

Minimal Flavor Violation in supersymmetry



Christopher Smith

- Outline

Introduction: Flavors in the SM

- I. *New Physics flavor puzzle(s)*
- II. *Minimal Flavor Violation (MFV)*
- III. *CP-violation under MFV*
- IV. *RGE behavior of MFV*
- V. *MFV and proton decay*

Conclusion

Introduction

A. The Standard Model flavor symmetry

The three generations of quarks/leptons have *identical gauge interactions*

$$\mathcal{L}_{Kin} = \sum_{k,I=1,2,3} \bar{\psi}_k^I i\mathbb{D}_k \psi_k^I, \quad D_k^\mu \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

where $\psi_k : Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad U = u_R^\dagger, \quad D = d_R^\dagger, \quad L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad E = \ell_R^\dagger$

As a result, the SM gauge interactions exhibit the $U(3)^5$ flavor symmetry:

$$G_f = U(3)^5 = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$$

Chivukula,
Georgi '87

With one $U(3)$ per fermion species, since under $g_k \in U(3)_k$

$$\psi_k^I \rightarrow (g_k)^{IJ} \psi_k^J \quad \Rightarrow \quad \mathcal{L}_{Kin} \rightarrow \sum_{k,I,J,K} \bar{\psi}_k^J (g_k^\dagger)^{JI} i\mathbb{D}_k (g_k)^{IK} \psi_k^K = \mathcal{L}_{Kin}$$

B. In the SM, the flavor symmetry is broken in a very special way:

- *The only sources of breaking are the Yukawa couplings:*

$$\mathcal{L}_{\text{Yukawa}} = U^I \mathbf{Y}_u^{IJ} (Q^J H) + D^I \mathbf{Y}_d^{IJ} (Q^J H^\dagger) + E^I \mathbf{Y}_e^{IJ} (L^J H^\dagger)$$

which themselves are also *very special*:

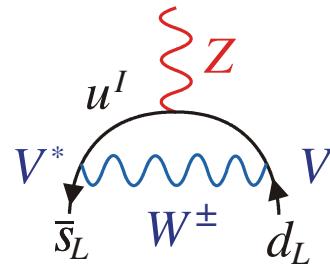
- The *fermion masses* are highly hierarchical ($m_t \gg m_c \gg m_u$)
- The *CKM matrix* is highly hierarchical (close to unit matrix),
- The CKM phase is the *unique source for all CP-violation*.
- Essential feature of *flavor physics* & *FCNC processes*:

$$B \rightarrow X_{d,s} \ell^+ \ell^- ,$$

$$B \rightarrow X_{d,s} \nu \bar{\nu} ,$$

$$K \rightarrow \pi \nu \bar{\nu} ,$$

$$K_L \rightarrow \pi^0 \ell^+ \ell^- , \dots$$



$\sim m_{u^I}^2$
 \Rightarrow top quark
dominates

$$b \rightarrow s : V_{tb}^* V_{ts} \sim 10^{-2}$$

$$b \rightarrow d : V_{tb}^* V_{td} \sim 10^{-3}$$

$$s \rightarrow d : V_{ts}^* V_{td} \sim 10^{-4}$$

C. Warm-up: “MFV” in the Standard Model

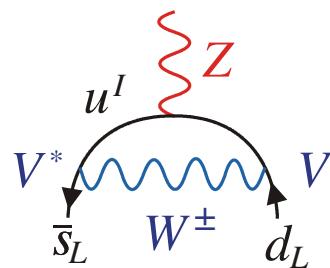
- The SM is made *artificially invariant under* G_f by forcing $\mathbf{Y}_{u,d,e}$ to transform as:

$$\mathbf{Y}_u \rightarrow g_U \mathbf{Y}_u g_Q^\dagger, \quad \mathbf{Y}_d \rightarrow g_D \mathbf{Y}_d g_Q^\dagger, \quad \mathbf{Y}_e \rightarrow g_E \mathbf{Y}_e g_L^\dagger$$

since then $\mathcal{L}_{Yukawa} = U \mathbf{Y}_u Q H + D \mathbf{Y}_d Q H^\dagger + E \mathbf{Y}_e L H^\dagger \xrightarrow{U(3)^5} \mathcal{L}_{Yukawa}$

Background values: $v \mathbf{Y}_u = m_u V_{CKM}$, $v \mathbf{Y}_d = m_d$, $v \mathbf{Y}_e = m_e$.

- All *SM amplitudes must then be invariant under* G_f , at all orders.



Example: The Z penguin:

$$\rightarrow \mathcal{O}_Z \sim \bar{Q}^I \gamma_\mu Q^I H^\dagger \underbrace{D^\mu H}_{\sim v^2 Z^\mu}$$

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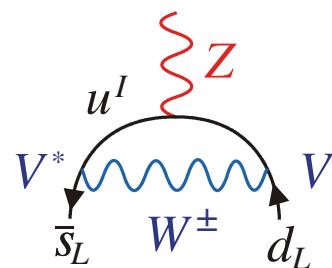
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Predicts the CKM & quadratic GIM:

$$v^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \approx m_t^2 \begin{pmatrix} |V_{td}|^2 & V_{td}^* V_{ts} & V_{td}^* V_{tb} \\ V_{ts}^* V_{td} & |V_{ts}|^2 & V_{ts}^* V_{tb} \\ V_{tb}^* V_{td} & V_{tb}^* V_{ts} & |V_{tb}|^2 \end{pmatrix}$$

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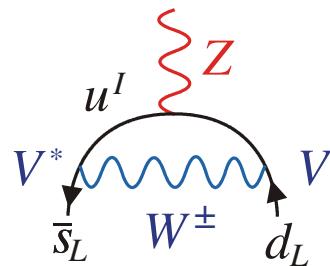
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Suppressed by
 $\sim \frac{m_{d^I} m_{d^J}}{v^2}$

Right-handed currents? $\mathcal{O}_Z \sim D \gamma_\mu \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \bar{D} v^2 Z^\mu$

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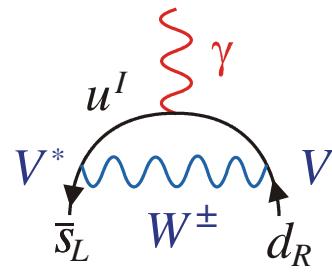
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Example: The EM operator:

$$\rightarrow \begin{cases} \mathcal{O}_\gamma \sim D^I \sigma_{\mu\nu} Q^I H^\dagger F^{\mu\nu} \\ \mathcal{O}_\gamma \sim E^I \sigma_{\mu\nu} L^I H^\dagger F^{\mu\nu} \end{cases}$$

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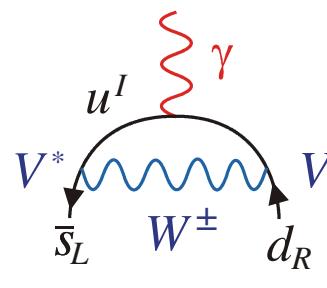
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No LFV, since \mathbf{Y}_e is diagonal: $\mu \not\rightarrow e\gamma$, $\mu \not\rightarrow eee$, ...

Experimentally, $m_\nu \neq 0$ but extremely small $\Rightarrow B(\mu \rightarrow e\gamma) < 10^{-50}$.

I. The MSSM flavor puzzles

A. Flavors and New Physics

- There is some *New Physics* (dark matter, m_ν , unification, EW stability, gravity,...)
- Most New Physics models have either *new flavored particles*, or *new flavor-breaking interactions* between quarks and leptons.
- The Lagrangian of NP can always be made *$U(3)^5$ symmetric*, but at the cost of allowing for *new spurions* (= NP flavor-breaking couplings).

$$\text{Ex: } \mathbf{X}_Q \rightarrow g_Q \mathbf{X}_Q g_Q^\dagger \quad \Rightarrow \quad \mathcal{O}_Z \sim \frac{1}{\Lambda_{NP}^2} \bar{Q}^I \gamma_\mu (\mathbf{X}_Q)^{IJ} Q^J v^2 Z^\mu$$

- *Flavor experiments* \Rightarrow either spurions non-natural, or NP scale very high.
- Ex: $\mathcal{O}_Z \Rightarrow K \rightarrow \pi \nu \bar{\nu}$. With $(X_Q)^{12} \approx 1 \Rightarrow \Lambda \gtrsim 75 \text{ TeV}$.
- Flavor structures of TeV-scale NP necessarily fine-tuned: *NP flavor puzzle*.

A. Flavors and New Physics: Situation in the MSSM

Essentially one superpartner for every SM particle, same gauge group.

Squarks and sleptons are *scalar flavored particles*.

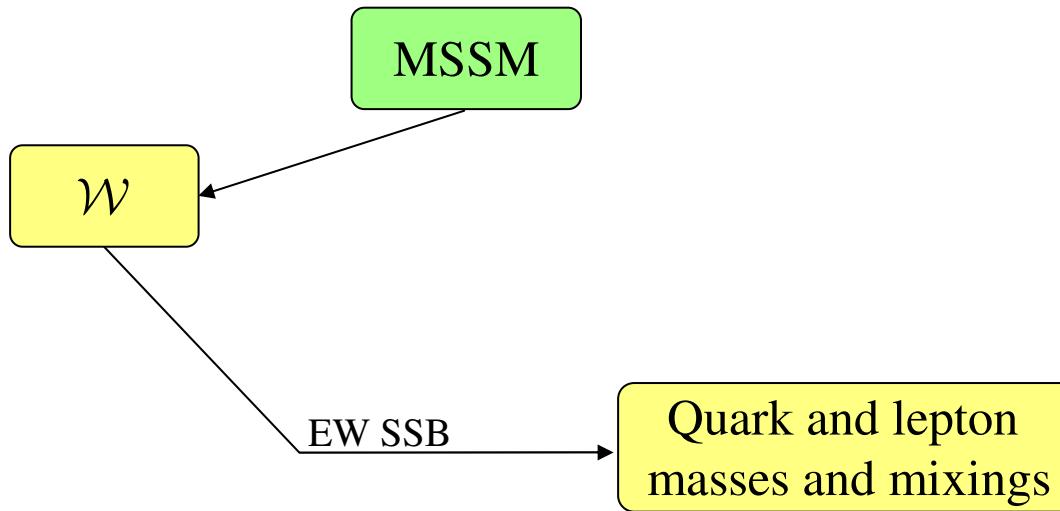
- MSSM gauge interactions still exhibit the $U(3)^5$ flavor symmetry.
- *Many new flavor couplings* \Leftrightarrow new spurions, a priori not hierarchical.
- *New contributions* to flavor transitions

$$\text{e.g.: } \mathcal{L}_{MSSM} \supset \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} \rightarrow \mathcal{O}_Z \sim \frac{1}{\Lambda_{SUSY}^4} (\bar{Q} \gamma_\mu \mathbf{m}_Q^2 Q) v^2 Z^\mu$$

$$\mathcal{L}_{MSSM} \supset \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} \rightarrow \mathcal{O}_\gamma \sim \frac{1}{\Lambda_{SUSY}^4} (E \mathbf{Y}_e \mathbf{m}_L^2 \sigma_{\mu\nu} L) H_d F^{\mu\nu}$$

- Experimental data impose to *fine-tune those additional spurions*:

Approx. alignment with SM: $\mathbf{m}_Q^2 \sim \Lambda_{SUSY}^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u$, $\mathbf{m}_L^2 \sim \Lambda_{SUSY}^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e$.



1. *Superpotential Yukawa couplings*: set fermion masses and mixings.

$$\mathcal{W} = U \mathbf{Y}_u (Q H_u) - D \mathbf{Y}_d (Q H_d) - E \mathbf{Y}_e (L H_d) + \mu (H_u H_d)$$

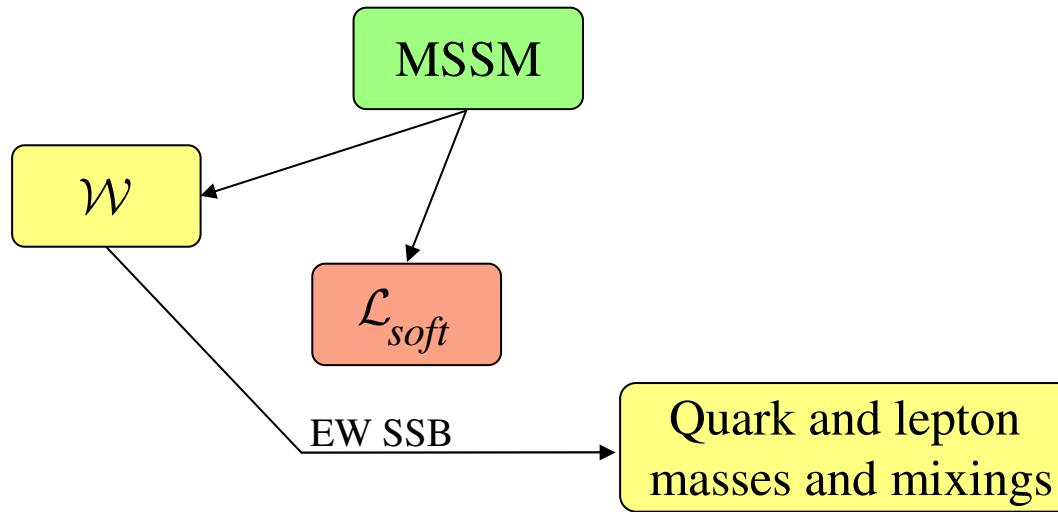
(Q, U, D, \dots now denote *superfields*, with fermion & scalar components)

Analogues of the SM Yukawa couplings (but with two Higgs doublets).

→ *same hierarchical fermion masses & CKM couplings.*

At this stage, perfect *alignment* of squarks with quarks, sleptons with leptons.

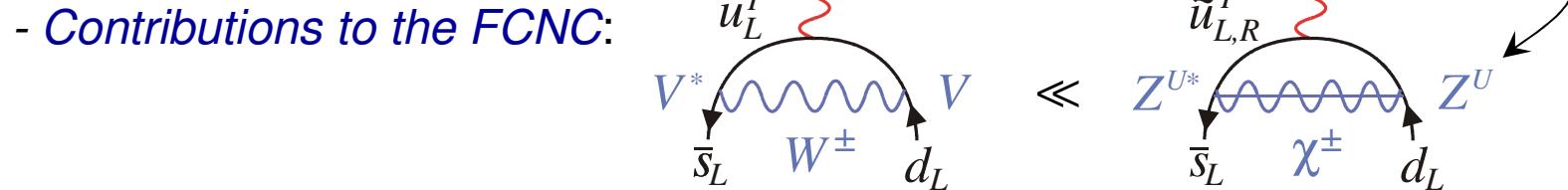
→ *same masses, same mixings.*



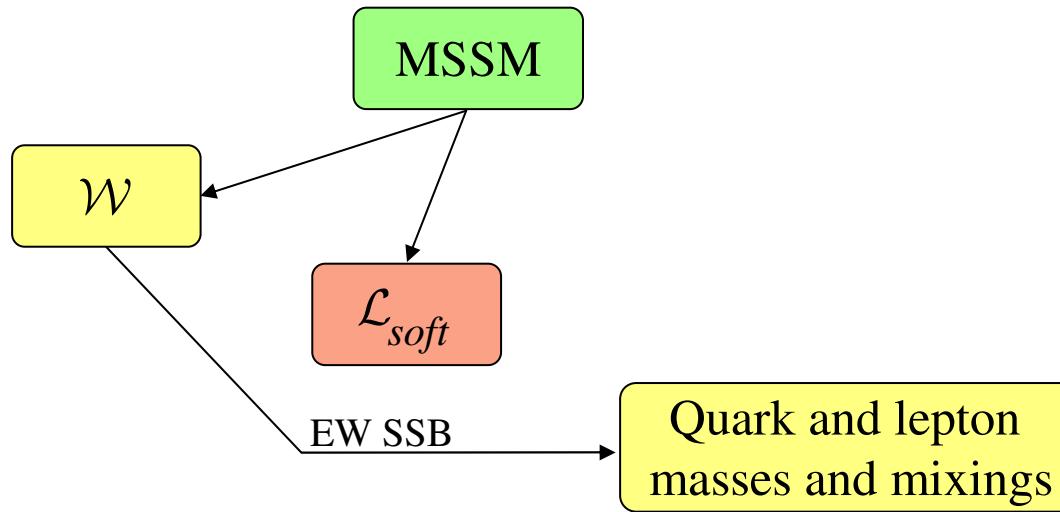
2. *Squark soft-breaking terms*: SUSY is broken, but the *exact mechanism is unclear*

- *Effective description*: $\mathcal{L}_{soft} \ni -\tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{U} \mathbf{A}_u (\tilde{Q} H_u) + \dots$

- *Squark mass terms*: $\delta M_{\tilde{u}}^2 = \begin{pmatrix} \mathbf{m}_Q^2 & v_u \mathbf{A}_u^\dagger \\ v_u \mathbf{A}_u & \mathbf{m}_U^2 \end{pmatrix}$ Large mass and gauge eigenstate mismatch?



With sparticle masses < 1 TeV, the squark flavor mixings must be small.



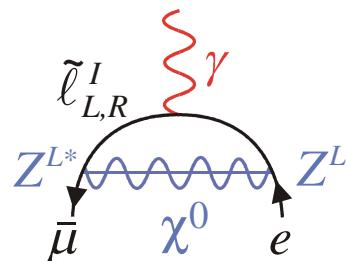
3. Slepton soft-breaking terms: similar situation as for squarks

- *Effective description:* $\mathcal{L}_{soft} \ni -\tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} - \tilde{E} \mathbf{m}_E^2 \tilde{E}^\dagger - \tilde{E} \mathbf{A}_e (\tilde{L} H_d) + \dots$

- *Slepton mass terms:* $\delta M_{\tilde{\ell}}^2 = \begin{pmatrix} \mathbf{m}_L^2 & v_d \mathbf{A}_e^\dagger \\ v_d \mathbf{A}_e & \mathbf{m}_E^2 \end{pmatrix}$

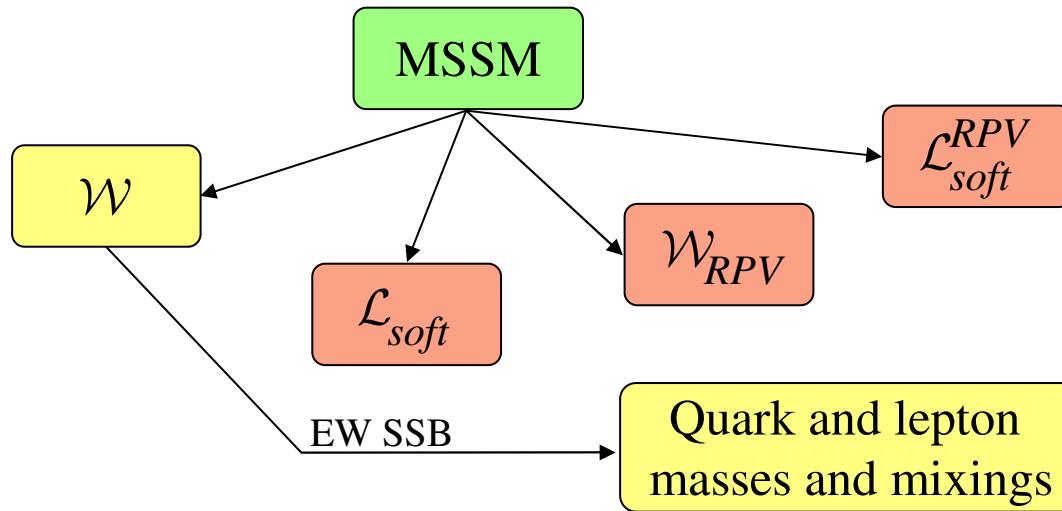
Large mass and gauge eigenstate mismatch?

- *New FCNC:*



With generic mixings, LFV much too large compared to experimental bounds.

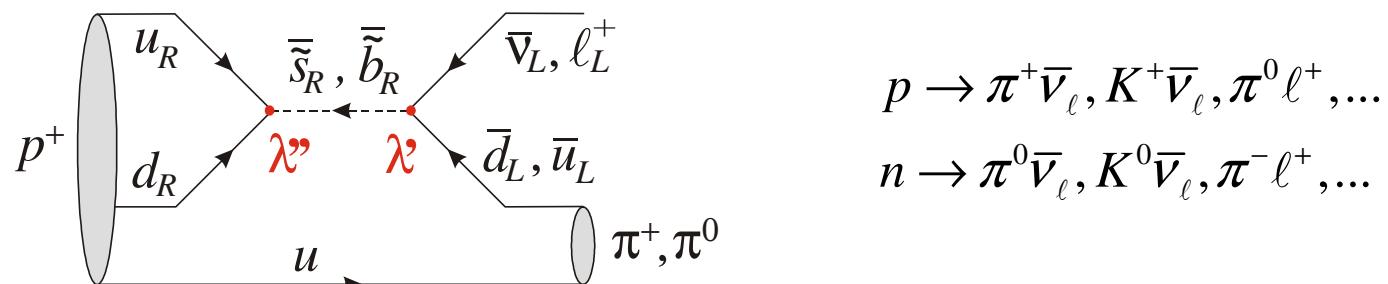
Again, sleptons and leptons must not be too misaligned.



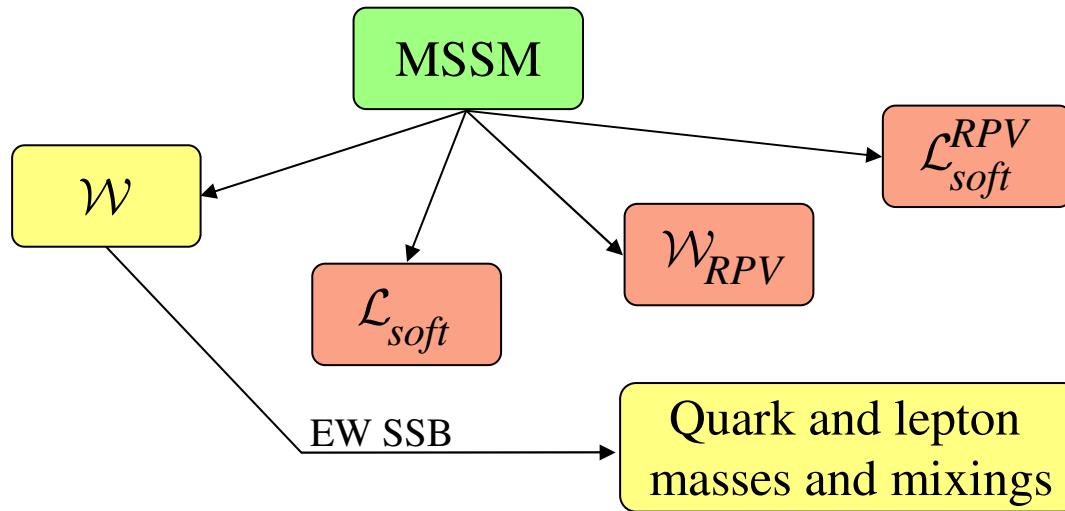
4. \mathcal{B} or \mathcal{L} violating couplings are allowed (both supersymmetric and not):

$$\mathcal{W}_{RPV} = \lambda^{IJK} (L^I L^J) E^K + \lambda'^{IJK} (L^I Q^J) D^K + \lambda''^{IJK} U^I D^J D^K + \mu'^I (L^I H_d)$$

These couplings induce *proton decay* (and associated) at tree-level:



But experimentally, $\tau_{p^+} > 10^{30}$ years : $\Gamma_{p^+} \sim \frac{m_p^5}{M_{\tilde{d}}^4} |\lambda'' \lambda'|^2 \Rightarrow |\lambda' \lambda''| \leq 10^{-27}$?



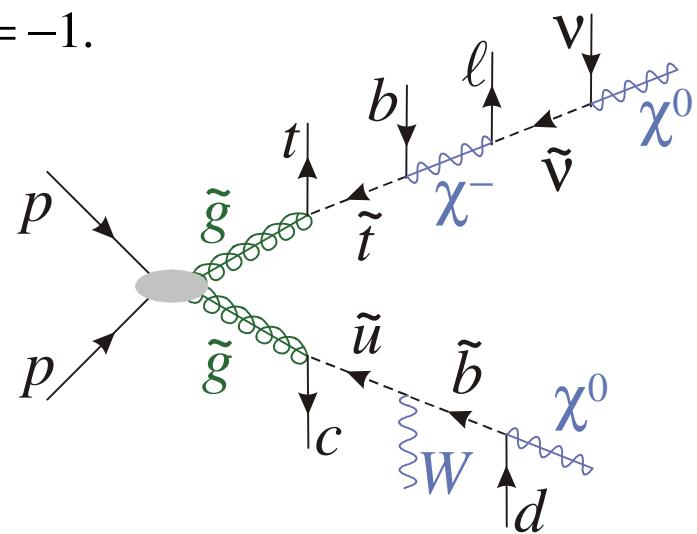
Usual escape route is to *impose R-parity*:

Farrar,Fayet '78

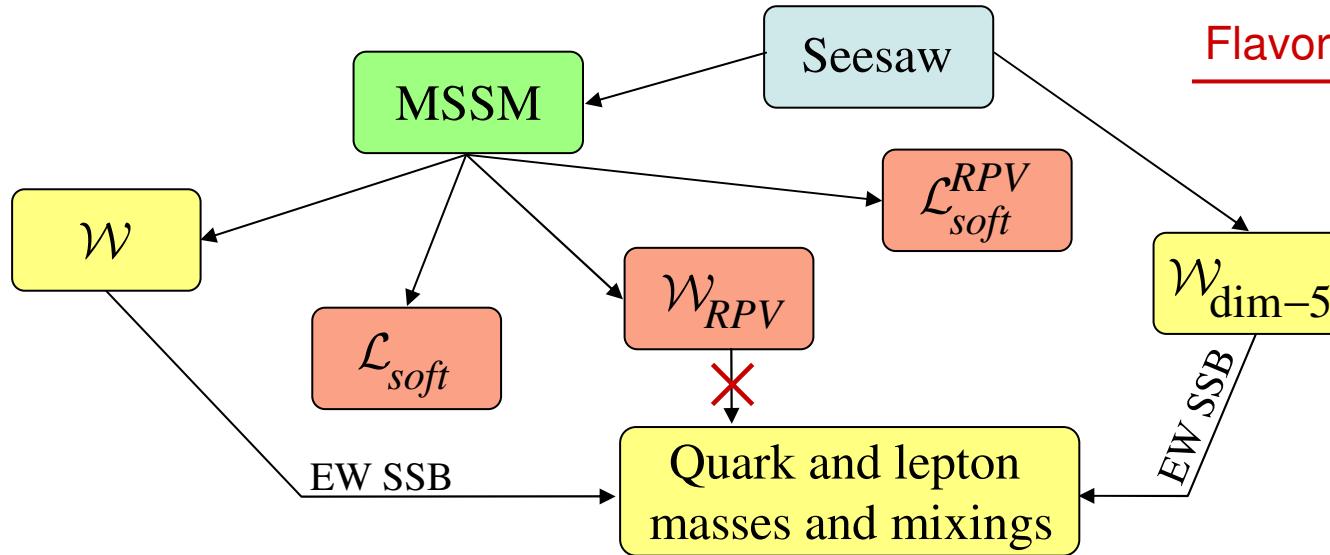
Assign $R(\text{Particles}) = +1$ and $R(\text{Sparticles}) = -1$.

→ \mathcal{W}_{RPV} and \mathcal{L}_{soft}^{RPV} couplings forbidden.

But also: sparticles produced in pairs, stable LSP (hence neutral LSP),...

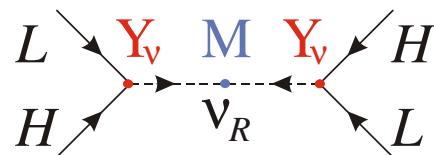


R-parity is a very (too?) tough constraint!



5. *Seesaw mechanism* to account for neutrino masses (not from \mathcal{W}_{RPV}):

- *Right-handed (s)neutrinos* are added: $\mathcal{W}_N = N\mathbf{M}N + N\mathbf{Y}_\nu(LH_u)$
- *Large \mathcal{L} violating mass \mathbf{M}* allowed $\rightarrow \nu_R$ are integrated out:

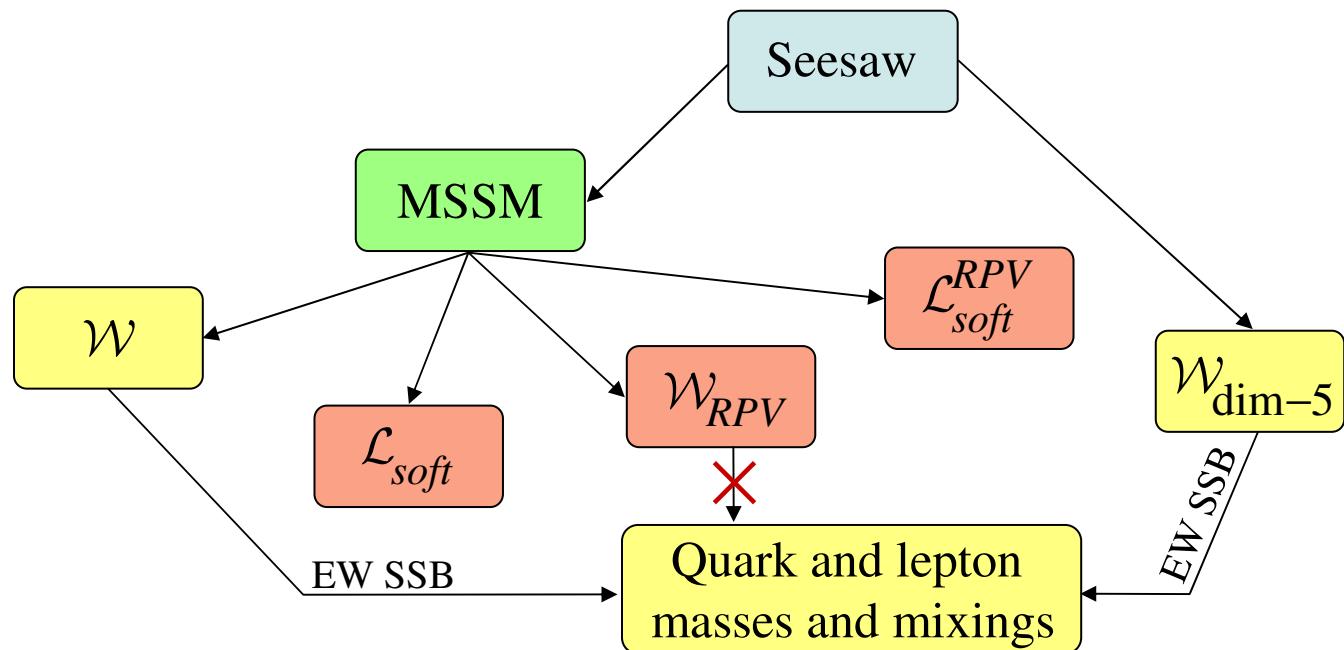


$$\mathcal{W}_{\text{dim-5}} = (\mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu)^{IJ} (L^I H_u)(L^J H_u)$$

- *Effective Majorana mass term for ν_L* : $\nu_u^2 \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu = U_{PMNS}^* \cdot \mathbf{m}_\nu \cdot U_{PMNS}^\dagger$
Then, $\mathbf{m}_\nu \sim 1 \text{ eV}$ with $\mathbf{Y}_\nu \sim \mathcal{O}(1)$ when $\mathbf{M} \sim 10^{13} \text{ GeV}$.

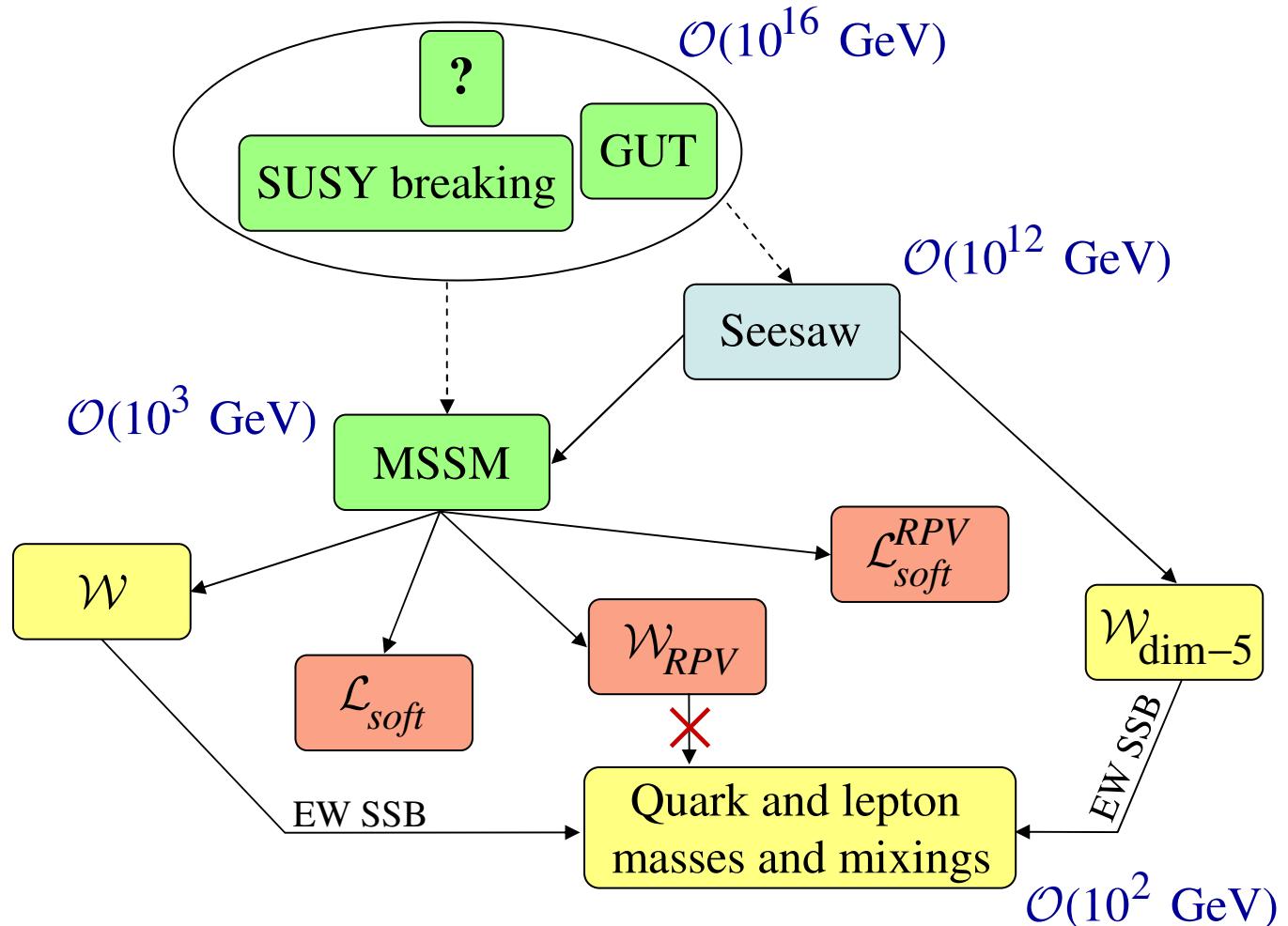
II. The MFV hypothesis

A. MFV and the origin of the flavor structures:



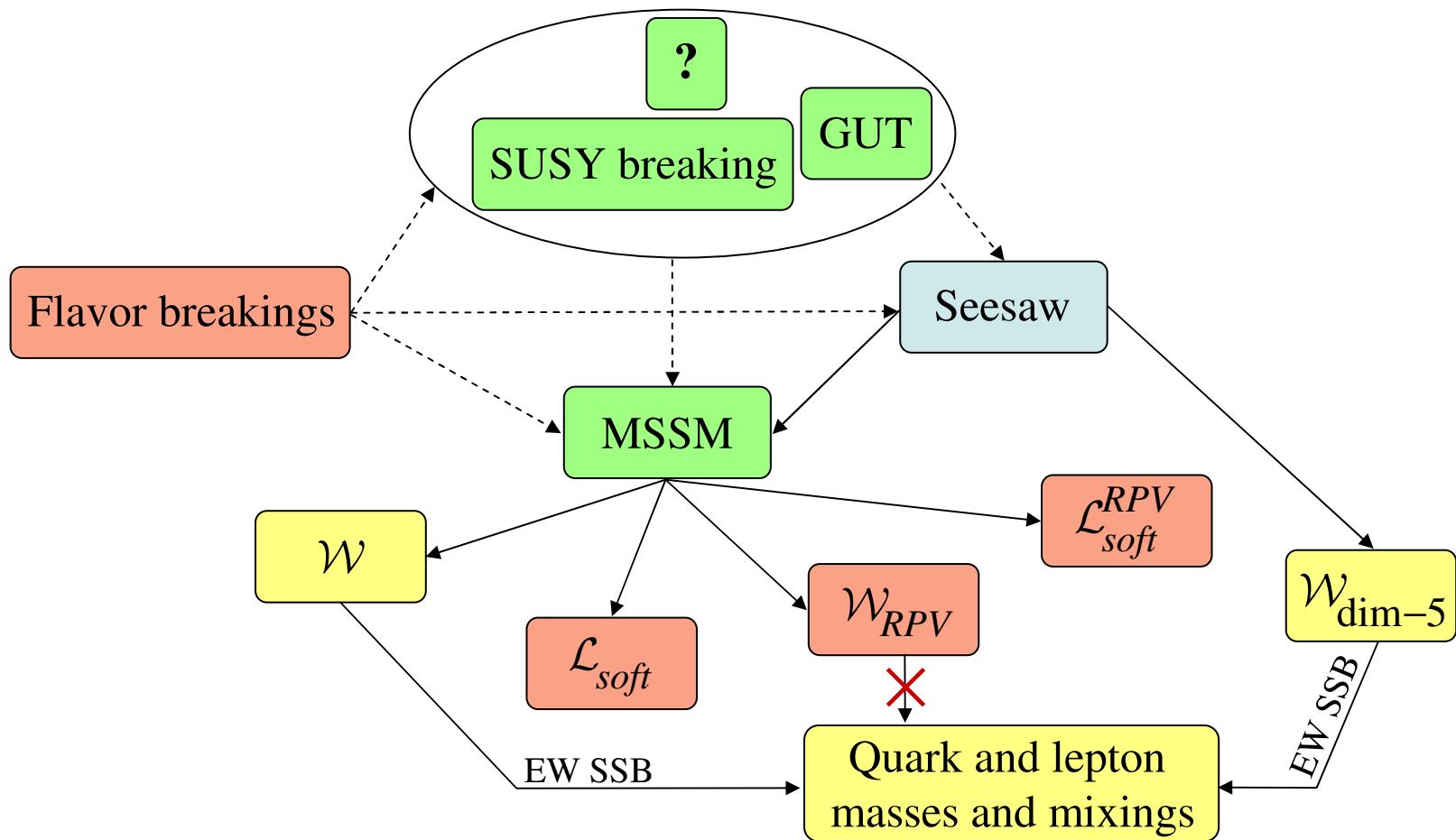
Only the flavor-breakings in the SM fermionic sector have been probed experimentally.

A. MFV and the origin of the flavor structures:



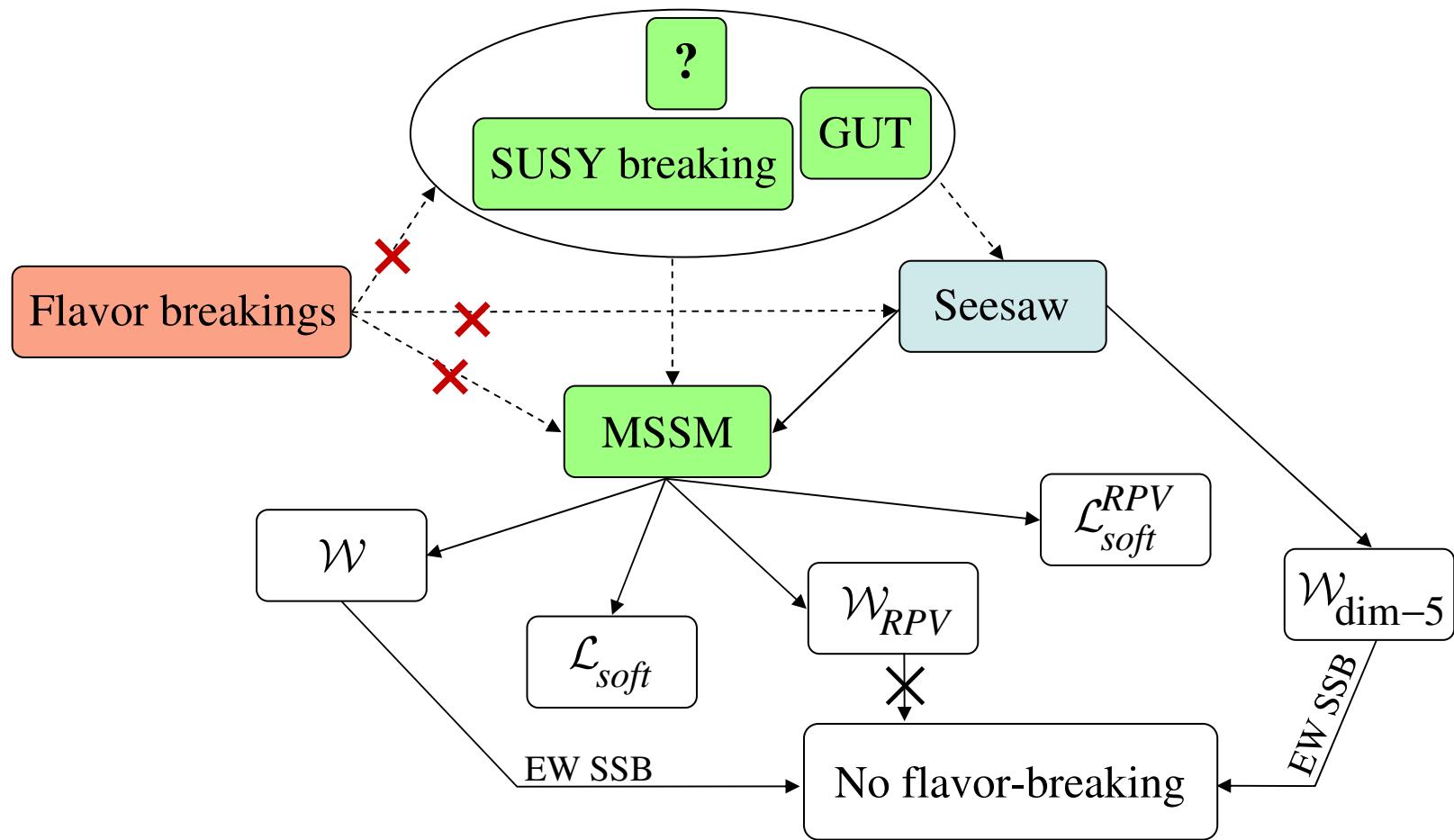
The MSSM is not the ultimate theory, but only a “low-energy” effective theory.

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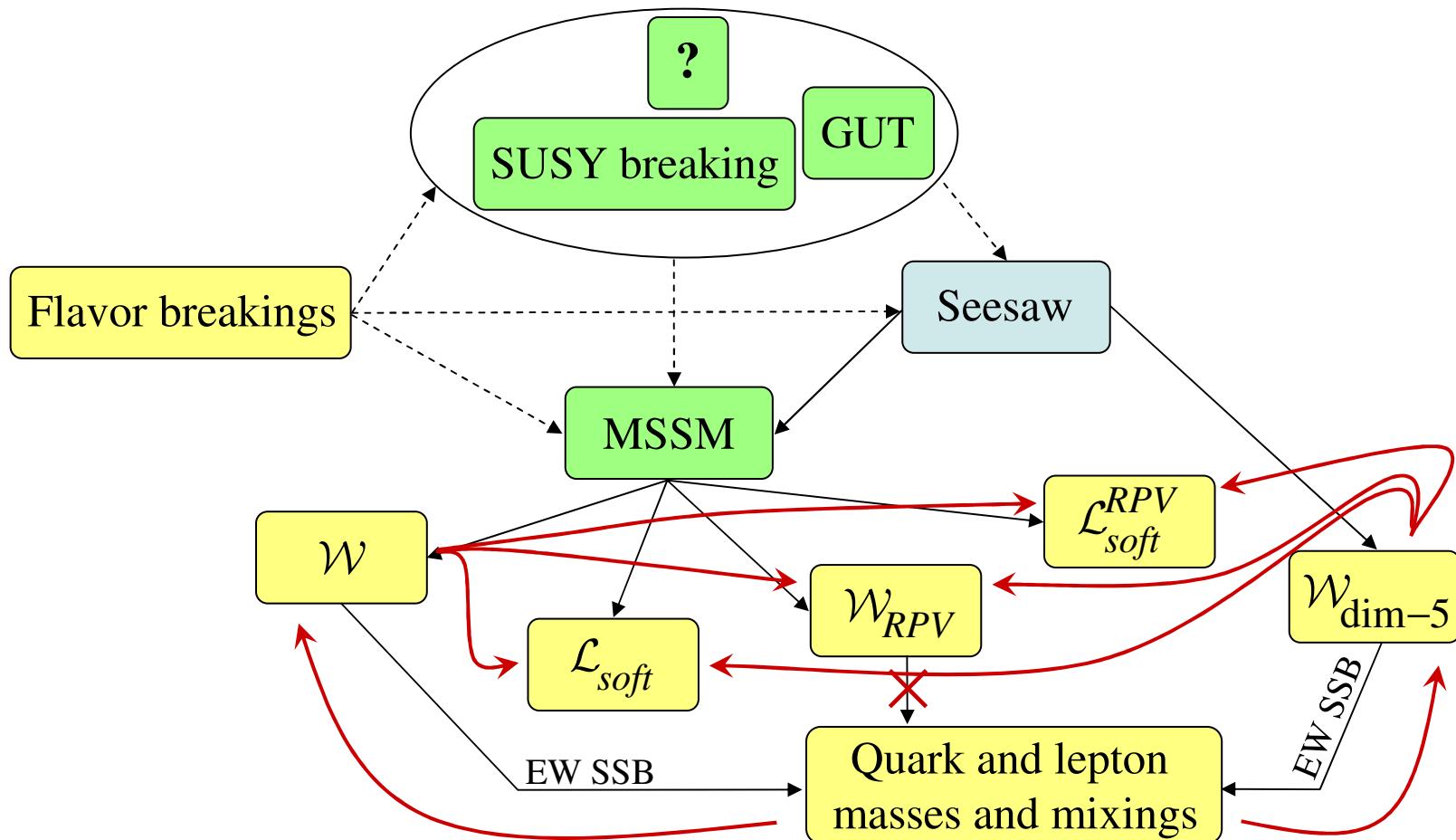
Some mechanism beyond the MSSM must explain the *origin of the flavor structures*.

A. MFV and the origin of the flavor structures:



If this mechanism is turned off, flavor-breaking terms become forbidden.

A. MFV and the origin of the flavor structures:



With MFV, all the flavor-breaking couplings are reconstructed in terms of the fermion masses and mixings, and become *naturally hierarchical*.

B. In practice:

- *Minimality hypothesis*: Minimal spurion content allowing for the known fermion masses and mixing - *this is the essence of MFV!*

Essentially, the Yukawas \mathbf{Y}_u , \mathbf{Y}_d , \mathbf{Y}_e plus a few seesaw spurions.

- *Symmetry principle*: All Lagrangian couplings written as formal \mathbf{G}_f -invariants

$$\mathbf{m}_Q^2 = m_0^2 (a_0 \mathbf{1} + a_1 \mathbf{Y}_u^\dagger \mathbf{Y}_u + a_2 \mathbf{Y}_d^\dagger \mathbf{Y}_d + \dots) \quad \text{with} \quad a_i \sim \mathcal{O}(1) \quad \leftarrow \text{naturality}$$

- *Freezing of the spurions* at their physical values:

Hall, Randall '90
D'Ambrosio, Giudice,
Isidori, Strumia '02

$$\mathbf{m}_Q^2 \sim m_0^2 \left(\begin{pmatrix} 1 & 10^{-4} & 10^{-3} \\ 10^{-4} & 1 & 10^{-2} \\ 10^{-3} & 10^{-2} & 1 \end{pmatrix} + i \begin{pmatrix} 0 & 10^{-4} & 10^{-3} \\ 10^{-4} & 0 & 10^{-4} \\ 10^{-3} & 10^{-4} & 0 \end{pmatrix} \right)$$

These hierarchies come entirely from those of \mathbf{Y}_u , \mathbf{Y}_d .

C. MFV expansions in the quark sector

Hall, Randall '90, D'Ambrosio, Giudice, Isidori, Strumia '02, Colangelo, Nikolidakis, CS '08

- Only a *finite number* of terms thanks to Cayley-Hamilton identity:

$$\mathbf{X}^3 - \langle \mathbf{X} \rangle \mathbf{X}^2 + \frac{1}{2} \mathbf{X} (\langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle) = \frac{1}{3} \langle \mathbf{X}^3 \rangle - \frac{1}{2} \langle \mathbf{X} \rangle \langle \mathbf{X}^2 \rangle + \frac{1}{6} \langle \mathbf{X} \rangle^3$$

- Use the large *mass hierarchy* to set $(\mathbf{Y}_i^\dagger \mathbf{Y}_i)^2 \sim \mathbf{Y}_i^\dagger \mathbf{Y}_i$, leaving:

$$\mathbf{m}_Q^2 = m_0^2 (a_1 \mathbf{1} + a_2 \mathbf{A} + a_3 \mathbf{B} + a_4 \{\mathbf{A}, \mathbf{B}\} + b_1 i[\mathbf{A}, \mathbf{B}])$$

$$\mathbf{A} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u$$

$$\mathbf{B} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d$$

$$\mathbf{m}_U^2 = m_0^2 (a_5 \mathbf{1} + \mathbf{Y}_u (a_6 \mathbf{1} + a_7 \mathbf{B} + a_8 \{\mathbf{A}, \mathbf{B}\} + b_2 i[\mathbf{A}, \mathbf{B}] \mathbf{Y}_u^\dagger))$$

$$\mathbf{m}_D^2 = m_0^2 (a_9 \mathbf{1} + \mathbf{Y}_d (a_{10} \mathbf{1} + a_{11} \mathbf{A} + a_{12} \{\mathbf{A}, \mathbf{B}\} + b_3 i[\mathbf{A}, \mathbf{B}] \mathbf{Y}_d^\dagger))$$

$$\mathbf{A}_u = A_0 \mathbf{Y}_u (c_1 \mathbf{1} + c_2 \mathbf{A} + c_3 \mathbf{B} + c_4 \{\mathbf{A}, \mathbf{B}\} + d_1 i[\mathbf{A}, \mathbf{B}])$$

$$\mathbf{A}_d = A_0 \mathbf{Y}_d (c_5 \mathbf{1} + c_6 \mathbf{A} + c_7 \mathbf{B} + c_8 \{\mathbf{A}, \mathbf{B}\} + d_2 i[\mathbf{A}, \mathbf{B}])$$

Using CH identities, all operators can be written as hermitian, hence $a_i, b_i \in \mathbb{R}$, $c_i, d_i \in \mathbb{C}$ since scalar mass terms are hermitian.

D. MFV expansions in the lepton sector

- Integrating out the right-handed neutrinos:

Cirigliano, Grinstein
Isidori, Wise '05

$$\begin{array}{cccc}
 \mathbf{Y}_e, & \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu, & \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu, & \mathbf{Y}_\nu^\dagger \mathbf{M}^{-1*} \mathbf{M}^{-1} \mathbf{Y}_\nu, \dots \\
 \downarrow & \downarrow & \searrow & \\
 \text{Lepton masses:} & & & \text{Neutrino masses:} \\
 v_d \mathbf{Y}_e = \mathbf{m}_e & & & v_u^2 \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu = \mathbf{U}^* \mathbf{m}_\nu \mathbf{U}^\dagger
 \end{array}$$

Not completely fixed (we take $\mathbf{M} = \mathbf{M}_R \mathbf{1}$):

$$v_u^2 \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu = \mathbf{M}_R \mathbf{U}^* \mathbf{m}_\nu^{1/2} e^{2i\Phi} \mathbf{m}_\nu^{1/2} \mathbf{U}^\dagger, \quad \Phi^{IJ} = \epsilon^{IJK} \phi_K$$

Casas, Ibarra '01,
Pascoli, Petcov,
Yaguna '03,...

- More terms remain since there is no third-generation dominance for \mathbf{v}_L :

$$\begin{aligned}
 \mathbf{m}_L^2 = & m_0^2 (a_1 \mathbf{1} + a_2 \mathbf{A} + a_3 \mathbf{B} + a_4 \mathbf{B}^2 + a_5 \{\mathbf{A}, \mathbf{B}\} + a_6 \mathbf{B} \mathbf{A} \mathbf{B} \\
 & + b_1 i[\mathbf{A}, \mathbf{B}] + b_2 i[\mathbf{A}, \mathbf{B}^2] + b_3 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B}))
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{A} &\equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e \\
 \mathbf{B} &\equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu
 \end{aligned}$$

Similar for \mathbf{m}_E^2 and \mathbf{A}_e .

Mercolli, CS '09

E. How to test MFV?

Colangelo, Nikolidakis, CS '08
Nikolidakis '08, Mercolli, CS '09

Generically, all flavor couplings expanded under MFV involve:

$$Q = x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} \\ + x_7 i[\mathbf{A}, \mathbf{B}] + x_8 i[\mathbf{A}, \mathbf{B}^2] + x_9 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B})$$

(A $\equiv Y_e^\dagger Y_e$, B $\equiv Y_\nu^\dagger Y_\nu$)
(A $\equiv Y_d^\dagger Y_d$, B $\equiv Y_u^\dagger Y_u$)

The MFV operators form a *complete basis* for the soft-breaking terms.

Allowing the coefficients to take any value \rightarrow *full MSSM*.

However, the MFV basis is made of *nearly parallel operators*.

A generic matrix expanded in the MFV basis requires *huge coefficients*!

E. How to test MFV?

Generically, all flavor couplings expanded under MFV involve:

$$Q = x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} \\ + x_7 i[\mathbf{A}, \mathbf{B}] + x_8 i[\mathbf{A}, \mathbf{B}^2] + x_9 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B})$$

$(\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$
 $(\mathbf{A} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d, \mathbf{B} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u)$

The MFV operators form a *complete basis* for the soft-breaking terms.

Allowing the coefficients to take any value \rightarrow *full MSSM*.

MFV expansion coefficients versus Mass Insertions:

Same number of free parameters (choice of basis).

BUT: to each coefficient corresponds a *whole set of mass insertions*, with a *definite flavor pattern* inherited from those of the spurions.



Permits to *test the naturality* of soft-breaking terms.

E. How to test MFV?

Generically, all flavor couplings expanded under MFV involve:

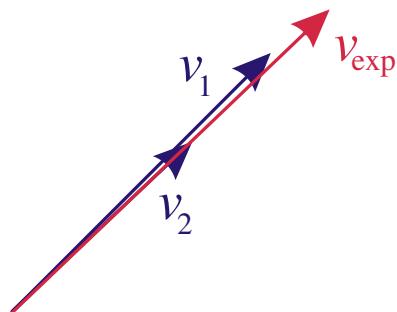
$$Q = x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} \\ + x_7 i[\mathbf{A}, \mathbf{B}] + x_8 i[\mathbf{A}, \mathbf{B}^2] + x_9 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B})$$

$(\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)$
 $(\mathbf{A} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d, \mathbf{B} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u)$

Imagine Q is constrained by experiment (collider + flavor).

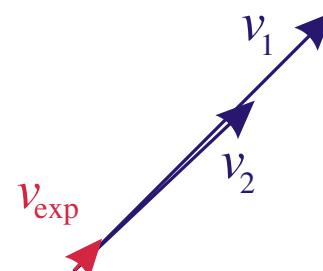
Three possible situations can arise when projecting Q in the MFV basis:

All the $x_i \sim O(1)$



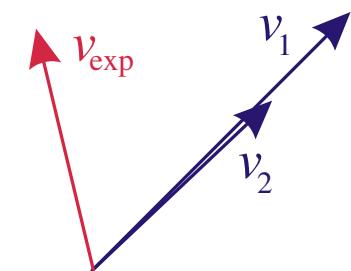
MFV flavor structure

Some of the $x_i \ll 1$



Fine-tuned flavor structure

Some of the $x_i \gg 1$



Generic flavor structure

E. How to test MFV?

Mercolli, C.S. '09

Current experimental constraints on the generic MSSM slepton sector:

m_L^2	(x_i / a_1)	m_R^2	(x_i / a_7)	$\text{Re } A_e$	$(x_i / a_1 a_7)$	$\text{Im } A_e$	$(x_i / a_1 a_7)$
a_1	<i>free</i>	a_7	<i>free</i>	$\text{Re } c_1 \leq 10^2$	stab.	$\text{Im } c_1 \leq 2$	d_e
$a_2 \leq 10^3$	<i>masses</i>	$a_8 \leq 10^3$	<i>masses</i>	$\text{Re } c_2 \leq 10^3$	stab.	$\text{Im } c_2 \leq 10^3$	stab.
$a_3 \leq 10$	$\mu \rightarrow e\gamma$	$a_9 \leq 10^6$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_3 \leq 10^3$	$\mu \rightarrow e\gamma$	$\text{Im } c_3 \leq 10^3$	$\mu \rightarrow e\gamma$
$a_4 \leq 10^4$	$\mu \rightarrow e\gamma$	$a_{10} \leq 10^9$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_4 \leq 10^6$	$\mu \rightarrow e\gamma$	$\text{Im } c_4 \leq 10^6$	$\mu \rightarrow e\gamma$
$a_5 \leq 10^3$	$\tau \rightarrow \mu\gamma$	$a_{11} \leq 10^7$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_5 \leq 10^5$	$\tau \rightarrow \mu\gamma$	$\text{Im } c_5 \leq 10^5$	$\tau \rightarrow \mu\gamma$
$a_6 \leq 10^4$	$\mu \rightarrow e\gamma$	$a_{12} \leq 10^{11}$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_6 \leq 10^7$	$\mu \rightarrow e\gamma$	$\text{Im } c_6 \leq 10^7$	$\mu \rightarrow e\gamma$
$b_1 \leq 10^3$	$\tau \rightarrow \mu\gamma$	$b_4 \leq 10^7$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_1 \leq 10^5$	$\tau \rightarrow \mu\gamma$	$\text{Im } d_1 \leq 10^5$	$\tau \rightarrow \mu\gamma$
$b_2 \leq 10^6$	$\tau \rightarrow \mu\gamma$	$b_5 \leq 10^{10}$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_2 \leq 10^8$	$\tau \rightarrow \mu\gamma$	$\text{Im } d_2 \leq 10^8$	$\tau \rightarrow \mu\gamma$
$b_3 \leq 10^8$	$\mu \rightarrow e\gamma$	$b_6 \leq 10^{13}$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_3 \leq 10^{10}$	$\mu \rightarrow e\gamma$	$\text{Im } d_3 \leq 10^{10}$	$\mu \rightarrow e\gamma$

$$M_{SUSY} \approx 500 \text{ GeV}, \tan \beta = 20, M_R = 10^{12} \text{ GeV}, m_{L,R} \leq 4 \text{ TeV}$$

CPC

CPV

F. Beyond MFV?

Within MFV, all flavor structures are related to that of the Yukawas.

Open questions:

- Why are the *Yukawa couplings so hierarchical?*
- Is there a *dynamical mechanism* behind MFV?

There is certainly something behind the Yukawa.

Explicit symmetry breaking



Spontaneous symmetry breaking

The approach followed here.

We assume a *minimal number of explicit breaking terms*.

Goldstone bosons?

Albrecht, Feldmann, Mannel '10

Discrete flavor symmetries?

Zwicky, Fischbacher '09

III. CP-violation under MFV

A. *CP-violating phases in the MFV approach*

Mercolli, C.S. '09

In the SM, CP-violation comes entirely from the phases in the spurions.

One in Y_u (Dirac), six in $Y_\nu^\dagger Y_\nu$ (1 Dirac, 2 Majorana, 3 from the ϕ_K)

Within MFV, there are several reasons for expecting *additional CP-phases*:

- The $U(3)^5$ does not say anything about CP-violating phases,
All the *MFV coefficients are free complex parameters*.
- There can be new *CP-violating phases in other sectors*,
CP-violation is a flavored phenomenon only in the SM!
- Potentially complex traces $\langle A^l B^m A^n \dots \rangle$ are $U(3)^5$ singlets,
Absorbed in the coefficients: forcing them to stay real is a *fine-tuning*!
(and is not RGE invariant)

B. Consequence: Is MFV breaking down?

MFV is very effective to *constrain flavor transitions* like $\ell^I \rightarrow \ell^J$ or $d^I \rightarrow d^J$.

But for *flavor-diagonal* operators, there is not much restriction.

- Complex coefficients can induce additional *flavor-diagonal CP-phases*.
- Is this compatible with bounds on EDMs?

$$H_{eff} = \mathbf{C}^{IJ} \bar{\Psi}_L^I \sigma_{\mu\nu} \Psi_R^J F^{\mu\nu} + \mathbf{C}^{IJ*} \bar{\Psi}_R^J \sigma_{\mu\nu} \Psi_L^I F^{\mu\nu}$$

$$I = J$$

$$I \neq J$$

$$H_{eff} = \mathbf{C} \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R F^{\mu\nu} + \mathbf{C}^* \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R F^{\mu\nu}$$

$$B(\Psi^I \rightarrow \Psi^J \gamma) \sim |\mathbf{C}^{IJ}|^2$$

$$= \text{Re } \mathbf{C} \bar{\Psi} \sigma_{\mu\nu} \Psi F^{\mu\nu} + i \text{Im } \mathbf{C} \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi F^{\mu\nu}$$

$$\equiv e \frac{a}{4m}$$

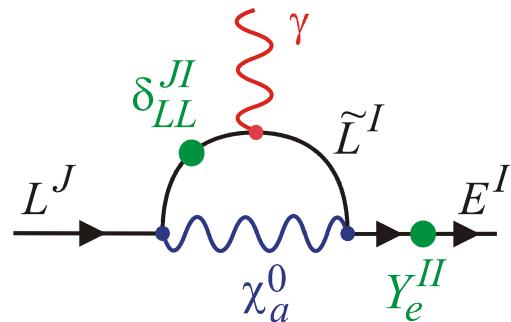
$$\equiv \frac{d}{2}$$

B. Consequence: Is MFV breaking down?

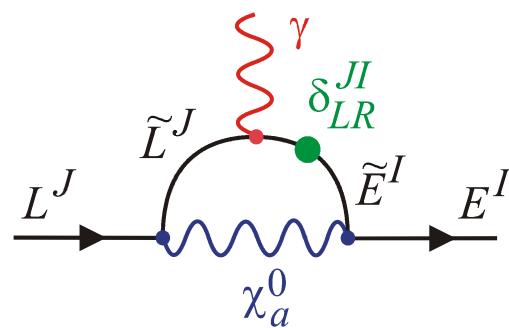
In the MSSM, the flavor-breaking & helicity flip come as

$$H_{\text{eff}} \sim E^I \sigma_{\mu\nu} (\mathbf{Y}_e, \mathbf{A}_e, \mathbf{Y}_e \mathbf{m}_L^2, \dots)^{IJ} L^J H_d F^{\mu\nu}$$

Further, this operator arises at one loop:



$$B(\ell^I \rightarrow \ell^J \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{\text{SUSY}}^8} |(\mathbf{m}_L^2)^{JI} + \dots|^2$$



Beyond MFV

$$\frac{d_I}{e} \sim \frac{\alpha}{M_{\text{SUSY}}^3} \text{Im} \left(\mathbf{m}_\ell^I \mu \tan \beta - v_d \mathbf{A}_e^{*II} \right) + \dots$$

Diagonal part of the trilinear terms.

C. Classification of the CP-phases

$$A \equiv Y_e^\dagger Y_e$$

$$B \equiv Y_\nu^\dagger Y_\nu$$

MFV expansions, with $a_i, b_i \in \mathbb{R}$, $c_i, d_i \in \mathbb{C}$:

$$\mathbf{m}_L^2 = m_0^2 (a_1 \mathbf{1} + a_2 \mathbf{A} + a_3 \mathbf{B} + a_5 \{\mathbf{A}, \mathbf{B}\} + a_6 \mathbf{B} \mathbf{A} \mathbf{B} + b_1 i[\mathbf{A}, \mathbf{B}] + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2)),$$

$$\mathbf{m}_E^2 = m_0^2 (a_7 \mathbf{1} + Y_e (a_8 \mathbf{1} + a_9 \mathbf{B} + a_{11} \{\mathbf{A}, \mathbf{B}\} + b_4 i[\mathbf{A}, \mathbf{B}]) Y_e^\dagger + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2)),$$

$$\mathbf{A}_e = A_0 Y_e (c_1 \mathbf{1} + c_2 \mathbf{A} + c_3 \mathbf{B} + c_5 \{\mathbf{A}, \mathbf{B}\} + d_1 i[\mathbf{A}, \mathbf{B}] + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2))$$

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$$m_L^2 = m_0^2 (a_1 1 + a_2 A + a_3 B + a_5 \{A, B\} + a_6 B A B + b_1 i[A, B] + \mathcal{O}(A^2, B^2)),$$

$$m_E^2 = m_0^2 (a_7 1 + Y_e (a_8 1 + a_9 B + a_{11} \{A, B\} + b_4 i[A, B]) Y_e^\dagger + \mathcal{O}(A^2, B^2)),$$

$$A_e = A_0 Y_e (c_1 1 + c_2 A + c_3 B + c_5 \{A, B\} + d_1 i[A, B] + \mathcal{O}(A^2, B^2))$$

In the slepton sector: 15 CP-violating coefficients + 6 spurion phases

In the squark sector: 13 CP-violating coefficients + 1 spurion phase

⇒ *Plenty of new CP-phases in MFV!*

C. Classification of the CP-phases

$$\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e$$

$$\mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$$

MFV expansions, with $a_i, b_i \in \mathbb{R}$, $c_i, d_i \in \mathbb{C}$:

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$$\mathbf{m}_E^2 = m_0^2 (a_7 \mathbf{1} + \mathbf{Y}_e (a_8 \mathbf{1} + a_9 \mathbf{B} + a_{11} \{\mathbf{A}, \mathbf{B}\} + b_4 i[\mathbf{A}, \mathbf{B}]) \mathbf{Y}_e^\dagger + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2)),$$

$$\mathbf{A}_e = A_0 \mathbf{Y}_e (c_1 \mathbf{1} + c_2 \mathbf{A} + c_3 \mathbf{B} + c_5 \{\mathbf{A}, \mathbf{B}\} + d_1 i[\mathbf{A}, \mathbf{B}] + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2))$$

Flavor-blind phase: $\text{Im } c_1$ (remember $d_I \sim \text{Im } \mathbf{A}_e^{*II} \sim \text{Im } c_1$)

Defined relative to the flavor-blind parameters of the MSSM (μ, M_1, M_2, \dots)

C. Classification of the CP-phases

$$\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e$$

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$$\mathbf{m}_E^2 = m_0^2 (a_7 \mathbf{1} + \mathbf{Y}_e (a_8 \mathbf{1} + a_9 \mathbf{B} + a_{11} \{\mathbf{A}, \mathbf{B}\} + b_4 i[\mathbf{A}, \mathbf{B}]) \mathbf{Y}_e^\dagger + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2)),$$

$$\mathbf{A}_e = A_0 \mathbf{Y}_e (c_1 \mathbf{1} + c_2 \mathbf{A} + c_3 \mathbf{B} + c_5 \{\mathbf{A}, \mathbf{B}\} + d_1 i[\mathbf{A}, \mathbf{B}] + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2))$$

Flavor-blind phase: $\text{Im } c_1$

Defined relative to the flavor-blind parameters of the MSSM (μ, M_1, M_2, \dots)

Flavor-diagonal phases: $\text{Im } c_{2-6}$ (remember $d_I \sim \text{Im } \mathbf{A}_e^{*II} \sim \text{Im } c_{2-6}$)

Contribute to EDMs at leading order in the MIA.

C. Classification of the CP-phases

$$\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e$$

$$\mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$$

MFV expansions, with $a_i, b_i \in \mathbb{R}$, $c_i, d_i \in \mathbb{C}$:

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$$\mathbf{m}_E^2 = m_0^2 (a_7 \mathbf{1} + \mathbf{Y}_e (a_8 \mathbf{1} + a_9 \mathbf{B} + a_{11} \{\mathbf{A}, \mathbf{B}\} + b_4 i[\mathbf{A}, \mathbf{B}]) \mathbf{Y}_e^\dagger + \mathcal{O}(\mathbf{A}^2, \mathbf{B}^2)),$$

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Flavor-blind phase: $\text{Im } c_1$

Defined relative to the flavor-blind parameters of the MSSM (μ, M_1, M_2, \dots)

Flavor-diagonal phases: $\text{Im } c_{2-6}$

Contribute to EDMs at leading order in the MIA.

Flavor off-diagonal phases: $b_i, \text{Re } d_i$, six phases of $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ \leftarrow (hermitian op.)

Start to contribute to EDMs at 2nd order in the MIA ($d_I \sim \text{Im}(\mathbf{m}_L^2)^{IK} (\mathbf{A}_e)^{KI}$).

D. Impact on the EDMs and LFV processes

A single operator dominates for $\mu \rightarrow e \gamma$ (coming from δ_{LL}):

$$B(\mu \rightarrow e \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} (Y_v^\dagger Y_v)^{12} \right|^2$$

A single operator per type of phases dominates for d_e :

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left(\frac{\text{Im } c_1}{a_1 a_7} + \frac{\text{Im } c_3}{a_1 a_7} Y_v^\dagger Y_v - \frac{b_1 \text{Re } c_3}{a_1^2 a_7} [Y_v^\dagger Y_v, Y_e^\dagger Y_e] Y_v^\dagger Y_v + \dots \right)^{11}$$



Flavor-blind



Flavor-diagonal



Flavor off-diagonal
(\geq neutrino phases)

Remark: $m_L^2 \approx m_0^2 a_1$, $m_R^2 \approx m_0^2 a_7$

D. Impact on the EDMs and LFV processes

A single operator dominates for $\mu \rightarrow e \gamma$ (coming from δ_{LL}):

$$B(\mu \rightarrow e \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} \frac{M_R \Delta m_{21}}{v_u^2} \right|^2$$

A single operator per type of phases dominates for d_e :

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Flavor-blind



Flavor-diagonal



Flavor off-diagonal
(\geq neutrino phases)



$$M_{SUSY} \approx 500 \text{ GeV}$$

D. Impact on the EDMs and LFV processes

A single operator dominates for $\mu \rightarrow e \gamma$ (coming from δ_{LL}):

$$B(\mu \rightarrow e \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} \frac{M_R \Delta m_{21}}{v_u^2} \right|^2 \quad \Rightarrow M_R \leq 10^{13} \text{ GeV}$$

A single operator per type of phases dominates for d_e :

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left(\frac{\text{Im } c_1}{a_1 a_7} + \frac{\text{Im } c_3}{a_1 a_7} \frac{M_R \Delta m_{21}}{v_u^2} - \frac{b_1 \text{Re } c_3}{a_1^2 a_7} \frac{m_\tau^2}{v_d^2} \left(\frac{M_R \Delta m_{21}}{v_u^2} \right)^2 + \dots \right)^{11}$$



 Flavor-blind Flavor-diagonal Flavor off-diagonal
 (≥ neutrino phases)

$$M_{SUSY} \approx 500 \text{ GeV}$$

$$\Delta m_{21} \approx \sqrt{\Delta m_\odot^2} \approx 10^{-9} - 10^{-11} \text{ GeV}$$

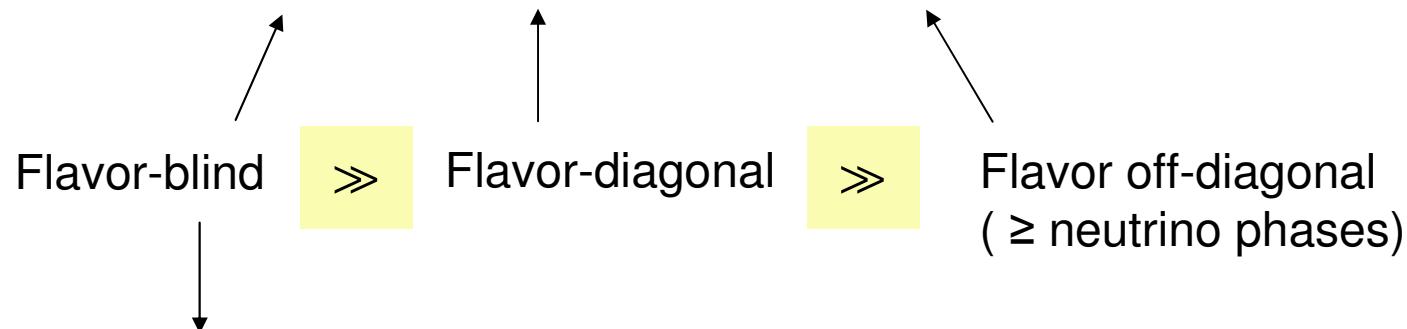
D. Impact on the EDMs and LFV processes

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A single operator per type of phases dominates for d_e :

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left(\frac{\text{Im } c_1}{a_1 a_7} + \frac{\text{Im } c_3}{a_1 a_7} \frac{M_R \Delta m_{21}}{v_u^2} - \frac{b_1 \text{Re } c_3}{a_1^2 a_7} \frac{m_\tau^2}{v_d^2} \left(\frac{M_R \Delta m_{21}}{v_u^2} \right)^2 + \dots \right)^{11}$$

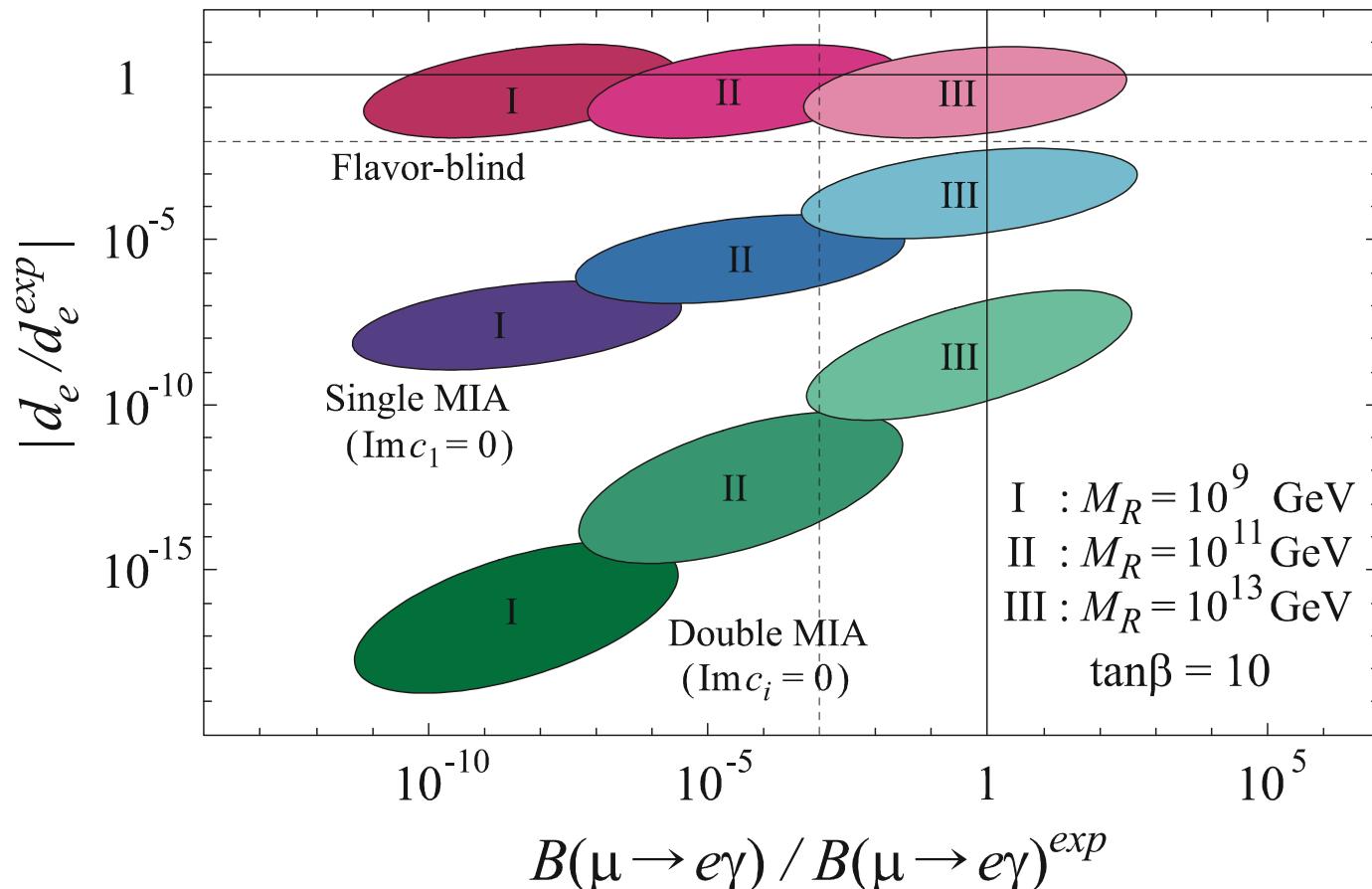


$$M_{SUSY} \approx 500 \text{ GeV}$$

$$\Delta m_{21} \approx \sqrt{\Delta m_\odot^2} \approx 10^{-9} - 10^{-11} \text{ GeV}$$

D. Phenomenological impacts

Mercolli, C.S. '09



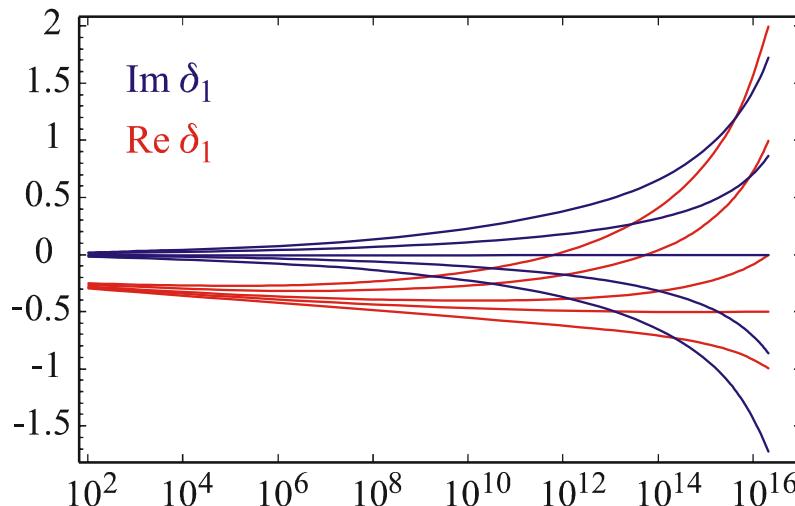
$$M_2 = \pm \mu = 2M_1 = \frac{2}{3}m_0 = A_0 = 400 \text{ GeV}, \quad a_i, b_i, c_i, d_i \in \pm[0.1, 8]$$

IV. RGE behavior

The MFV expansions are RGE invariant, but to get the RGE invariance of MFV itself requires in addition that the *coefficients must remain of $\mathcal{O}(1)$ at all scales*.

Running down from MFV at the GUT scale:

- *IR fixed-points* for ratios of coefficients \leftrightarrow predictions for *mass insertions*.
- In particular, all *CP-violating phases* run towards zero (in the quark sector).



$$\delta_1 \equiv \frac{(\delta_{RL}^U)^{32}}{V_{ts}} = \frac{(\delta_{RL}^U)^{31}}{V_{td}}$$

Paradisi, Ratz, Schieren, Simonetto '08
Colangelo, Nikolaidakis, C.S. '08

Running up from MFV at the EW scale:

- MFV is lost at the GUT scale if one starts far enough from the fixed points.
(*some ratios of coefficients explode*)

IV. MFV and proton decay

A. MFV expansions and the flavor $U(1)$ symmetries

Assume that the high-energy dynamics violates \mathcal{B} and/or \mathcal{L} .

We want to parametrize the RPV couplings in terms of the spurions:

$$\mathcal{W}_{RPV} = \underbrace{\mu'{}^I L^I H_d + \lambda^{IJK} L^I L^J E^K + \lambda'{}^{IJK} L^I Q^J D^K}_{\Delta \mathcal{L} = 1} + \underbrace{\lambda''{}^{IJK} U^I D^J D^K}_{\Delta \mathcal{B} = 1}$$

Odd number of flavor indices \rightarrow MFV under $SU(3)^5$ instead of $U(3)^5$,
and *use ε -tensors to form invariants.*

Expected since \mathcal{B} and \mathcal{L} are combinations of the flavor $U(1)$'s:

$$\begin{aligned} G_f &= SU(3)^5 \times U(1)_Q \times U(1)_U \times U(1)_D \times U(1)_L \times U(1)_E \\ &= SU(3)^5 \times U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}} \times U(1)_Y \times U(1)_{PQ} \times U(1)_E \end{aligned}$$

But note: It is not needed to break all five $U(1)$'s!

B. Intrinsic difference between $\Delta\mathcal{L}=1$ and $\Delta\mathcal{B}=1$ couplings

- The \mathcal{B} violating couplings can be constructed using $\Delta\mathcal{B}=0$ quark Yukawas:

$$\lambda''^{IJK} = \epsilon^{LJK} (Y_u Y_d^\dagger)^{IL} \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_D^\dagger) \lambda''^{IJK} U^I D^J D^K$$

$$\lambda''^{IJK} = \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN} \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_Q^\dagger) \lambda''^{IJK} U^I D^J D^K$$

...

- But \mathcal{L} violating couplings are strictly forbidden as long as $m_\nu = 0$:

The SU(3) combinatorics demand a spurion transforming like a six.

The only spurion available is the suppressed $\Delta\mathcal{L}=2$ Majorana mass term:

$$\Upsilon_\nu \equiv v_u Y_\nu^T M^{-1} Y_\nu \rightarrow g_L^* \Upsilon_\nu g_L^\dagger \sim 6_{SU(3)_L} \otimes 1_{SU(3)_E}$$

All $\Delta\mathcal{L}=1$ couplings are suppressed by neutrino masses!!!

C. What happens in the SM?

- *No renormalizable interaction* can break \mathcal{B} or \mathcal{L} .

- Model-independent dimension-six $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ operators:

$$\mathcal{L}_{\Delta(\mathcal{B}+\mathcal{L})} = \frac{\mathcal{E}_{abc}}{\Lambda^2} \left(\textcolor{red}{c_1^{IJKL}} L^I Q_a^J Q_b^K Q_c^L + \textcolor{red}{c_2^{IJKL}} E^I U_a^J U_b^K D_c^L \right. \\ \left. + \textcolor{red}{c_3^{IJKL}} E^I U_a^J Q_b^{\dagger K} Q_c^{\dagger L} + \textcolor{red}{c_4^{IJKL}} L^I Q_a^J D_b^{\dagger K} U_c^{\dagger L} \right) \quad \text{Weinberg '79}$$

→ Under MFV, forbidden when $m_\nu = 0$ and thus *very suppressed!*

- Highly-suppressed instanton effects break $\mathcal{B}+\mathcal{L}$:

$$\mathcal{L}_{\Delta(\mathcal{B}+\mathcal{L})} \sim e^{-4\pi \sin^2 \theta_W / \alpha} (\varepsilon_{IJK} L^I L^J L^K) (\varepsilon_{IJK} Q^I Q^J Q^K)^3 \quad t'Hooft '76$$

Of course, it *respects MFV!* But under $\textcolor{green}{G}_f = SU(3)^5 \times \textcolor{blue}{U}(1)_U \times U(1)_D \times \textcolor{blue}{U}(1)_E$.

D. MFV for the R-parity violating terms of the MSSM:

Nikolidakis, C.S. '07

Structures ($\mathcal{W}_{RPV} = \mu' LH_d + \lambda' LLE + \lambda' LQD + \lambda'' UDD$)		Scaling	Breaking
μ'_1^I	$\mu \epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST}, \dots$	$\tan^2 \beta$	$U(1)_L$
λ'_1^{IJK}	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} (Y_e)^{KJ}, \dots$	$\tan^3 \beta$	$U(1)_L$
λ'_2^{IJK}	$\epsilon^{IMJ} (Y_e^\dagger Y_\nu^\dagger)^{KM}, \dots$	$\tan \beta$	$U(1)_L$
λ'_3^{IJK}	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} \epsilon^{LMJ} \epsilon^{ABK} (Y_e^\dagger)^{LA} (Y_e^\dagger)^{MB}, \dots$	$\tan^4 \beta$	$U(1)_{L,E}$
λ'_1^{IJK}	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} (Y_d)^{KJ}, \dots$	$\tan^3 \beta$	$U(1)_L$
λ'_2^{IJK}	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} \epsilon^{LMJ} \epsilon^{ABK} (Y_d^\dagger)^{LA} (Y_d^\dagger)^{MB}, \dots$	$\tan^4 \beta$	$U(1)_{L,D,Q}$
λ''_1^{IJK}	$\epsilon^{LJK} (Y_u Y_d^\dagger)^{IL}, \dots$	$\tan \beta$	$U(1)_D$
λ''_2^{IJK}	$\epsilon^{IMN} (Y_d Y_u^\dagger)^{JM} (Y_d Y_u^\dagger)^{KN}, \dots$	$\tan^2 \beta$	$U(1)_U$
λ''_3^{IJK}	$\epsilon^{LMN} (Y_u)^{IL} (Y_d)^{JM} (Y_d)^{KN}, \dots$	$\tan^2 \beta$	$U(1)_Q$
λ''_4^{IJK}	$\epsilon^{LMN} \epsilon^{ABI} \epsilon^{CJK} (Y_d^\dagger)^{LC} (Y_u^\dagger)^{MA} (Y_u^\dagger)^{NB}, \dots$	$\tan \beta$	$U(1)_{Q,U,D}$

(Similar expansions for R-parity violating soft-breaking terms)

D. Check of the bounds on R-parity violating couplings

In addition to the *neutrino mass factor* $\Upsilon_\nu \sim \mathcal{O}(m_\nu / v_u) \sim \mathcal{O}(10^{-12})$, ε -tensor antisymmetry forces all couplings to be proportional to *light-fermion masses*:

$$\text{Ex: } \varepsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN} \rightarrow \varepsilon^{123} Y_u^{I1} Y_d^{J2} Y_d^{K3} + \dots \rightarrow \frac{m_u}{v_u} \frac{m_s}{v_d} \frac{m_b}{v_d} + \dots$$

Are these two mechanisms sufficient to pass experimental bounds ?

Hundreds of bounds, most rather weak and immediately satisfied.

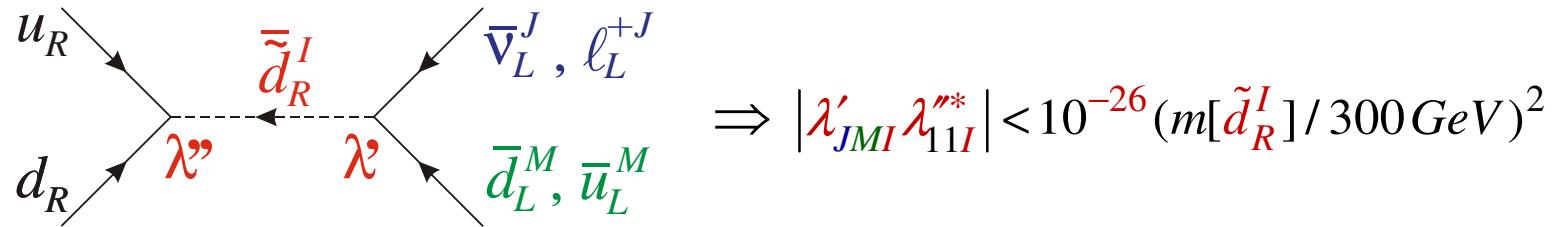
(LFV & FCNC, EDM's, $n - \bar{n}$ oscillations, EWPO from LEP, Tevatron,...)

Toughest constraints from $\Delta\mathcal{B} = 1$ *nucleon decays*, i.e. $p, n \rightarrow \pi\nu, \pi\ell, K\nu, K\ell, \dots$

Bounds on various combinations $|\langle \mu', \lambda, \lambda' \rangle \times \lambda''|$,

For some $IJK, I'J'K'$, as constraining as $|\lambda'_{IJK} \lambda''_{I'J'K'}| < 10^{-25} - 10^{-27}$.

Example of MFV suppression for a specific proton decay mechanism



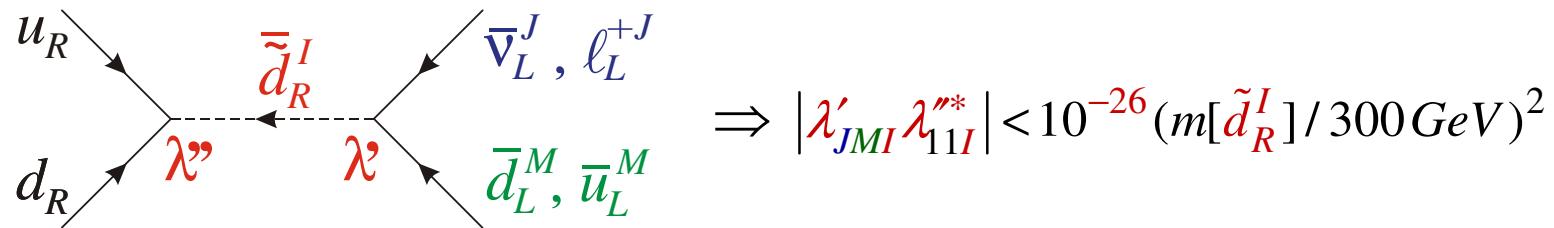
$$\Rightarrow |\lambda'_{JMI} \lambda''^*_{11I}| < 10^{-26} (m[\tilde{d}_R^I] / 300 \text{GeV})^2$$

If the leading operators are: $\lambda' : (a_0 \epsilon^{LMI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{LM} (Y_d^\dagger Y_u^\dagger Y_u)^{KJ}) L^I Q^J D^K$
 $\lambda'' : (a_1 \epsilon^{LJK} (Y_u^\dagger Y_d^\dagger)^{IL} + a_2 \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN}) U^I D^J D^K$

The MFV prediction is then

$$|\lambda'_{JMI} \lambda''^*_{11I}| \approx \frac{\Delta m_{31}}{v_u} \frac{m_\tau^2}{v_d^2} \lambda^3 \frac{m_b m_t^2 m_u}{v_d v_u^3} \left(a_0 a_1 \frac{m_s}{v_d} + a_0 a_2 \frac{m_d m_b}{v_d^2} \right)$$

Example of MFV suppression for a specific proton decay mechanism



If the leading operators are:

$$\lambda' : (a_0 \epsilon^{LMI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{LM} (Y_d Y_u^\dagger Y_u)^{KJ}) L^I Q^J D^K$$

$$\lambda'' : (a_1 \epsilon^{LJK} (Y_u Y_d^\dagger)^{IL} + a_2 \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN}) U^I D^J D^K$$

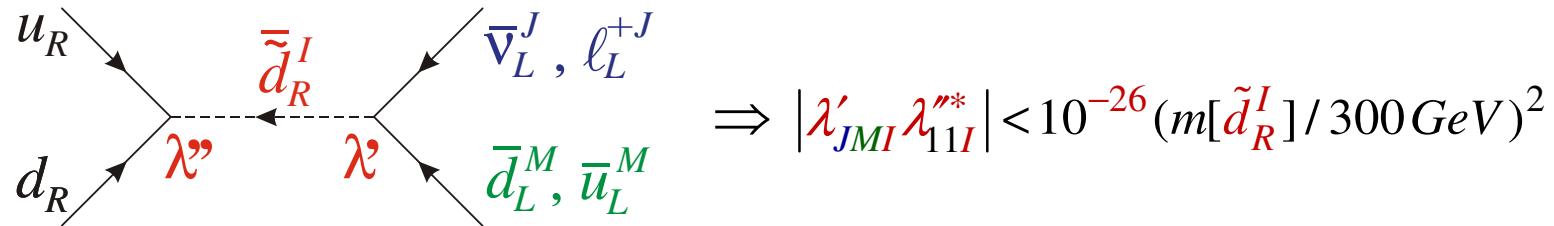
The MFV prediction is then

$$|\lambda'_{JMI} \lambda''^*_{11I}| \approx \frac{\Delta m_{31}}{v_u} \frac{m_\tau^2}{v_d^2} \lambda^3 \frac{m_b m_t^2 m_u}{v_d v_u^3} \left(a_0 a_1 \frac{m_s}{v_d} + a_0 a_2 \frac{m_d m_b}{v_d^2} \right)$$

Annotations pointing to the equation:

- Neutrino mass, related to $\Delta m_{atm}^2 \approx 3 \cdot 10^{-3} \text{ eV}^2$
- Charged lepton mass
- CKM factors
- Light-quark masses
- Symmetry of Y_ν , the Majorana mass term
- Antisymmetry of ϵ tensors

Example of MFV suppression for a specific proton decay mechanism



If the leading operators are: $\lambda' : (a_0 \epsilon^{LMI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{LM} (Y_d Y_u^\dagger Y_u)^{KJ}) L^I Q^J D^K$
 $\lambda'' : (a_1 \epsilon^{LJK} (Y_u Y_d^\dagger)^{IL} + a_2 \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN}) U^I D^J D^K$

The MFV prediction is then

$$|\lambda'_{JMI} \lambda''''^*_{11I}| \approx \frac{\Delta m_{31}}{v_u} \frac{m_\tau^2}{v_d^2} \lambda^3 \frac{m_b m_t^2 m_u}{v_d v_u^3} \left(a_0 a_1 \frac{m_s}{v_d} + a_0 a_2 \frac{m_d m_b}{v_d^2} \right)$$

$$\approx a_0 a_1 10^{-28} \tan^4 \beta + a_0 a_2 10^{-31} \tan^5 \beta \quad (\text{for } m_\nu^{\text{lightest}} = 0)$$

Conservatively, MFV can account for the necessary suppression.

- MFV coefficients of $\mathcal{O}(1)$, while $\mathcal{O}(\lambda)$ or $\mathcal{O}(g^2 / 4\pi)$ also natural,
- No GIM-like interferences, no cancellations among processes,

E. Where to expect significant experimental signals ?

1. Proton decay could be close to current bounds (worthy to pursue the search!)

2. Except for proton decay, lepton-number effectively conserved.
(since $\mu', \lambda, \lambda' < \mathcal{O}(10^{-12})$)

3. MFV predictions for the baryonic couplings $\varepsilon^{abc} \lambda''^{IJK} U_a^I D_b^J D_c^K$:

Structure	λ''_1	λ''_2	λ''_3	$\lambda''_{4,5}$
Broken $U(1)$	$U(1)_D$	$U(1)_U$	$U(1)_Q$	$U(1)_{U,D,Q}$
$\tan \beta = 5$	$\begin{pmatrix} 8 & 8 & 8 \\ 4 & 6 & 5 \\ 1 & 6 & 4 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 7 \\ 12 & 9 & 9 \\ 13 & 12 & 13 \end{pmatrix}$	$\begin{pmatrix} 13 & 8 & 10 \\ 10 & 6 & 7 \\ 6 & 5 & 6 \end{pmatrix}$	$\begin{pmatrix} 5 & 5 & 5 \\ 7 & 9 & 7 \\ 7 & 12 & 10 \end{pmatrix}$
$\tan \beta = 50$	$\begin{pmatrix} 7 & 7 & 7 \\ 3 & 5 & 4 \\ 0 & 5 & 3 \end{pmatrix}$	$\begin{pmatrix} 9 & 4 & 5 \\ 10 & 7 & 7 \\ 11 & 10 & 11 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 8 \\ 8 & 4 & 5 \\ 4 & 3 & 4 \end{pmatrix}$	$\begin{pmatrix} 4 & 4 & 4 \\ 6 & 8 & 6 \\ 6 & 11 & 9 \end{pmatrix}$

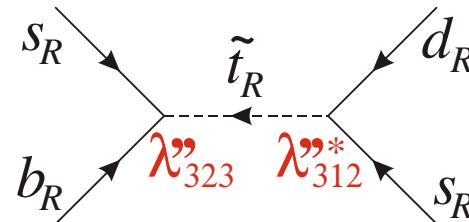
Notations :
 $x \equiv \mathcal{O}(10^{-x})$

$\begin{pmatrix} 112 & 123 & 131 \\ 212 & 223 & 231 \\ 312 & 323 & 331 \end{pmatrix}$

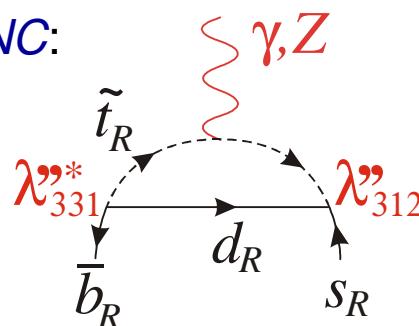
$\curvearrowleft \lambda''_{312} \longrightarrow$ Sizeable $\tilde{t}_R d_R s_R, t_R \tilde{d}_R s_R, t_R d_R \tilde{s}_R$ couplings.

4. Probing $\Delta\mathcal{B} = 1$ interactions at low-energy:

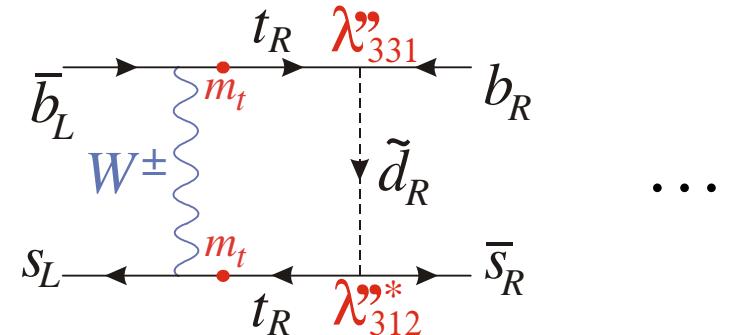
- Squarks as *diquark currents*:



- Induce *new FCNC*:



Chakraverty, Choudhury '01, ...



Barbieri, Masiero '86, Slavich '00, ...

- *With MFV*, these are *typically small* compared to the SM contributions:

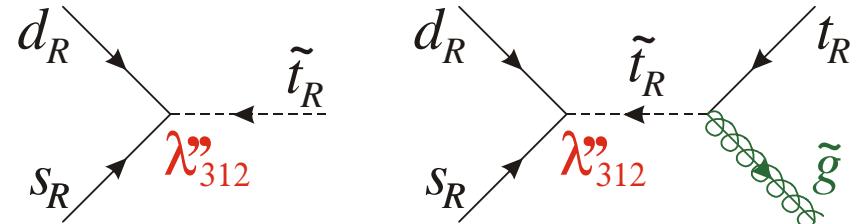
$$b \rightarrow s : |\lambda_{312}'' \lambda_{331}''*| < 10^{-3}, \quad b \rightarrow d : |\lambda_{312}'' \lambda_{323}''*| < 10^{-5}, \quad s \rightarrow d : |\lambda_{313}'' \lambda_{323}''*| < 10^{-8}$$

$$b \rightarrow s : |V_{tb}^* V_{ts}| \sim 10^{-2}, \quad b \rightarrow d : |V_{tb}^* V_{td}| \sim 10^{-3}, \quad s \rightarrow d : |V_{ts}^* V_{td}| \sim 10^{-4}$$

5. Probing $\Delta\mathcal{B} = 1$ effects at colliders: drastic changes for the phenomenology.

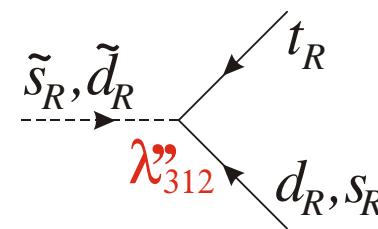
- Single stop resonant production and associated *single gluino* production:

Dimopoulos, Hall '88, Dreiner, Ross '91,
Chaichan et al. '00, Allanach et al. '01, ...

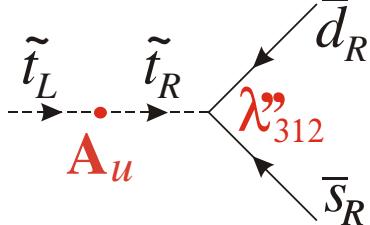


- *Top production*, from squark decay:

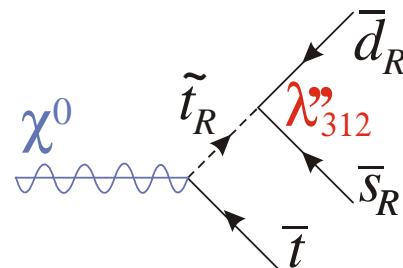
Berger et al. '99, Chiappetta et al. '99, ...



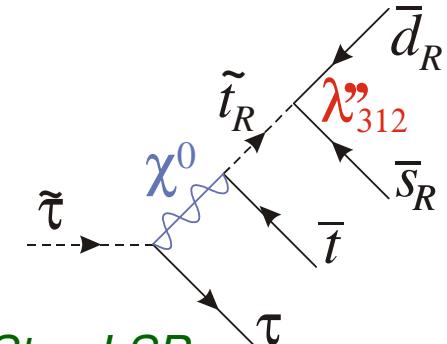
- *LSP* not necessarily colorless & neutral, and will *decay*, maybe in the detector:



Stop LSP



Neutralino LSP



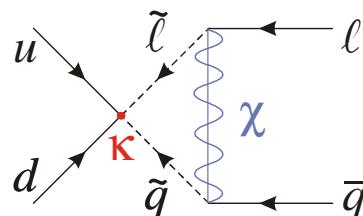
Stau LSP

F. R-parity or not R-parity ?

- *Avoiding proton decay* is no longer a good motivation for R-parity. 

- *Dim-5 R-parity conserving operators* can also induce proton decay: 

Ibanez,
Ross '92



$$\mathcal{W}_{\text{dim-5}} \ni \frac{\kappa_1^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}} (Q^I Q^J) (Q^K L^L) + \frac{\kappa_2^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}} (D^I U^J U^K) E^L$$

MFV separately suppresses $\Delta\mathcal{L} = 1$ and $\Delta\mathcal{B} = 1$ effects.

- *GUT*: R-parity often built in (*SO(10)*-GUT) or required (*SU(5)*-GUT). 

Example: $G_f = U(3)_{\bar{5}} \times U(3)_{10}$: $\mathbf{Y}_{\bar{5}} \sim (\bar{3}, \bar{3})$, $\mathbf{Y}_{10} \sim (1, \bar{6})$

Cirigliano, Grinstein,
Isidori, Wise '05

Seesaw spurion not required for $\mathcal{W}_{RPV} = \Lambda^{IJK} \bar{5}^I \bar{5}^J 10^K + \dots$

- *Cosmology*:  MSSM-LSP not stable \rightarrow nature of dark matter still to be resolved.
 Baryon asymmetry generated from CPV, $\Delta\mathcal{B} = 1$ couplings?

Should experimentalists accept the burden of R-parity “only” for dark matter???

Conclusion

MFV, as a phenomenological hypothesis on the elementary flavor structures:

A single mechanism explaining:

- *Smallness of susy effects in FCNC*
- *Extremely long proton lifetime*

Consequences of the *Yukawa hierarchies* and of the *small neutrino masses*.

MFV, as a window into physics beyond the MSSM:

It permits to *identify the flavor couplings which are fine-tuned* (none at present) out of those which are as “natural” as the SM Yukawas.

In particular, *the proton lifetime does not require fine-tuned RPV couplings!*

Since a *consistent picture emerges with only a few spurions*, the mechanism behind all the flavor structures could be relatively simple.

CP-violation is controlled by non-MFV physics, as expected from $\text{Arg}(\mu) \ll 1$.