Dark matter as higher-form fields

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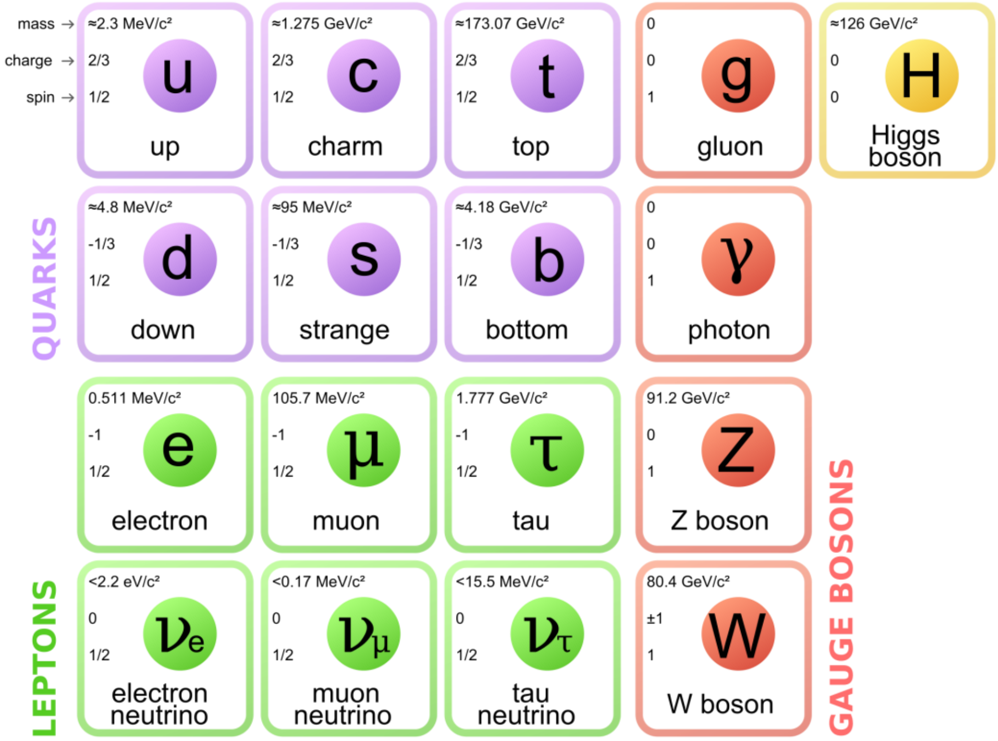
Sous la supervision de Christopher Smith, équipe Physique Théorique

Présentations doctorants - 2026

Overview

- Introduction and motivations: Going beyond the Standard Model with unusual fields
- A dictionary of higher-forms: promises and limits
- The quest for a suitable source
- More formal implications in QFT: higher-forms and anomalies
- Conclusion

The standard Model in a nutshell



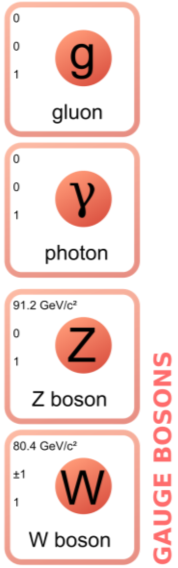
The known part of the universe content is described by the Standard Model of Particle Physics:

- The quarks and leptons compose most of the matter
- The gauge bosons mediate their interactions
- The Higgs field provides the masses

The particles are represented by fields that evolve in a (3+1)D space-time.

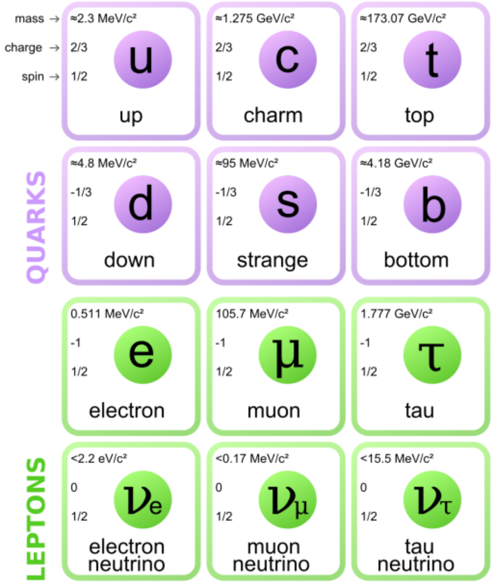
Boson of spin 0: Scalar particle

$$\phi$$



Boson of spin 1: Vector in spacetime

$$A^{\mu} = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}$$



Fermions: represented by spinors ψ.
Come by pairs spinors – anti spinors $\bar{\psi}$.

$\bar{\psi}\psi$ Behaves as a scalar

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We focus on the bosonic part of the SM

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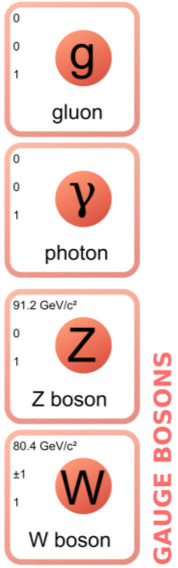

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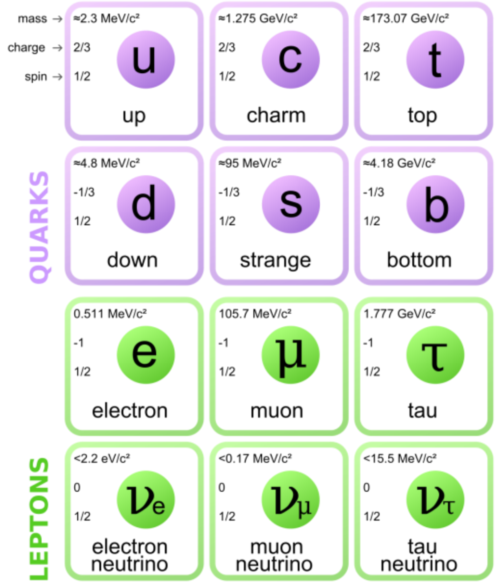

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From the field, we define the *strength-tensor* :

$$F_{\phi}^{\mu} = \frac{\partial \phi}{\partial x_{\mu}} \equiv \partial^{\mu} \phi$$

$x = (t, x, y, z)$ spacetime vector

$$F_{A}^{\mu\nu} = \frac{\partial A^{\nu}}{\partial x_{\mu}} - \frac{\partial A^{\mu}}{\partial x_{\nu}} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

Antisymmetric in $\mu \leftrightarrow \nu$

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The strength-tensors are useful to describe the fields dynamic, as the free-fields obey the Klein-Gordon equation:

$$\partial_\mu F_\phi^\mu + m^2 \phi = 0$$

m is the mass of the field

$$\partial_\mu F_A^{\mu\nu} + m^2 A^\nu = 0$$

With $m^2 = 0$ if the field is massless

These equations of motions are encoded in a Lagrangian,

$$L_\phi = \frac{1}{2} F^\phi_\mu F^\mu_\phi - \frac{m^2}{2}\phi^2$$

$$L_A = -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + \frac{m^2}{2} A_\mu A^\mu$$

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The fields can also interact with an external source or with other fields. In this case, the Lagrangian picks up an extra term corresponding to the interaction:

$$L_\phi = \frac{1}{2} F^\phi_\mu F^\mu_\phi - \frac{m^2}{2}\phi^2 + \phi J \longrightarrow \partial_\mu F^\mu_\phi + m^2\phi = J$$

$$L_A = -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + \frac{m^2}{2} A_\mu A^\mu - A_\mu J^\mu \longrightarrow \partial_\mu F^{\mu\nu}_A + m^2 A^\nu = J^\nu$$

The standard Model in terms of fields

The Lagrangian formulation is taken as fundamental. It is easy to distinguish the free propagation of the fields from their interactions.

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For example, in the QED sector of the Standard Model:

$$L_{QED} = \underbrace{\bar{\psi}(i\partial^{\mu}\gamma_{\mu} - m)\psi}_{\text{Fermions free propagation with mass term}} - \underbrace{\frac{1}{4}F^{A}_{\mu\nu}F^{\mu\nu}_{A}}_{\text{Photon free propagation (massless)}} - \underbrace{Qe\bar{\psi}\gamma^{\mu}\psi A_{\mu}}_{\text{Photon – fermions coupling}}$$

The QED sector exacerbates a $U(1)$ symmetry, meaning that the Lagrangian does not change under the gauge transformation:

$$\psi \rightarrow e^{iQe\alpha(x)}\psi \qquad A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

Gauge parameter

Specifically, the strength-tensor of the photon $F_A^{\mu\nu}$ is in itself invariant under the transformation.

The standard Model in terms of symmetries

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This symmetry forbids a mass term for the photon, as

$$L_{mass} = \frac{m^2}{2} A_\mu A^\mu$$

Is not gauge invariant

The standard Model in terms of symmetries

The entire Standard model respects three symmetries:

- A $SU(3)$ color symmetry, describing the strong interaction
- A $SU(2) \times U(1)$ electroweak symmetry

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- A $SU(2) \times U(1)$ electroweak symmetry

At low energy, the $SU(2) \times U(1)$ symmetry is broken thanks to the presence of the Higgs field

There is no symmetry to prevent the W and Z bosons to acquire a mass.

The residual symmetry is the one of QED: only the photon remains massless.

The standard Model: long story short

The fundamental interactions are related to symmetry groups and are mediated by vector fields:

$$A^\mu = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix}$$

- The vector fields are all massless as long as the symmetries are exact
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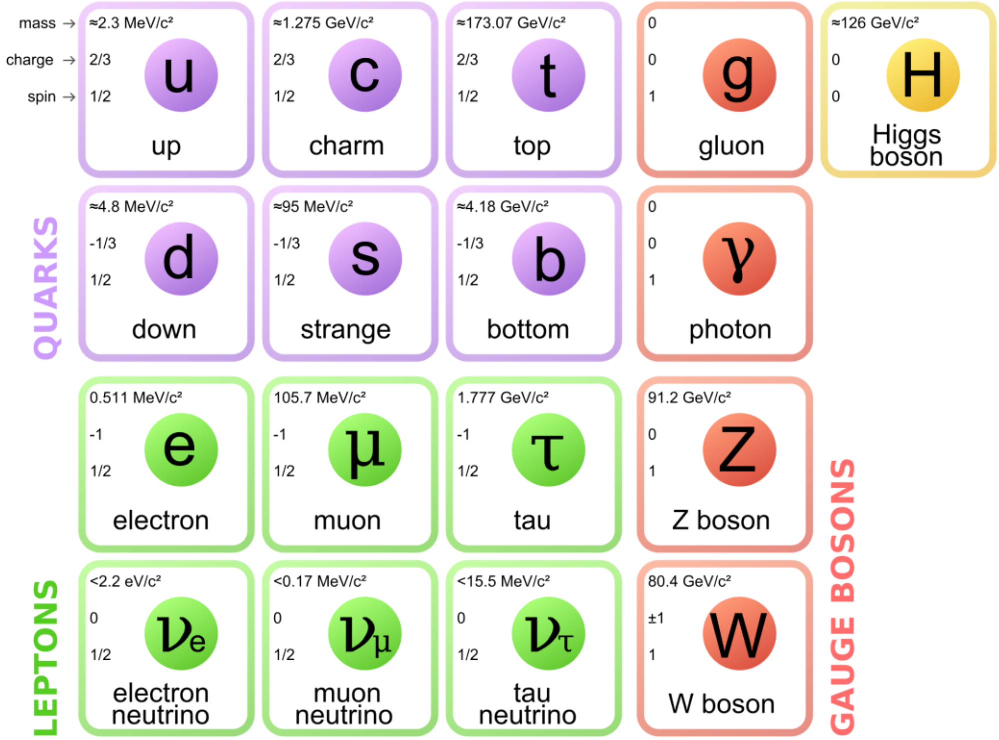
The Higgs mechanism is a miracle: this is the only conventional way for a vector to acquire a mass from a fundamentally symmetric theory

The standard Model is not the end of the story



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Dark matter is needed to fit astrophysical and cosmological data

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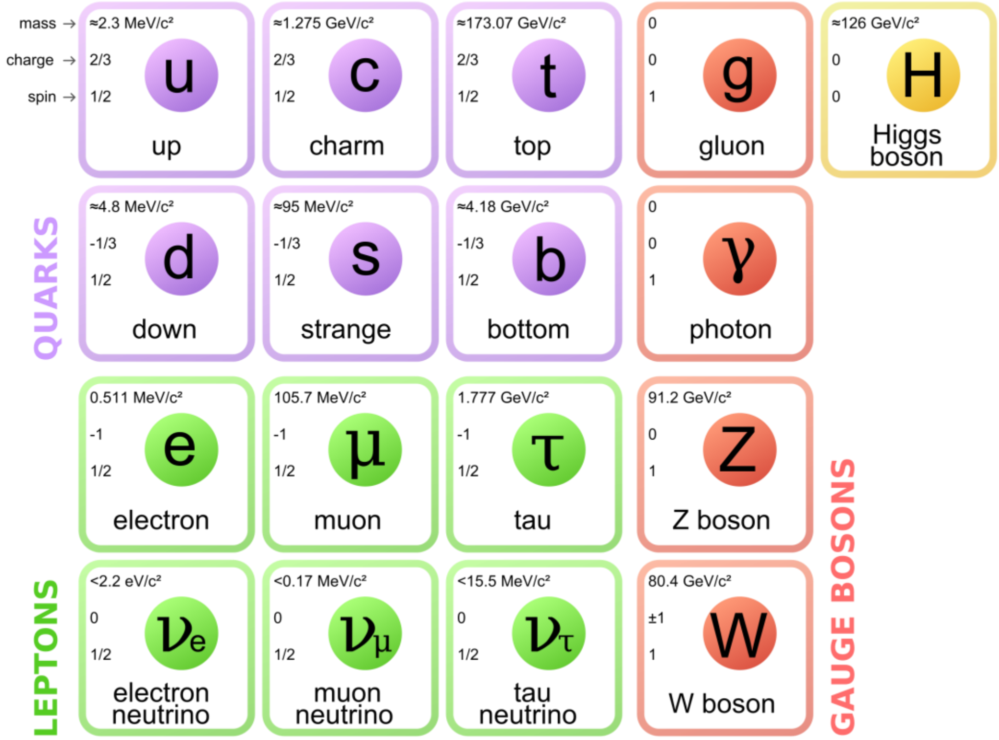

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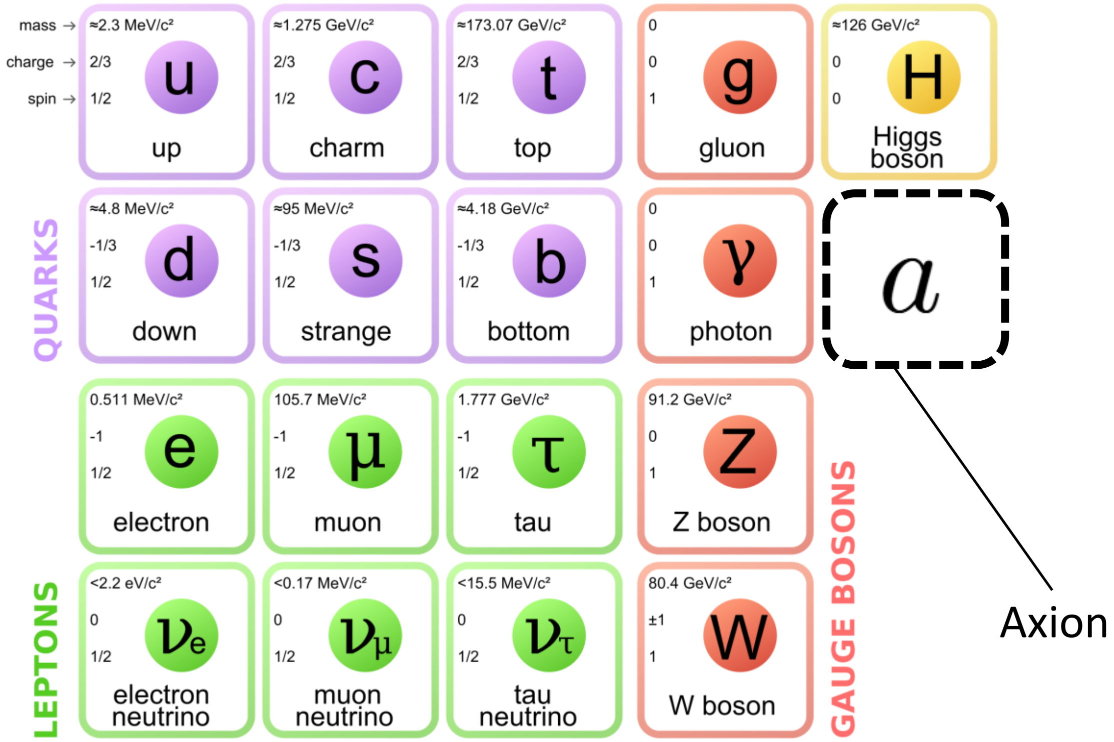

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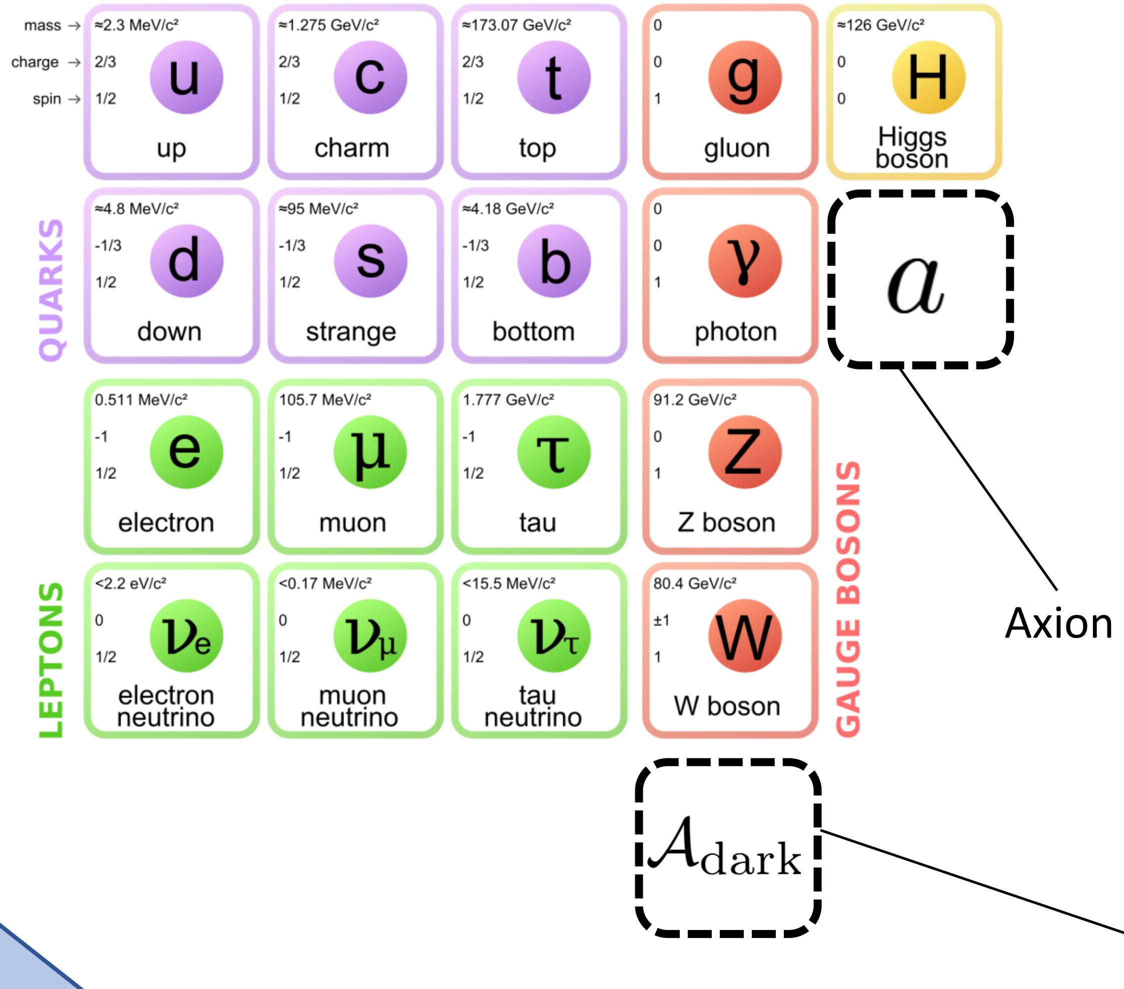

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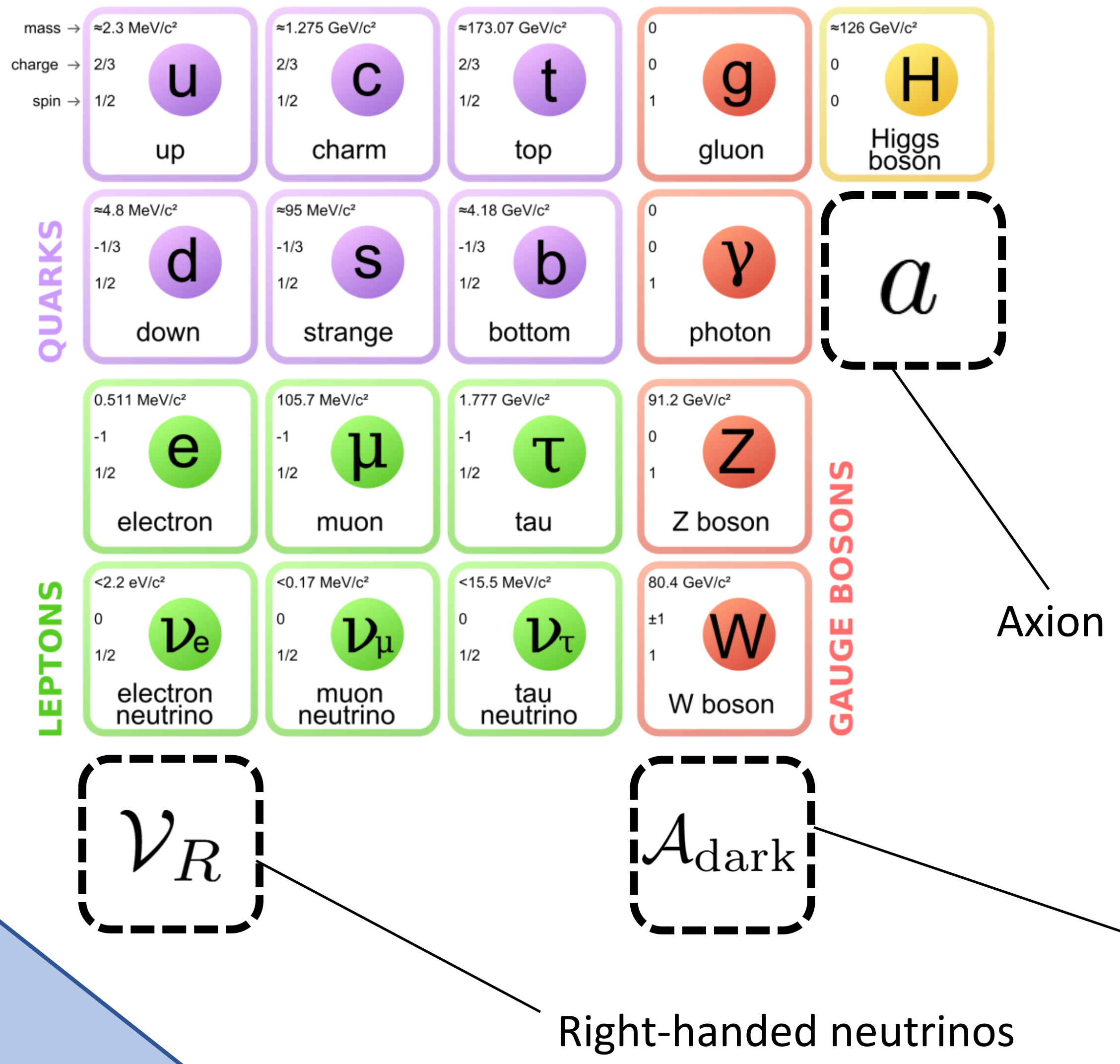

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Dark photons

Here comes a new challenger

Scalar fields or vector fields are the common candidates:

$$\phi$$

0 spacetime index

1 spacetime index

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We propose to consider the *Kalb-Ramond* field as Dark Matter candidate:

- Two spacetime indices
- Antisymmetric in $\mu \leftrightarrow \nu$
- Represented by a matrix

$$B^{\mu\nu} = \begin{pmatrix} 0 & B^{01} & B^{02} & B^{03} \\ -B^{01} & 0 & B^{12} & B^{13} \\ -B^{02} & -B^{12} & 0 & B^{23} \\ -B^{03} & -B^{13} & -B^{23} & 0 \end{pmatrix}$$

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We also regard a three-indices antisymmetric tensor field:

$$C^{\mu\nu\rho}$$

3 spacetime indices

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Kalb-Ramond field and the three-form field $\Longrightarrow$ Higher-form fields

Propagation? Symmetries? Masses? Couplings?

Overview

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We know the Lagrangians of free scalar and vector fields

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We define the Strength-tensors of Kalb-Ramond field and three-form field:

$$F^{\mu\nu\rho}_B = \partial^\mu B^{\nu\rho} + \partial^\nu B^{\rho\mu} + \partial^\rho B^{\mu\nu}$$

$$F^{\mu\nu\rho\sigma}_C = \partial^\mu C^{\nu\rho\sigma} + \partial^\nu C^{\rho\mu\sigma} + \partial^\rho C^{\mu\nu\sigma} + \partial^\sigma C^{\nu\mu\rho}$$

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And we build the Lagrangians describing their free propagation:

$$L_B = \frac{1}{12} F^B_{\mu\nu\rho} F_B^{\mu\nu\rho} - \frac{m^2}{4} B_{\mu\nu} B^{\mu\nu}$$

$$L_C = -\frac{1}{48} F^C_{\mu\nu\rho\sigma} F_C^{\mu\nu\rho\sigma} + \frac{m^2}{12} C_{\mu\nu\rho} C^{\mu\nu\rho}$$

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Symmetries of higher-forms

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2-indices parameter

As they leave invariant the strength tensors $F_B^{\mu\nu\rho}$ and $F_C^{\mu\nu\rho\sigma}$.

Adding mass terms would break these symmetries

Degrees of freedom of the higher-forms

We can count the number of degrees of freedom propagated by each form in the massive and massless case.
We take in account the constraints given by:

- The equations of motion
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Some fields propagate the same number of DoFs

This agreement in the number of DoFs is more than a coincidence, its an equivalence: we talk about duality.

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Higher forms and duality

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! Dualities are valid a priori only for free theories (in absence of interactions or external sources)

- Massive A^{μ} and $B^{\mu\nu}$
- Massive ϕ and $C^{\mu\nu\rho}$
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Describe the same physics (as long as they are uncoupled)

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Can we break this duality? Are dual fields always equivalent?

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However, the two operators describing the interactions are not equivalent.

Interactions break duality

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Example: duality between the massive vector and KR fields : couplings to fermions

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Consequently:

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dominant $\bar{\psi}\gamma^\mu\psi A_\mu$

subdominant $\frac{1}{m}\bar{\psi}(\gamma^\mu\gamma^\nu\gamma^5)\psi F^A_{\mu\nu}$

$\frac{1}{m}\varepsilon_{\mu\nu\rho\sigma}\bar{\psi}\gamma^\mu\psi F_B^{\nu\rho\sigma}$ subdominant

$\bar{\psi}(\gamma^\mu\gamma^\nu)\psi B_{\mu\nu}$ dominant

Two dual forms have different dominant operator

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- ❑ Imposing gauge invariance discriminates between the vector and KR theory

$$F^{\gamma}_{\mu\nu} F^{A\,\mu\nu}$$ ✓

Kinetic mixing: mixing of the dark vector with the photon

$$F^{\gamma}_{\mu\nu} B^{\mu\nu}$$ ✗

Mixing of the KR field with the photon

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$$\frac{1}{m}\bar{\psi}(\gamma^{\mu}\gamma^{\nu})\psi F^{A}_{\mu\nu}$$ ✔

Coupling with the magnetic dipole moment operator

$$\bar{\psi}(\gamma^{\mu}\gamma^{\nu})\psi B_{\mu\nu}$$ ✘

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$$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi A_{\mu} \quad ✗$$

Coupling with the axial current

$$\frac{1}{m}\bar{\psi}\gamma^{\mu}\gamma^{5}\psi\varepsilon_{\mu\nu\rho\sigma}F_{B}^{\nu\rho\sigma} \quad ✓$$

Coupling with the axial current

Different three-body decays

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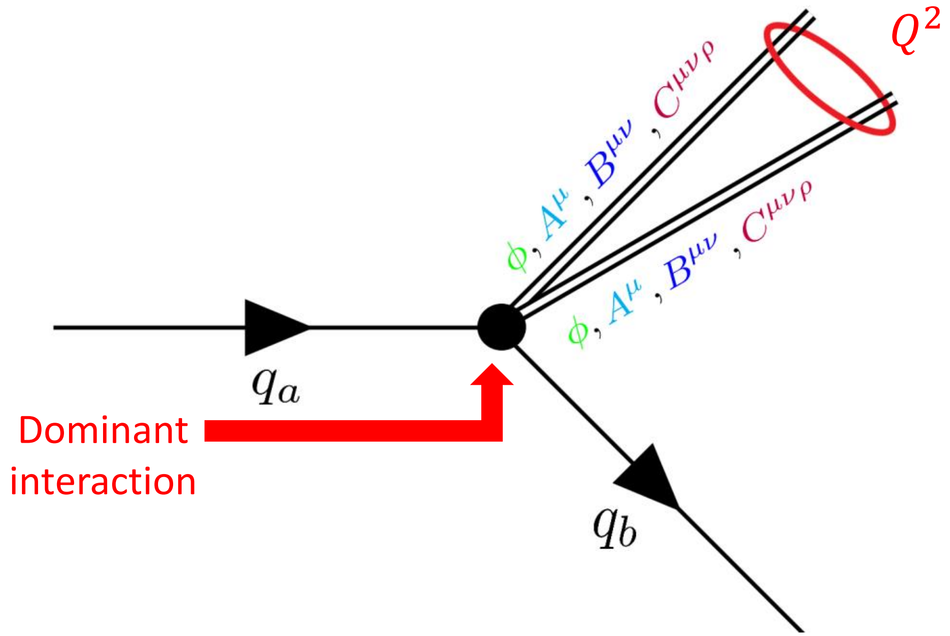

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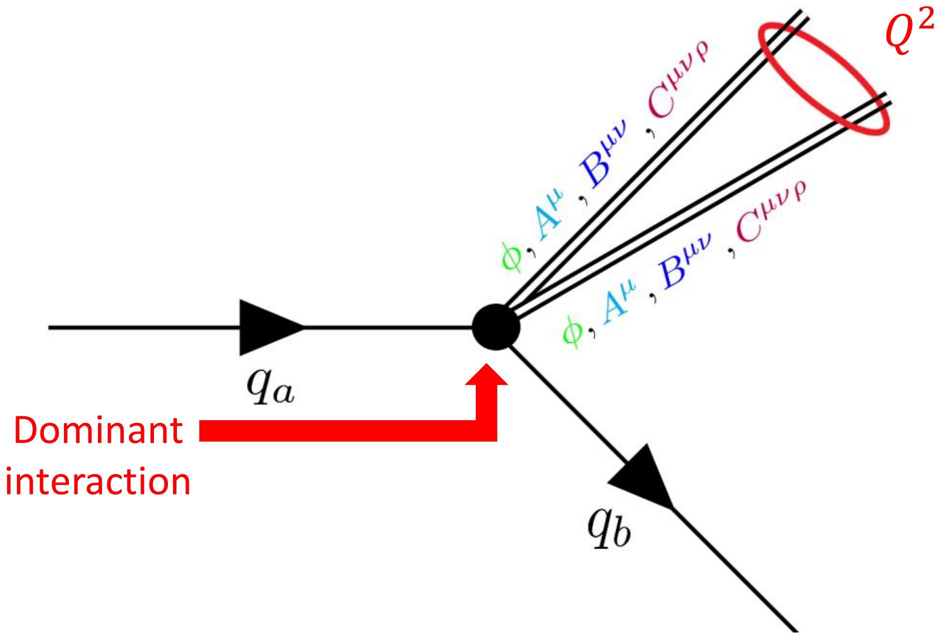

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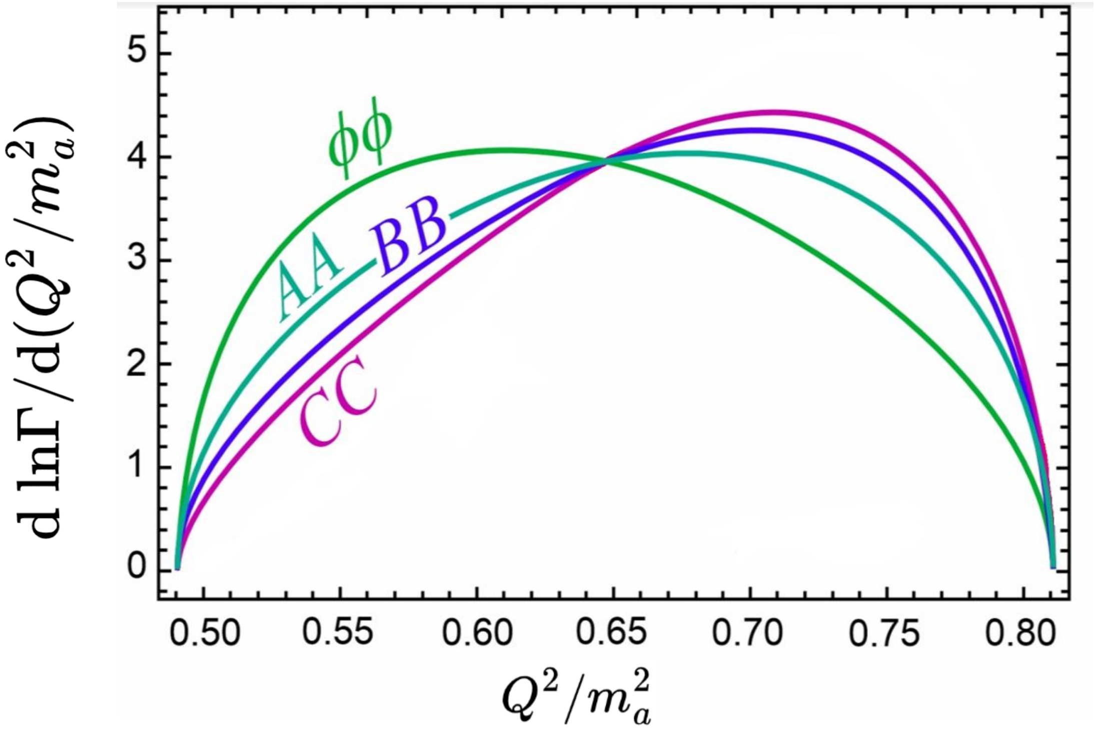

Normalized differential decay rates for $q_a \to q_b XX$, in function of the squared impulsion Q^2 normalized by m_a^2. Here, $\frac{m_b}{m_a} = 0{,}1, \frac{m_X}{m_a} = 0{,}35$.

A dictionary for higher-forms

Using the higher-forms from a phenomenological point-of-view had never been done before, so we needed to establish from scratch a complete dictionary, including

A dictionary for higher-forms

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Subject of:

Dark Higher-form portals and dualities, Cypris Plantier and Christopher Smith, arXiv:2506.04795 [hep-ph], Published in *PR.D 112, 075043* (2025)

Higher-forms: limits and perspectives

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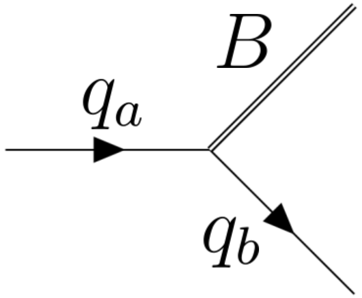

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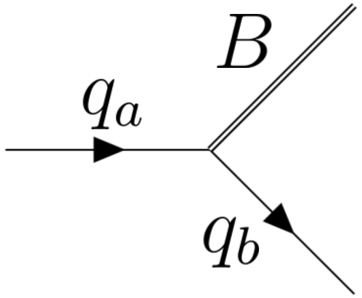

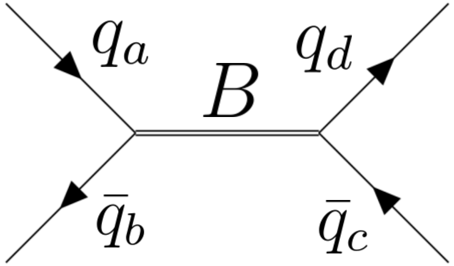

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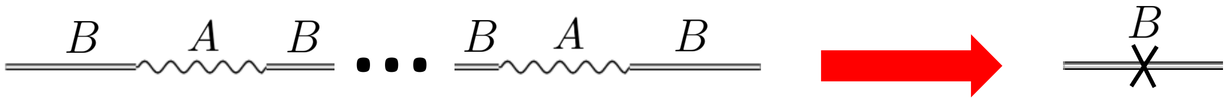
  The most promising mechanism is based on oscillations that reduce to an effective mass



An infinite series of oscillations of a KR field in a vector field creates an effective mass

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- If we hope to find a fundamental, well-behaving theory for an interacting Kalb-Ramond field, we need an appropriate source: two-indices, conserved and with the good mass dimension.

We want $$B_{\mu\nu}J^{\mu\nu}$$ With $$\partial_\mu J^{\mu\nu} = 0$$

Overview

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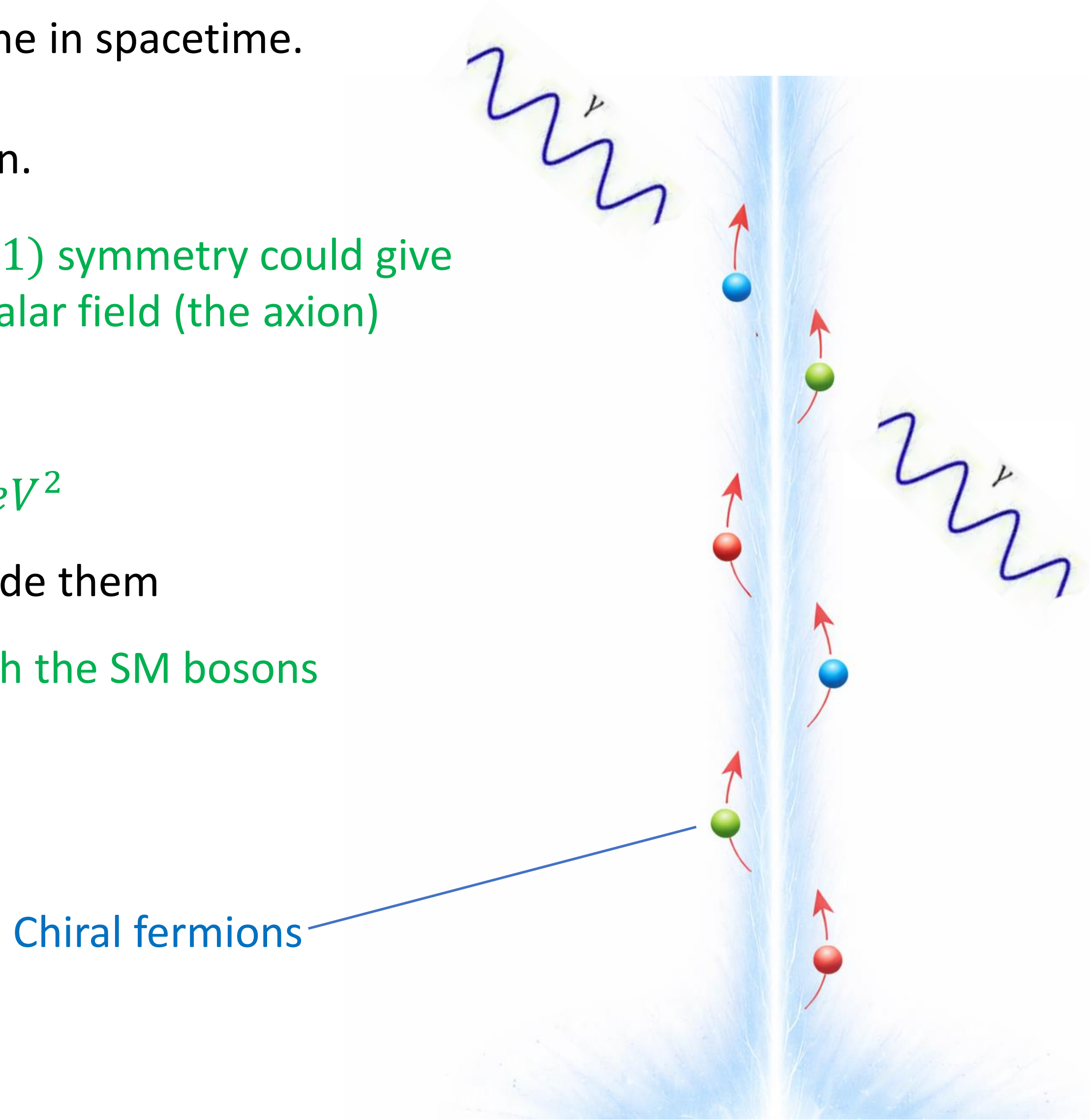

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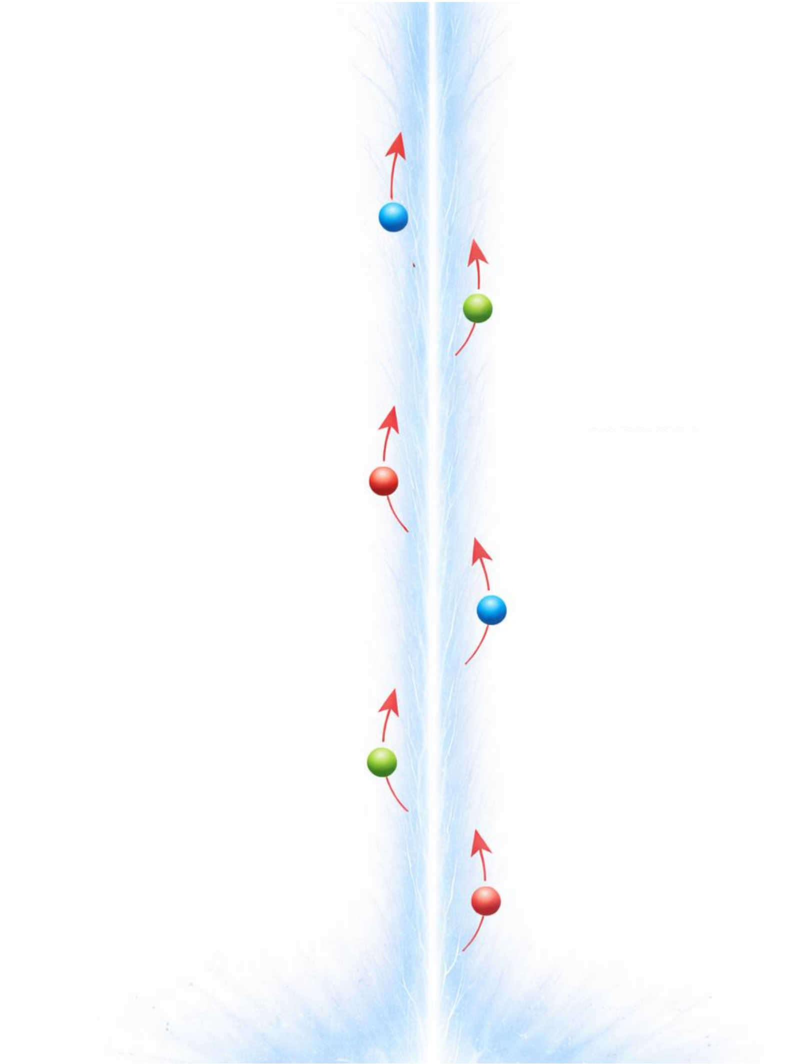

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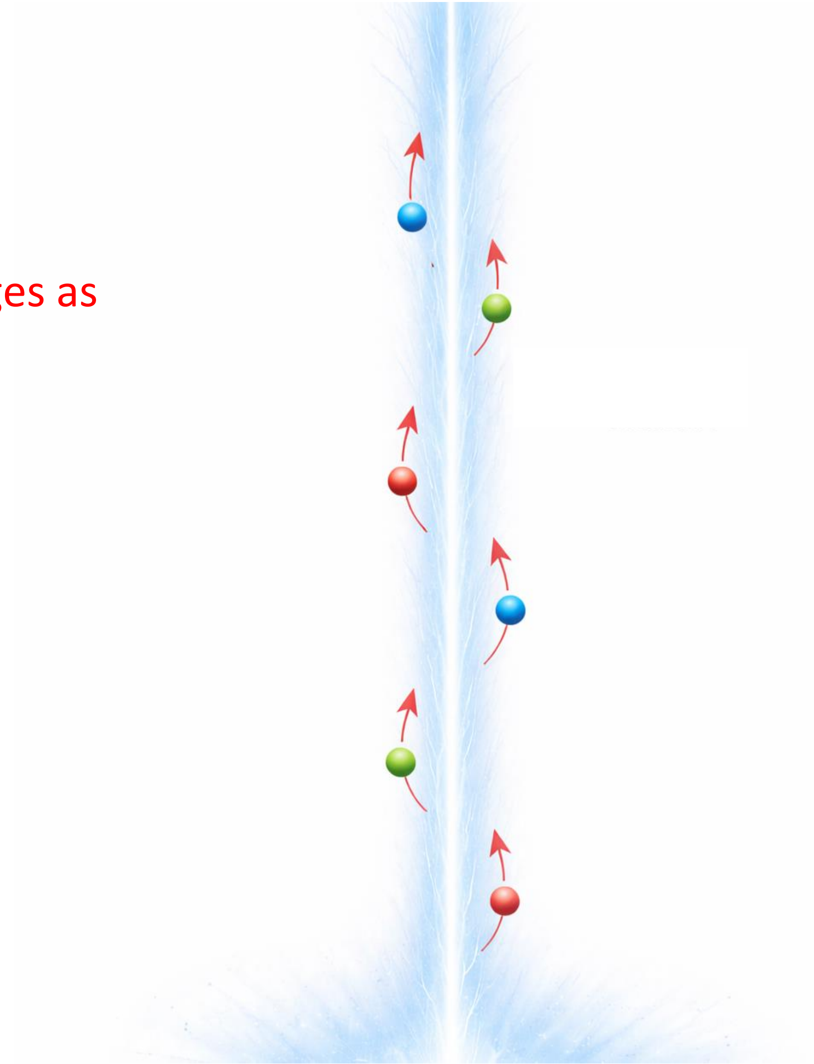

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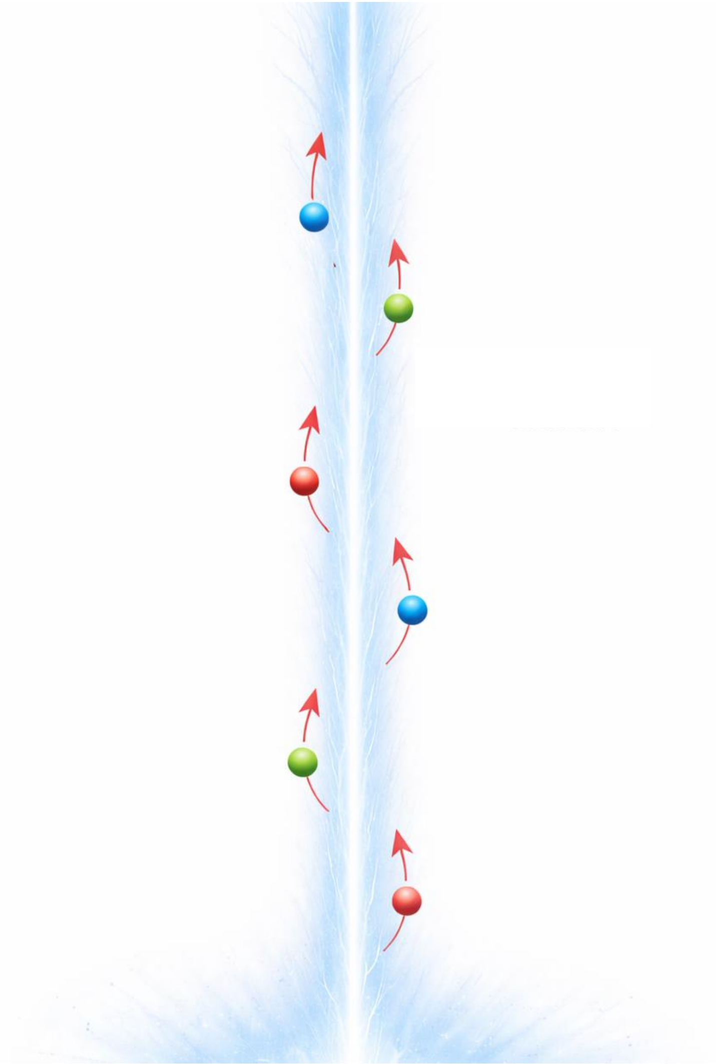

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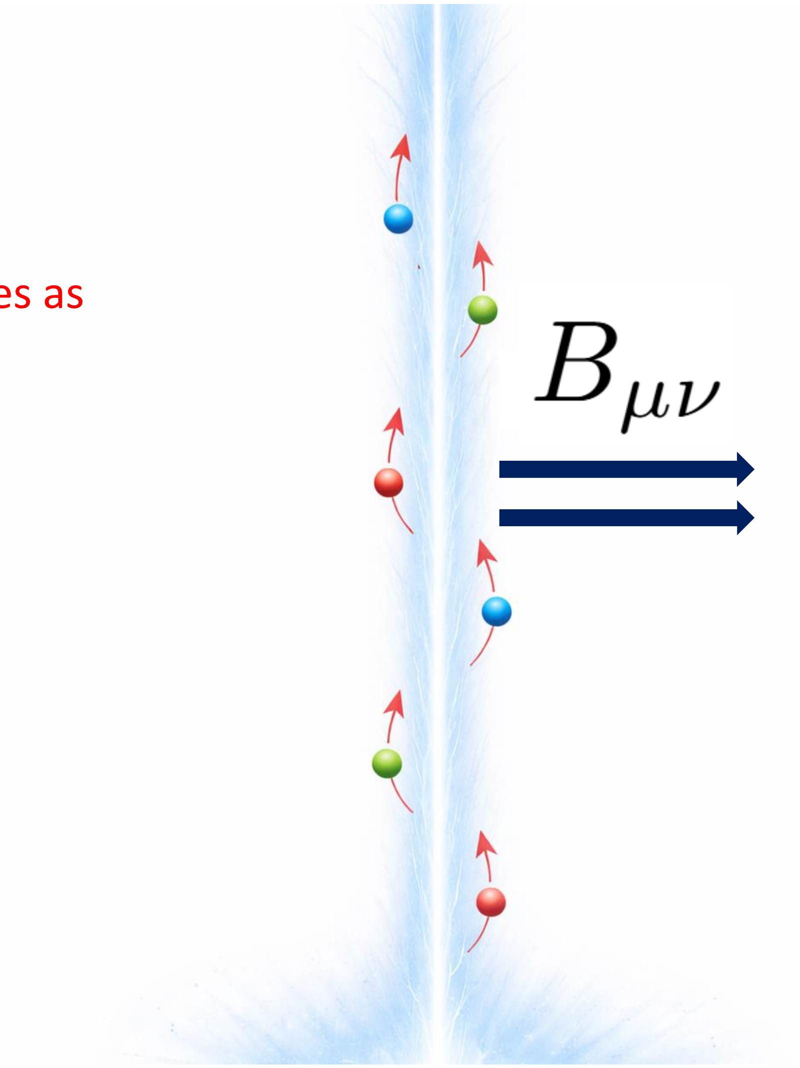
The string resumes in a conserved source $J^{\mu\nu}$ for the KR field

A cosmic string could emit Kalb-Ramond fields!



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This is the axial anomaly!

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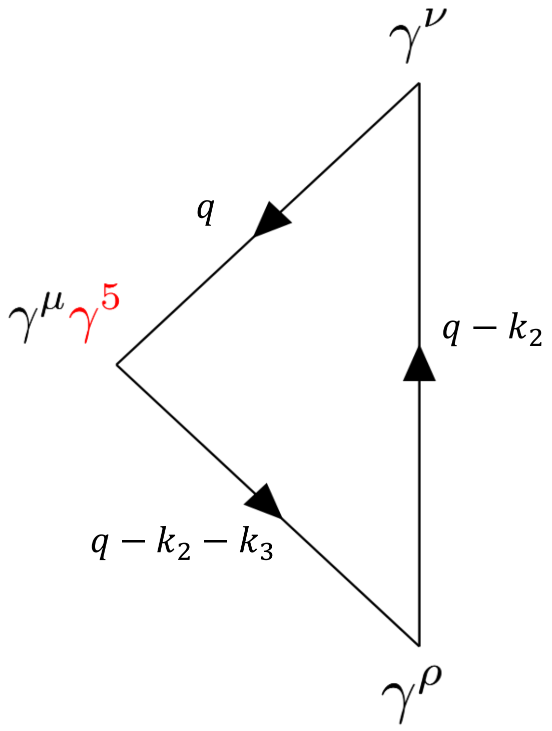

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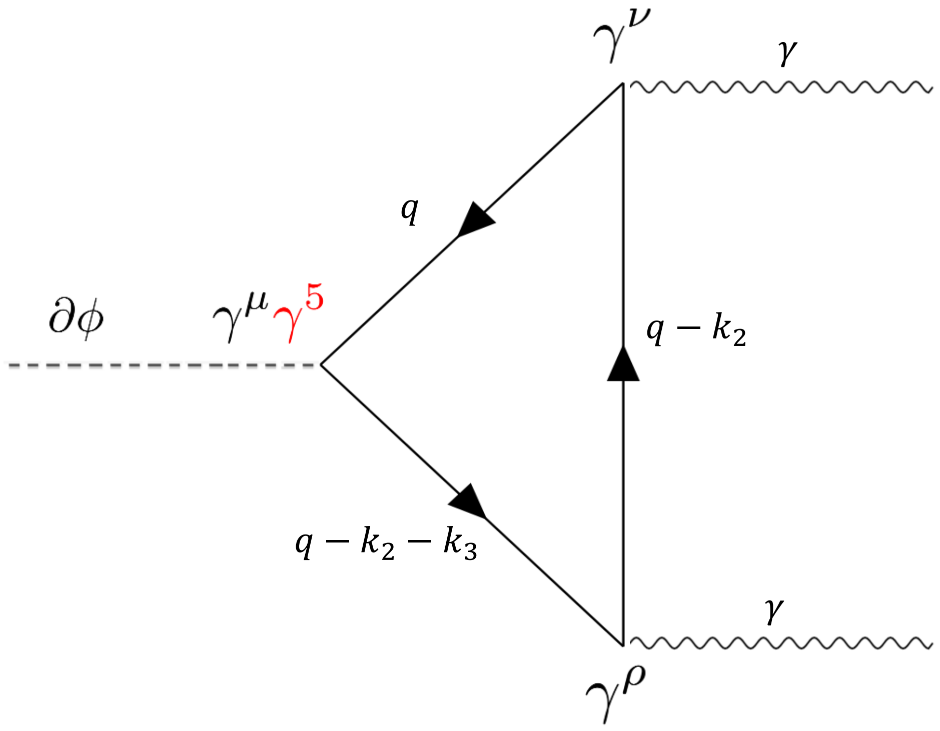

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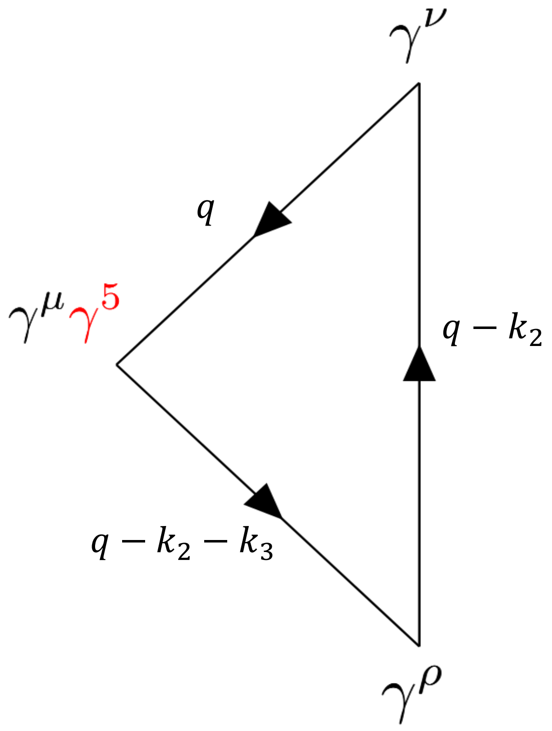

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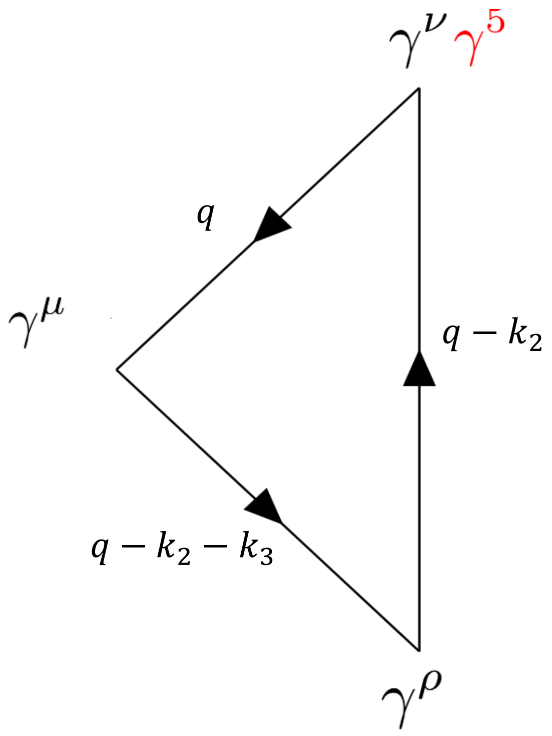

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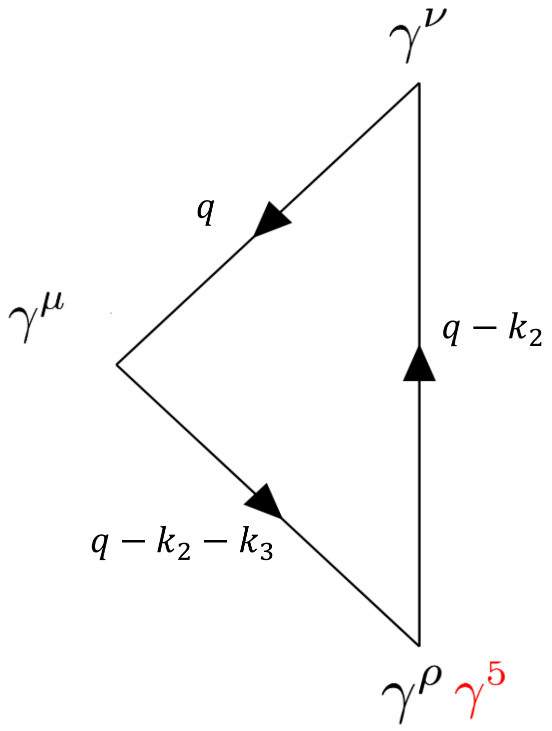

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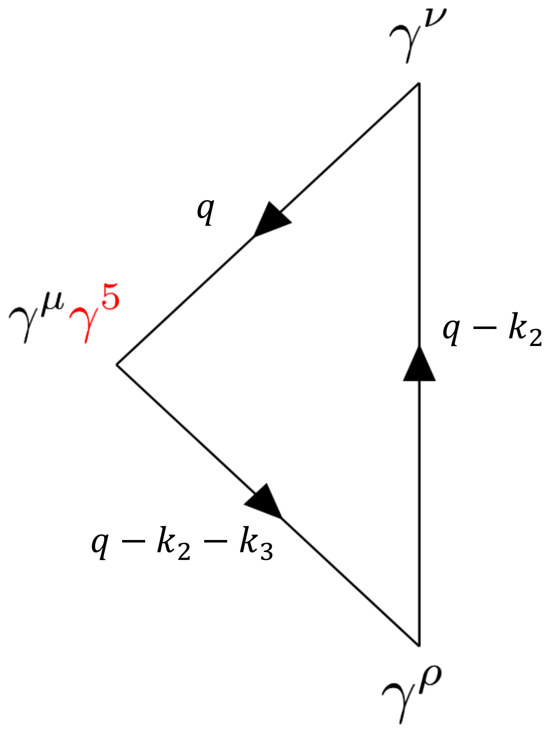

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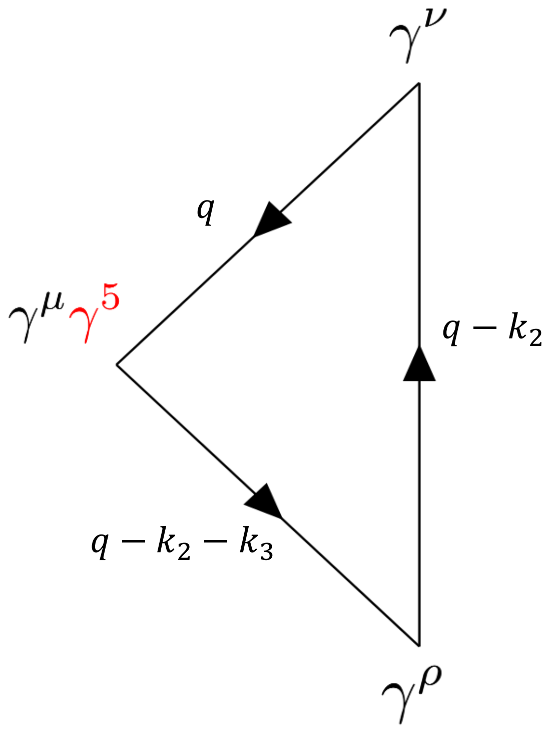
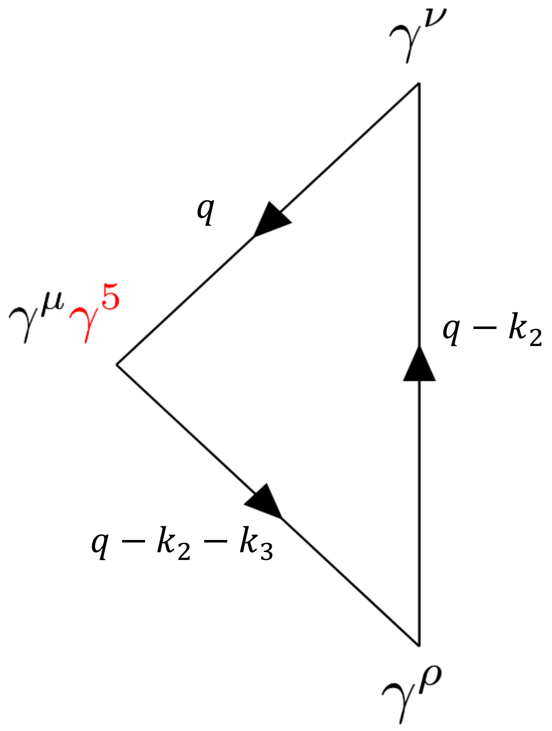
We have to keep track of the ambiguities related to γ^5. They give birth to an extra term: the anomaly.



The AVV triangle

The anomaly manifests itself at loop level through the AVV triangle. It takes the form of a term that

- Is independent of the fermion mass m_f
- Depends on the regularization scheme employed to treat γ^5
- Cannot be renormalized by adding a local counterterm in the lagrangian



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Kalb-Ramond fields and three-form fields couple preferentially to other currents:

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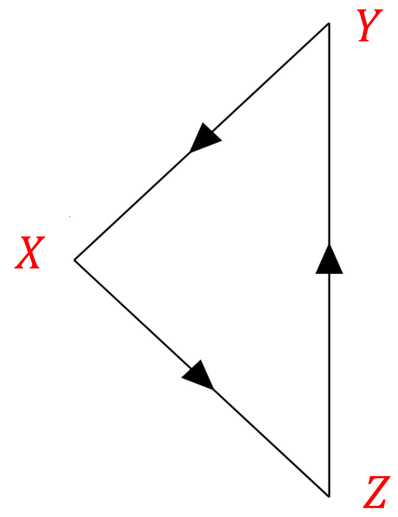
It opens the gate to a host of new triangles to consider:

- Do they exist
- If so, do they exacerbate anomalies?



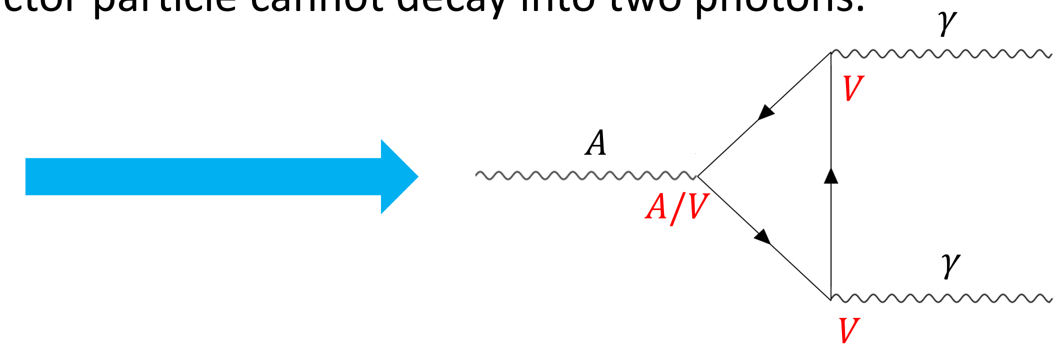
$X, Y, Z = V, A, T, \tilde{T}, \varepsilon, \tilde{\varepsilon}$

Duality and triangle diagrams

These triangles can help us testing dualities.

The Landau-Yang theorem states that a dark vector particle cannot decay into two photons.

$$\boxed{A \longrightarrow \gamma\gamma = 0}$$



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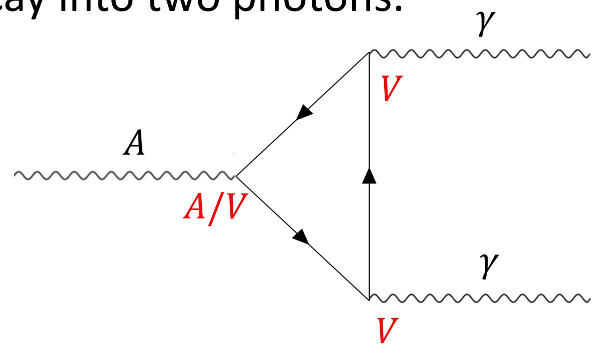
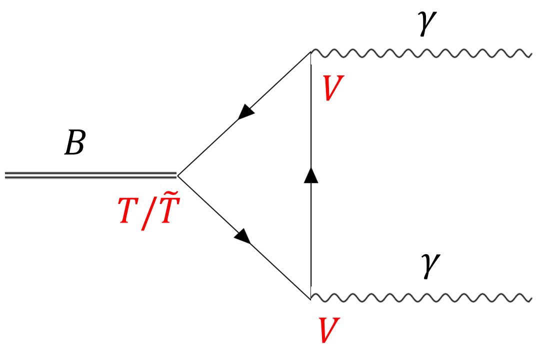


$= 0$

The Kalb-Ramond field is dual to the vector field. Does Landau-Yand still holds for it?



$=?$

$$\boxed{B \longrightarrow \gamma\gamma =?}$$

Higher forms and fermionic loops

- We calculated and classified all the triangle diagrams
- We commented on their possible anomalous natures
- We tested how the A/B and ϕ/C dualities adapt in the context of processes involving triangle loops
- We derived phenomenological consequences

Subject of:

Dark higher-form fields and triangle anomalies, Cypris Plantier and Christopher Smith, arXiv:2602.14839 [hep-ph], Accepted for publication in JHEP, march 2026.

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Thank you for your attention and feel free to ask questions!