CALCULATIONS OF COUPLING IN MULTI-CAVITY SCRF STRUCTURES

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MULTI-CAVITY COUPLING

A single cell has the usual mode spectrum

TM_{mnp} , TE_{mnp}

Coupled cell (e.g. in a multi-cell cavity)

Modes split into passbands

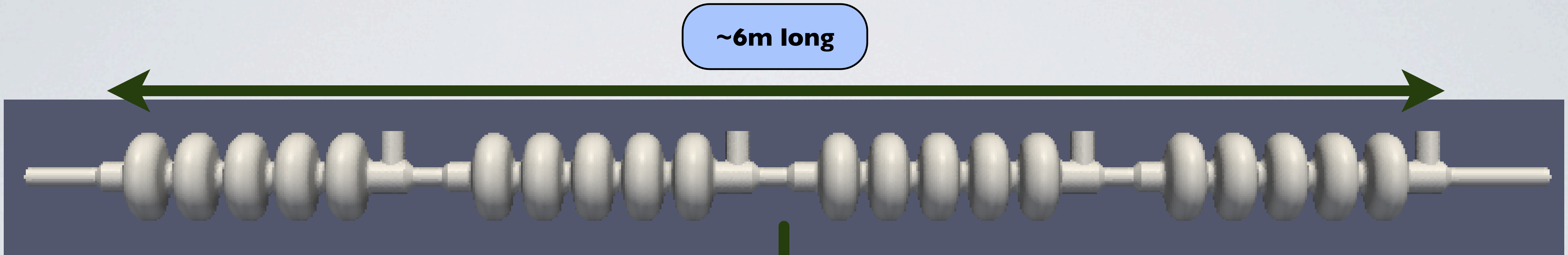
Each oscillation characterised by a phase advance per cell

Multi-cavity installations (i.e. a cryomodule)

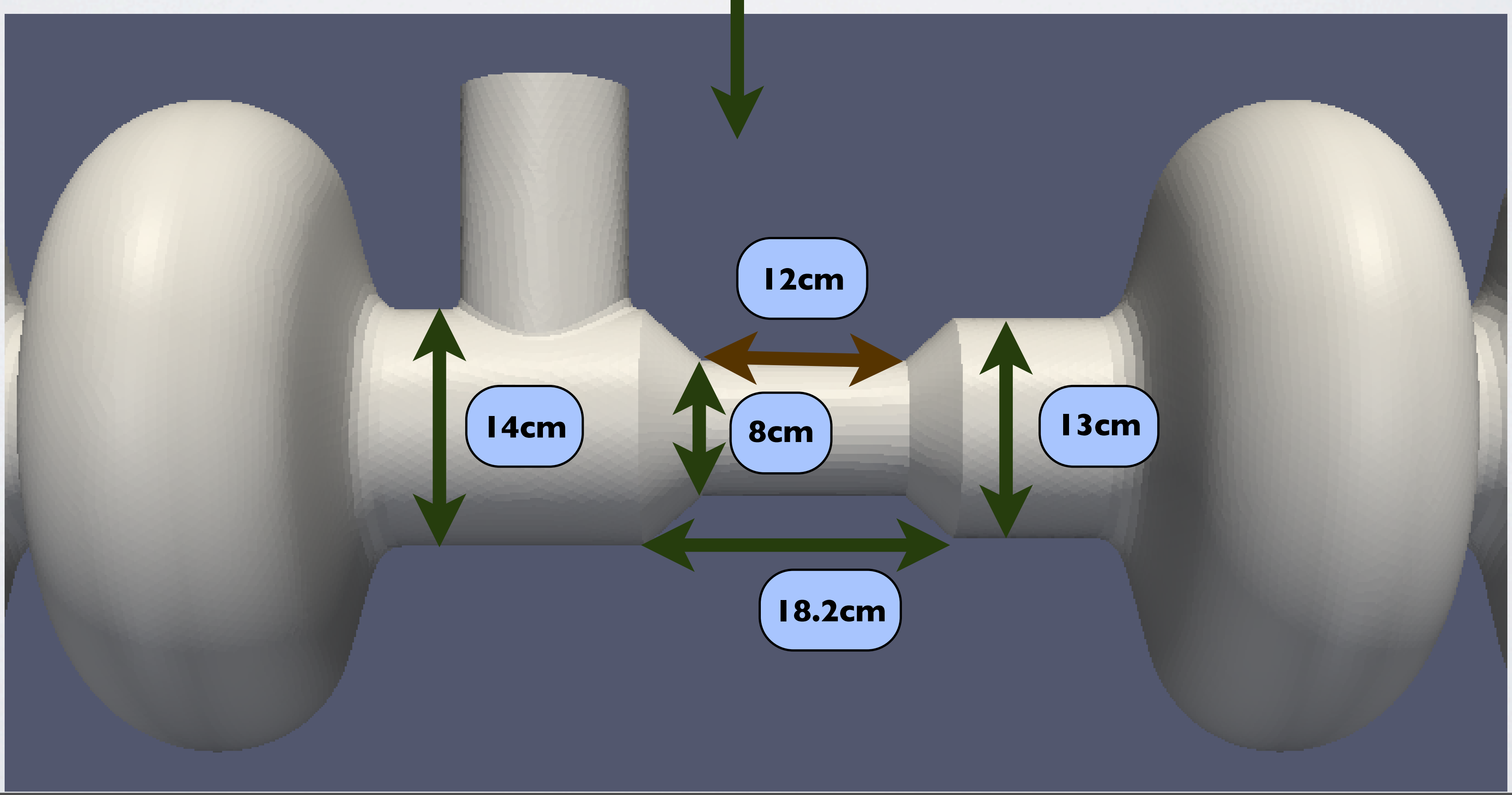
Modes below cutoff so disregarded

But this neglects evanescent coupling!

EIGENSOLVE 4 FULL CAVITIES



~880k elements
Average volume = 1.96 x 10^{-7} m^3
Min edge length = 2mm
Max edge length = 24mm
Magnetic symmetry plane



OMEGA3P SIM

Eigensolver in frequency domain

Part of the ACE3P suite developed at SLAC

- Highly parallelised EM codes

Franklin supercomputer at NERSC

- 38000 computer cores

INTRA-CAVITY COUPLING

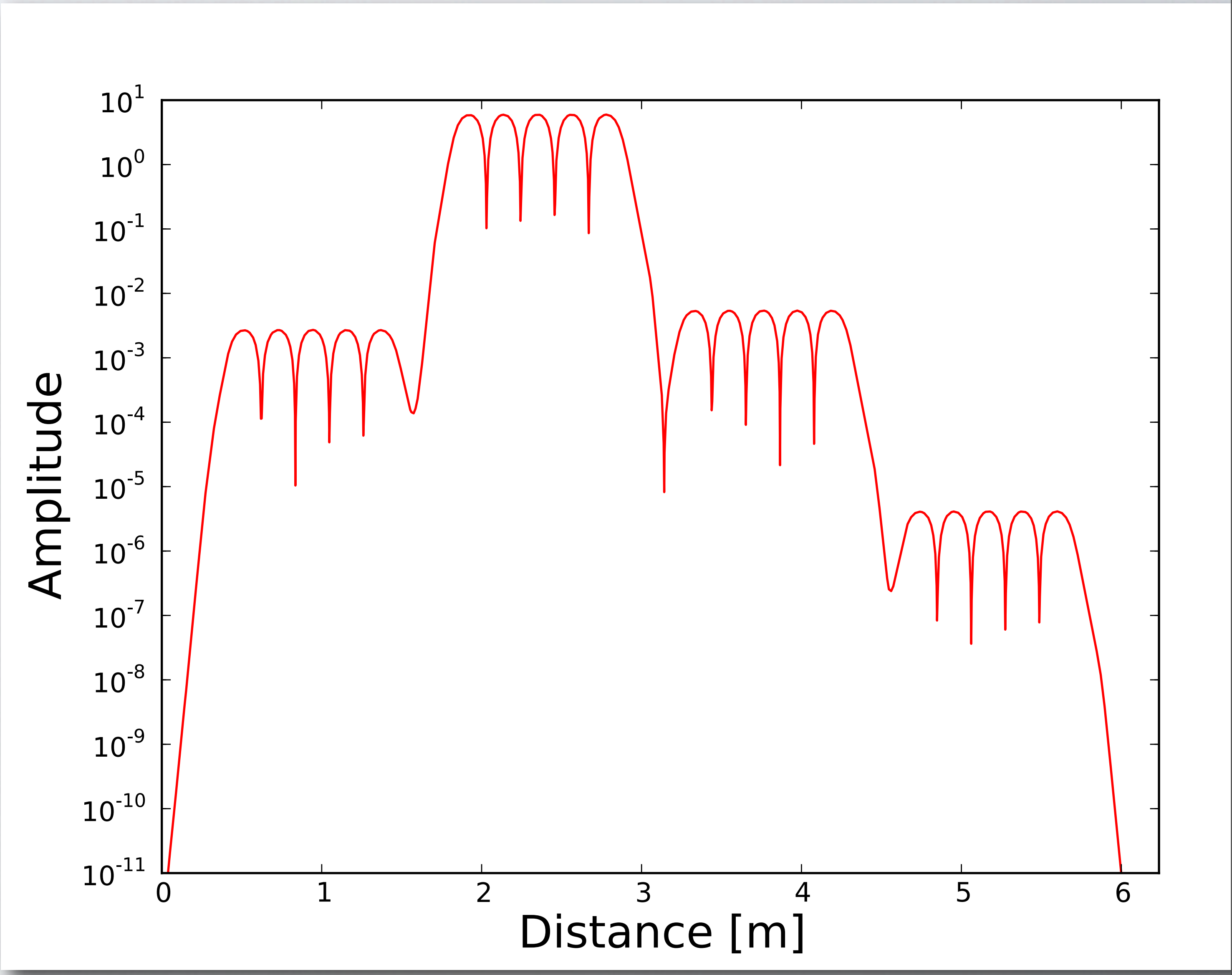
Finding the first 100 modes of a four cavity sim uses ~2000 cpu hrs

Each cavity mode will be found four times

one for each cavity

A single cavity will dominate each mode however the evanescent field allows coupling

Need to extract coupling from simulations



COUPLED OSCILLATORS

Eigenmodes of coupled oscillators split according to the phase difference

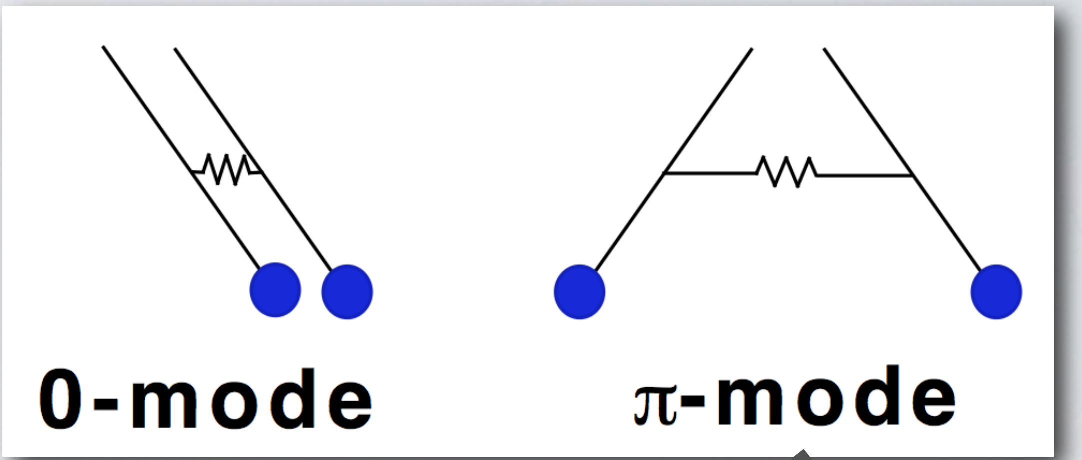
'0'-mode, 'π'-mode, etc.

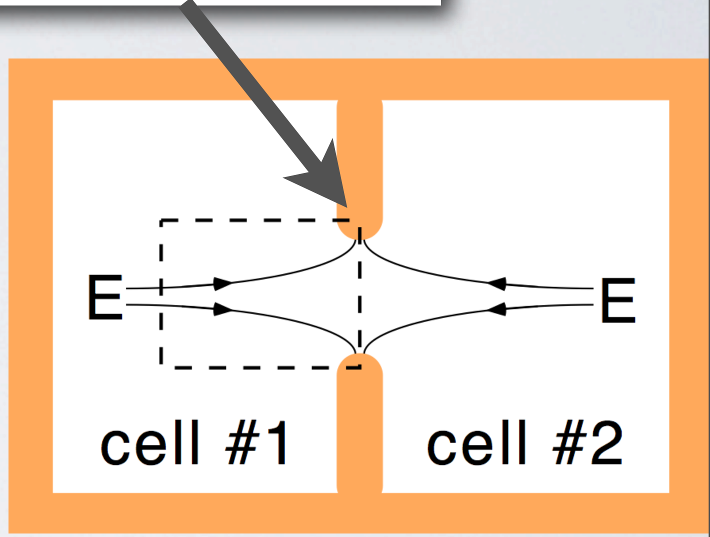
For N+1 coupled oscillators

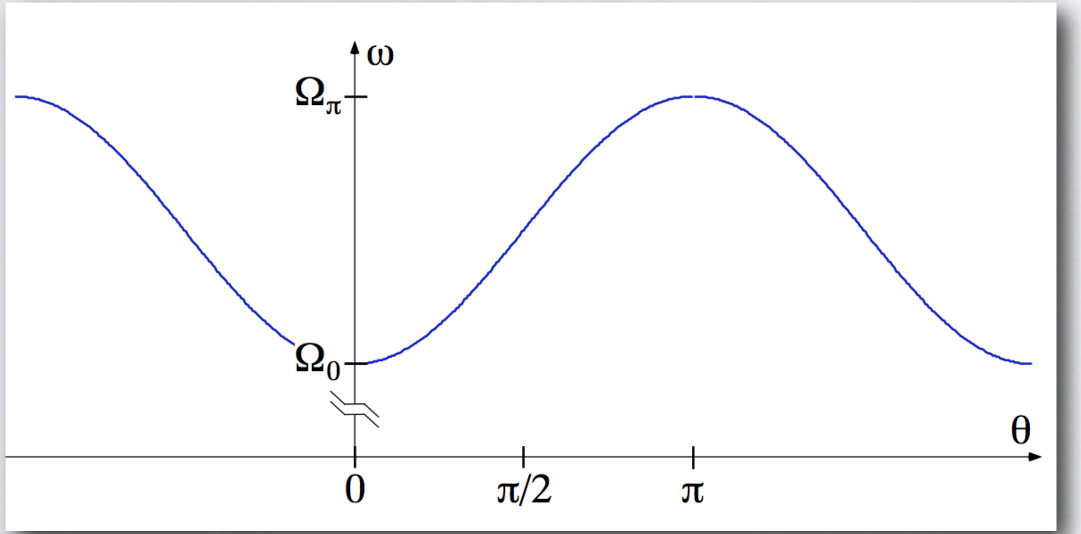
iπ/N radians phase advance (i=0,1,2,..N)

Frequency also splits

- Dependant of coupling strength
- Each new mode may be plotted on a Brillouin curve
  - For N < ∞ the modes are equally spaced along the curve



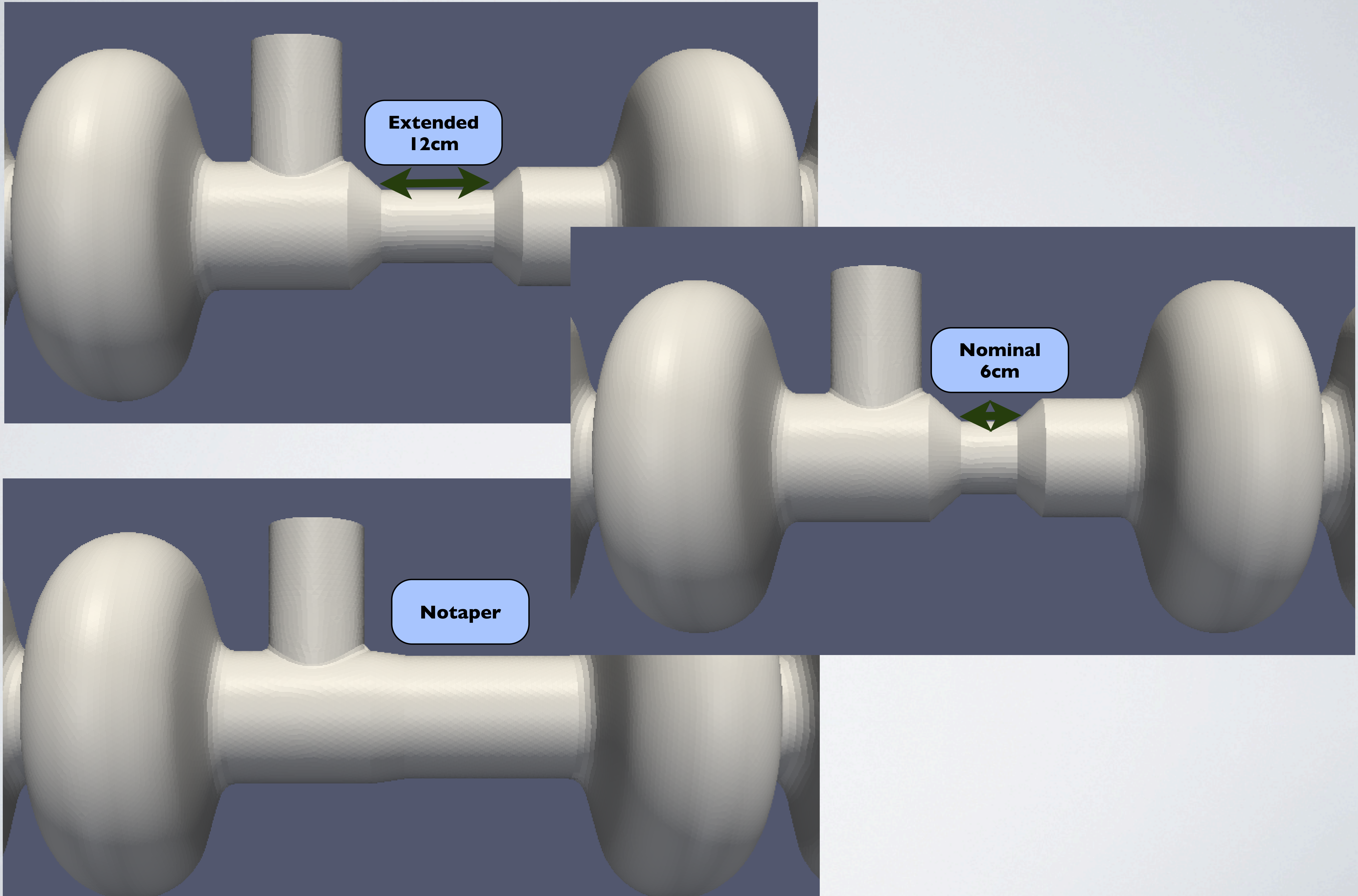


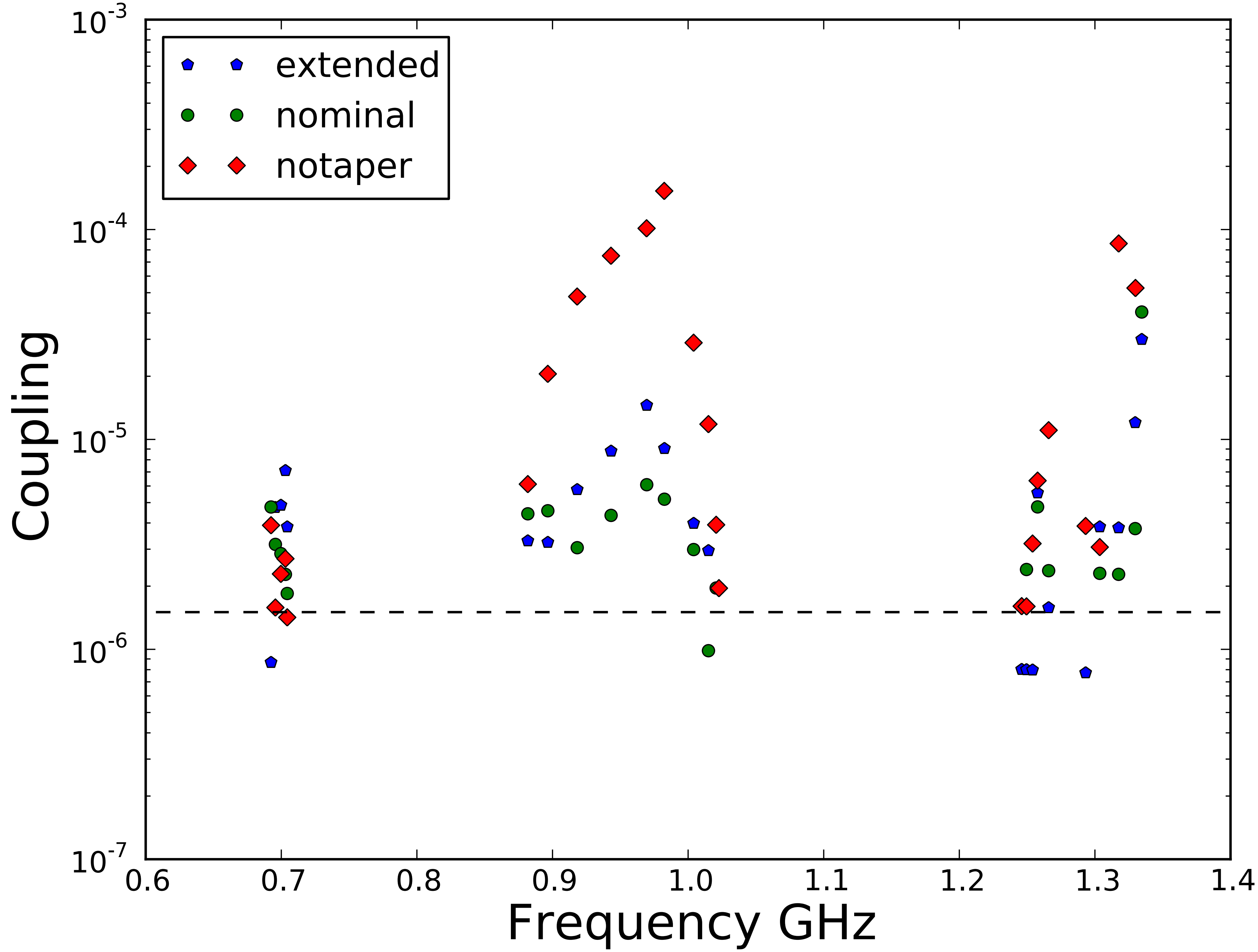


Dispersion Relation $\omega_\theta^2 = \omega_{\frac{\pi}{2}}^2 (1 - k\cos\theta)$

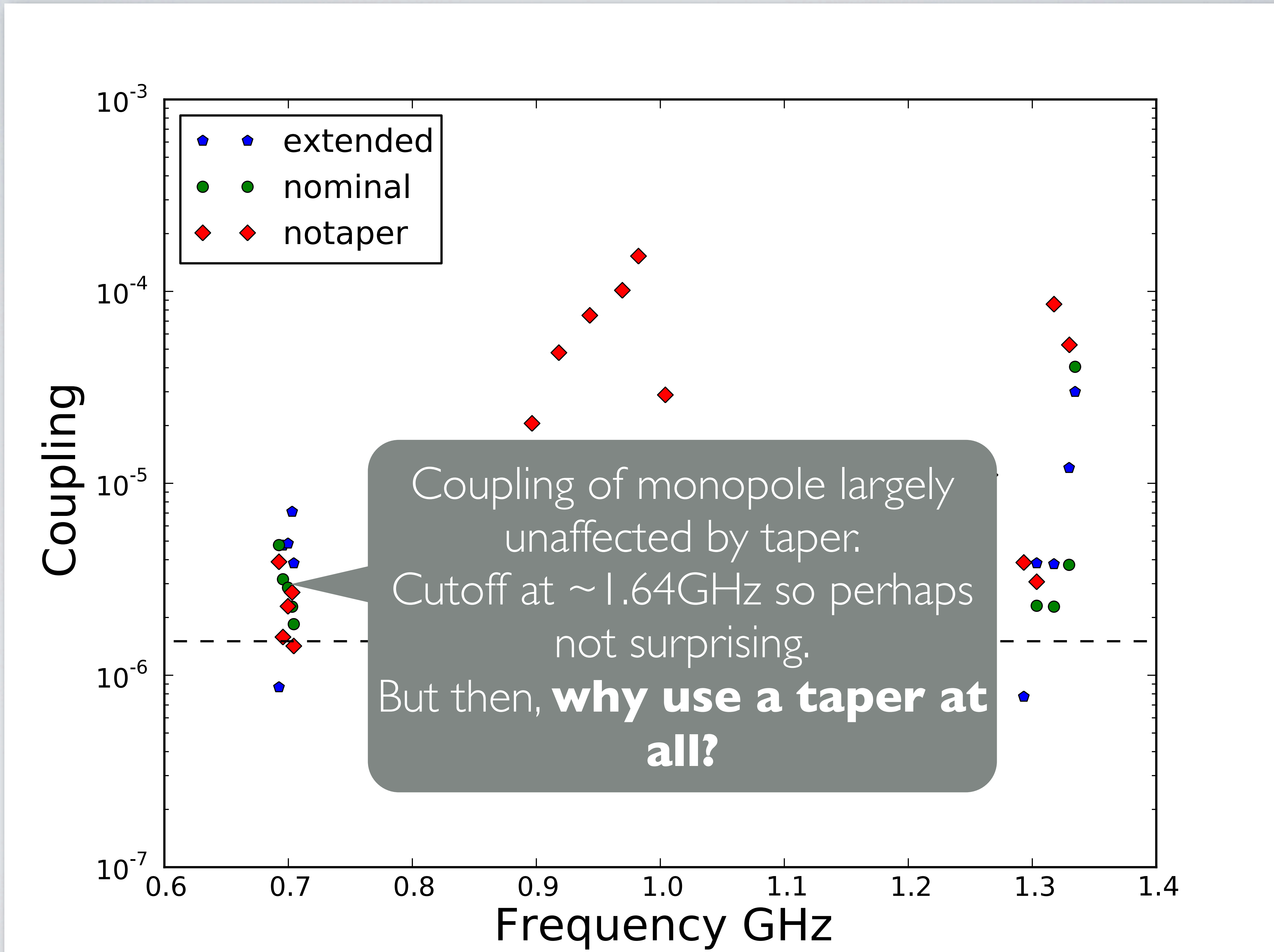
Coupling $k = \dfrac{\omega_\pi^2 - \omega_0^2}{\omega_\pi^2 + \omega_0^2}$

THREE GEOMETRIES





extended
nominal
notaper
Coupling
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
Frequency GHz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1.2
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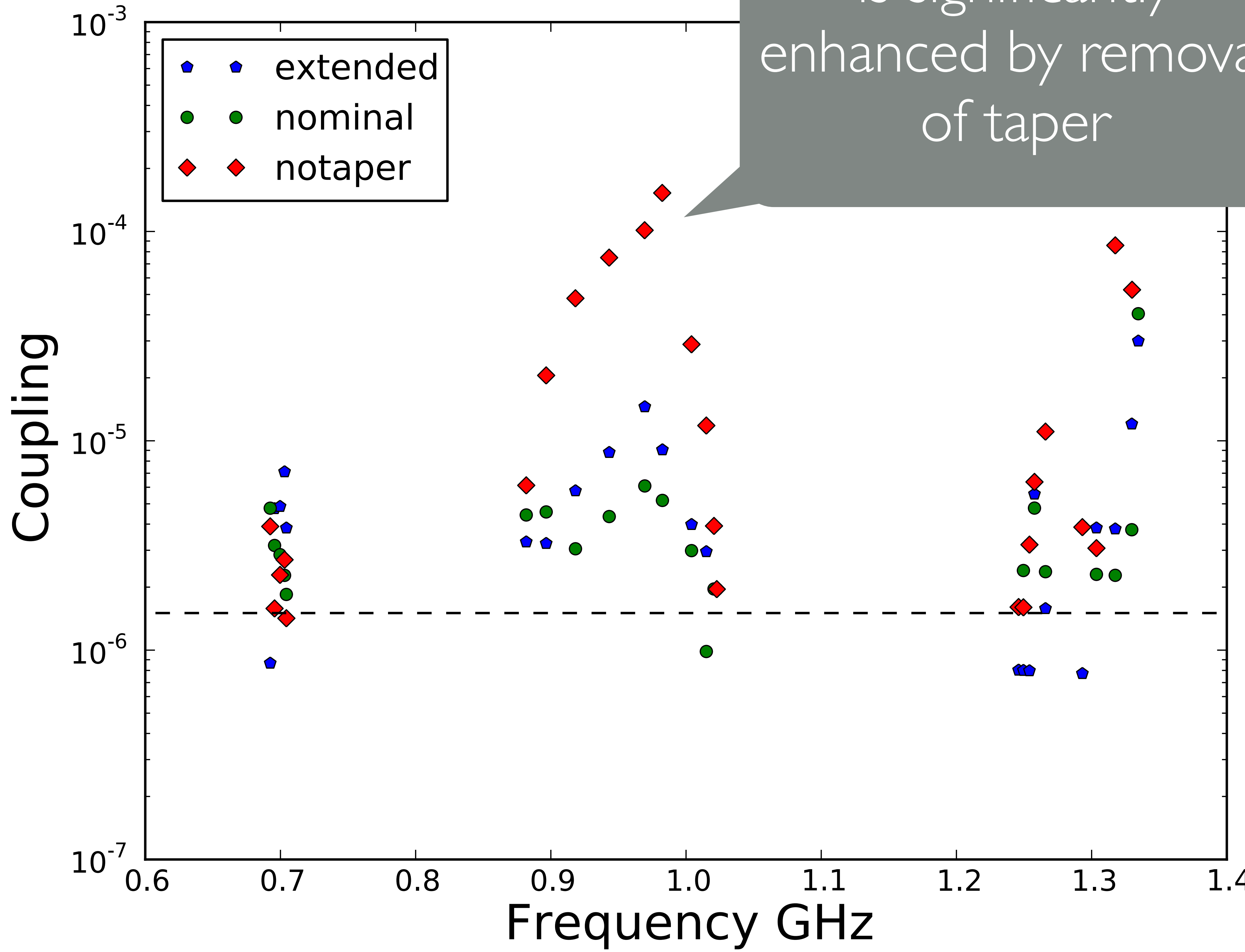
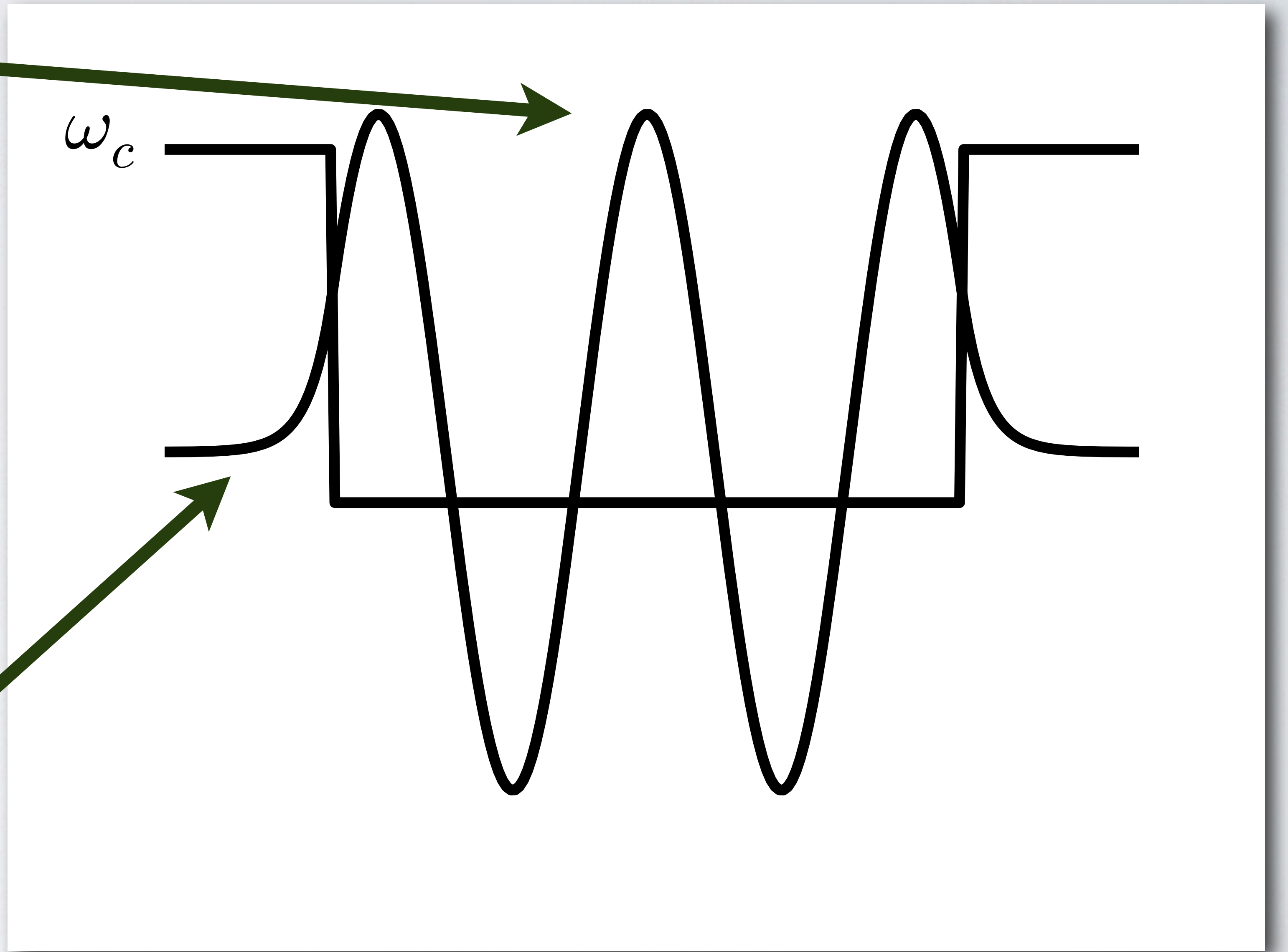
Coupling of monopole largely unaffected by taper.
Cutoff at ~1.64GHz so perhaps not surprising.
But then, **why use a taper at all?**



Note: Dipole coupling is significantly enhanced by removal of taper

SIMPLIFIED MODEL

Oscillation inside cavity



Decays exponentially inside beam pipe

FINITE POTENTIAL WELL

$$E > V$$

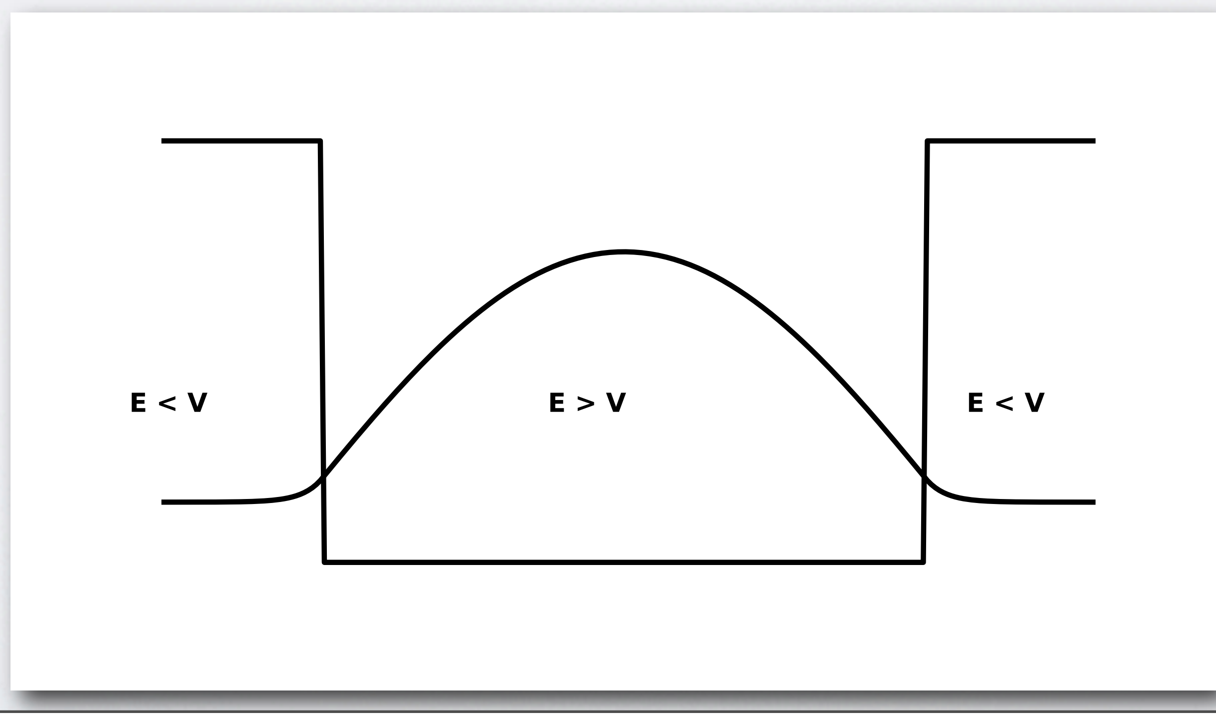
$$\psi_j = A_j \cos(k_j z) + B_j \sin(k_j z)$$

$$k = \frac{\sqrt{2m_j E}}{\hbar^2}$$

$$E < V$$

$$\psi_j = A_j e^{k_j z} + B_j e^{-k_j z}$$

$$k = \frac{\sqrt{2m_j (V - E)}}{\hbar^2}$$



FINITE POTENTIAL WELL

$\psi, \frac{d\psi}{dz}$ must be continuous at each boundary

Rewrite in terms of matrices

$$^{m}M_j = \begin{pmatrix} e^{k_j z_m} & e^{-k_j z_m} \\ k_j e^{k_j z_m} & -k_j e^{-k_j z_m} \end{pmatrix} \qquad E < V$$

$$^{m}M_j = \begin{pmatrix} \cos(k_j z_m) & \sin(k_j z_m) \\ -k_j \sin(k_j z_m) & k_j \cos(k_j z_m) \end{pmatrix} \qquad E > V$$

Therefore at each boundary

$$^{j}M_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} =^{j} M_{j+1} \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix}$$

FINITE POTENTIAL WELL

At boundary I

$$^{0}M_0 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = ^{0} M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

At boundary II

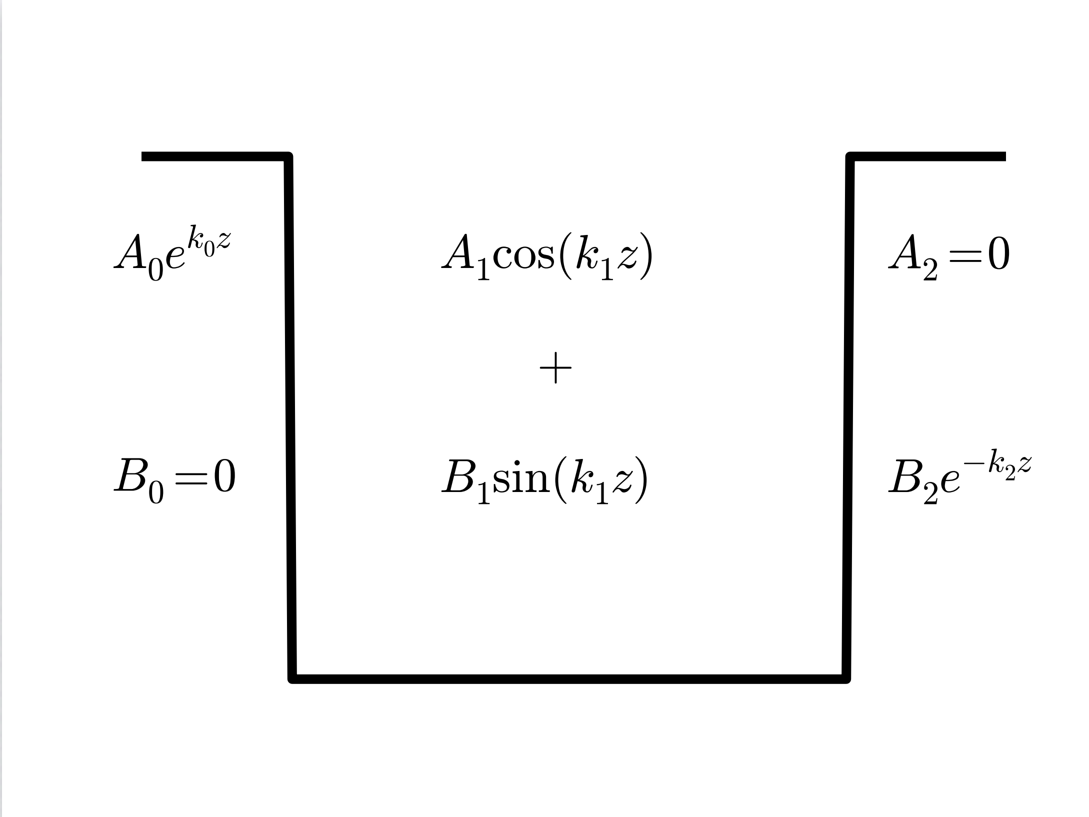
$$^{1}M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = ^{1} M_2 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

Therefore

$$[(^{1}M_2)^{-1} * {}^{1}M_1 * (^{0}M_1)^{-1} * {}^{0}M_0] \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

Need to find bound state!
Therefore, set **A_0 = 1** and **B_0 = 0**
No backward wave in first region

Solve to find where **A_2 = 0**
No forward wave in last region

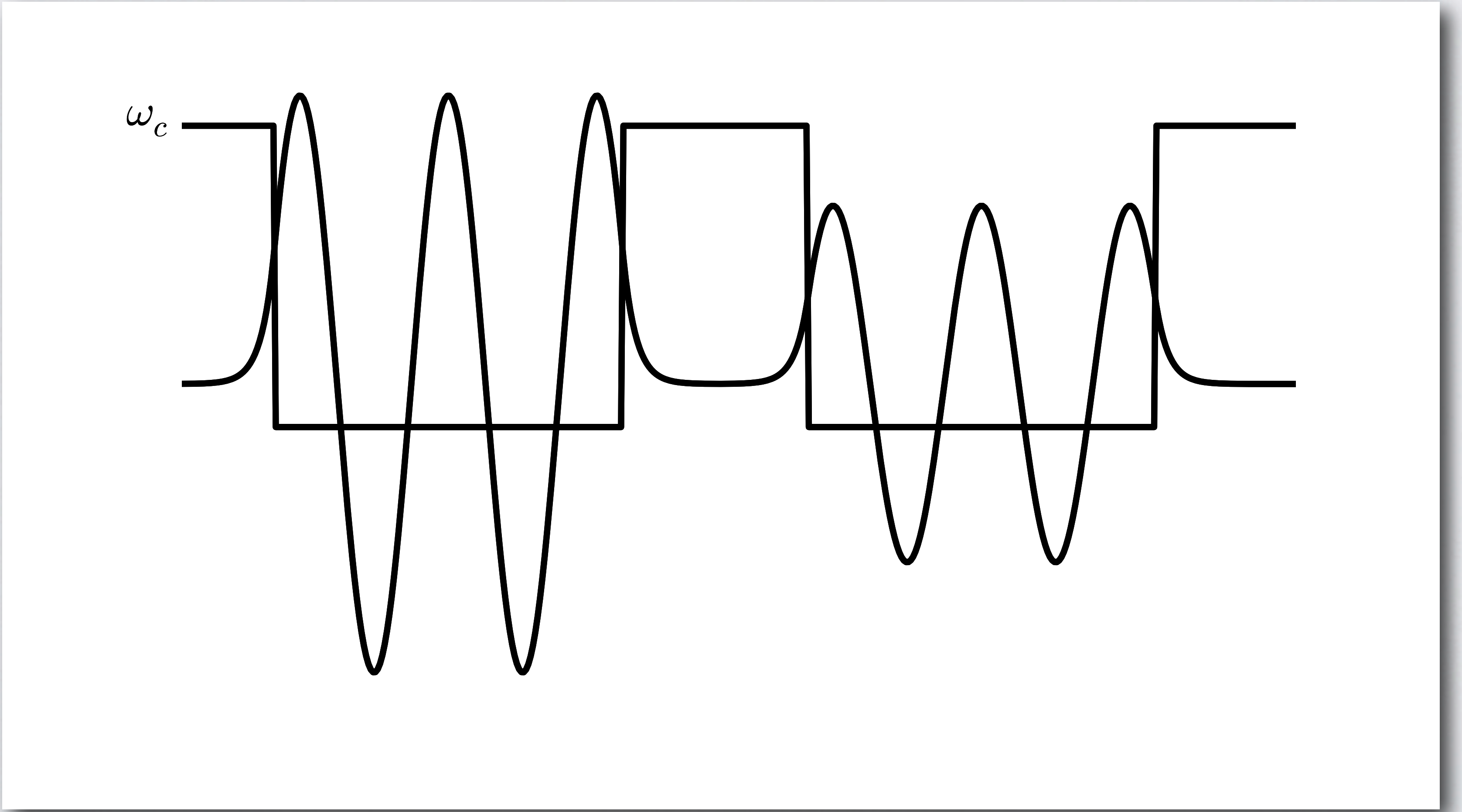


N COUPLED WELLS

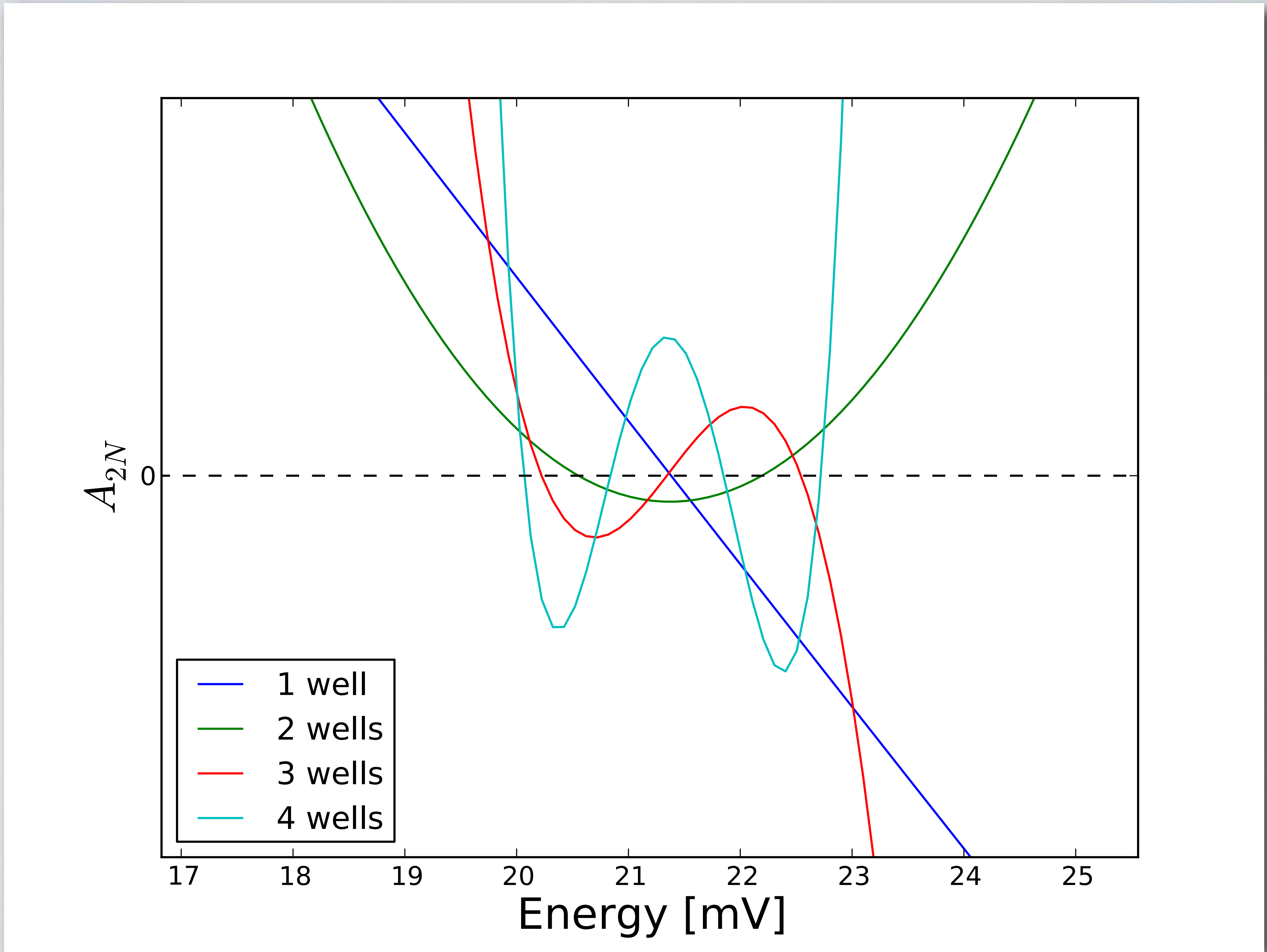
For N coupled wells

$$\left(\prod_{2N-1}^{0}\left[({}^{j}M_{j+1})^{-1} * {}^{j}M_{j}\right]\right)\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_{2N} \\ B_{2N} \end{pmatrix}$$

Again, solve for $\mathbf{A_{2N}=0}$ if $\mathbf{A_0=1}$,$\mathbf{B_0=0}$



DISCRETE ENERGY LEVELS



POTENTIAL WELL TO CAVITY

$$k = \frac{\omega}{c} \qquad \omega > \omega_c$$

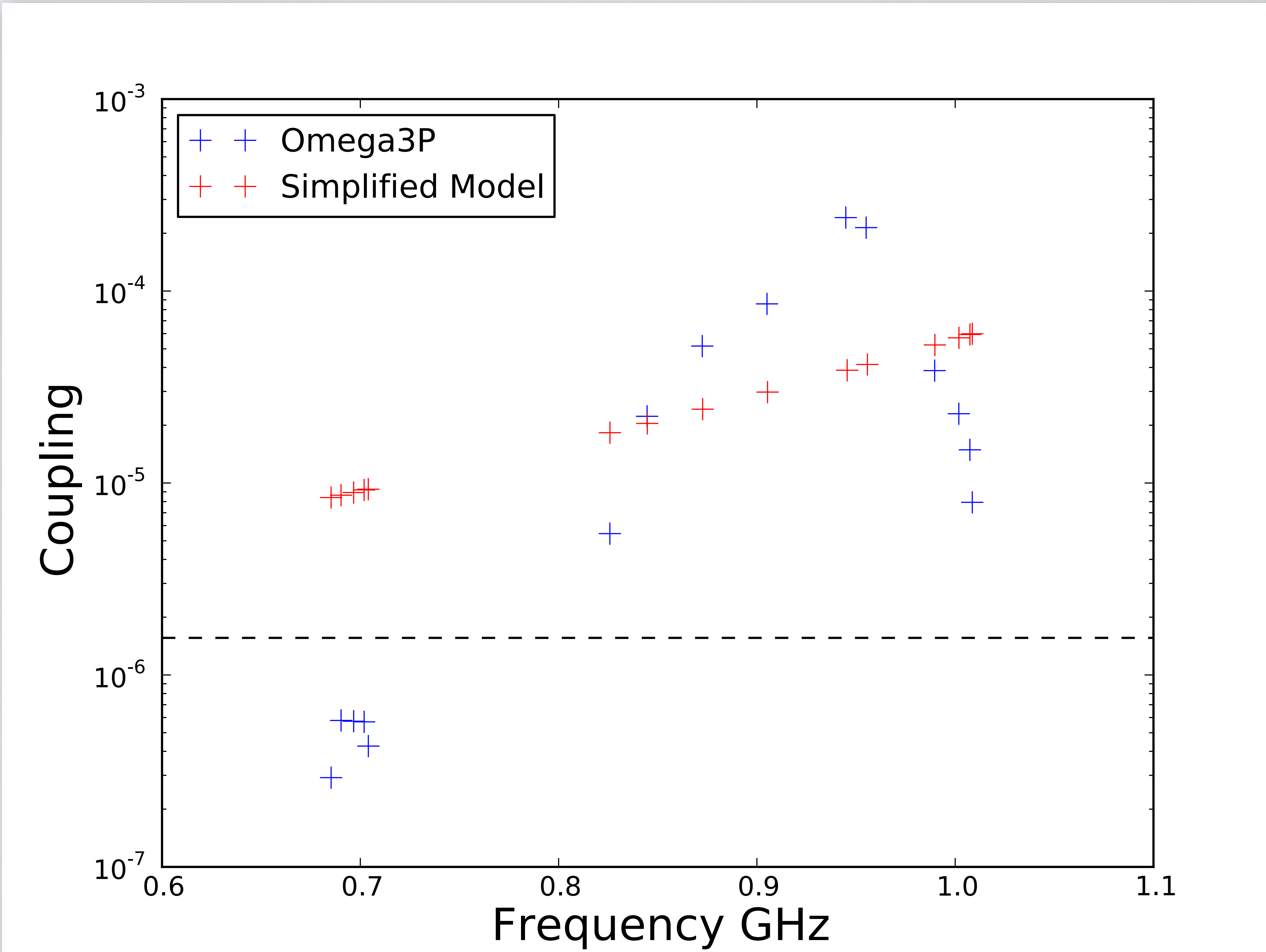
$$k = \sqrt{\left(\frac{p_{nm}}{a}\right)^2 - \left(\frac{\omega}{c}\right)^2} \qquad \omega < \omega_c$$

Does k need to change depending on phase advance?

To create cavity, set up a well where the lowest eigenvalue is the resonant frequency using

$$z = \frac{2\tan^{-1}(\frac{K1}{K0})}{K0}$$

COMPARISON OF RESULTS



SUMMARY

Cavity to cavity coupling - is a taper necessary?

- Negligible effect on monopole coupling
- Increases loss factor

Calculations using simplified model

- Preliminary results show rough agreement for dipole
- Can model be improved?
  - Change k according the phase advance?
  - Are we severely limited by only 1 dimension?
  - What about modes above cut-off?