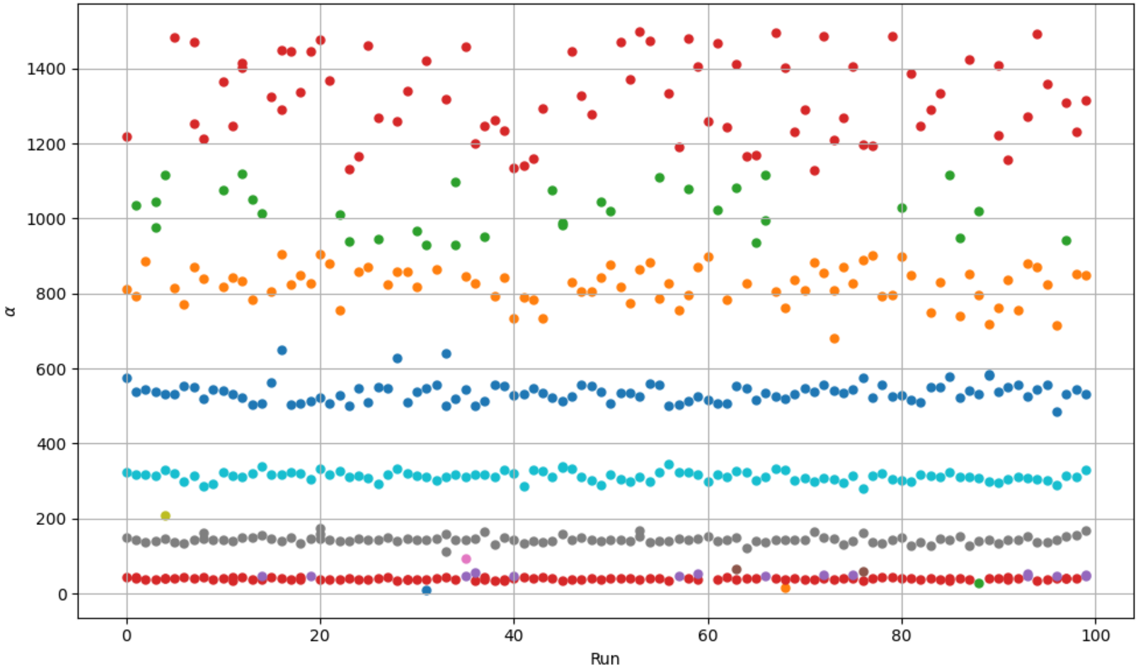
Identification of Physical Modes

Stabilization diagram



Stable poles form continuous tracks across increasing model order.

Repeatability across runs



The same modes are recovered independently from multiple noise realizations.

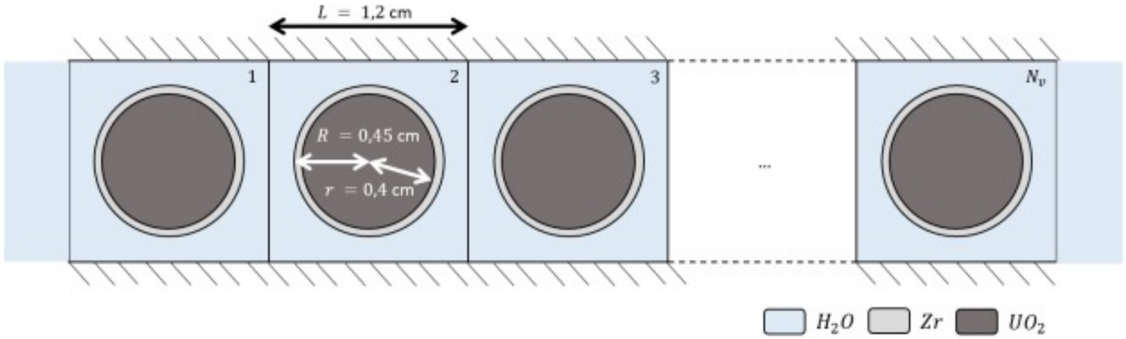
Physical modes remain stable with model order and are consistently observed across independent runs.

2.2. Reactor models

Several 1D reactor are studied to show the impact of spatial effects in neutron noise measurement, the length of the reactor is modified : 61.2, 122.4, 183.6, 244.8 and 306 cm.

The Kij matrices are obtained using TRIPOLI4 with 102 volumes (mesh) for each lengths.

➔ Kinetic parameters are obtained by fitting for each combination of detector positions.

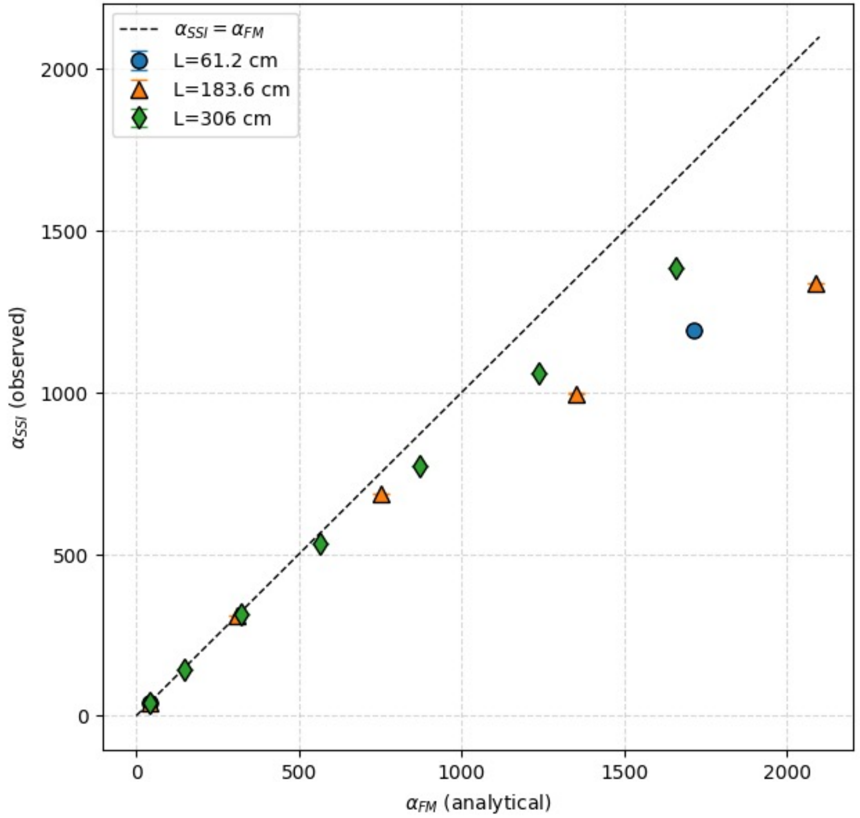


Parameters	Values
Reactivity (pcm)	-400
Neutron multiplicity probabilities per fission p(v = 0, 1, 2, 3, 4, 5)	0.0, 0.2, 0.4, 0.2, 0.1, 0.1
Source intensity (neutron/s)	10^6
Sampling time T_e (µs)	100
Mean generation time Λ_{eff} (µs)	100
Simulation time Tsimu (s)	200
Number of simulation	100

3. Results

An evaluation of several α_k has been grouped into a table for the 61,2, 183,6 and 306 cm reactor configurations. The signals has been taken from all over the reactor (number of detector considered : 102)

Mode k	$L = 61.2$ cm		$L = 183.6$ cm		$L = 306$ cm	
	α^{SSI}	α^{FM}	α^{SSI}	α^{FM}	α^{SSI}	α^{FM}
1	39.17 ± 0.32	39.92	39.05 ± 0.31	39.92	40.03 ± 0.30	39.92
2	1193.10 ± 3.36	1713.92	311.55 ± 2.39	311.14	143.73 ± 1.02	145.99
3	–	–	686.28 ± 6.63	752.82	315.64 ± 2.88	321.71
4	–	–	997.47 ± 5.53	1350.74	531.93 ± 3.96	563.80
5	–	–	1338.22 ± 7.89	2088.50	772.00 ± 6.02	869.95
6	–	–	–	–	1060.97 ± 8.95	1236.89
7	–	–	–	–	1385.56 ± 9.52	1658.78



As the core size increases, more higher-order modes become dynamically significant and observable in neutron noise. Consequently, SSI is able to reconstruct an increasing number of physical α-modes in agreement with the fission matrix reference.

3. Results

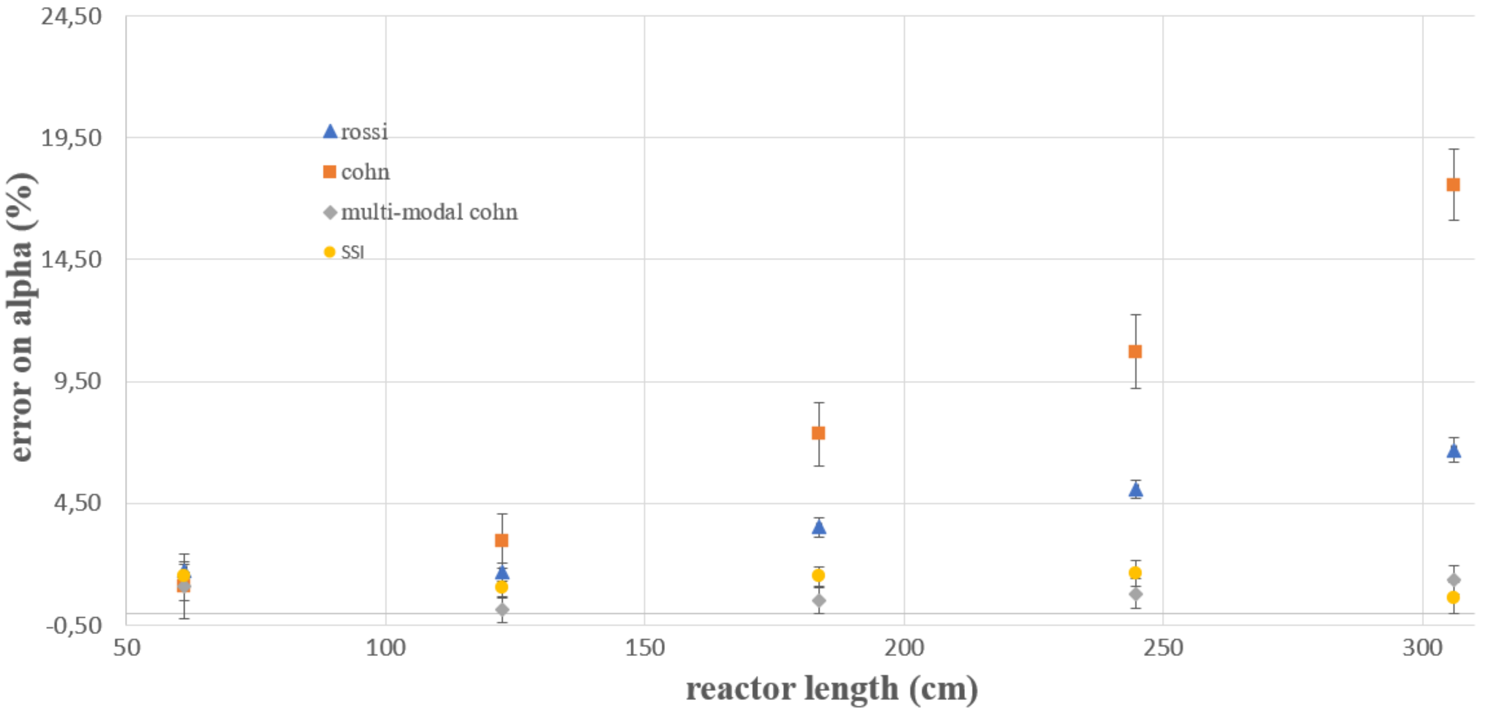
Identification of Physical Modes in all reactor size studied

Cohn-α :
- Single lorentzian spectrum
- Fundamental mode only
- Frequency domain fit
- Higer order modes distort spectral shape

Rossi-α :
- Single exponential decay
- Single dominant mode
- Time domain fit
- Less sensitive to higher modes through time domain correlation

Cohn-α multi-modal:
- Includes higher order modes
- Accurate fundamental mode estimates
- Require prior knowledge of the fission matrix

SSI :
- No predefined spectral model
- No prior modal information
- Estimates fundamental and higher order modes simultaneously

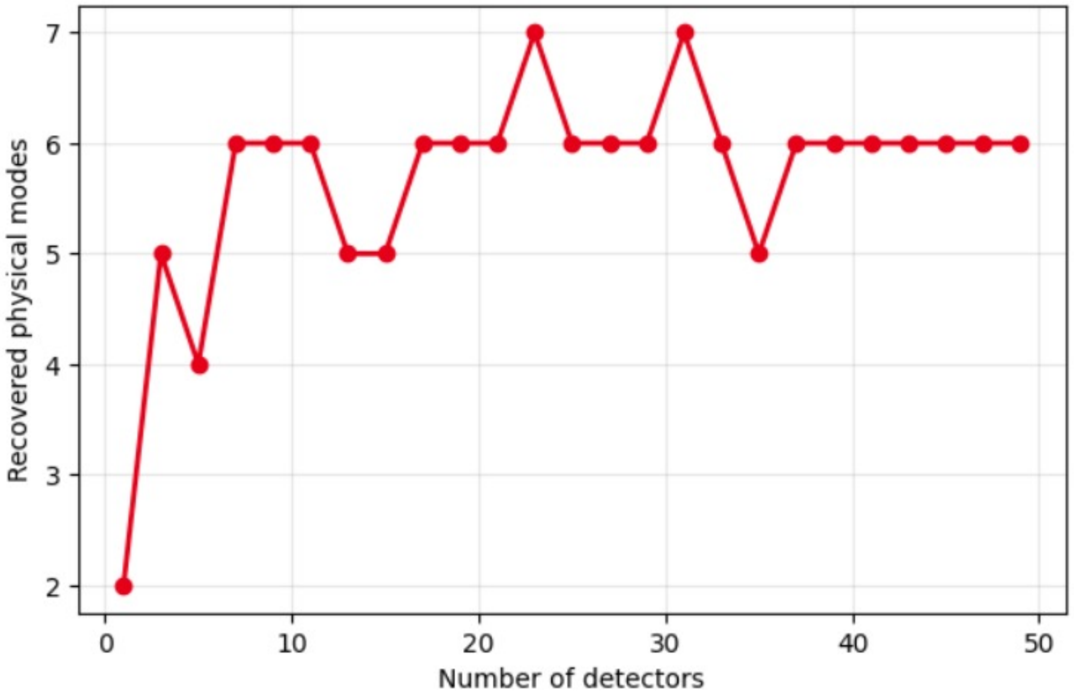


SSI preserves accurate α estimates without prior knowledge of the reactor dynamics, matching the performance of the multimodal Cohn-α model.

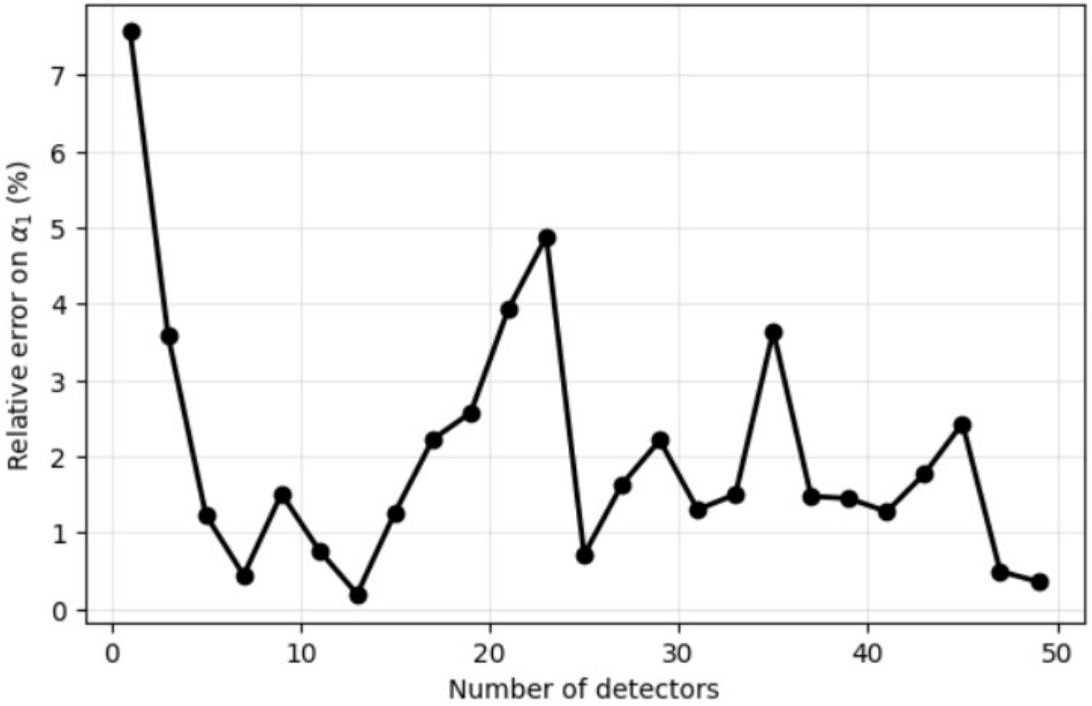
3. Results

Influence of detector number on SSI observability

Goal : Quantify how increasing the number of detectors improves the number and quality of reconstructed neutron noise modes.



Increasing detector redundancy enhances system rank and reveals higher-order α-modes.



Increasing detector redundancy improves the evaluation stability

SSI benefits directly from spatial information: increasing detector coverage improves both modal richness and statistical robustness of identified neutron noise dynamics.

4. Conclusion and Outlook

SSI Method, Main Assumptions and Limitations :

- **Linearity and Time Invariance:** The reactor is assumed to behave as a linear, time-invariant system whose physical properties remain constant during the measurement.
- **Stationarity:** The statistical properties of the measured signals (mean and variance) are constant over time.
- **Input–State Independence:** The excitation noise is assumed to be independent of the system state.
- **Zero-Mean Gaussian Noise:** The stochastic excitation is assumed to be zero-mean and Gaussian. In practice, Poisson noise provides a very good approximation.
- **Transient Dynamics:** The reactor is assumed to be subcritical with a constant noise source, so that SSI identifies only stable, decaying modes .
- **Observability (Key Limitation):** Sensors must be placed so that all modes of interest are observable.

Optimal sensor placement requires prior knowledge of the system, making observability the main practical limitation of the SSI method.

4. Conclusion and Outlook

SSI reconstructs reactor dynamics directly from neutron noise measurements :

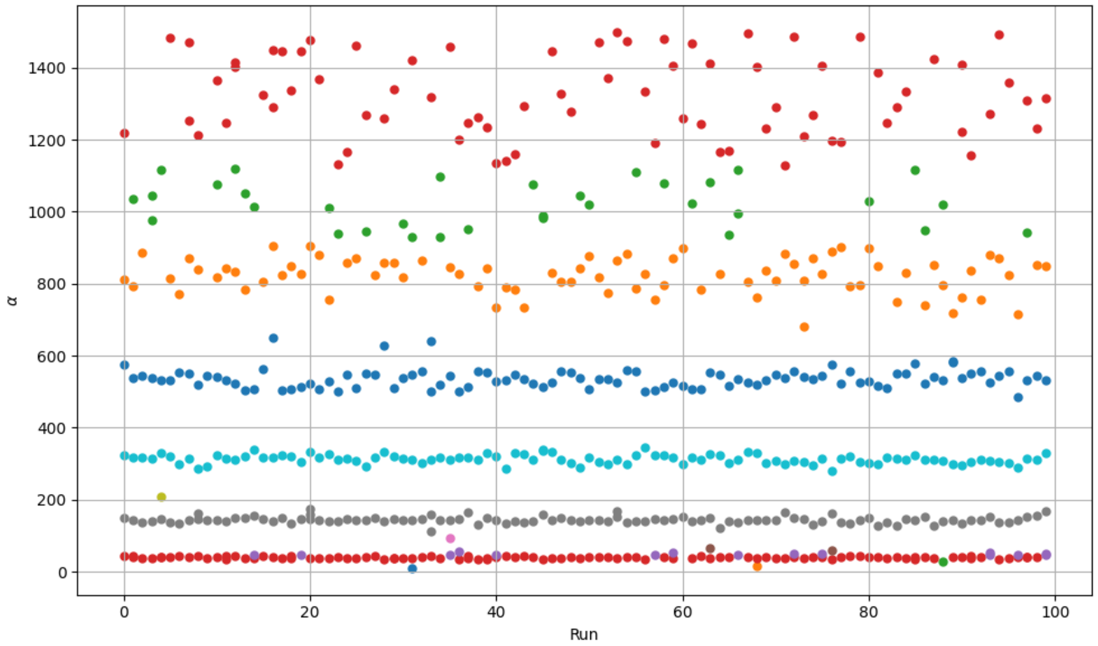
- No prior spectral model
- No prior knowledge of the transport matrix
- Direct access to state operator

Main findings :

- Robust identification of fundamental mode
- Access to higher order modes
- Stability accross independant Monte-Carlo realisation

Outlook :

- Taking into account delayed neutron
- Detailed study of detector position
- Publish a paper on ANE



Method	Number of modes	Requires fission matrix
Rossi-α	1	No
Classical Cohn-α	1	No
Multi-modal Cohn-α	Multiple	Yes
SSI	Multiple	No

SSI provides a scalable, data-driven access to the full neutron noise spectrum, revealing modes that are inaccessible to classical methods

Thank you

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