

# Minimal Flavor Violation in supersymmetry



Christopher Smith

- Outline

*Introduction: Flavors in the SM*

*I. New Physics flavor puzzle(s)*

*II. Minimal Flavor Violation (MFV)*

*III. CP-violation under MFV*

*IV. RGE behavior of MFV*

*V. MFV and proton decay*

*Conclusion*

# Introduction

## A. The Standard Model flavor symmetry

The three generations of quarks/leptons have *identical gauge interactions*

$$\mathcal{L}_{Kin} = \sum_{k,I=1,2,3} \bar{\psi}_k^I i \not{D}_k \psi_k^I, \quad D_k^\mu \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\text{where } \psi_k : Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad U = u_R^\dagger, \quad D = d_R^\dagger, \quad L = \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}, \quad E = \ell_R^\dagger$$

As a result, the SM gauge interactions exhibit the  $U(3)^5$  *flavor symmetry*:

$$G_f = U(3)^5 = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$$

Chivukula,  
Georgi '87

With one  $U(3)$  per fermion species, since under  $g_k \in U(3)_k$

$$\psi_k^I \rightarrow (g_k)^{IJ} \psi_k^J \quad \Rightarrow \quad \mathcal{L}_{Kin} \rightarrow \sum_{k,I,J,K} \bar{\psi}_k^J (g_k^\dagger)^{JI} i \not{D}_k (g_k)^{IK} \psi_k^K = \mathcal{L}_{Kin}$$

*B. In the SM, the flavor symmetry is broken in a very special way:*

- *The only sources of breaking are the Yukawa couplings:*

$$\mathcal{L}_{Yukawa} = U^I \mathbf{Y}_u^{IJ} (Q^J H) + D^I \mathbf{Y}_d^{IJ} (Q^J H^\dagger) + E^I \mathbf{Y}_e^{IJ} (L^J H^\dagger)$$

which themselves are also *very special*:

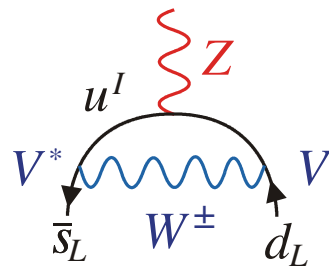
- The *fermion masses* are highly hierarchical ( $m_t \gg m_c \gg m_u$ )
- The *CKM matrix* is highly hierarchical (close to unit matrix),
- The CKM phase is the *unique source for all CP-violation*.
- Essential feature of *flavor physics* & *FCNC processes*:

$$B \rightarrow X_{d,s} \ell^+ \ell^- ,$$

$$B \rightarrow X_{d,s} \nu \bar{\nu} ,$$

$$K \rightarrow \pi \nu \bar{\nu} ,$$

$$K_L \rightarrow \pi^0 \ell^+ \ell^- , \dots$$



$\sim m_{u^I}^2$   
 $\Rightarrow$  top quark  
 dominates

$$b \rightarrow s : V_{tb}^* V_{ts} \sim 10^{-2}$$

$$b \rightarrow d : V_{tb}^* V_{td} \sim 10^{-3}$$

$$s \rightarrow d : V_{ts}^* V_{td} \sim 10^{-4}$$

### C. Warm-up: “MFV” in the Standard Model

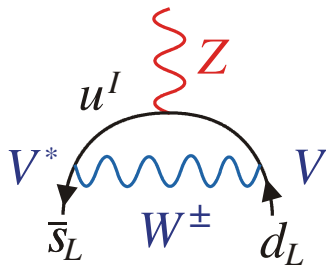
- The SM is made *artificially invariant under*  $G_f$  by forcing  $\mathbf{Y}_{u,d,e}$  to transform as:

$$\mathbf{Y}_u \rightarrow g_U \mathbf{Y}_u g_Q^\dagger, \quad \mathbf{Y}_d \rightarrow g_D \mathbf{Y}_d g_Q^\dagger, \quad \mathbf{Y}_e \rightarrow g_E \mathbf{Y}_e g_L^\dagger$$

$$\text{since then } \mathcal{L}_{Yukawa} = U \mathbf{Y}_u Q H + D \mathbf{Y}_d Q H^\dagger + E \mathbf{Y}_e L H^\dagger \xrightarrow{U(3)^5} \mathcal{L}_{Yukawa}$$

Background values:  $v \mathbf{Y}_u = m_u V_{CKM}$ ,  $v \mathbf{Y}_d = m_d$ ,  $v \mathbf{Y}_e = m_e$ .

- All *SM amplitudes must then be invariant under*  $G_f$ , at all orders.



Example: The Z penguin:

$$\rightarrow \mathcal{O}_Z \sim \bar{Q}^I \gamma_\mu Q^I \underbrace{H^\dagger D^\mu H}_{\sim v^2 Z^\mu}$$

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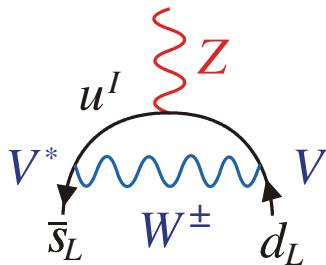
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Predicts the CKM & quadratic GIM:  $v^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u \approx m_t^2 \begin{pmatrix} |V_{td}|^2 & V_{td}^* V_{ts} & V_{td}^* V_{tb} \\ V_{ts}^* V_{td} & |V_{ts}|^2 & V_{ts}^* V_{tb} \\ V_{tb}^* V_{td} & V_{tb}^* V_{ts} & |V_{tb}|^2 \end{pmatrix}$

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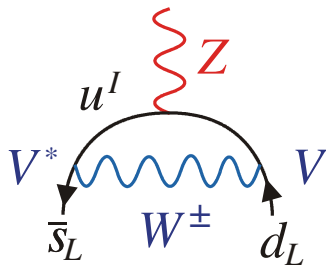
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Right-handed currents?  $\mathcal{O}_Z \sim D \gamma_\mu \mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \bar{D} v^2 Z^\mu$

Suppressed by

$$\sim \frac{m_{d^I} m_{d^J}}{v^2}$$

### C. Warm-up: “MFV” in the Standard Model

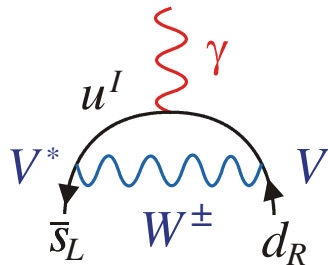
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Example: The EM operator:

$$\rightarrow \begin{cases} \mathcal{O}_\gamma \sim D^I \sigma_{\mu\nu} Q^I H^\dagger F^{\mu\nu} \\ \mathcal{O}_\gamma \sim E^I \sigma_{\mu\nu} L^I H^\dagger F^{\mu\nu} \end{cases}$$

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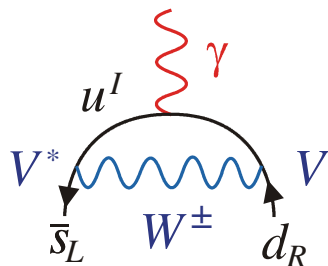
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Example: The EM operator:

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No LFV, since  $\mathbf{Y}_e$  is diagonal:  $\mu \not\rightarrow e\gamma$ ,  $\mu \not\rightarrow eee$ , ...

Experimentally,  $m_\nu \neq 0$  but extremely small  $\Rightarrow B(\mu \rightarrow e\gamma) < 10^{-50}$ .

# I. The MSSM flavor puzzles

## A. Flavors and New Physics

- There is some *New Physics* (dark matter,  $m_\nu$ , unification, EW stability, gravity,...)
- Most New Physics models have either *new flavored particles*, or *new flavor-breaking interactions* between quarks and leptons.
- The Lagrangian of NP can always be made  $U(3)^5$  *symmetric*, but at the cost of allowing for *new spurions* (= NP flavor-breaking couplings).

$$\text{Ex: } \mathbf{X}_Q \rightarrow g_Q \mathbf{X}_Q g_Q^\dagger \Rightarrow \mathcal{O}_Z \sim \frac{1}{\Lambda_{NP}^2} \bar{Q}^I \gamma_\mu (\mathbf{X}_Q)^{IJ} Q^J v^2 Z^\mu$$

- *Flavor experiments*  $\Rightarrow$  either spurions non-natural, or NP scale very high.

$$\text{Ex: } \mathcal{O}_Z \Rightarrow K \rightarrow \pi \nu \bar{\nu}. \text{ With } (X_Q)^{12} \approx 1 \Rightarrow \Lambda \gtrsim 75 \text{ TeV.}$$

- Flavor structures of TeV-scale NP necessarily fine-tuned: *NP flavor puzzle*.

## A. Flavors and New Physics: Situation in the MSSM

Essentially one superpartner for every SM particle, same gauge group.

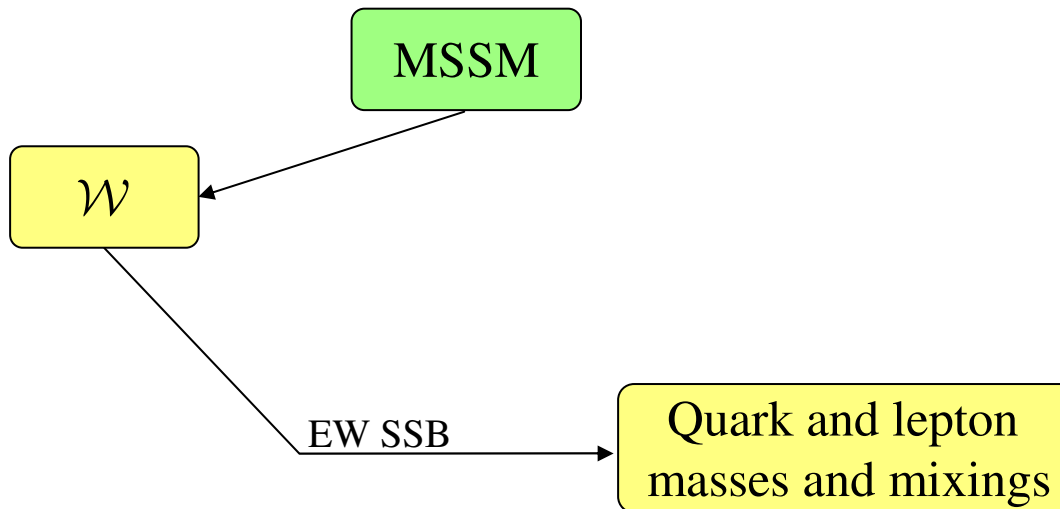
Squarks and sleptons are *scalar flavored particles*.

- MSSM gauge interactions still exhibit the  $U(3)^5$  *flavor symmetry*.
- *Many new flavor couplings*  $\Leftrightarrow$  *new spurions*, a priori not hierarchical.
- *New contributions* to flavor transitions

$$\begin{aligned} \text{e.g.: } \mathcal{L}_{MSSM} \supset \tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} &\rightarrow \mathcal{O}_Z \sim \frac{1}{\Lambda_{SUSY}^4} (\bar{Q} \gamma_\mu \mathbf{m}_Q^2 Q) v^2 Z^\mu \\ \mathcal{L}_{MSSM} \supset \tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} &\rightarrow \mathcal{O}_\gamma \sim \frac{1}{\Lambda_{SUSY}^4} (E \mathbf{Y}_e \mathbf{m}_L^2 \sigma_{\mu\nu} L) H_d F^{\mu\nu} \end{aligned}$$

- Experimental data impose to *fine-tune those additional spurions*:

$$\text{Approx. alignment with SM: } \mathbf{m}_Q^2 \sim \Lambda_{SUSY}^2 \mathbf{Y}_u^\dagger \mathbf{Y}_u, \quad \mathbf{m}_L^2 \sim \Lambda_{SUSY}^2 \mathbf{Y}_e^\dagger \mathbf{Y}_e.$$



1. *Superpotential Yukawa couplings*: set fermion masses and mixings.

$$\mathcal{W} = U\mathbf{Y}_u(QH_u) - D\mathbf{Y}_d(QH_d) - E\mathbf{Y}_e(LH_d) + \mu(H_u H_d)$$

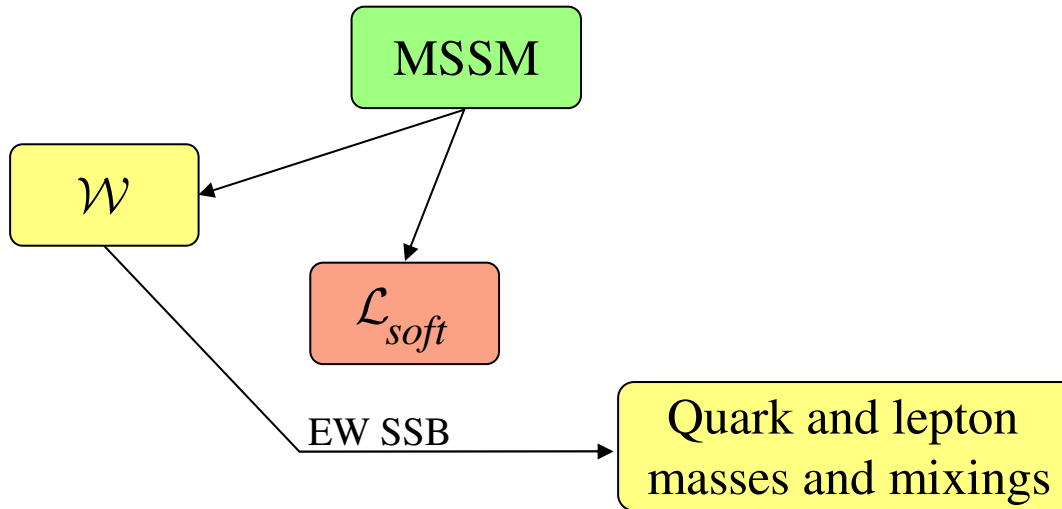
$(Q, U, D, \dots$  now denote *superfields*, with fermion & scalar components)

Analogues of the SM Yukawa couplings (but with two Higgs doublets).

→ *same hierarchical fermion masses & CKM couplings*.

At this stage, perfect *alignment* of squarks with quarks, sleptons with leptons.

→ *same masses, same mixings*.



2. *Squark soft-breaking terms*: SUSY is broken, but the *exact mechanism is unclear*

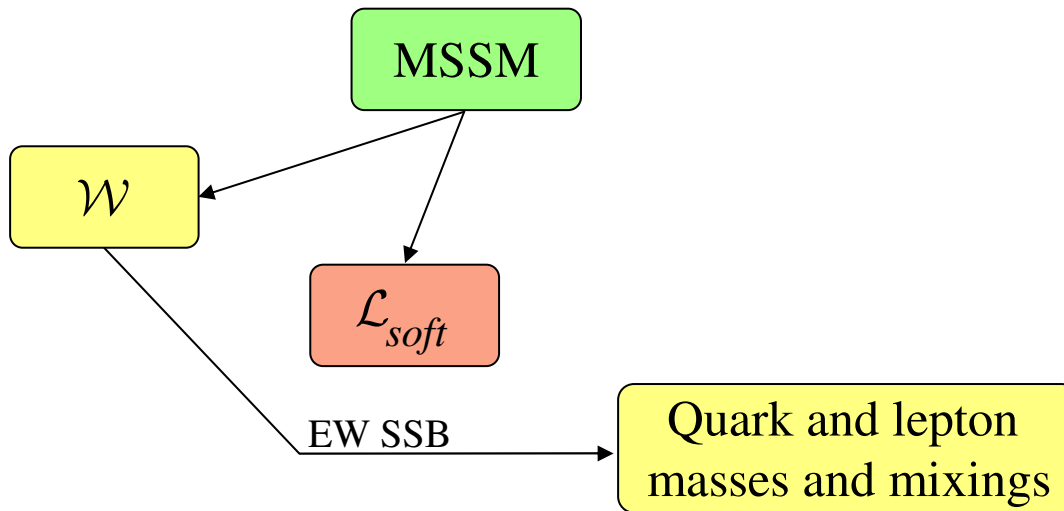
- *Effective description*:  $\mathcal{L}_{soft} \ni -\tilde{Q}^\dagger \mathbf{m}_Q^2 \tilde{Q} - \tilde{U} \mathbf{m}_U^2 \tilde{U}^\dagger - \tilde{U} \mathbf{A}_u (\tilde{Q} H_u) + \dots$

- *Squark mass terms*:  $\delta M_{\tilde{u}}^2 = \begin{pmatrix} \mathbf{m}_Q^2 & v_u \mathbf{A}_u^\dagger \\ v_u \mathbf{A}_u & \mathbf{m}_U^2 \end{pmatrix}$  Large mass and gauge eigenstate mismatch?

- *Contributions to the FCNC*:

The diagram shows two Feynman diagrams representing contributions to Flavor Changing Neutral Currents (FCNC). The left diagram shows a loop involving a  $W^\pm$  boson and a  $Z$  boson, with external lines for  $u_L^I$ ,  $\bar{s}_L$ , and  $d_L$ . The right diagram shows a loop involving a  $\chi^\pm$  chargino and a  $Z$  boson, with external lines for  $\tilde{u}_{L,R}^I$ ,  $\bar{s}_L$ , and  $d_L$ . The diagrams are connected by a double arrow indicating a comparison or relationship.

With sparticle masses  $< 1$  TeV, the squark flavor mixings must be small.



3. *Slepton soft-breaking terms*: similar situation as for squarks

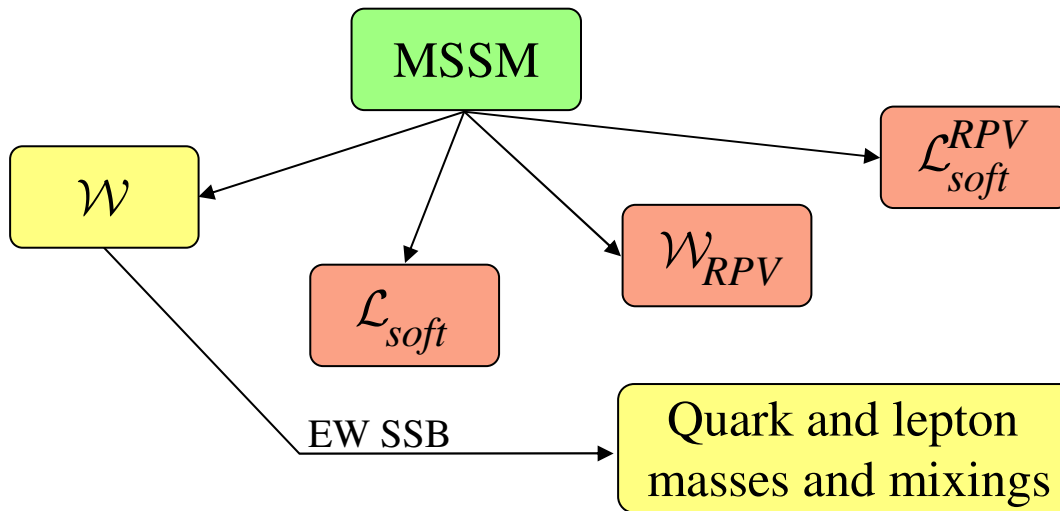
- *Effective description*:  $\mathcal{L}_{soft} \ni -\tilde{L}^\dagger \mathbf{m}_L^2 \tilde{L} - \tilde{E} \mathbf{m}_E^2 \tilde{E}^\dagger - \tilde{E} \mathbf{A}_e (\tilde{L} H_d) + \dots$

- *Slepton mass terms*:  $\delta M_{\tilde{\ell}}^2 = \begin{pmatrix} \mathbf{m}_L^2 & v_d \mathbf{A}_e^\dagger \\ v_d \mathbf{A}_e & \mathbf{m}_E^2 \end{pmatrix}$  Large mass and gauge eigenstate mismatch?

- *New FCNC*: With generic mixings, LFV much too large compared to experimental bounds.

The Feynman diagram shows a loop process. A slepton ( $\tilde{\ell}_{L,R}^I$ ) is in the loop, interacting with a photon ( $\gamma$ ) and a neutralino ( $\chi^0$ ). The external lines are a muon ( $\bar{\mu}$ ) and an electron ( $e$ ), with a  $Z^L$  boson exchange. This process represents a flavor-changing neutral current (FCNC).

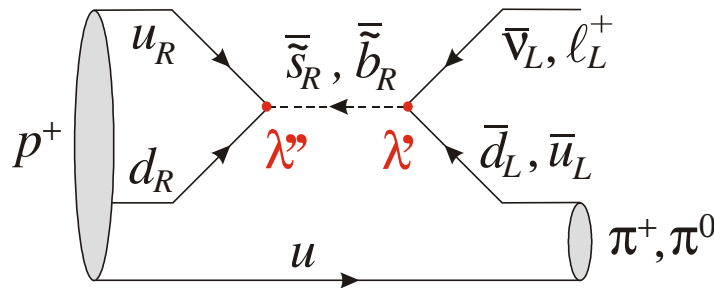
Again, sleptons and leptons must not be too misaligned.



4.  $\mathcal{B}$  or  $\mathcal{L}$  violating couplings are allowed (both supersymmetric and not):

$$\mathcal{W}_{RPV} = \lambda^{IJK} (L^I L^J) E^K + \lambda'^{IJK} (L^I Q^J) D^K + \lambda''^{IJK} U^I D^J D^K + \mu'^I (L^I H_d)$$

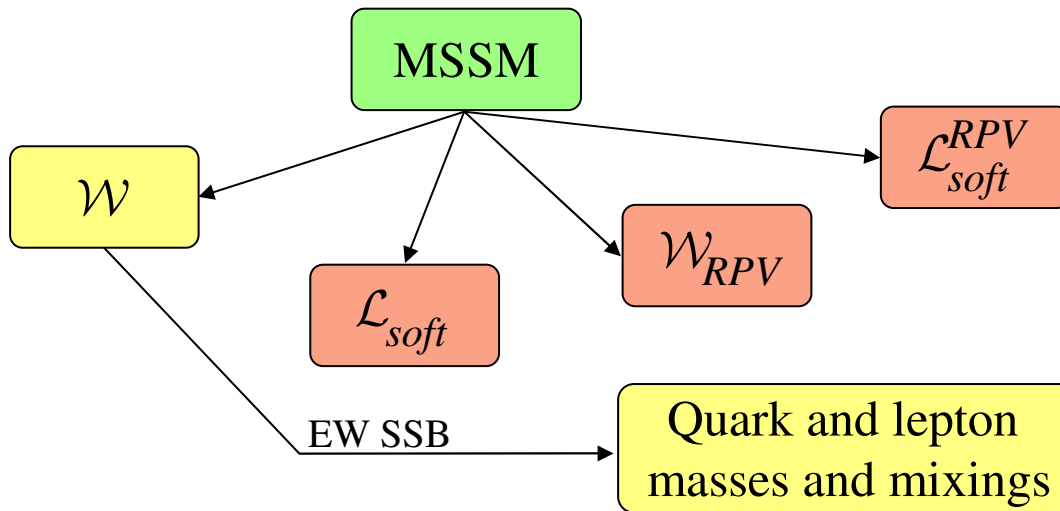
These couplings induce *proton decay* (and associated) at tree-level:



$$p \rightarrow \pi^+ \bar{\nu}_\ell, K^+ \bar{\nu}_\ell, \pi^0 \ell^+, \dots$$

$$n \rightarrow \pi^0 \bar{\nu}_\ell, K^0 \bar{\nu}_\ell, \pi^- \ell^+, \dots$$

But experimentally,  $\tau_{p^+} > 10^{30}$  years :  $\Gamma_{p^+} \sim \frac{m_p^5}{M_{\tilde{d}}^4} |\lambda'' \lambda'|^2 \Rightarrow |\lambda' \lambda''| \leq 10^{-27}$  ?



Usual escape route is to *impose  $R$ -parity*:

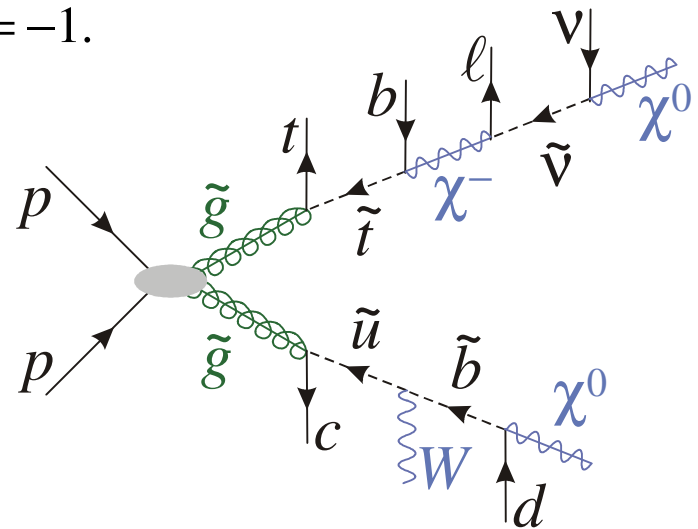
*Farrar, Fayet '78*

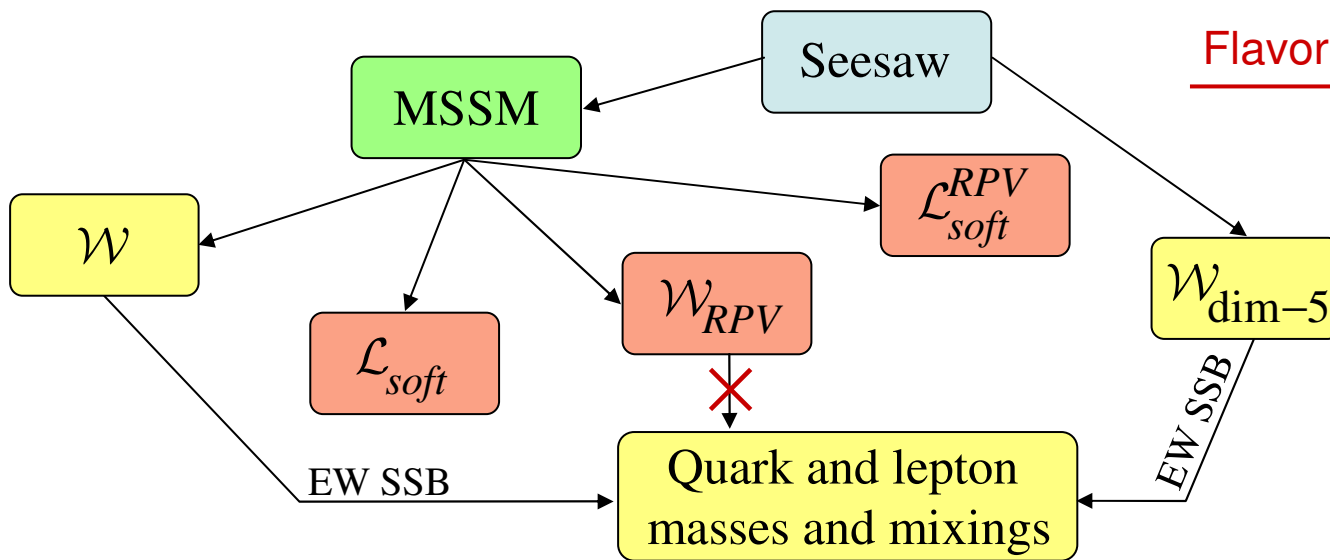
Assign  $R(Particles) = +1$  and  $R(Sparticles) = -1$ .

→  $\mathcal{W}_{RPV}$  and  $\mathcal{L}_{soft}^{RPV}$  couplings forbidden.

*But also:* sparticles produced in pairs,  
stable LSP (hence neutral LSP),...

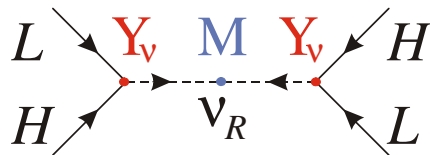
R-parity is a very (too?) tough constraint!





5. *Seesaw mechanism* to account for neutrino masses (not from  $\mathcal{W}_{RPV}$ ):

- *Right-handed (s)neutrinos* are added:  $\mathcal{W}_N = N\mathbf{M}N + N\mathbf{Y}_\nu(LH_u)$
- *Large  $\mathcal{L}$  violating mass  $\mathbf{M}$*  allowed  $\rightarrow \nu_R$  are integrated out:



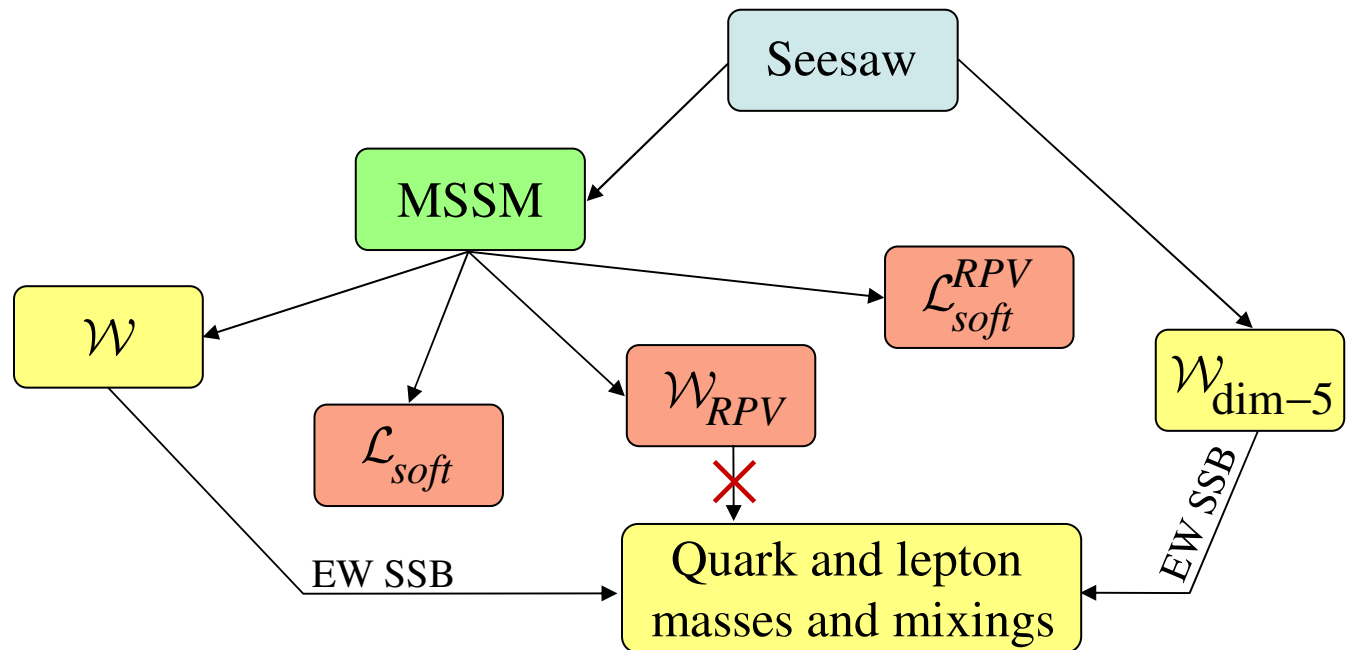
$$\mathcal{W}_{\text{dim-5}} = (\mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu)^{IJ} (L^I H_u)(L^J H_u)$$

- *Effective Majorana mass term for  $\nu_L$* :  $\nu_u^2 \mathbf{Y}_\nu^T \mathbf{M}^{-1} \mathbf{Y}_\nu = U_{PMNS}^* \cdot \mathbf{m}_\nu \cdot U_{PMNS}^\dagger$

Then,  $\mathbf{m}_\nu \sim 1\text{eV}$  with  $\mathbf{Y}_\nu \sim \mathcal{O}(1)$  when  $\mathbf{M} \sim 10^{13}\text{GeV}$ .

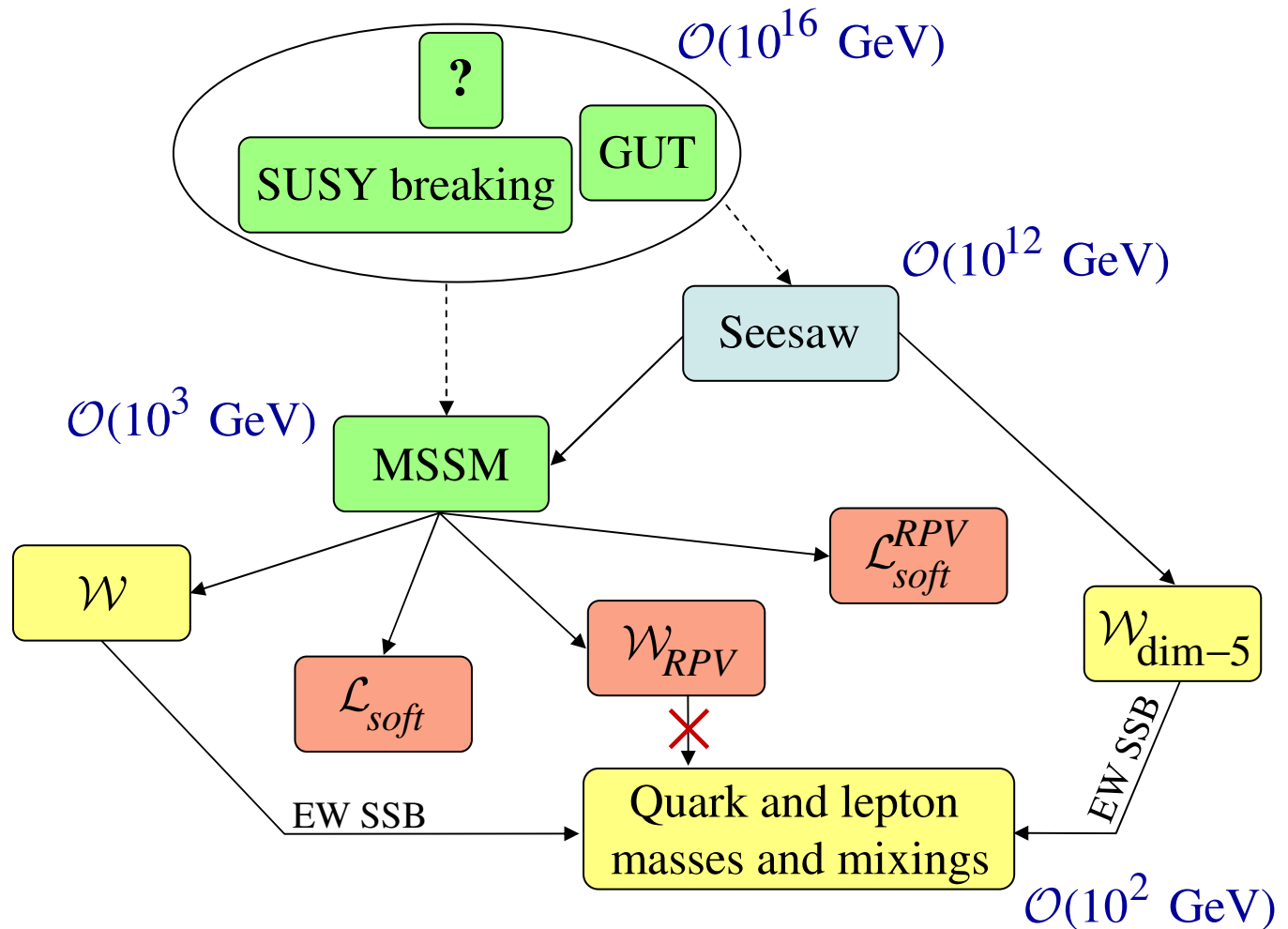
## II. The MFV hypothesis

# A. MFV and the origin of the flavor structures:



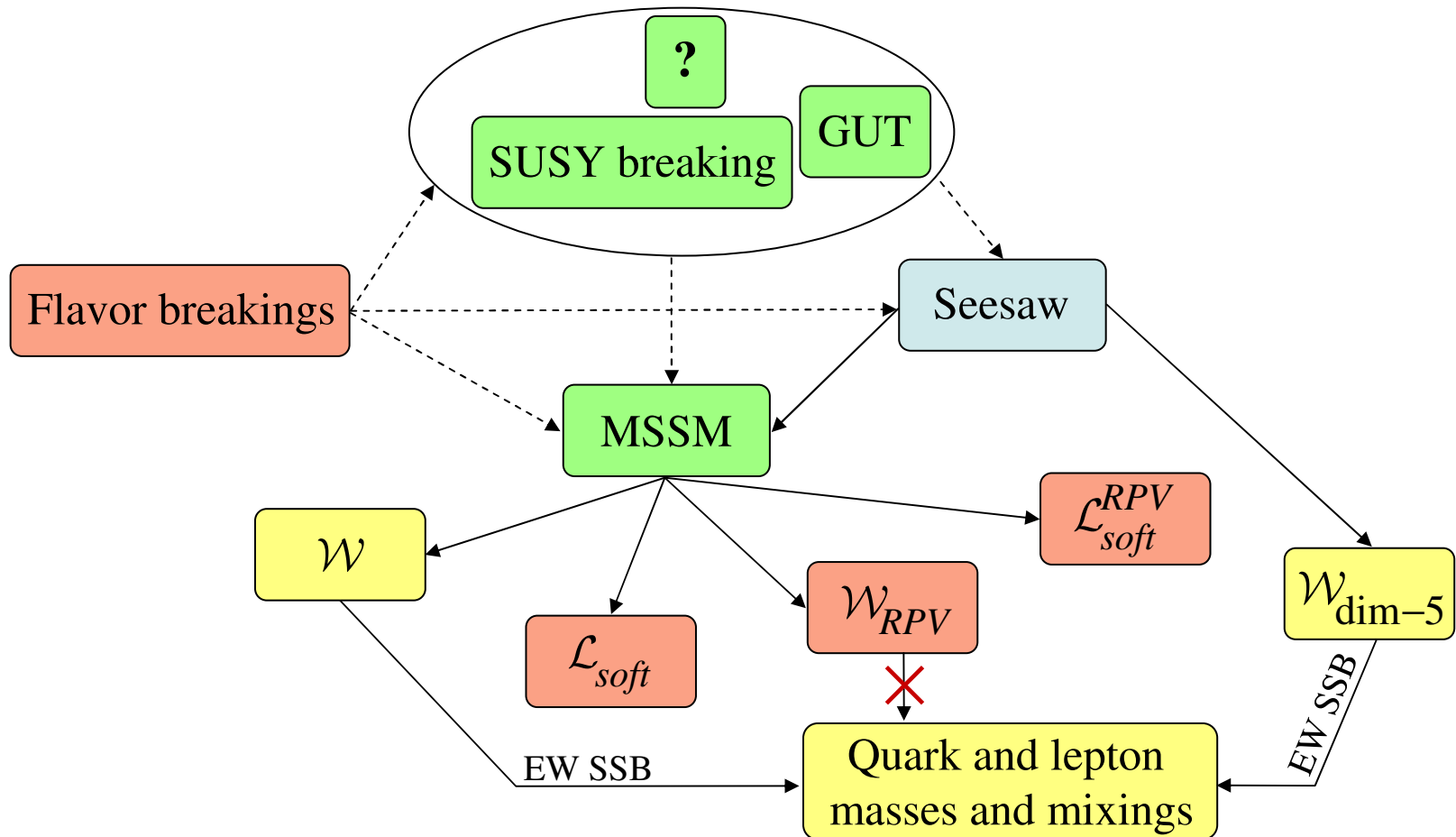
Only the flavor-breakings in the SM fermionic sector have been probed experimentally.

# A. MFV and the origin of the flavor structures:



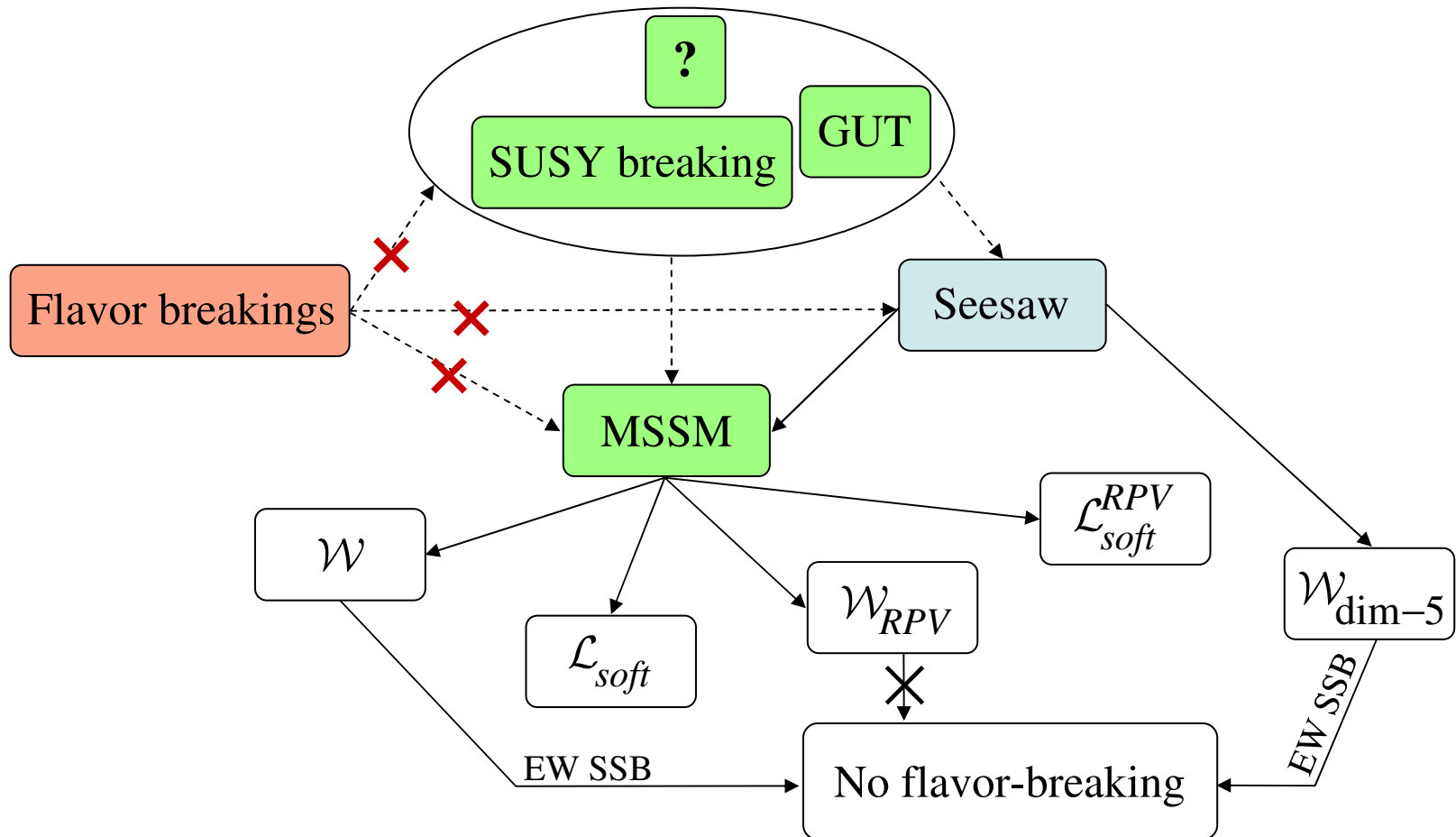
The MSSM is not the ultimate theory, but only a “low-energy” effective theory.

# A. MFV and the origin of the flavor structures:



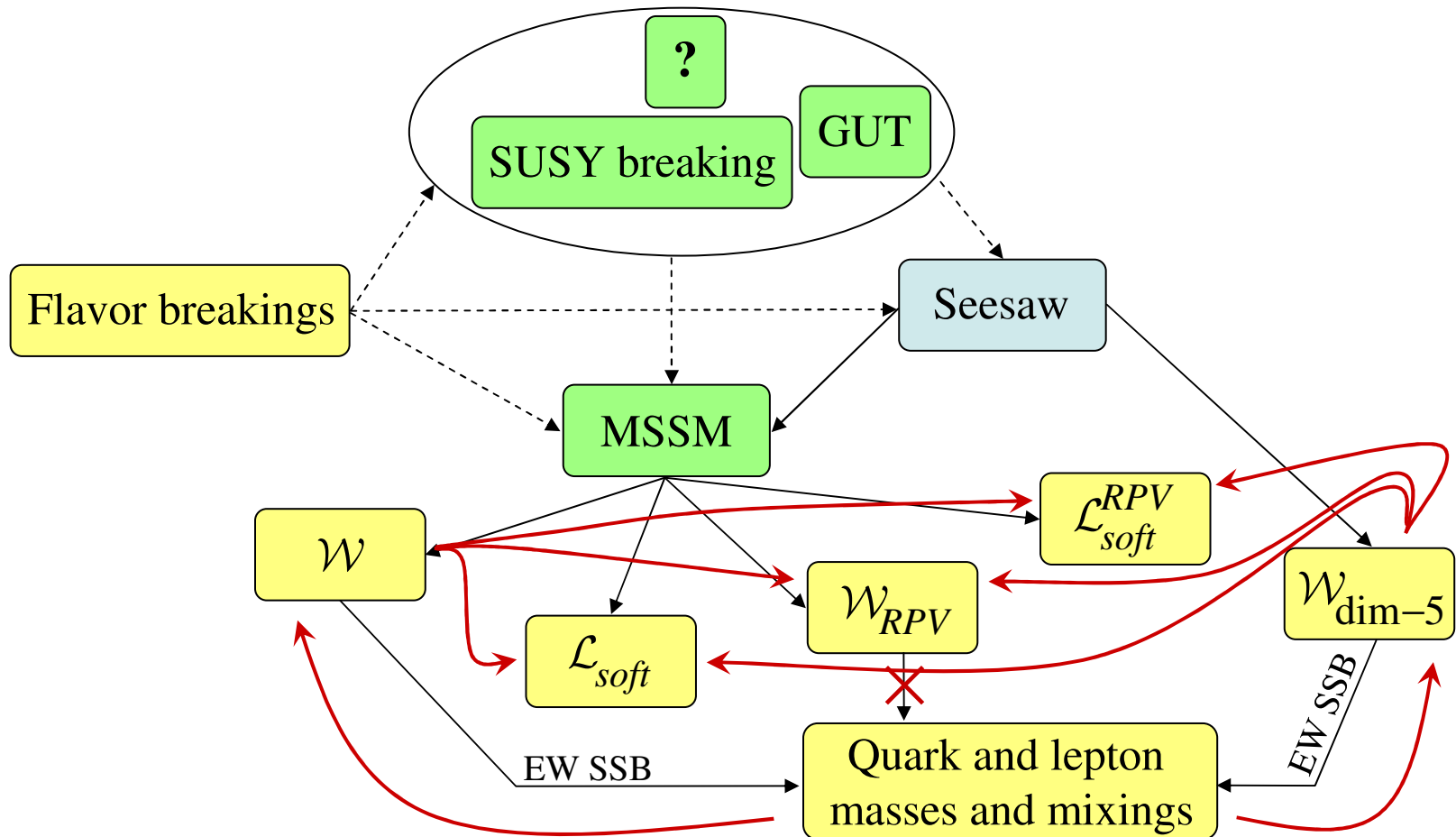
Some mechanism beyond the MSSM must explain the *origin of the flavor structures*.

# A. MFV and the origin of the flavor structures:



If this mechanism is turned off, flavor-breaking terms become forbidden.

# A. MFV and the origin of the flavor structures:



With MFV, all the flavor-breaking couplings are reconstructed in terms of the fermion masses and mixings, and become *naturally hierarchical*.

*B. In practice:*

- *Minimality hypothesis*: Minimal spurion content allowing for the known fermion masses and mixing - *this is the essence of MFV!*

Essentially, the Yukawas  $Y_u, Y_d, Y_e$  plus a few seesaw spurions.

- *Symmetry principle*: All Lagrangian couplings written as formal  $G_f$ -invariants

$$\mathbf{m}_Q^2 = m_0^2 (a_0 \mathbf{1} + a_1 Y_u^\dagger Y_u + a_2 Y_d^\dagger Y_d + \dots) \quad \text{with } a_i \sim \mathcal{O}(1) \quad \leftarrow \text{naturalness}$$

- *Freezing of the spurions* at their physical values:

Hall, Randall '90  
D'Ambrosio, Giudice,  
Isidori, Strumia '02

$$\mathbf{m}_Q^2 \sim m_0^2 \left( \begin{pmatrix} 1 & 10^{-4} & 10^{-3} \\ 10^{-4} & 1 & 10^{-2} \\ 10^{-3} & 10^{-2} & 1 \end{pmatrix} + i \begin{pmatrix} 0 & 10^{-4} & 10^{-3} \\ 10^{-4} & 0 & 10^{-4} \\ 10^{-3} & 10^{-4} & 0 \end{pmatrix} \right)$$

These hierarchies come entirely from those of  $Y_u, Y_d$ .

### C. MFV expansions in the quark sector

Hall, Randall '90, D'Ambrosio, Giudice, Isidori, Strumia '02, Colangelo, Nikolidakis, CS '08

- Only a *finite number* of terms thanks to Cayley-Hamilton identity:

$$\mathbf{X}^3 - \langle \mathbf{X} \rangle \mathbf{X}^2 + \frac{1}{2} \mathbf{X} (\langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle) = \frac{1}{3} \langle \mathbf{X}^3 \rangle - \frac{1}{2} \langle \mathbf{X} \rangle \langle \mathbf{X}^2 \rangle + \frac{1}{6} \langle \mathbf{X} \rangle^3$$

- Use the large *mass hierarchy* to set  $(\mathbf{Y}_i^\dagger \mathbf{Y}_i)^2 \sim \mathbf{Y}_i^\dagger \mathbf{Y}_i$ , leaving:

$$\mathbf{m}_Q^2 = m_0^2 (a_1 \mathbf{1} + a_2 \mathbf{A} + a_3 \mathbf{B} + a_4 \{ \mathbf{A}, \mathbf{B} \} + b_1 i [\mathbf{A}, \mathbf{B}])$$

$$\mathbf{m}_U^2 = m_0^2 (a_5 \mathbf{1} + \mathbf{Y}_u (a_6 \mathbf{1} + a_7 \mathbf{B} + a_8 \{ \mathbf{A}, \mathbf{B} \} + b_2 i [\mathbf{A}, \mathbf{B}]) \mathbf{Y}_u^\dagger)$$

$$\mathbf{m}_D^2 = m_0^2 (a_9 \mathbf{1} + \mathbf{Y}_d (a_{10} \mathbf{1} + a_{11} \mathbf{A} + a_{12} \{ \mathbf{A}, \mathbf{B} \} + b_3 i [\mathbf{A}, \mathbf{B}]) \mathbf{Y}_d^\dagger)$$

$$\mathbf{A}_u = A_0 \mathbf{Y}_u (c_1 \mathbf{1} + c_2 \mathbf{A} + c_3 \mathbf{B} + c_4 \{ \mathbf{A}, \mathbf{B} \} + d_1 i [\mathbf{A}, \mathbf{B}])$$

$$\mathbf{A}_d = A_0 \mathbf{Y}_d (c_5 \mathbf{1} + c_6 \mathbf{A} + c_7 \mathbf{B} + c_8 \{ \mathbf{A}, \mathbf{B} \} + d_2 i [\mathbf{A}, \mathbf{B}])$$

$$\mathbf{A} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u$$

$$\mathbf{B} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d$$

Using CH identities, all operators can be written as hermitian,

hence  $a_i, b_i \in \mathbb{R}$ ,  $c_i, d_i \in \mathbb{C}$  since scalar mass terms are hermitian.

## D. MFV expansions in the lepton sector

Cirigliano, Grinstein  
Isidori, Wise '05

- Integrating out the right-handed neutrinos:

$$Y_e, \quad Y_\nu^\dagger Y_\nu, \quad Y_\nu^T M^{-1} Y_\nu, \quad Y_\nu^\dagger M^{-1*} M^{-1} Y_\nu, \quad \dots$$

Lepton masses:  $v_d Y_e = m_e$

Neutrino masses:  $v_u^2 Y_\nu^T M^{-1} Y_\nu = U^* m_\nu U^\dagger$

Not completely fixed (we take  $M = M_R 1$ ):

$$v_u^2 Y_\nu^\dagger Y_\nu = M_R U^* m_\nu^{1/2} e^{2i\Phi} m_\nu^{1/2} U^\dagger, \quad \Phi^{IJ} = \varepsilon^{IJK} \phi_K$$

Casas, Ibarra '01,  
Pascoli, Petcov,  
Yaguna '03,...

- More terms remain since there is no third-generation dominance for  $v_L$ :

$$\mathbf{m}_L^2 = m_0^2 (a_1 1 + a_2 A + a_3 B + a_4 B^2 + a_5 \{A, B\} + a_6 BAB + b_1 i[A, B] + b_2 i[A, B^2] + b_3 i(BAB^2 - B^2 AB))$$

$$A \equiv Y_e^\dagger Y_e$$

$$B \equiv Y_\nu^\dagger Y_\nu$$

Similar for  $\mathbf{m}_E^2$  and  $A_e$ .

Mercolli, CS '09

*E. How to test MFV?*

Colangelo, Nikolidakis, CS '08  
 Nikolidakis '08, Mercolli, CS '09

Generically, all flavor couplings expanded under MFV involve:

$$\begin{aligned}
 Q = & x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} & (\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
 & + x_7 i[\mathbf{A}, \mathbf{B}] + x_8 i[\mathbf{A}, \mathbf{B}^2] + x_9 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B}) & (\mathbf{A} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d, \mathbf{B} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u)
 \end{aligned}$$

The MFV operators form a *complete basis* for the soft-breaking terms.

Allowing the coefficients to take any value  $\rightarrow$  *full MSSM*.

However, the MFV basis is made of *nearly parallel operators*.

A generic matrix expanded in the MFV basis requires *huge coefficients*!

### E. How to test MFV?

Generically, all flavor couplings expanded under MFV involve:

$$\begin{aligned}
 Q = & x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} & (\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
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 \end{aligned}$$

The MFV operators form a *complete basis* for the soft-breaking terms.

Allowing the coefficients to take any value  $\rightarrow$  *full MSSM*.

MFV expansion coefficients versus Mass Insertions:

Same number of free parameters (choice of basis).

**BUT:** to each coefficient corresponds a *whole set of mass insertions*, with a *definite flavor pattern* inherited from those of the spurions.



Permits to *test the naturality* of soft-breaking terms.

## E. How to test MFV?

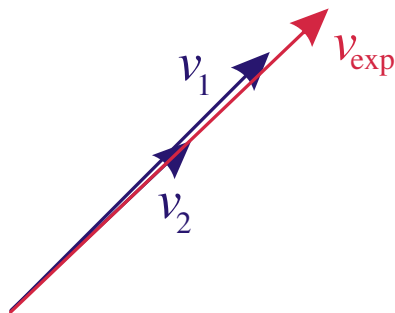
Generically, all flavor couplings expanded under MFV involve:

$$\begin{aligned}
 Q = & x_1 \mathbf{1} + x_2 \mathbf{A} + x_3 \mathbf{B} + x_4 \mathbf{B}^2 + x_5 \{\mathbf{A}, \mathbf{B}\} + x_6 \mathbf{B} \mathbf{A} \mathbf{B} \\
 & + x_7 i[\mathbf{A}, \mathbf{B}] + x_8 i[\mathbf{A}, \mathbf{B}^2] + x_9 i(\mathbf{B} \mathbf{A} \mathbf{B}^2 - \mathbf{B}^2 \mathbf{A} \mathbf{B})
 \end{aligned}
 \quad
 \begin{aligned}
 (\mathbf{A} \equiv \mathbf{Y}_e^\dagger \mathbf{Y}_e, \mathbf{B} \equiv \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu) \\
 (\mathbf{A} \equiv \mathbf{Y}_d^\dagger \mathbf{Y}_d, \mathbf{B} \equiv \mathbf{Y}_u^\dagger \mathbf{Y}_u)
 \end{aligned}$$

Imagine  $Q$  is constrained by experiment (collider + flavor).

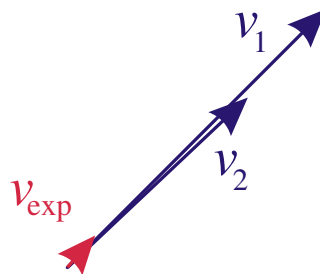
Three possible situations can arise when projecting  $Q$  in the MFV basis:

All the  $x_i \sim O(1)$



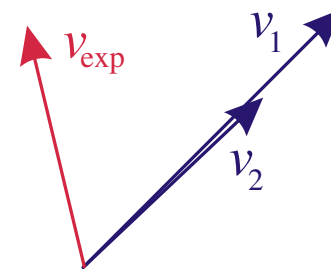
MFV flavor structure

Some of the  $x_i \ll 1$



Fine-tuned flavor structure

Some of the  $x_i \gg 1$



Generic flavor structure

## E. How to test MFV?

Mercolli, C.S. '09

Current experimental constraints on the generic MSSM slepton sector:

$m_L^2$	$(x_i / a_1)$	$m_R^2$	$(x_i / a_7)$	$\text{Re } A_e$	$(x_i / a_1 a_7)$	$\text{Im } A_e$	$(x_i / a_1 a_7)$
$a_1$	<i>free</i>	$a_7$	<i>free</i>	$\text{Re } c_1 \leq 10^2$	<i>stab.</i>	$\text{Im } c_1 \leq 2$	$d_e$
$a_2 \leq 10^3$	<i>masses</i>	$a_8 \leq 10^3$	<i>masses</i>	$\text{Re } c_2 \leq 10^3$	<i>stab.</i>	$\text{Im } c_2 \leq 10^3$	<i>stab.</i>
$a_3 \leq 10$	$\mu \rightarrow e\gamma$	$a_9 \leq 10^6$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_3 \leq 10^3$	$\mu \rightarrow e\gamma$	$\text{Im } c_3 \leq 10^3$	$\mu \rightarrow e\gamma$
$a_4 \leq 10^4$	$\mu \rightarrow e\gamma$	$a_{10} \leq 10^9$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_4 \leq 10^6$	$\mu \rightarrow e\gamma$	$\text{Im } c_4 \leq 10^6$	$\mu \rightarrow e\gamma$
$a_5 \leq 10^3$	$\tau \rightarrow \mu\gamma$	$a_{11} \leq 10^7$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_5 \leq 10^5$	$\tau \rightarrow \mu\gamma$	$\text{Im } c_5 \leq 10^5$	$\tau \rightarrow \mu\gamma$
$a_6 \leq 10^4$	$\mu \rightarrow e\gamma$	$a_{12} \leq 10^{11}$	$\tau \rightarrow \mu\gamma$	$\text{Re } c_6 \leq 10^7$	$\mu \rightarrow e\gamma$	$\text{Im } c_6 \leq 10^7$	$\mu \rightarrow e\gamma$
$b_1 \leq 10^3$	$\tau \rightarrow \mu\gamma$	$b_4 \leq 10^7$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_1 \leq 10^5$	$\tau \rightarrow \mu\gamma$	$\text{Im } d_1 \leq 10^5$	$\tau \rightarrow \mu\gamma$
$b_2 \leq 10^6$	$\tau \rightarrow \mu\gamma$	$b_5 \leq 10^{10}$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_2 \leq 10^8$	$\tau \rightarrow \mu\gamma$	$\text{Im } d_2 \leq 10^8$	$\tau \rightarrow \mu\gamma$
$b_3 \leq 10^8$	$\mu \rightarrow e\gamma$	$b_6 \leq 10^{13}$	$\tau \rightarrow \mu\gamma$	$\text{Re } d_3 \leq 10^{10}$	$\mu \rightarrow e\gamma$	$\text{Im } d_3 \leq 10^{10}$	$\mu \rightarrow e\gamma$

$$M_{\text{SUSY}} \approx 500 \text{ GeV}, \tan \beta = 20, M_R = 10^{12} \text{ GeV}, m_{L,R} \leq 4 \text{ TeV}$$

CPC

CPV

## F. Beyond MFV?

Within MFV, all flavor structures are related to that of the Yukawas.

### Open questions:

- Why are the *Yukawa couplings so hierarchical*?
- Is there a *dynamical mechanism* behind MFV?

There is certainly something behind the Yukawa.

### Explicit symmetry breaking



### Spontaneous symmetry breaking

The approach followed here.

We assume a *minimal number of explicit breaking terms*.

Goldstone bosons?

*Albrecht, Feldmann, Mannel '10*

Discrete flavor symmetries?

*Zwicky, Fischbacher '09*

### III. CP-violation under MFV

## A. CP-violating phases in the MFV approach

Mercolli, C.S. '09

In the SM, CP-violation comes entirely from the phases in the spurions.

One in  $\mathbf{Y}_u$  (Dirac), six in  $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$  (1 Dirac, 2 Majorana, 3 from the  $\phi_K$ )

Within MFV, there are several reasons for expecting *additional CP-phases*:

- The  $U(3)^5$  does not say anything about CP-violating phases,  
All the *MFV coefficients are free complex parameters*.

- There can be new *CP-violating phases in other sectors*,  
*CP-violation is a flavored phenomenon only in the SM!*

- Potentially complex traces  $\langle \mathbf{A}^l \mathbf{B}^m \mathbf{A}^n \dots \rangle$  are  $U(3)^5$  singlets,

Absorbed in the coefficients: forcing them to stay real is a *fine-tuning!*  
(and is not RGE invariant)

## B. Consequence: Is MFV breaking down?

MFV is very effective to *constrain flavor transitions* like  $\ell^I \rightarrow \ell^J$  or  $d^I \rightarrow d^J$ .

But for *flavor-diagonal* operators, there is not much restriction.

- Complex coefficients can induce additional *flavor-diagonal CP-phases*.
- Is this compatible with bounds on EDMs?

$$H_{eff} = C^{IJ} \bar{\Psi}_L^I \sigma_{\mu\nu} \Psi_R^J F^{\mu\nu} + C^{IJ*} \bar{\Psi}_R^J \sigma_{\mu\nu} \Psi_L^I F^{\mu\nu}$$

$I = J$



$$\begin{aligned} H_{eff} &= C \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R F^{\mu\nu} + C^* \bar{\Psi}_R \sigma_{\mu\nu} \Psi_L F^{\mu\nu} \\ &= \text{Re } C \bar{\Psi} \sigma_{\mu\nu} \Psi F^{\mu\nu} + i \text{Im } C \bar{\Psi} \sigma_{\mu\nu} \gamma_5 \Psi F^{\mu\nu} \\ &\equiv e \frac{a}{4m} \qquad \qquad \qquad \equiv \frac{d}{2} \end{aligned}$$

$I \neq J$



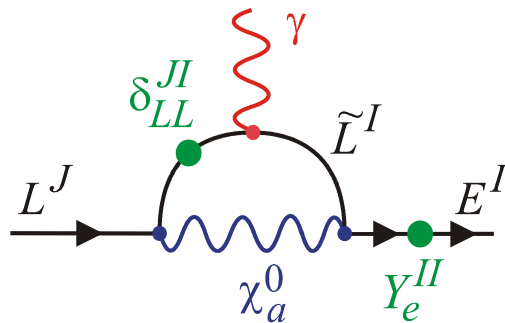
$$B(\Psi^I \rightarrow \Psi^J \gamma) \sim |C^{IJ}|^2$$

## B. Consequence: Is MFV breaking down?

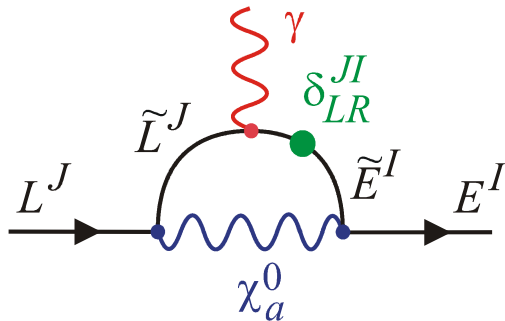
In the MSSM, the flavor-breaking & helicity flip come as

$$H_{eff} \sim E^I \sigma_{\mu\nu} (\mathbf{Y}_e, \mathbf{A}_e, \mathbf{Y}_e \mathbf{m}_L^2, \dots)^{IJ} L^J H_d F^{\mu\nu}$$

Further, this operator arises at one loop:



$$B(\ell^I \rightarrow \ell^J \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{SUSY}^8} |(\mathbf{m}_L^2)^{JI} + \dots|^2$$



Beyond MFV

$$\frac{d_I}{e} \sim \frac{\alpha}{M_{SUSY}^3} \text{Im} \left( m_\ell^I \mu \tan \beta - v_d \mathbf{A}_e^{*II} \right) + \dots$$

Diagonal part of the trilinear terms.

### C. Classification of the CP-phases

$$A \equiv Y_e^\dagger Y_e$$

$$B \equiv Y_\nu^\dagger Y_\nu$$

MFV expansions, with  $a_i, b_i \in \mathbb{R}$ ,  $c_i, d_i \in \mathbb{C}$  :

$$\mathbf{m}_L^2 = m_0^2 (a_1 \mathbf{1} + a_2 A + a_3 B + a_5 \{A, B\} + a_6 BAB + b_1 i[A, B] + \mathcal{O}(A^2, B^2)),$$

$$\mathbf{m}_E^2 = m_0^2 (a_7 \mathbf{1} + Y_e (a_8 \mathbf{1} + a_9 B + a_{11} \{A, B\} + b_4 i[A, B]) Y_e^\dagger + \mathcal{O}(A^2, B^2)),$$

$$A_e = A_0 Y_e (c_1 \mathbf{1} + c_2 A + c_3 B + c_5 \{A, B\} + d_1 i[A, B] + \mathcal{O}(A^2, B^2))$$

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In the slepton sector: 15 CP-violating coefficients + 6 spurion phases

In the squark sector: 13 CP-violating coefficients + 1 spurion phase

$\Rightarrow$  *Plenty of new CP-phases in MFV!*

### C. Classification of the CP-phases

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*Flavor-blind phase:*  $\text{Im } c_1$  (remember  $d_I \sim \text{Im } A_e^{*II} \sim \text{Im } c_1$ )

Defined relative to the flavor-blind parameters of the MSSM ( $\mu, M_1, M_2, \dots$ )

### C. Classification of the CP-phases

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Defined relative to the flavor-blind parameters of the MSSM ( $\mu, M_1, M_2, \dots$ )

*Flavor-diagonal phases:*  $\text{Im } c_{2-6}$  (remember  $d_I \sim \text{Im } A_e^{*II} \sim \text{Im } c_{2-6}$  )

Contribute to EDMs at leading order in the MIA.

### C. Classification of the CP-phases

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*Flavor-blind phase:*  $\text{Im } c_1$

Defined relative to the flavor-blind parameters of the MSSM ( $\mu, M_1, M_2, \dots$ )

*Flavor-diagonal phases:*  $\text{Im } c_{2-6}$

Contribute to EDMs at leading order in the MIA.

*Flavor off-diagonal phases:*  $b_i, \text{Re } d_i$ , six phases of  $Y_\nu^\dagger Y_\nu$   $\leftarrow$  (hermitian op.)

Start to contribute to EDMs at 2<sup>nd</sup> order in the MIA ( $d_I \sim \text{Im}(\mathbf{m}_L^2)^{IK} (A_e)^{KI}$ ).

### D. Impact on the EDMs and LFV processes

A single operator dominates for  $\mu \rightarrow e \gamma$  (coming from  $\delta_{LL}$ ):

$$B(\mu \rightarrow e \gamma) \sim \frac{\alpha M_W^4 \tan^2 \beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} (\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu)^{12} \right|^2$$

A single operator per type of phases dominates for  $d_e$ :

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left( \frac{\text{Im } c_1}{a_1 a_7} + \frac{\text{Im } c_3}{a_1 a_7} \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu - \frac{b_1 \text{Re } c_3}{a_1^2 a_7} [\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu, \mathbf{Y}_e^\dagger \mathbf{Y}_e] \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \dots \right)^{11}$$

Flavor-blind

Flavor-diagonal

Flavor off-diagonal  
( $\geq$  neutrino phases)

Remark:  $m_L^2 \approx m_0^2 a_1$  ,  $m_R^2 \approx m_0^2 a_7$

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Flavor-blind

Flavor-diagonal

Flavor off-diagonal  
( $\geq$  neutrino phases)

$$M_{SUSY} \approx 500 \text{ GeV}$$

### D. Impact on the EDMs and LFV processes

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Flavor-blind

Flavor-diagonal

Flavor off-diagonal  
( $\geq$  neutrino phases)

$$M_{SUSY} \approx 500 \text{ GeV}$$

$$\Delta m_{21} \approx \sqrt{\Delta m_\odot^2} \approx 10^{-9} - 10^{-11} \text{ GeV}$$

## D. Impact on the EDMs and LFV processes

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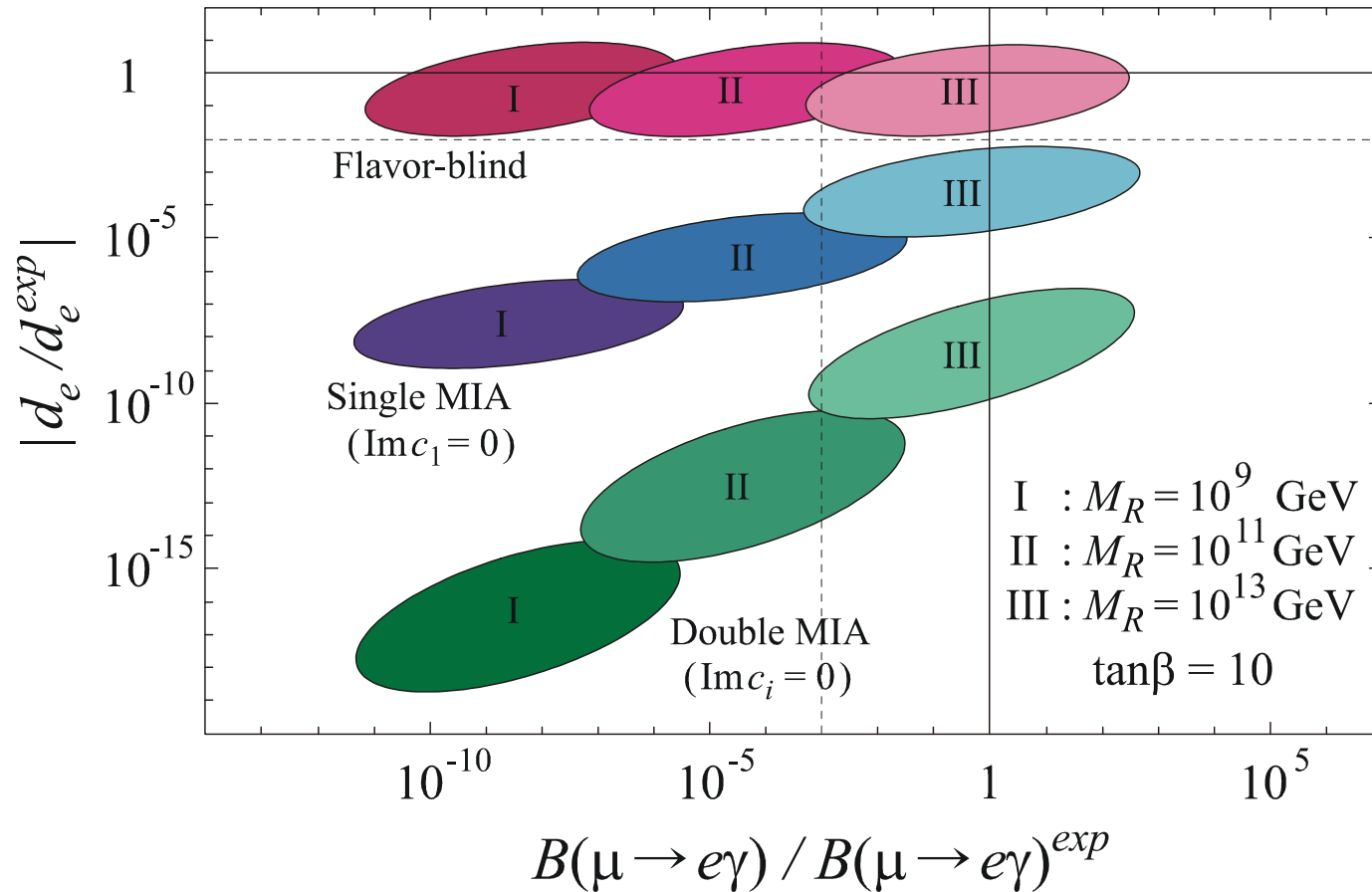
Flavor-blind  $\gg$  Flavor-diagonal  $\gg$  Flavor off-diagonal ( $\geq$  neutrino phases)

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$$\Delta m_{21} \approx \sqrt{\Delta m_\odot^2} \approx 10^{-9} - 10^{-11} \text{ GeV}$$

## D. Phenomenological impacts

Mercolli, C.S. '09



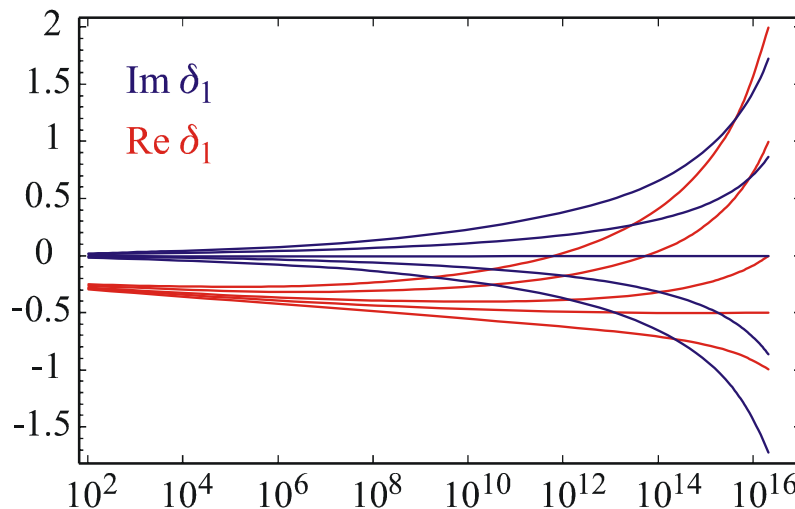
$$M_2 = \pm\mu = 2M_1 = \frac{2}{3}m_0 = A_0 = 400 \text{ GeV}, \quad a_i, b_i, c_i, d_i \in \pm[0.1, 8]$$

## IV. RGE behavior

The MFV expansions are RGE invariant, but to get the RGE invariance of MFV itself requires in addition that the *coefficients must remain of  $\mathcal{O}(1)$  at all scales*.

*Running down* from MFV at the GUT scale:

- *IR fixed-points* for ratios of coefficients  $\leftrightarrow$  predictions for *mass insertions*.
- In particular, all *CP-violating phases* run towards zero (in the quark sector).



$$\delta_1 \equiv \frac{(\delta_{RL}^U)^{32}}{V_{ts}} = \frac{(\delta_{RL}^U)^{31}}{V_{td}}$$

Paradisi, Ratz, Schieren, Simonetto '08  
Colangelo, Nikolidakis, C.S. '08

*Running up* from MFV at the EW scale:

- MFV is lost at the GUT scale if one starts far enough from the fixed points.  
(*some ratios of coefficients explode*)

## IV. MFV and proton decay

## A. MFV expansions and the flavor $U(1)$ symmetries

Assume that the high-energy dynamics violates  $\mathcal{B}$  and/or  $\mathcal{L}$ .

We want to parametrize the RPV couplings in terms of the spurions:

$$\mathcal{W}_{RPV} = \underbrace{\mu'^I L^I H_d + \lambda^{IJK} L^I L^J E^K + \lambda'^{IJK} L^I Q^J D^K}_{\Delta\mathcal{L}=1} + \underbrace{\lambda''^{IJK} U^I D^J D^K}_{\Delta\mathcal{B}=1}$$

Odd number of flavor indices  $\rightarrow$  *MFV under  $SU(3)^5$  instead of  $U(3)^5$ ,*  
and *use  $\varepsilon$ -tensors to form invariants.*

Expected since  $\mathcal{B}$  and  $\mathcal{L}$  are combinations of the flavor  $U(1)$ 's:

$$\begin{aligned} G_f &= SU(3)^5 \times U(1)_Q \times U(1)_U \times U(1)_D \times U(1)_L \times U(1)_E \\ &= SU(3)^5 \times U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}} \times U(1)_Y \times U(1)_{PQ} \times U(1)_E \end{aligned}$$

But note: It is not needed to break all five  $U(1)$ 's!

## B. Intrinsic difference between $\Delta\mathcal{L}=1$ and $\Delta\mathcal{B}=1$ couplings

- The  $\mathcal{B}$  violating couplings can be constructed using  $\Delta\mathcal{B}=0$  quark Yukawas:

$$\begin{aligned}\lambda''^{IJK} &= \epsilon^{LJK} (Y_u Y_d^\dagger)^{IL} \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_D^\dagger) \lambda''^{IJK} U^I D^J D^K \\ \lambda''^{IJK} &= \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN} \Rightarrow \lambda''^{IJK} U^I D^J D^K \rightarrow \det(g_Q^\dagger) \lambda''^{IJK} U^I D^J D^K \\ &\dots\end{aligned}$$

- But  $\mathcal{L}$  violating couplings are strictly forbidden as long as  $m_\nu = 0$ :

The SU(3) combinatorics demand a spurion transforming like a six.

The only spurion available is the suppressed  $\Delta\mathcal{L}=2$  Majorana mass term:

$$\Upsilon_\nu \equiv v_u Y_\nu^T M^{-1} Y_\nu \rightarrow g_L^* \Upsilon_\nu g_L^\dagger \sim 6_{SU(3)_L} \otimes 1_{SU(3)_E}$$

*All  $\Delta\mathcal{L}=1$  couplings are suppressed by neutrino masses!!!*

### C. What happens in the SM?

- *No renormalizable interaction* can break  $\mathcal{B}$  or  $\mathcal{L}$ .
- Model-independent dimension-six  $\Delta\mathcal{B}$  and  $\Delta\mathcal{L}$  operators:

$$\mathcal{L}_{\Delta(\mathcal{B}+\mathcal{L})} = \frac{\epsilon_{abc}}{\Lambda^2} \left( c_1^{IJKL} L^I Q_a^J Q_b^K Q_c^L + c_2^{IJKL} E^I U_a^J U_b^K D_c^L \right. \\ \left. + c_3^{IJKL} E^I U_a^J Q_b^{\dagger K} Q_c^{\dagger L} + c_4^{IJKL} L^I Q_a^J D_b^{\dagger K} U_c^{\dagger L} \right) \quad \text{Weinberg '79}$$

→ Under MFV, forbidden when  $m_\nu = 0$  and thus *very suppressed!*

- Highly-suppressed instanton effects break  $\mathcal{B}+\mathcal{L}$  :

$$\mathcal{L}_{\Delta(\mathcal{B}+\mathcal{L})} \sim e^{-4\pi \sin^2 \theta_W / \alpha} \left( \epsilon_{IJK} L^I L^J L^K \right) \left( \epsilon_{IJK} Q^I Q^J Q^K \right)^3 \quad t'Hooft '76$$

Of course, it *respects MFV!* But under  $G_f = SU(3)^5 \times U(1)_U \times U(1)_D \times U(1)_E$ .

## D. MFV for the R-parity violating terms of the MSSM:

Nikolidakis, C.S. '07

Structures ( $\mathcal{W}_{RPV} = \mu' LH_d + \lambda LLE + \lambda' LQD + \lambda'' UDD$ )		Scaling	Breaking
$\mu_1^I$	$\mu \epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST}, \dots$	$\tan^2 \beta$	$U(1)_L$
$\lambda_1^{IJK}$	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} (Y_e)^{KJ}, \dots$	$\tan^3 \beta$	$U(1)_L$
$\lambda_2^{IJK}$	$\epsilon^{IMJ} (Y_e Y_\nu^\dagger)^{KM}, \dots$	$\tan \beta$	$U(1)_L$
$\lambda_3^{IJK}$	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} \epsilon^{LMJ} \epsilon^{ABK} (Y_e^\dagger)^{LA} (Y_e^\dagger)^{MB}, \dots$	$\tan^4 \beta$	$U(1)_{L,E}$
$\lambda_1'^{IJK}$	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} (Y_d)^{KJ}, \dots$	$\tan^3 \beta$	$U(1)_L$
$\lambda_2'^{IJK}$	$\epsilon^{STI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{ST} \epsilon^{LMJ} \epsilon^{ABK} (Y_d^\dagger)^{LA} (Y_d^\dagger)^{MB}, \dots$	$\tan^4 \beta$	$U(1)_{L,D,Q}$
$\lambda_1''^{IJK}$	$\epsilon^{LJK} (Y_u Y_d^\dagger)^{IL}, \dots$	$\tan \beta$	$U(1)_D$
$\lambda_2''^{IJK}$	$\epsilon^{IMN} (Y_d Y_u^\dagger)^{JM} (Y_d Y_u^\dagger)^{KN}, \dots$	$\tan^2 \beta$	$U(1)_U$
$\lambda_3''^{IJK}$	$\epsilon^{LMN} (Y_u)^{IL} (Y_d)^{JM} (Y_d)^{KN}, \dots$	$\tan^2 \beta$	$U(1)_Q$
$\lambda_4''^{IJK}$	$\epsilon^{LMN} \epsilon^{ABI} \epsilon^{CJK} (Y_d^\dagger)^{LC} (Y_u^\dagger)^{MA} (Y_u^\dagger)^{NB}, \dots$	$\tan \beta$	$U(1)_{Q,U,D}$

(Similar expansions for R-parity violating soft-breaking terms)

### D. Check of the bounds on R-parity violating couplings

In addition to the *neutrino mass factor*  $\Upsilon_\nu \sim \mathcal{O}(m_\nu / v_u) \sim \mathcal{O}(10^{-12})$ ,  $\varepsilon$ -tensor antisymmetry forces all couplings to be proportional to *light-fermion masses*:

$$\text{Ex: } \varepsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN} \rightarrow \varepsilon^{123} Y_u^{I1} Y_d^{J2} Y_d^{K3} + \dots \rightarrow \frac{m_u}{v_u} \frac{m_s}{v_d} \frac{m_b}{v_d} + \dots$$

*Are these two mechanisms sufficient to pass experimental bounds ?*

Hundreds of bounds, most rather weak and immediately satisfied.

(LFV & FCNC, EDM's,  $n - \bar{n}$  oscillations, EWPO from LEP, Tevatron,...)

Toughest constraints from  $\Delta\mathcal{B} = 1$  *nucleon decays, i.e.*  $p, n \rightarrow \pi\nu, \pi\ell, K\nu, K\ell, \dots$

Bounds on various combinations  $|(\mu', \lambda, \lambda') \times \lambda''|$ ,

For some  $IJK, I'J'K'$ , as constraining as  $|\lambda'_{IJK} \lambda''_{I'J'K'}| < 10^{-25} - 10^{-27}$ .

*Example of MFV suppression for a specific proton decay mechanism*

$$\Rightarrow |\lambda'_{JMI} \lambda''_{11I}| < 10^{-26} (m[\tilde{d}_R^I] / 300 \text{ GeV})^2$$

If the leading operators are:  $\lambda' : (a_0 \epsilon^{LMI} (\mathbf{Y}_e^\dagger \mathbf{Y}_e \mathbf{Y}_\nu^\dagger)^{LM} (\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u)^{KJ}) L^I Q^J D^K$   
 $\lambda'' : (a_1 \epsilon^{LJK} (\mathbf{Y}_u \mathbf{Y}_d^\dagger)^{IL} + a_2 \epsilon^{LMN} \mathbf{Y}_u^{IL} \mathbf{Y}_d^{JM} \mathbf{Y}_d^{KN}) U^I D^J D^K$

The MFV prediction is then

$$|\lambda'_{JMI} \lambda''_{11I}| \approx \frac{\Delta m_{31}}{v_u} \frac{m_\tau^2}{v_d^2} \lambda^3 \frac{m_b m_t^2 m_u}{v_d v_u^3} \left( a_0 a_1 \frac{m_s}{v_d} + a_0 a_2 \frac{m_d m_b}{v_d^2} \right)$$

Example of MFV suppression for a specific proton decay mechanism

$$\Rightarrow |\lambda'_{JMI} \lambda_{11I}^{''*}| < 10^{-26} (m[\tilde{d}_R^I] / 300 \text{ GeV})^2$$

If the leading operators are:  $\lambda' : (a_0 \epsilon^{LMI} (\Upsilon_e^\dagger \Upsilon_e \Upsilon_\nu^\dagger)^{LM} (\Upsilon_d \Upsilon_u^\dagger \Upsilon_u)^{KJ}) L^I Q^J D^K$   
 $\lambda'' : (a_1 \epsilon^{LJK} (\Upsilon_u \Upsilon_d^\dagger)^{IL} + a_2 \epsilon^{LMN} \Upsilon_u^{IL} \Upsilon_d^{JM} \Upsilon_d^{KN}) U^I D^J D^K$

The MFV prediction is then

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Neutrino mass, related to  $\Delta m_{atm}^2 \approx 3 \cdot 10^{-3} \text{ eV}^2$

Charged lepton mass

CKM factors

Light-quark masses

Symmetry of  $\Upsilon_\nu$ , the Majorana mass term

Antisymmetry of  $\epsilon$  tensors

*Example of MFV suppression for a specific proton decay mechanism*

$$\Rightarrow |\lambda'_{JMI} \lambda''^{*}_{11I}| < 10^{-26} (m[\tilde{d}_R^I] / 300 \text{ GeV})^2$$

If the leading operators are:  $\lambda' : (a_0 \epsilon^{LMI} (Y_e^\dagger Y_e Y_\nu^\dagger)^{LM} (Y_d Y_u^\dagger Y_u)^{KJ}) L^I Q^J D^K$   
 $\lambda'' : (a_1 \epsilon^{LJK} (Y_u Y_d^\dagger)^{IL} + a_2 \epsilon^{LMN} Y_u^{IL} Y_d^{JM} Y_d^{KN}) U^I D^J D^K$

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$$\approx a_0 a_1 10^{-28} \tan^4 \beta + a_0 a_2 10^{-31} \tan^5 \beta \quad (\text{for } m_\nu^{\text{lightest}} = 0)$$

*Conservatively, MFV can account for the necessary suppression.*

- MFV coefficients of  $\mathcal{O}(1)$ , while  $\mathcal{O}(\lambda)$  or  $\mathcal{O}(g^2 / 4\pi)$  also natural,
- No GIM-like interferences, no cancellations among processes,

### E. Where to expect significant experimental signals ?

1. Proton decay could be close to current bounds (worthy to pursue the search!)
2. Except for proton decay, lepton-number effectively conserved.  
(since  $\mu', \lambda, \lambda' < \mathcal{O}(10^{-12})$ )
3. MFV predictions for the baryonic couplings  $\epsilon^{abc} \lambda''^{IJK} U_a^I D_b^J D_c^K$  :

Structure	$\lambda_1''$	$\lambda_2''$	$\lambda_3''$	$\lambda_{4,5}''$
Broken $U(1)$	$U(1)_D$	$U(1)_U$	$U(1)_Q$	$U(1)_{U,D,Q}$
$\tan \beta = 5$	$\begin{pmatrix} 8 & 8 & 8 \\ 4 & 6 & 5 \\ \mathbf{1} & 6 & 4 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 7 \\ 12 & 9 & 9 \\ 13 & 12 & 13 \end{pmatrix}$	$\begin{pmatrix} 13 & 8 & 10 \\ 10 & 6 & 7 \\ 6 & 5 & 6 \end{pmatrix}$	$\begin{pmatrix} 5 & 5 & 5 \\ 7 & 9 & 7 \\ 7 & 12 & 10 \end{pmatrix}$
$\tan \beta = 50$	$\begin{pmatrix} 7 & 7 & 7 \\ 3 & 5 & 4 \\ \mathbf{0} & 5 & 3 \end{pmatrix}$	$\begin{pmatrix} 9 & 4 & 5 \\ 10 & 7 & 7 \\ 11 & 10 & 11 \end{pmatrix}$	$\begin{pmatrix} 11 & 6 & 8 \\ 8 & 4 & 5 \\ 4 & 3 & 4 \end{pmatrix}$	$\begin{pmatrix} 4 & 4 & 4 \\ 6 & 8 & 6 \\ 6 & 11 & 9 \end{pmatrix}$

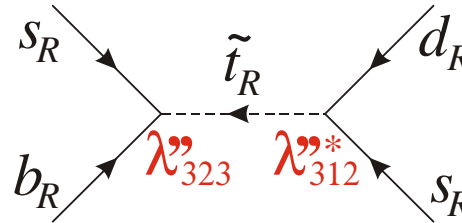
Notations :  
 $x \equiv \mathcal{O}(10^{-x})$

$$\begin{pmatrix} 112 & 123 & 131 \\ 212 & 223 & 231 \\ \mathbf{312} & 323 & 331 \end{pmatrix}$$

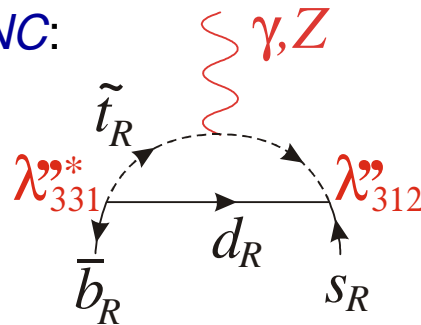
$\hookrightarrow \lambda_{312}'' \longrightarrow$  Sizeable  $\tilde{t}_R d_R s_R, t_R \tilde{d}_R s_R, t_R d_R \tilde{s}_R$  couplings.

#### 4. Probing $\Delta B = 1$ interactions at low-energy:

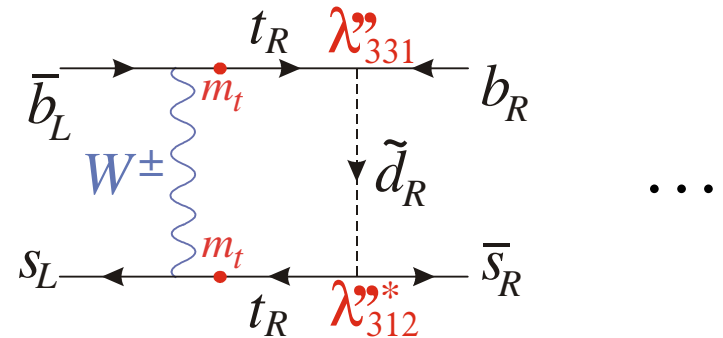
- Squarks as *diquark currents*:



- Induce *new FCNC*:



Chakraverty, Choudhury '01, ...



Barbieri, Masiero '86, Slavich '00, ...

- *With MFV*, these are *typically small* compared to the SM contributions:

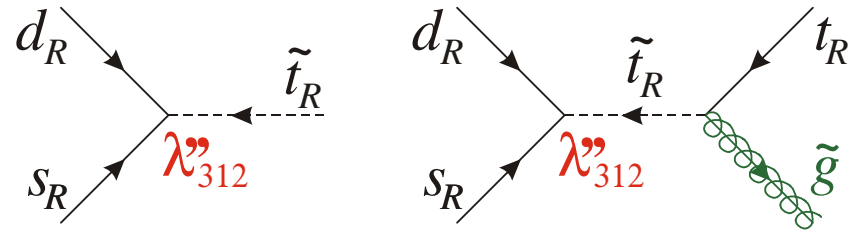
$$b \rightarrow s : |\lambda''_{312} \lambda''^*_{331}| < 10^{-3}, \quad b \rightarrow d : |\lambda''_{312} \lambda''^*_{323}| < 10^{-5}, \quad s \rightarrow d : |\lambda''_{313} \lambda''^*_{323}| < 10^{-8}$$

$$b \rightarrow s : |V_{tb}^* V_{ts}| \sim 10^{-2}, \quad b \rightarrow d : |V_{tb}^* V_{td}| \sim 10^{-3}, \quad s \rightarrow d : |V_{ts}^* V_{td}| \sim 10^{-4}$$

## 5. Probing $\Delta B = 1$ effects at colliders: drastic changes for the phenomenology.

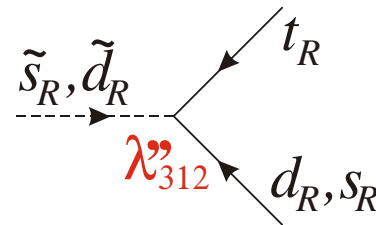
- *Single stop* resonant production and associated *single gluino* production:

*Dimopoulos, Hall '88, Dreiner, Ross '91, Chaichan et al. '00, Allanach et al. '01,...*

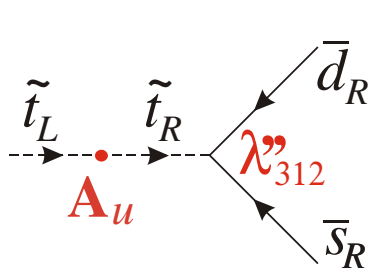


- *Top production*, from squark decay:

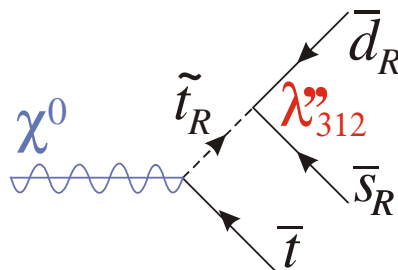
*Berger et al. '99, Chiappetta et al. '99,...*



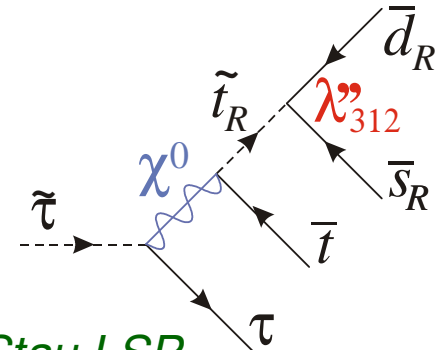
- *LSP* not necessarily colorless & neutral, and will *decay*, maybe in the detector:



*Stop LSP*



*Neutralino LSP*



*Stau LSP*

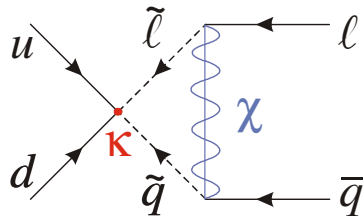
*For a review of these and other possible signals, see e.g. Barbier et al. '05.*

## F. R-parity ☹️ or not R-parity 😊?

- *Avoiding proton decay* is no longer a good motivation for R-parity. 😊

- *Dim-5 R-parity conserving operators* can also induce proton decay: 😊

Ibanez,  
Ross '92



$$\mathcal{W}_{\text{dim-5}} \ni \frac{\kappa_1^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}} (Q^I Q^J)(Q^K L^L) + \frac{\kappa_2^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}} (D^I U^J U^K) E^L$$

MFV separately suppresses  $\Delta\mathcal{L} = 1$  and  $\Delta\mathcal{B} = 1$  effects.

- *GUT*: R-parity often built in ( $SO(10)$ -GUT) or required ( $SU(5)$ -GUT). ☹️

Example:  $G_f = U(3)_{\bar{5}} \times U(3)_{10} : Y_{\bar{5}} \sim (\bar{3}, \bar{3}), Y_{10} \sim (1, \bar{6})$

Seesaw spurion not required for  $\mathcal{W}_{RPV} = \Lambda^{IJK} \bar{5}^I \bar{5}^J 10^K + \dots$

Cirigliano, Grinstein,  
Isidori, Wise '05

- *Cosmology*: ☹️ MSSM-LSP not stable  $\rightarrow$  nature of dark matter still to be resolved.  
😊 Baryon asymmetry generated from CPV,  $\Delta\mathcal{B} = 1$  couplings?

*Should experimentalists accept the burden of R-parity “only” for dark matter???*

Conclusion

*MFV, as a phenomenological hypothesis on the elementary flavor structures:*

A single mechanism explaining: - *Smallness of susy effects in FCNC*  
- *Extremely long proton lifetime*

Consequences of the *Yukawa hierarchies* and of the *small neutrino masses*.

*MFV, as a window into physics beyond the MSSM:*

It permits to *identify the flavor couplings which are fine-tuned* (none at present) out of those which are as “natural” as the SM Yukawas.

In particular, *the proton lifetime does not require fine-tuned RPV couplings!*

Since a *consistent picture emerges with only a few spurions*, the mechanism behind all the flavor structures could be relatively simple.

*CP-violation is controlled by non-MFV physics*, as expected from  $Arg(\mu) \ll 1$ .