On the Weight of the Heaviest Known Elementary Particle

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LPSC - Colloquium, May 26, 2011

On the Top Quark Mass

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Outline

- Standard Model and Top Quark
  - A brief history
  - Why knowing the top mass is relevant
  - Why this is conceptually non-trivial
  - How is the top mass determined?
- What top mass is measured (= m_t^{Pythia})
- What is the relation to any mass theorists know ?
  - Factorization Theorem in e+e-
  - First rough answer
  - Plans to go on …. toward LHC
- Outlook and Conclusions

Standard Model and Top

Materiebausteine:

		el. Ladung
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}$ $-\frac{1}{3}$
Leptonen	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	0 -1

$m_t \approx 175 \text{ GeV}$

$\rightarrow$ Wechselwirkungen bestimmt durch Eichsymmetrien

$SU(3)_C \times SU(2)_L \times U(1)_Y$ $\rightarrow$ 3 Eichkopplungen

Eichbosonen: $G, \quad W^\pm, \quad Z^0, \quad \gamma$

Standard Model and Top

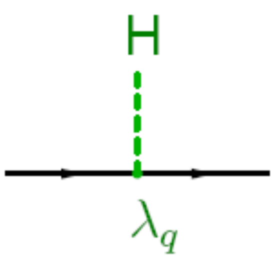
Materiebausteine:

		el. Ladung
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$+\frac{2}{3}$ $-\frac{1}{3}$
Leptonen	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	0 -1

$\rightarrow$ Higgsmechanismus **Higgsboson:** H

- Quarks koppeln an das Higgsfeld
- Massen durch spontane Symmetriebrechung

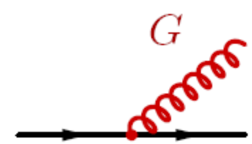
$$\langle 0 | H | 0 \rangle = V \neq 0 \quad \Rightarrow \quad \boxed{M_q = V \cdot \lambda_q}$$



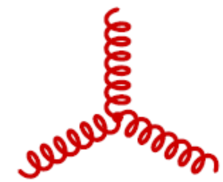
$$SU(3)_C \times SU(2)_L \times U(1)_I \quad \rightarrow \quad SU(3)_C \times U(1)_{em}$$

Standard Model and Top

Materiebausteine:

Quarks	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	
Leptonen	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	

red,green,blue



$\rightarrow$ Starke Wechselwirkung: Quanten-Chromo-Dynamik (QCD)

- hohe Energien: $\alpha_s \ll 1$ $\rightarrow$ Störungstheorie
  niedrige Energien: $\alpha_s \sim \mathcal{O}(1)$

Wilzek
Politzer
Gross
2004

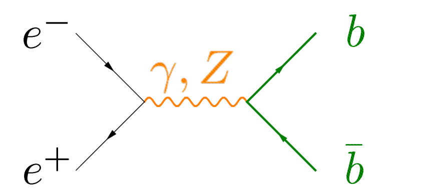
- **Knowing QCD is essential for conducting and interpreting collider experiments !!**

Hunting the Top

Discovery: **1995 (Tevatron)** $m_t = 176 \pm 13\ \mathrm{GeV}$

Start of the Hunt: **1977 Discovery of the bottom quark** $\Upsilon(b\bar{b})$ $m_b \approx 4.5\ \mathrm{GeV}$

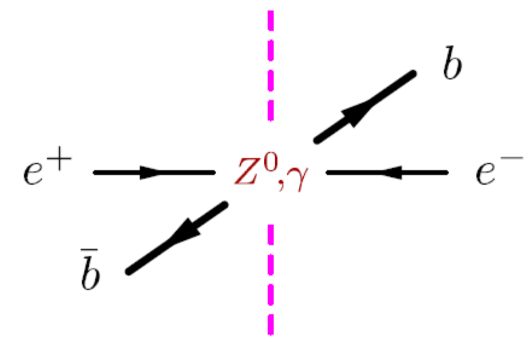
$$\mathrm{SU}(2)_L \quad \begin{matrix} +\frac{1}{2} \\ -\frac{1}{2} \end{matrix} \qquad \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \underbrace{\begin{pmatrix} t \\ b \end{pmatrix} \text{ oder } b}_{?} \quad 0$$



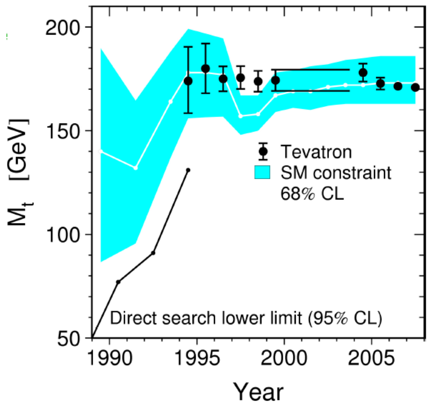
→ Measurement of bottom quark isospin: 1984 (PETRA / TRISTAN) $39 \pm 3\ \%$

$E_{\mathrm{cm}} = 36\ \mathrm{GeV}$

b-Isospin	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$
Bruchteil der b's in e^--Richtung	28 %	37 %	50 %	62 %



Hunting the Top

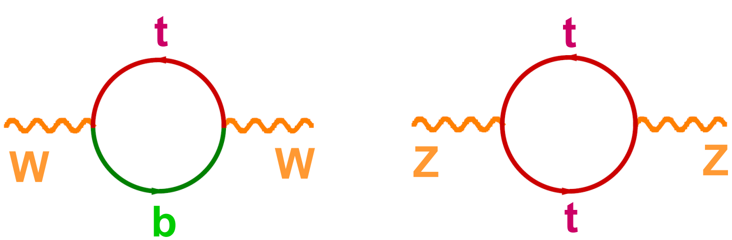


1995 (Tevatron) $m_t = 176 \pm 13\ \mathrm{GeV}$

2009 (Tevatron) $m_t = 173.1 \pm 1.3\ \mathrm{GeV}$

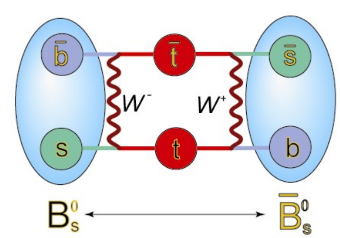
Indirect Massbounds:

Elektroweak Quanten Corrections:



$$m_Z^2 - m_W^2 \sim \alpha\, m_t^2$$

Flavor-Violating Processes:



$$\sim \alpha\, m_t^2 / m_W^2$$

$$1993/1994\text{: } m_t = 130 - 200\ \mathrm{GeV}$$

Hunting the Higgs

Direct search (still) unsuccessful !

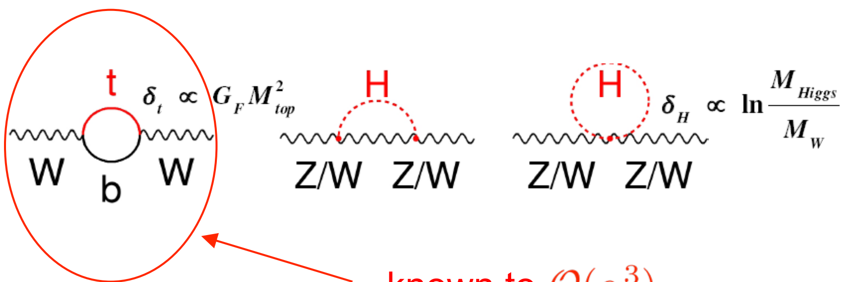
LEP-II $e^+ \to\leftarrow e^-, \;\; Q = 200 \text{ GeV}$ $m_H > 114.4 \text{ GeV} \text{ (95\%CL)}$

Tevatron Data not yet sufficient !

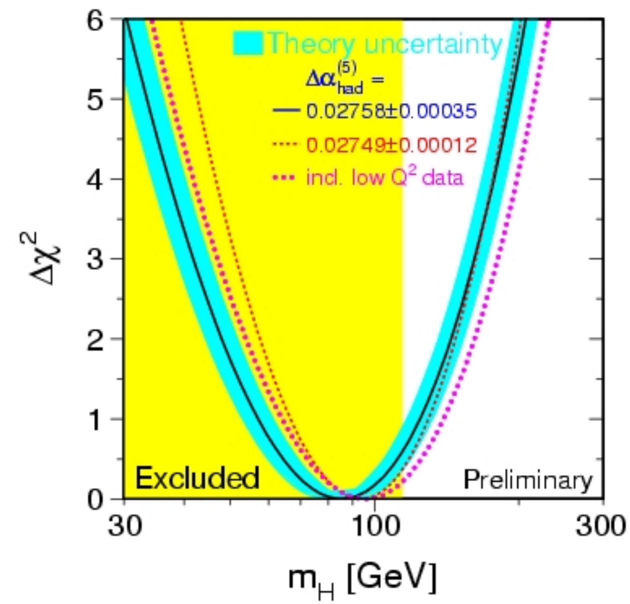
LHC Higgs search is primary task.

Need for a precise Top mass

Fit to electroweak precision observables



known to $\mathcal{O}(\alpha_s^3)$



$$\sin\theta_W \times \Big(1 + \delta(m_t, m_H, \ldots)\Big)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$

$$m_t = 170.9 \pm 1.8 \text{ GeV}$$

2 GeV error: 15% change in m_H

$$m_H = 76^{+33}_{-24} \text{ GeV}$$

$$m_H < 182 \text{ GeV} \text{ (95\%CL)}$$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\overline{m}_t)$$

Need for a precise Top mass

Blick in die Zukunft:

Minimales Supersymmetrisches Standard Model

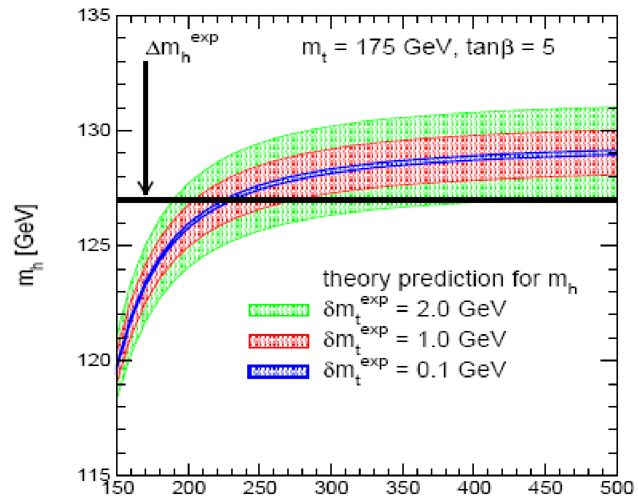
5 Higgs Bosonen:

m_h (skalar, neutral)

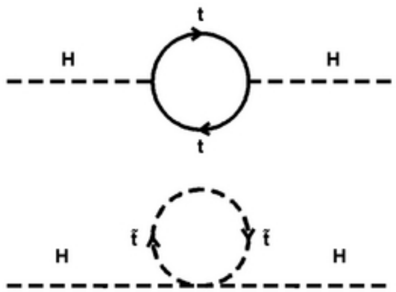
m_H (skalar, neutral)

m_A (speudoskalar, neutral)

$m_H^{\pm}$ (geladen)



Heinemeyer, Kraml, Porod, Weiglein



$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde t_1} m_{\tilde t_2}}{m_t^2}\right)$$

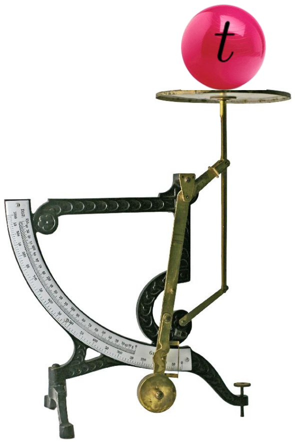
Corrections known to $\mathcal{O}(\alpha_s^3)$

Best convergence using the $\overline{\text{MS}}$ top scheme:

$$\overline{m}_t(\sqrt{M_{\text{SUSY}}\overline{m}_t})$$

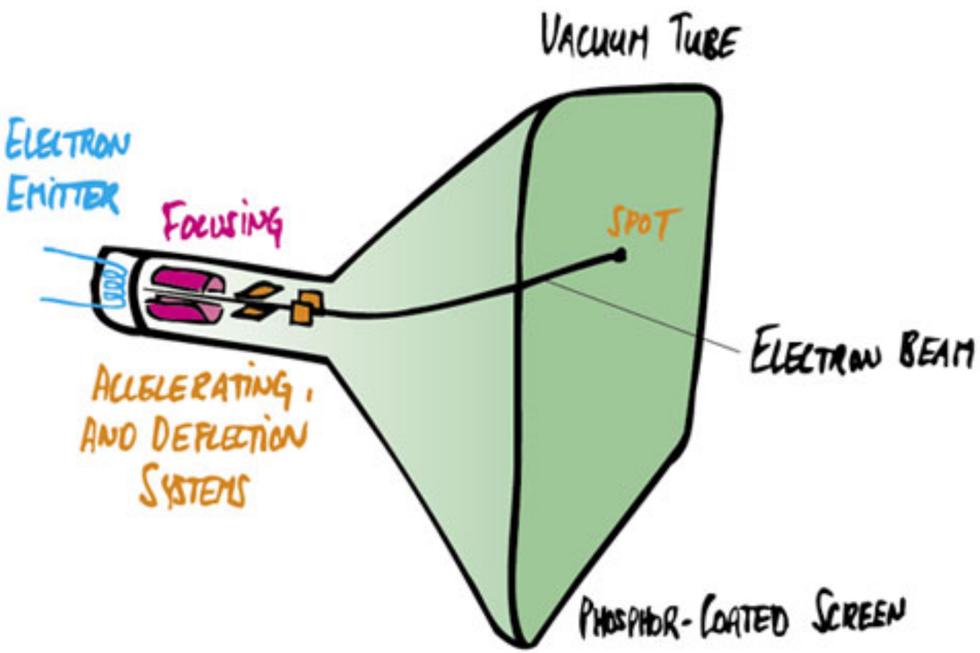
Haber, Hempfling, Hoang

Measuring the Mass



Measuring the Mass

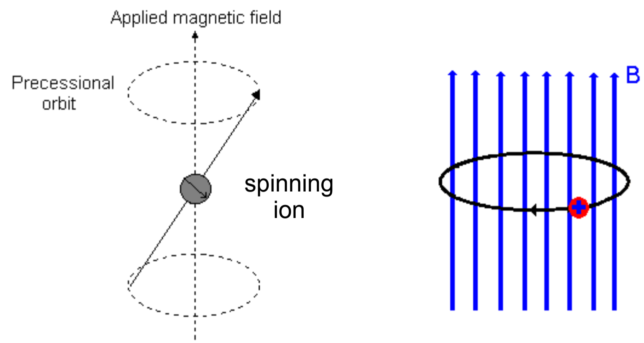
Masse des Elektrons:



Measuring the Mass

Mass of the electron:

$$m_\mathrm{e} = \frac{\omega_\mathrm{c}}{\omega_\mathrm{L}} \frac{g\,|e|}{2q}\, m_\mathrm{ion}$$



Larmor- and cyclotron frequency of electrons bound in an ion

Quantum electro dynamics (QED)

$$g(nS) = \underbrace{2 - \frac{2\,(Z\alpha)^2}{3\,n^2} + \frac{(Z\alpha)^4}{n^3}\left(\frac{1}{2n} - \frac{2}{3}\right) + \mathcal{O}(Z\alpha)^6}_{\text{Breit (1928), Dirac theory}}$$

$$+ \underbrace{\frac{\alpha}{\pi}\left\{2 \times \frac{1}{2}\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{a_{41}\ln[(Z\alpha)^{-2}] + a_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{one-loop correction}}$$

$$+ \underbrace{\left(\frac{\alpha}{\pi}\right)^2 \left\{-0.656958\left(1 + \frac{(Z\alpha)^2}{6n^2}\right) + \frac{(Z\alpha)^4}{n^3}\left\{b_{41}\ln[(Z\alpha)^{-2}] + b_{40}\right\} + \mathcal{O}(Z\alpha)^5\right\}}_{\text{two-loop correction}}$$

$$+ \mathcal{O}(\alpha^3)$$

$$\alpha = \frac{1}{137.035999679(94)} \ll 1$$

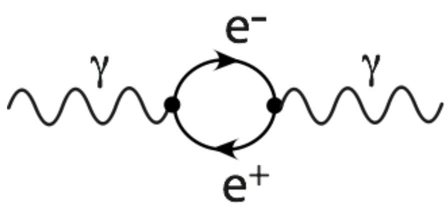
$$m_e = 0.51099892(4)\ \mathrm{MeV}$$

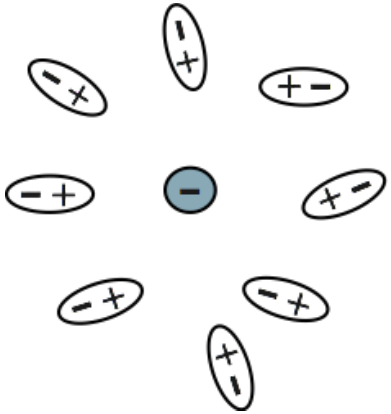
rest mass of the electron

Beier, Häffner etal. 2002

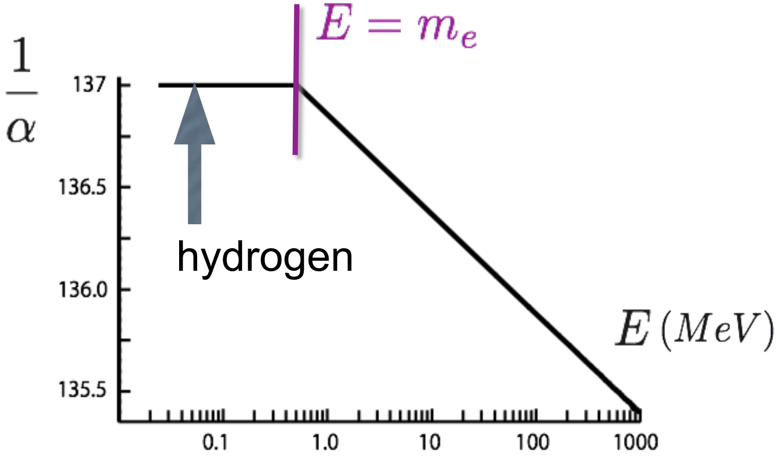
Measuring the Mass

Vacuum polarisation (elektron):





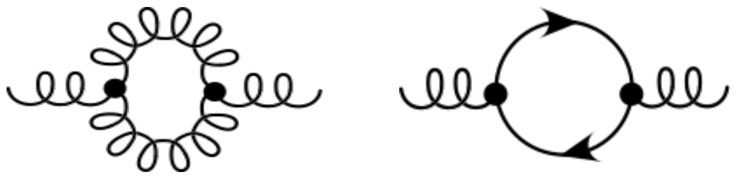
"Screening"

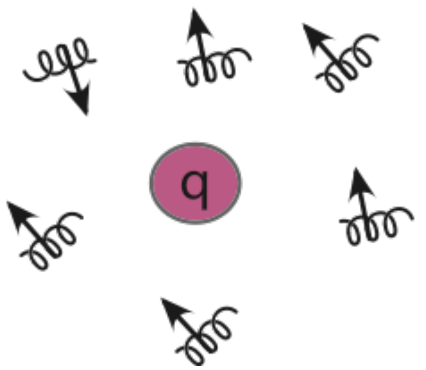


QED quantum corrections can be computed very well in perturbation theory.

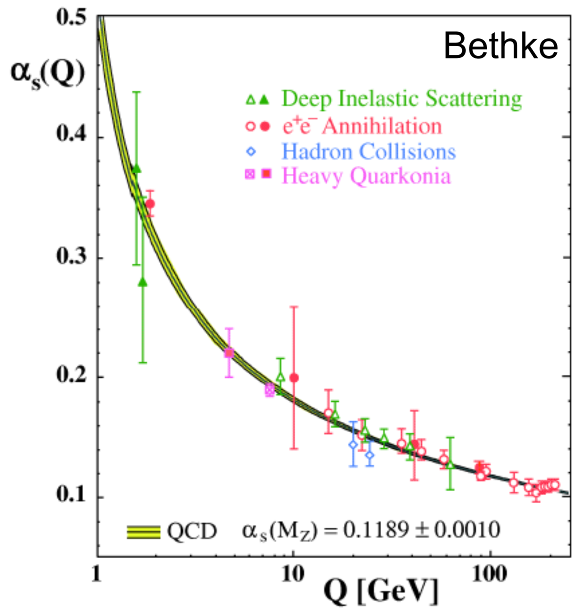
Measuring the Mass

Vacuum polarization (quarks):





"Anti-Screening"



High energies: QCD can be treated perturbatively: **"Asymtotic Freedom"**

Low energies: QCD quantum effects are nonperturbative

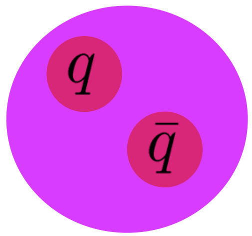
"Confinement" $(\Lambda_{\mathrm{QCD}} \approx 0.3\ \mathrm{GeV})$

Measuring the Mass

Confinement:

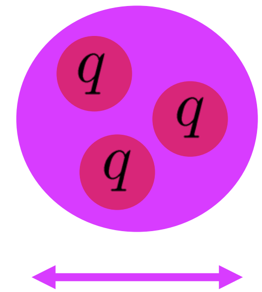
Mesons

$\pi, K, \rho, B, \ldots$



Baryons

$p, n, \Sigma, \Delta, \ldots$



$$r = \Lambda_{\mathrm{QCD}}^{-1} \simeq 1 \text{ fm}$$

Free quarks cannot be obeserved in our present day conditions.

Stable quarks only exist within bound states (hadrons).

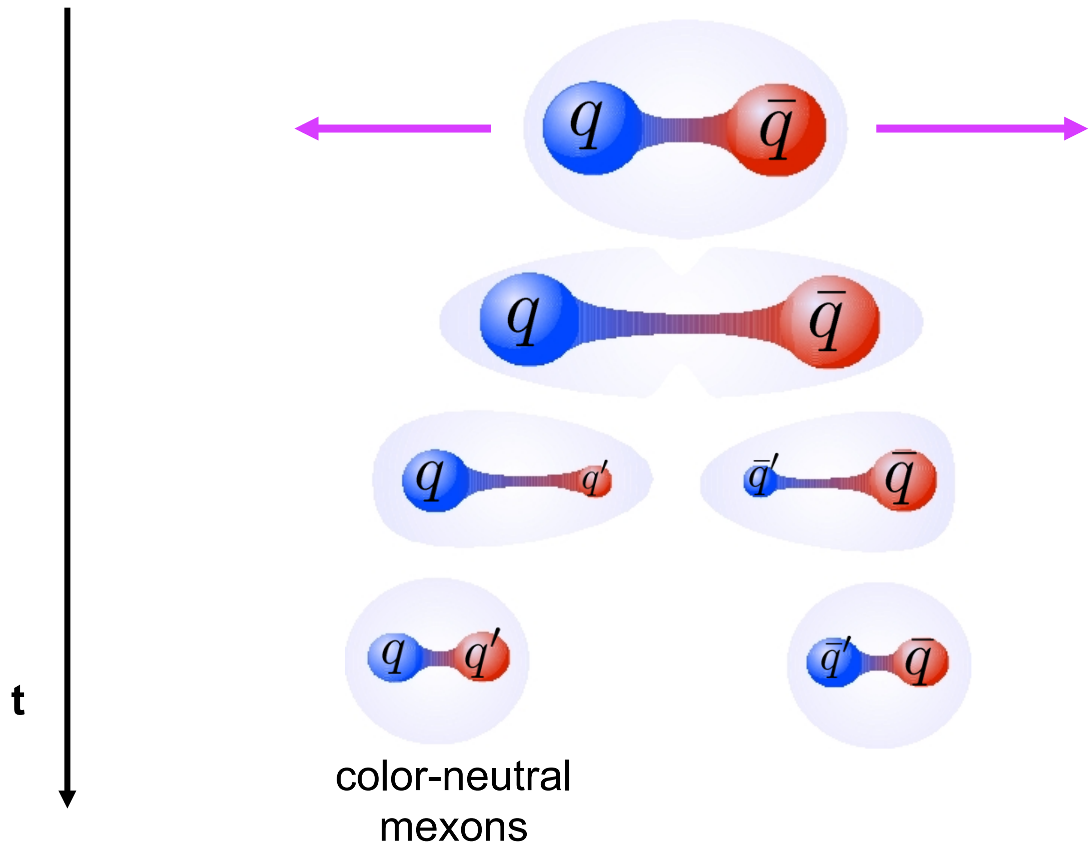
Proton mass (0.93 GeV) arises almost entirely from confinement dynamics

Hadronization time:

$$\tau_{\mathrm{had}} = 10^{-23} \text{ s}$$

Measuring the Mass

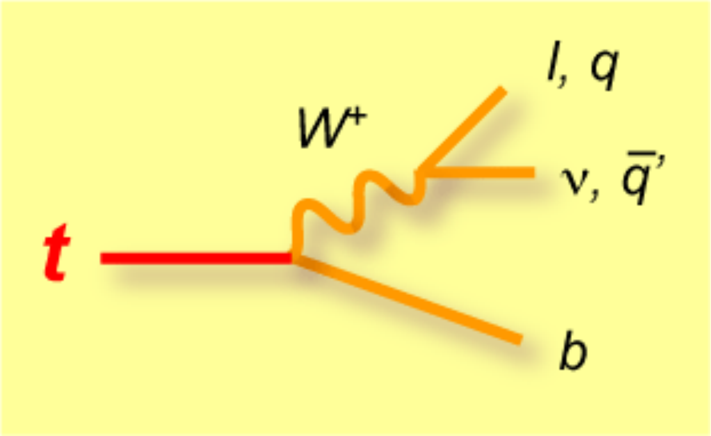
String Breaking: Confinement effect in particle pair production



Measuring the Mass

Weak decay of the top quark:

$$\Gamma(t \rightarrow bW) \approx 1.5 \text{ GeV}$$



Top quark decays before it can form hadrons

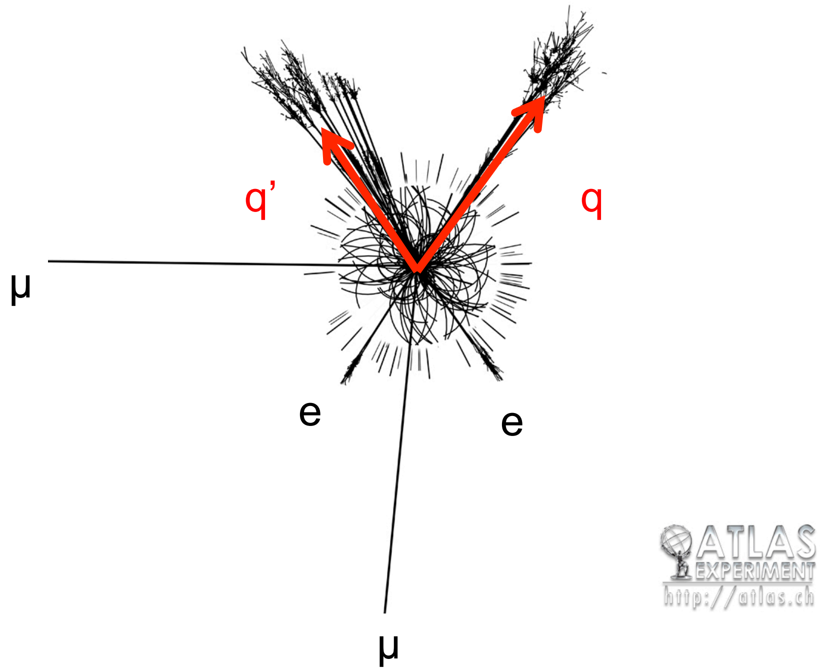
Lifetime:

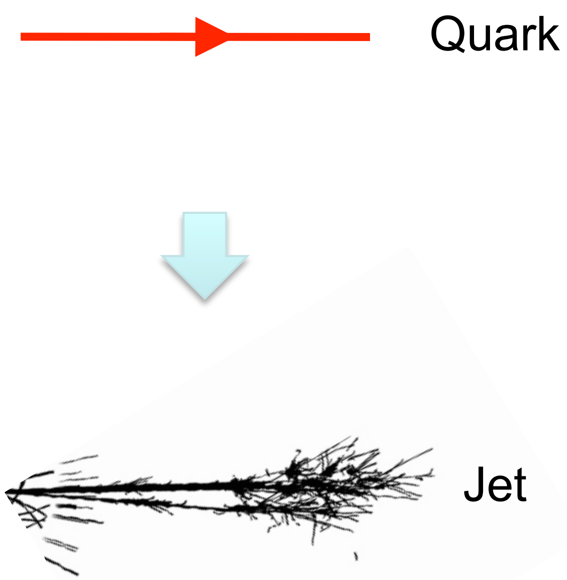
$$\tau_{\text{had}} = 10^{-24} \text{ s}$$

Hadronization time:

$$\tau_{\text{had}} = 7 \times 10^{-24} \text{ s}$$

Description of Jets



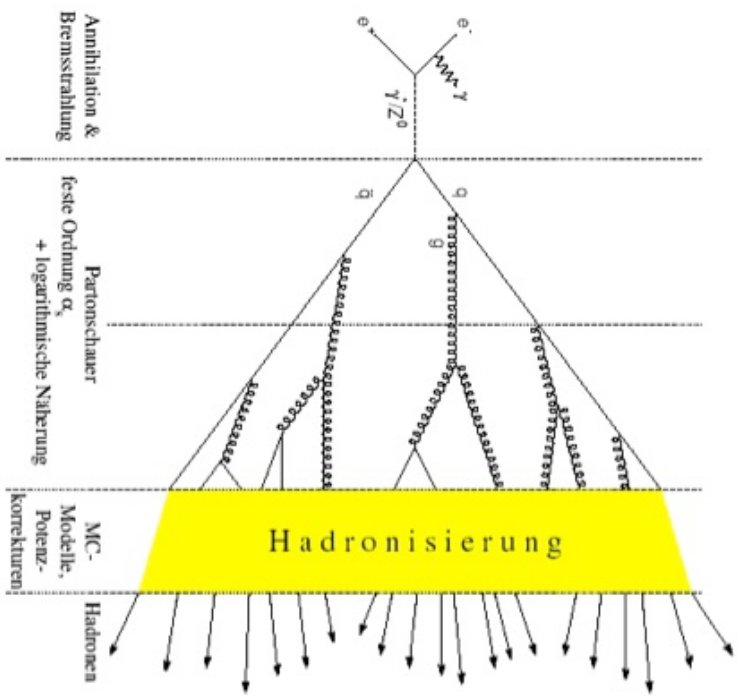


Description of Jets

Monte Carlo generators:

Universal instrument to describe hadronic final states.

- Hadronization models are “tuned” to experimental data.



- Parton-Shower: leading-log approximation
- Classic approximation
- No quantum interference
- Infrared regularization scheme in the parton showers is not specified.

Monte Carlo generators

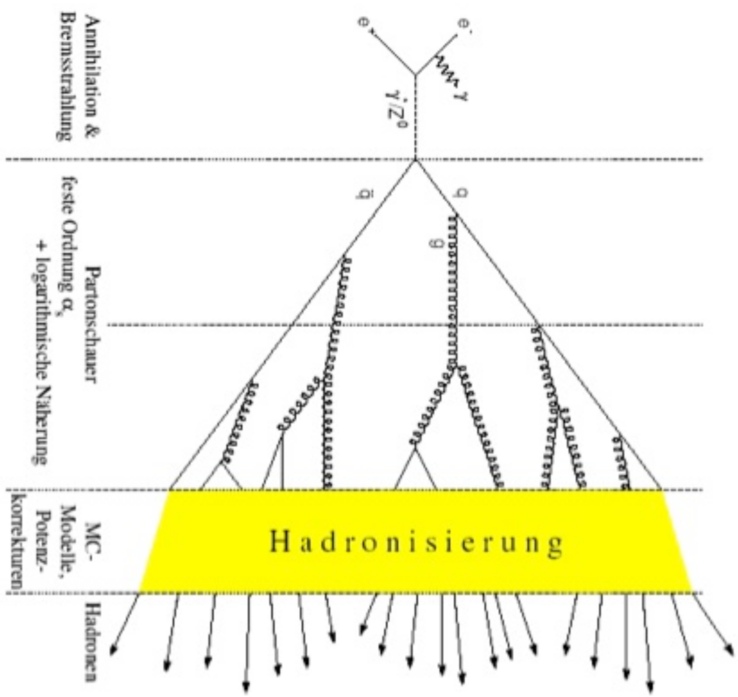
= QCD inspired model

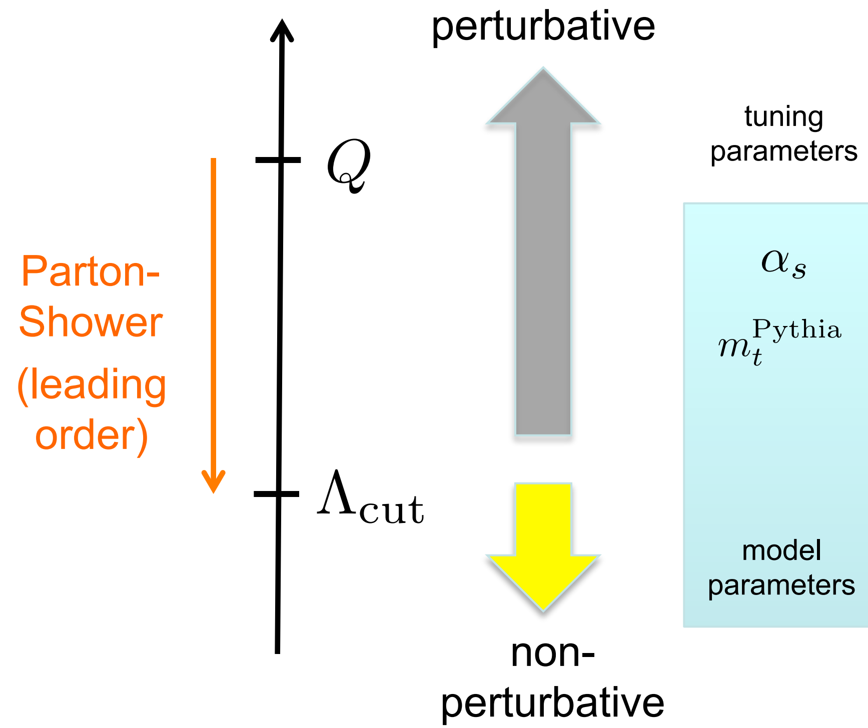
Description of Jets

Monte Carlo generators:

Universal instrument to describe hadronic final states.

- Hadronization model and α_s are "tuned" to experimental data.





Concept of a Quark Mass

Quantum Field Theory:

Particles: Field-valued operators made from creation and annihilation operators

Lagrangian operators constructed using correspondence principle

Classic action: m is the rest mass

No other mass concept exists at the classic level.

$$\mathcal{L}_{\rm QCD} = \mathcal{L}_{\rm classic} + \mathcal{L}_{\rm gauge-fix} + \mathcal{L}_{\rm ghost}$$

$$(p^2 - m^2)\, q(x) = 0$$

$$\mathcal{L}_{\rm classic} = -\frac{1}{4} F^A_{\alpha\beta} F^{\alpha\beta}_A + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta}\, q_b$$

$$D^\mu = \partial^\mu + igT^C A^{\mu C}$$

$$i\frac{p + m}{p^2 - m^2 + i\epsilon}$$

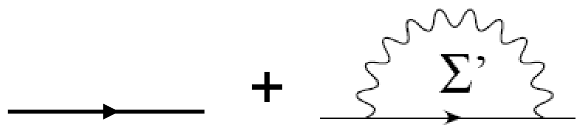
$$-i\frac{(g^{\mu\nu} + \frac{p^\mu p^\nu}{p^2}(\xi - 1))}{p^2 + i\epsilon}$$

classic particle poles

Concept of a Quark Mass

Renormalization: UV-divergences in quantum corrections

Fields, couplings, masses in classic action are bare quantities that need to be renormalized to have (any) physical relevance



$$= \not{p} - m^0 + \Sigma(p, m^0)$$

$$m^0 \, \frac{\alpha_s}{\pi} \left[-\frac{1}{\epsilon} + \text{finite stuff} \right]$$

Mass Renormalization Schemes you know:

Pole mass: mass = classic rest mass (concept sick in QCD!)

$$m^0 = m^{\text{pole}} + \delta m^{\text{pole}} \qquad \delta m^{\text{pole}} = \Sigma(m, m)$$

RENORMALONS !!

$\overline{\text{MS}}$ mass:

$$m^0 = \overline{m}(\mu) - \frac{\alpha_s}{\pi} \frac{1}{\epsilon}$$

(purely formal & unphysical)

What is the Pythia top mass ?

Concept of a Quark Mass

All mass schemes are related through a perturbative series.

$$m^{\text{schemeA}} - m^{\text{schemeB}} = \#\,\alpha_s + \#\,\alpha_s^2 + \#\,\alpha_s^3 + \dots$$

Lesson 1: Renormalization schemes are defined by what quantum fluctuations are kept in the dynamical matrix elements and by what quantum fluctuations are absorbed into the couplings and parameters.

Why do we have to care?

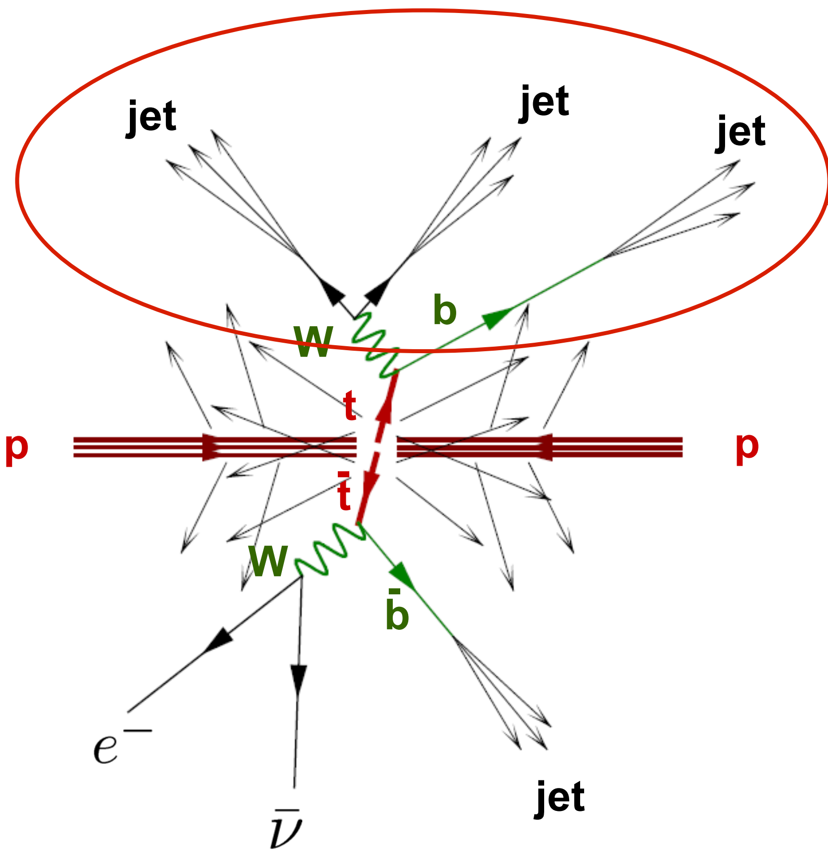
Different mass schemes are useful and appropriate for different applications.

Which is the best mass for a specific application?

Lesssson 2: A good scheme choice is one that gives systematically (not accidentally) good convergence. But there are almost always several alternatives one can use.

Description of Jets

LHC:

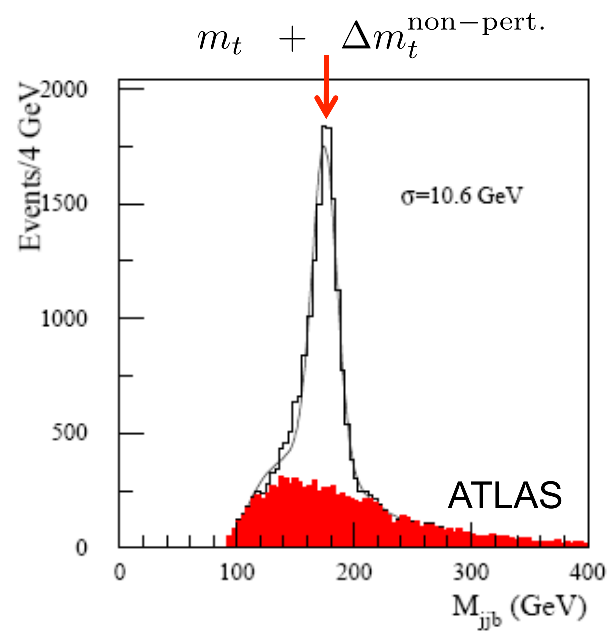


Principle of mass measurements:

Identification of the top decay products

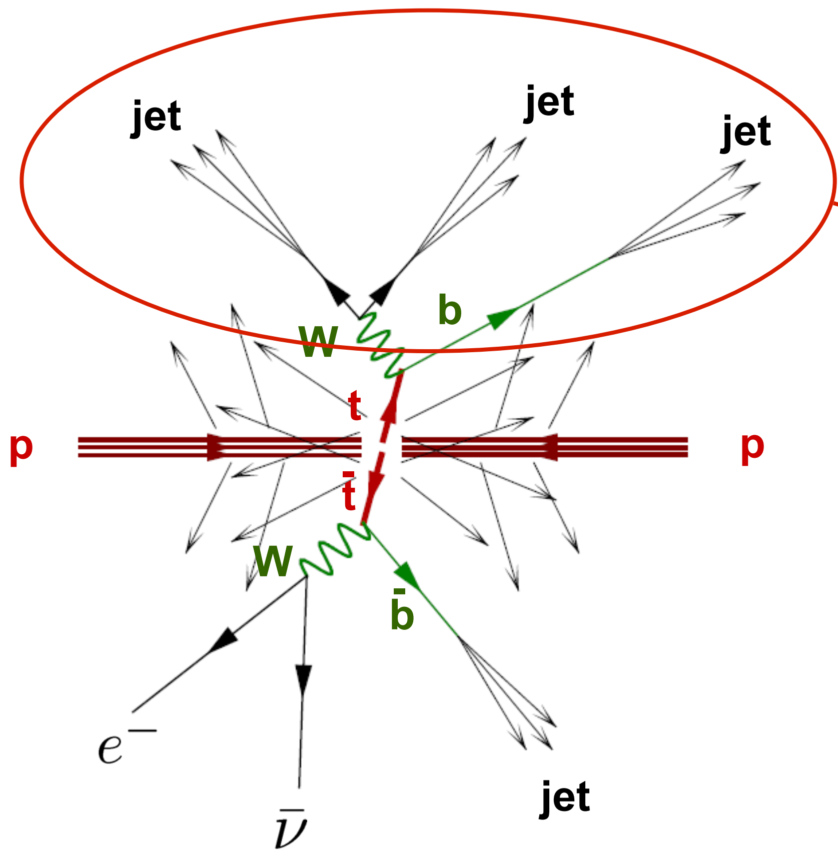
$$“\ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Invariant mass distribution



Description of Jets

LHC:



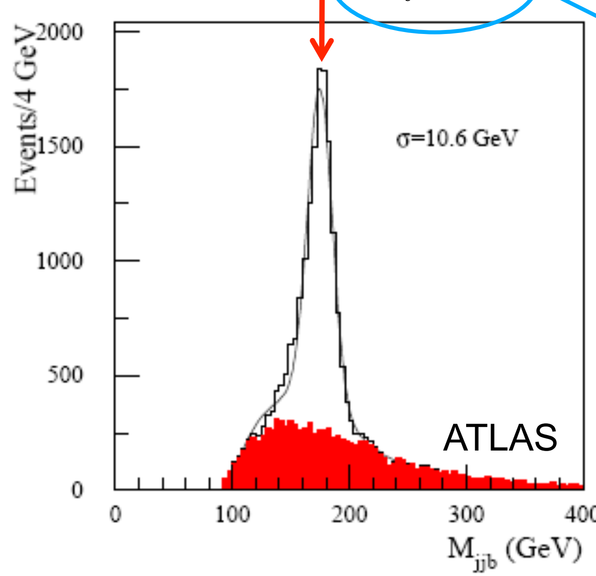
Principle of mass measurements:

Identification of the top decay products

$$\text{“}\ m_{\text{top}}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ \text{”}$$

Invariant mass distribution

$m_t + \Delta m_t^{\text{non-pert}}$

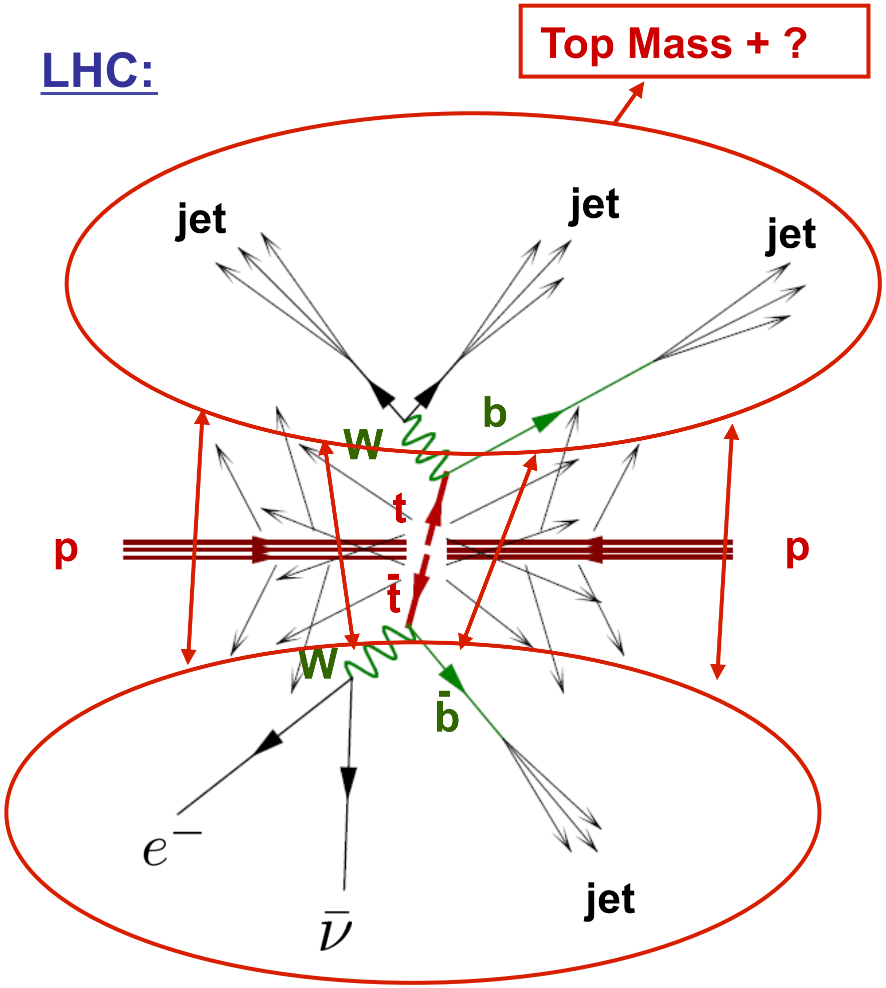


depends on:

- mass definition
- jet definition
- Monte Carlo

Description of Jets

LHC:



Principle of mass measurements:

Identification of the top decay products

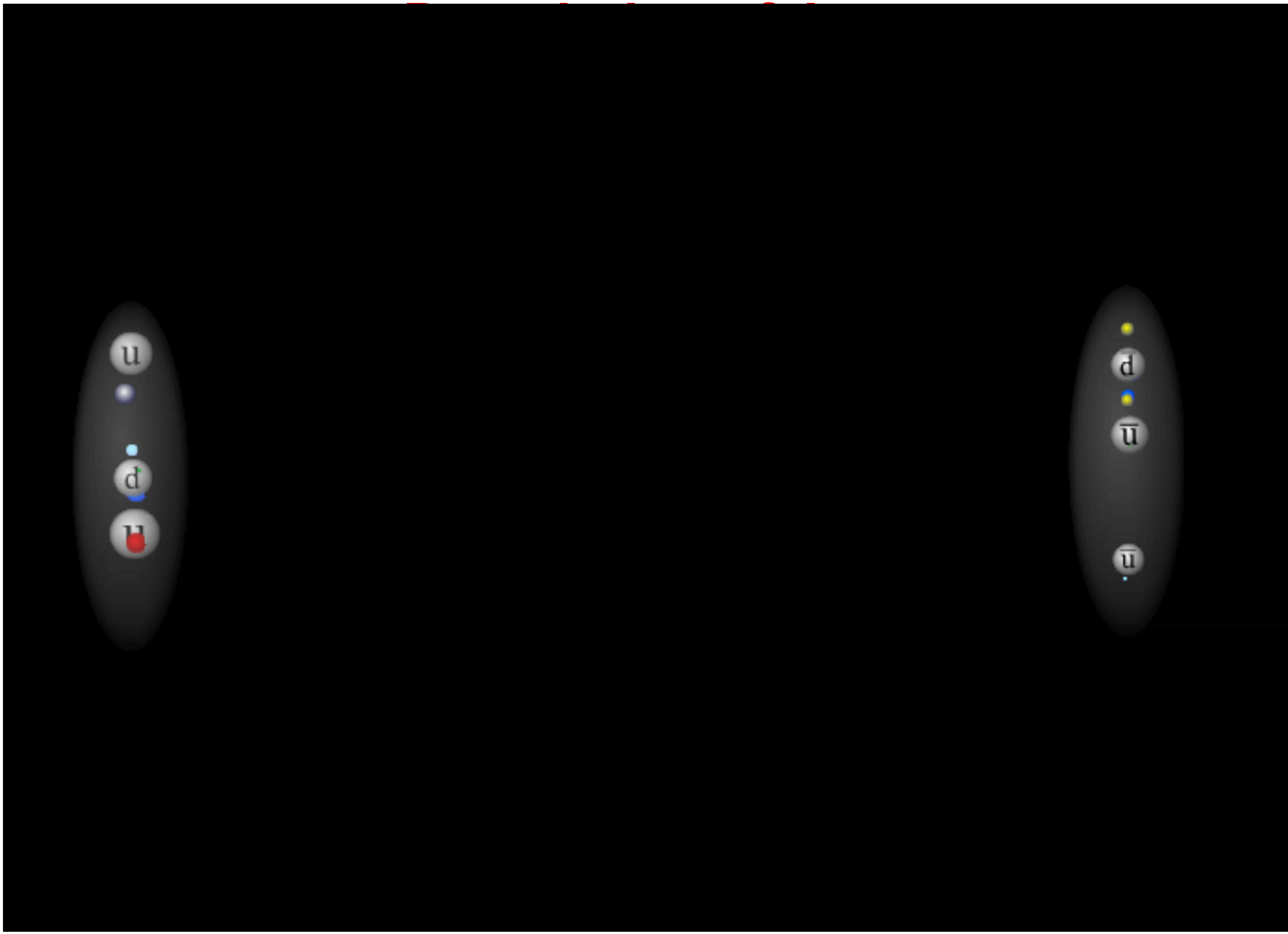
$$“\ m_{\rm top}^2 = p_t^2 = \left(\sum_i p_i^\mu\right)^2\ ”$$

Problem is non-trivial !

- Measured object does not exist a priori, but only through the experimental prescription for the measurement. **Quantum effects !!**

The idea of a - by itself - well defined object having a well defined mass is incorrect !!

Details and uncertainties of the parton shower and the hadronization models in den MC's influence the measured top quark mass.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Main Methods at Tevatron

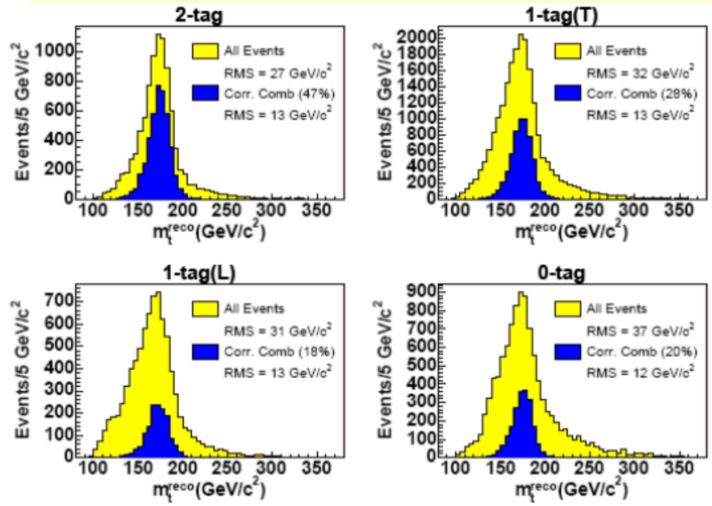
Template Method

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$
$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

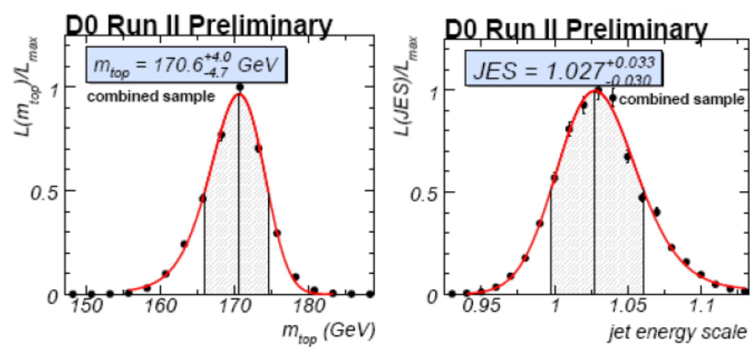
parton distribution functions

$$P(x;m_t) = \frac{1}{\sigma}\int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x|y)$$

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays "before hadronization" ($\Gamma_t \approx 1.5$ GeV)

$$m_t = 172.4 \pm 1.2 \text{ GeV}$$

Combination of CDF and DØ Results on the Mass of the Top Quark

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 1 fb^{-1} of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 170.9 \pm 1.1(\text{stat}) \pm 1.5(\text{syst})$ GeV/c^2, which corresponds to a total uncertainty of 1.8 GeV/c^2. The top-quark mass is now known with a precision of 1.1%.

FERMILAB-TM-2380-E
TEVEWWG/top 2007/01

$m_t = 172.6 \pm 1.4$ GeV

13th March 2007

$M_t = 170.9 \pm 1.8$ GeV/c^2

<1% precision !

How shall we theorists judge the error ?

What is the theoretical error ?

What mass is it ?

Main Methods at Tevatron

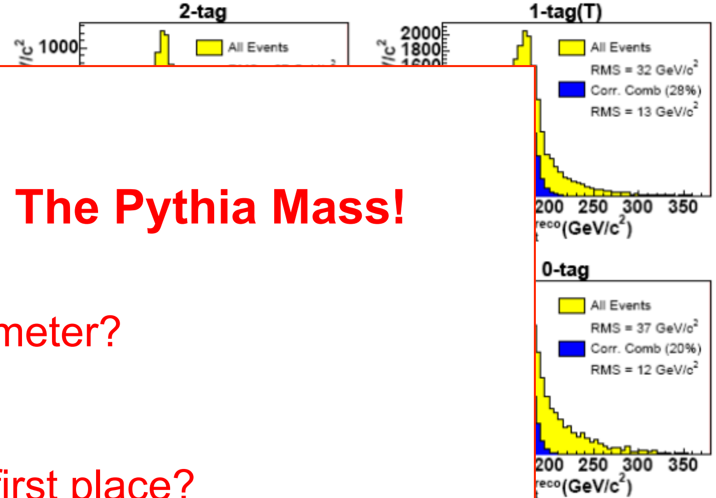
Template Method

- Principle: perform kinematic fit and reconstruct top
  mass event

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - \ldots}{\ldots} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} \ldots$$

Usually pick

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamic

- Principle: c
  a function c
  objects in t
  Maximize s

What mass is measured?: The Pythia Mass!

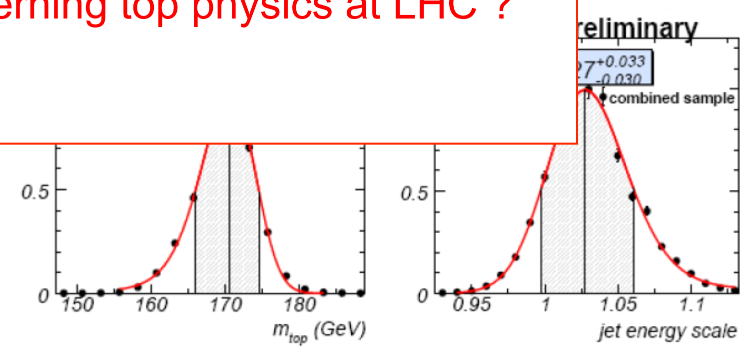
- What is the Pythia mass parameter?
- It's not the pole mass !
- How reliable is the MC in the first place?
- How can we approach the issue?
- Should we be worried concerning top physics at LHC ?

parton distribution functions

$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

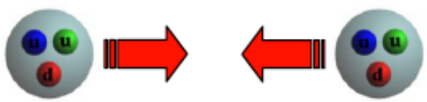


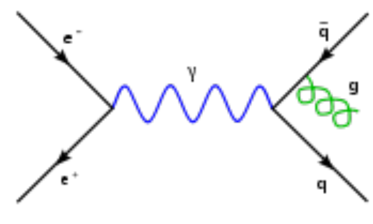
QCD Factorization

Drell-Yan: $pp \longrightarrow \ell^+\ell^- + X$ (inclusive)

Collins, Soper,Sterman; Bodwin

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q^2\mathrm{d}Y} = \sum_{i,j}\int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2} \boxed{f_i(x_1,\mu)\ f_j(x_2,\mu)} \boxed{H_{ij}^{\mathrm{incl}}(x_1,x_2,q^2,Y,\mu)}$$





non-perturbative
parton distribution function

(process independent)

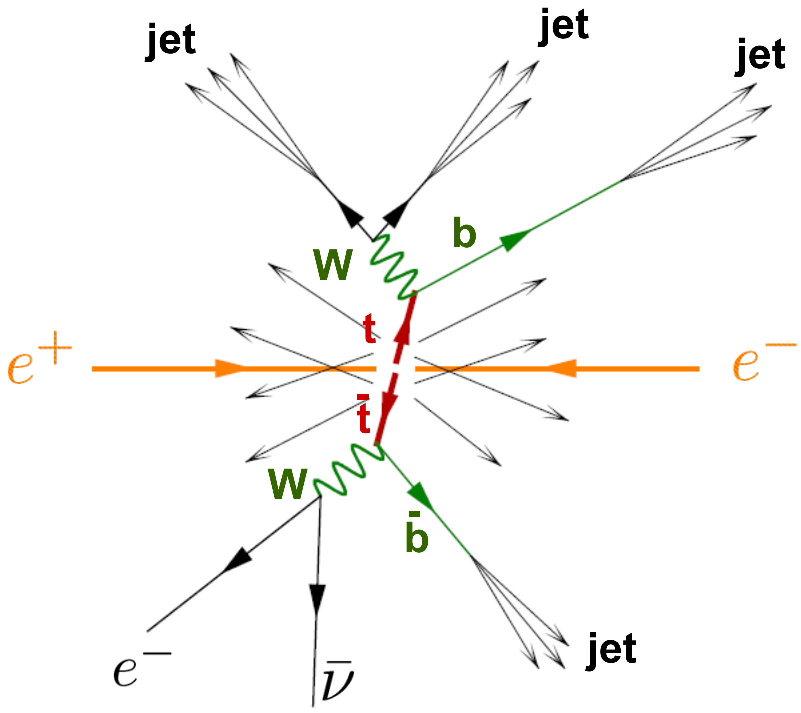
perturbative
hard cross section

(process dependent)

→ QCD factorization in the initial state

QCD Factorization

Top Invariant Mass Distribution:



$$e^+e^- \to t\bar{t}$$

$$Q \gg m_t \quad (p_T \gg m_t)$$

→ We need: QCD factorization in the final state

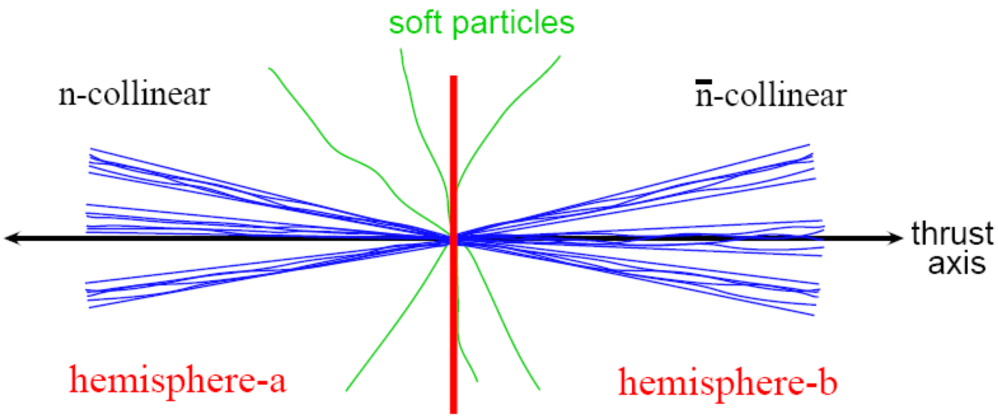
QCD Factorization

Top Invariant Mass Distribution:

Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008

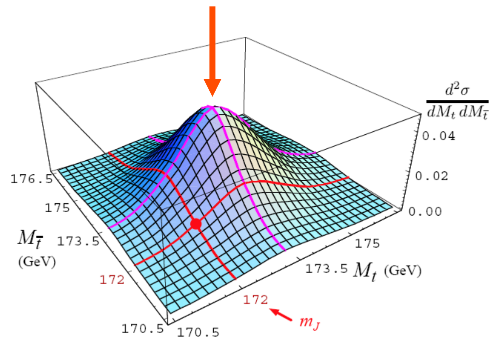
Definition of the observable



$$M_t^2 = \Big(\sum_{i\,\epsilon\, a} p_i^\mu\Big)^2 \qquad M_{\bar t}^2 = \Big(\sum_{i\,\epsilon\, b} p_i^\mu\Big)^2$$

resonance region:

$$M_{t,\bar t} - m_t \sim \Gamma$$



$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}}$$ Double differential hemisphere mass distribution

QCD Factorization

Fleming, Mantry, Stewart, AHH

Phys.Rev.D77:074010,2008
Phys.Rev.D77:114003,2008
Phys.Lett.B660:483-493,2008

$$Q \gg m_t \gg \Gamma_t > \Lambda_{\rm QCD}$$

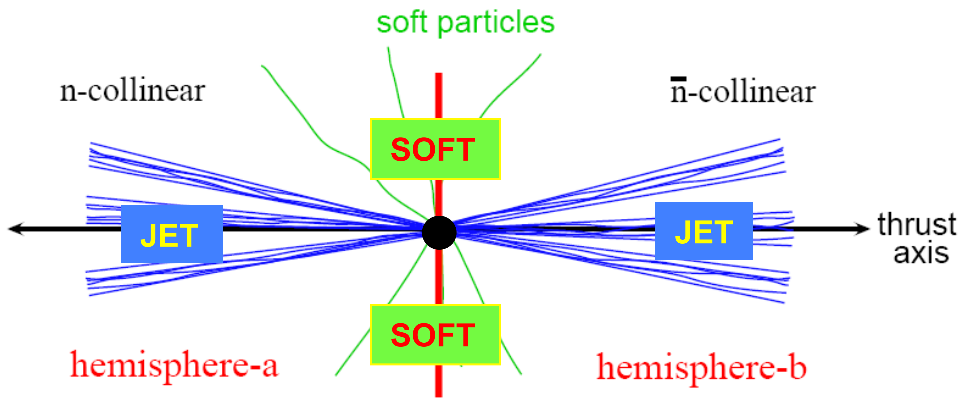
QCD

↓

Soft-Collinear-Effective-Theory

↓

Heavy-Quark-Effective-Theory + **Unstable-Particle-Effective-Theory**



Faktorization Formula

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

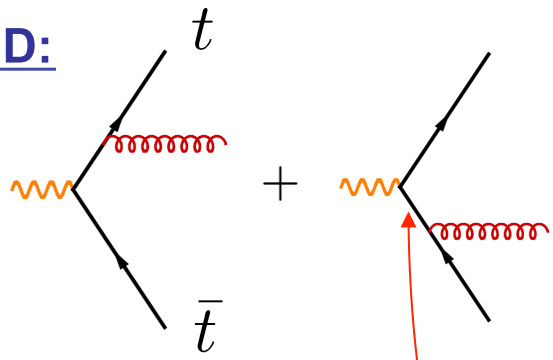
$$\hat s = \frac{M_t^2 - m_J^2}{m_J}$$

JET **JET** **SOFT**

$k_+ = k_0 - k_3$
$k_- = k_0 + k_3$

QCD Factorization

full QCD:



3 phase space regions: $\lambda \sim m_t/Q$

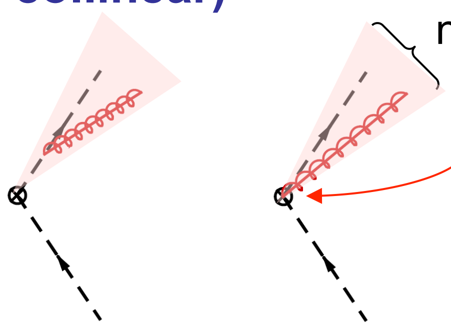
- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$\frac{1}{(p_{\bar{t}}+k)^2 - m_t^2} \qquad (p_{\bar{t}}^2 \approx m_t^2\,,\ \ \bar{n}^2 = 0)$$

$$\downarrow$$

$$\frac{1}{Q\ \bar{n}.k}$$

Gluon collinear to the top:
(n-collinear)



$$W_n^\dagger(\infty, x) = \mathrm{P}\ \exp\left(ig\int_0^\infty ds\bar{n}\cdot A_+(ns+x)\right)$$

$h_{v_+}(x)$ → **gauge dependent**

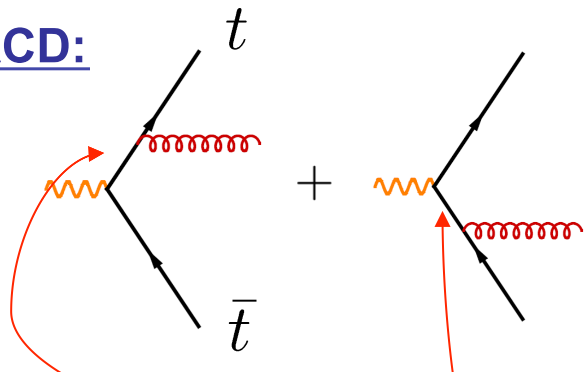
$W_n^\dagger(\infty, x)\, h_{v_+}(x)$ → **gauge independent**

$$B_+(\hat{s}, \Gamma_t, \mu) = \mathrm{Im}\left[\frac{-i}{12\pi m_J}\int d^4x\, e^{ir.x}\, \langle 0|\, T\, \{\bar{h}_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\,\}\, |0\rangle\right]$$

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

QCD Factorization

full QCD:



$$\frac{1}{(p_{t,\bar{t}}+k)^2-m_t^2}$$

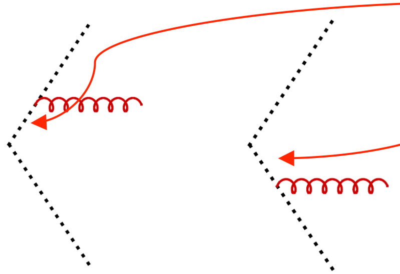
3 phase space regions: $\lambda \sim m_t/Q$

- n-collinear: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, 1, \lambda)$
- n̄-collinear: $(k_+, k_-, k_\perp) \sim Q(1, \lambda^2, \lambda)$
- soft: $(k_+, k_-, k_\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2)$

$$(p_{t,\bar{t}}^2 \approx m_t^2\,,\ n^2 = 0\,,\ \bar{n}^2 = 0)$$

Gluon soft:

$$\frac{1}{Q\ n.k} \qquad \frac{1}{Q\ \bar{n}.k}$$



$$Y_n(x) = \overline{\mathrm{P}}\ \exp\Big(-ig\int_0^\infty ds\, n\cdot A_s(ns+x)\Big)$$

$$\overline{Y_{\bar{n}}}(x) = \overline{\mathrm{P}}\ \exp\Big(-ig\int_0^\infty ds\, \bar{n}\cdot \overline{A}_s(\bar{n}s+x)\Big)$$

$$S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\langle 0|\,(\overline{Y}_{\bar{n}})^{cd}\,(Y_n)^{ce}(0)\,\delta(\ell^- - (\hat{P}_a^+)^\dagger)\delta(\ell^- - \hat{P}_b^-)\,(Y_n^\dagger)^{ef}(\overline{Y}_{\bar{n}}^\dagger)^{df}(0)\,|0\rangle$$

QCD Factorization

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

Jet functions: $$B_+(\hat s, \Gamma_t, \mu) = {\rm Im}\left[\frac{-i}{12\pi m_J} \int d^4x\, e^{ir.x} \langle 0|\, T\{\bar h_{v_+}(0) W_n(0)\, W_n^\dagger(x) h_{v_+}(x)\}\, |0\rangle\right]$$

- perturbative
- dependent on mass, width, color charge

$$B_\pm^{\rm Born}(\hat s, \Gamma_t) = \frac{1}{\pi m_t} \frac{\Gamma_t}{\hat s^2 + \Gamma_t^2} \qquad \hat s = \frac{M^2 - m_t^2}{m_t}$$

Soft function: $$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a}) \delta(\ell^- - k_s^{-b}) \langle 0| \overline{Y}_{\bar n}\, Y_n(0) |X_s\rangle \langle X_s| Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0)|0\rangle$$

- non-perturbative
- analogous to the pdf's
- dependent on color charge, kinematics

Independent of the mass !

NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172 \text{ GeV}$$

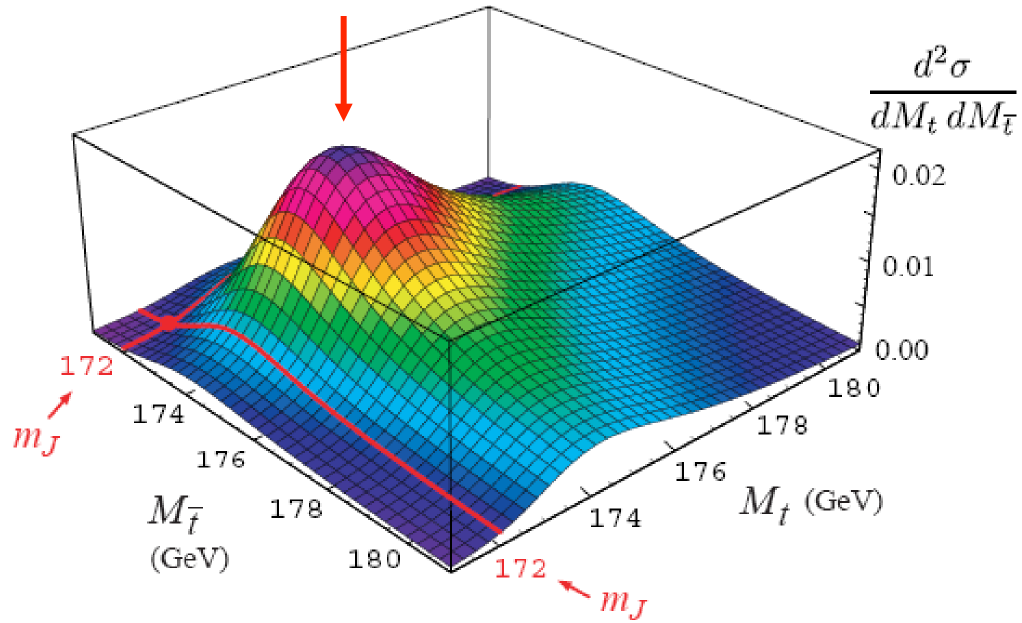
$$\Gamma = 1.43 \text{ GeV}$$

$$m_J(2\,\text{GeV}) = 172 \text{ GeV}$$

$$\mu_\Gamma = 5 \text{ GeV}$$

$$\mu_\Lambda = 1 \text{ GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55 \text{ GeV}$$

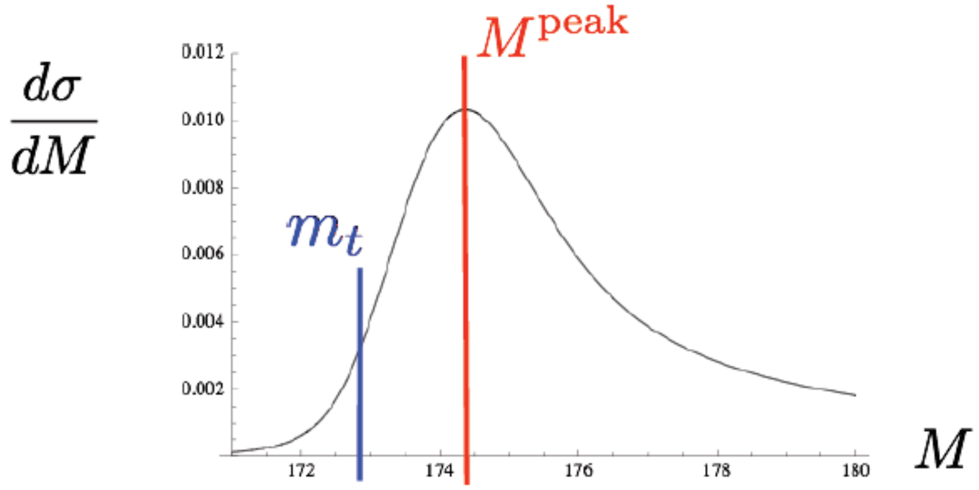


Non-perturbative effects shift the peak by +2.4 GeV and broaden the distribution.

$$k_+ = k_0 - k_3$$
$$k_- = k_0 + k_3$$

QCD Factorization

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008



$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q}{m_t}\,\Omega_1 + \mathcal{O}\Big(\frac{m_t \Lambda_{\rm QCD}}{Q}\Big)$$

first moment of the soft function: $\Omega_1 = \int d\ell\, \ell\, S(\ell, \mu)$

→ from event shape distributions

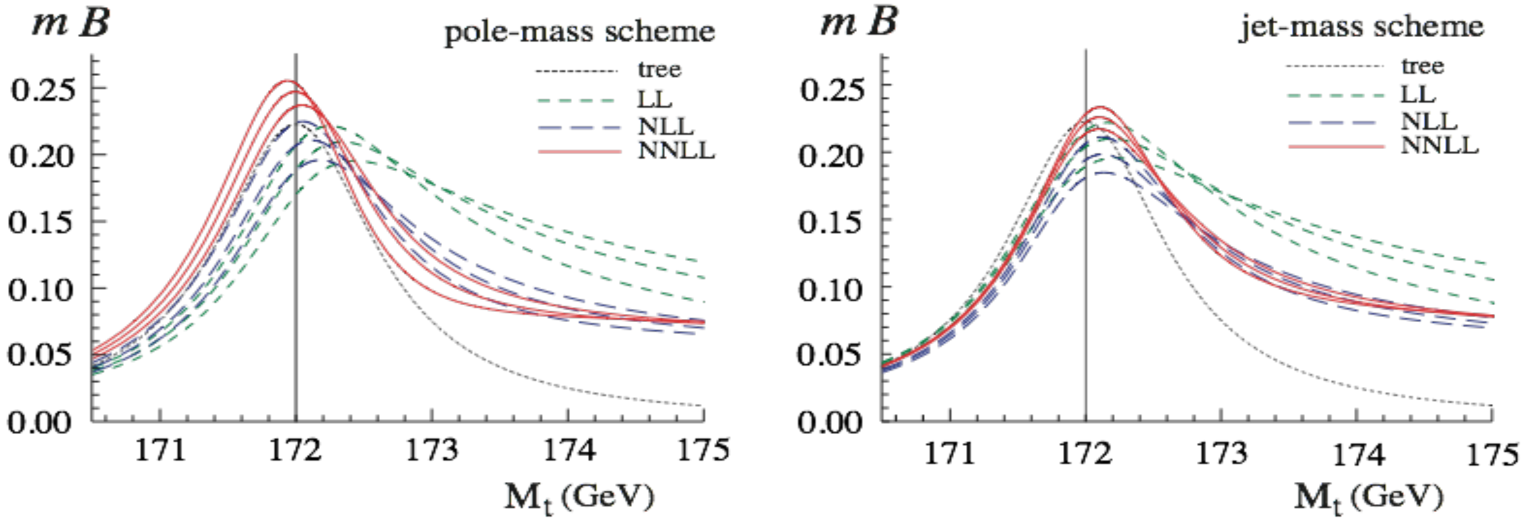
QCD Factorization

Higher Orders & Top Mass Scheme:

Fleming, Mantry, Stewart, AHH
Phys.Rev.D77:074010,2008

$$\mathcal{B}_{\pm}(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\,\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\}$$

Jain, Scimemi, Stewart
PRD77, 094008(2008)

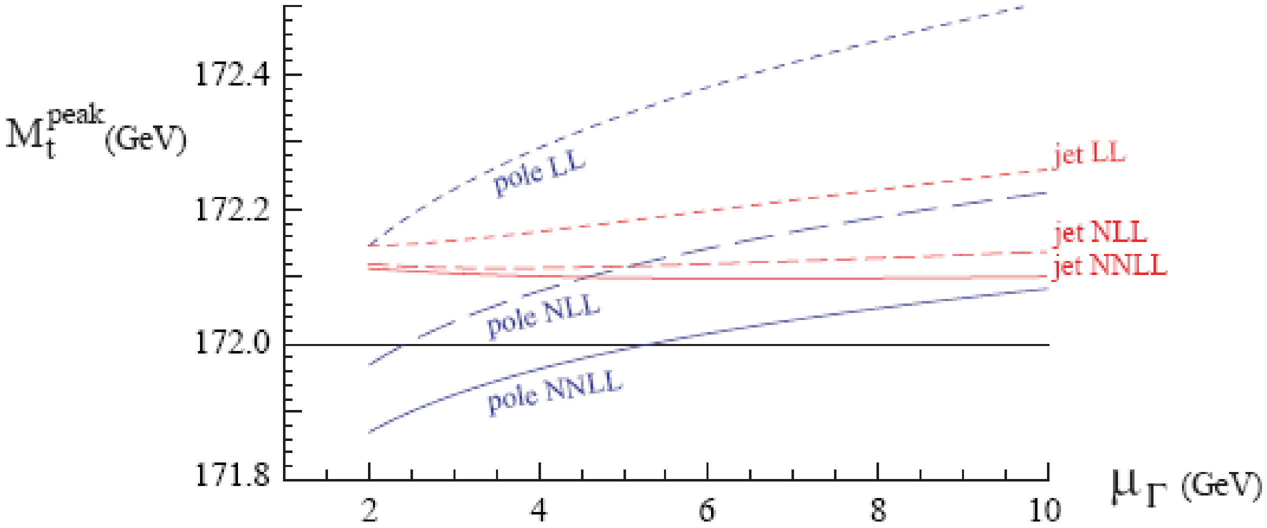


$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R \frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right] + \mathcal{O}(\alpha_s^2)$$

$$R \sim \Gamma_t$$

NLL Numerical Analysis

Scale-dependence of peak position



- Jet mass scheme: significanly better perturbative behavior.
- Renormalon problem of pole scheme already evident at NLL.

Theory Issues for $pp \longrightarrow t\bar{t} + X$

- ★ definition of jet observables → Hadron event shapes
- ★ initial state radiation
- ★ final state radiation
- • underlying events → Soft function ?
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

$$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}$$

Banfi, Salam, Zanderighi

★ **Can be addressed in the framework of a LC.**

★ **Requires extensions of LC concepts and other known concepts**

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Final State Shower

- Start: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1\ \mathrm{GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

Let's assume that these aspects are treated correctly in the MC

MC Top Mass

Conclusion (quick answer):

constant of order unity → c

$$m_t^{\mathrm{MC}}(R_{sc}) = m_t^{\mathrm{pole}} - R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right]$$

Determination of the MSbar mass:

$$m_t^{\mathrm{TeV}} = m_t^{\mathrm{MC}}(R_{sc}) = 172.6 \pm 0.8(stat) \pm 1.1(syst)$$

↓ 3-loop R-evolution equation

AHH,Jain, Scimemi, Stewart
PRL 101,151602(2008)

$$\overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3\,^{+0.6}_{-0.3}\ \mathrm{GeV} \qquad (c = 3^{+6}_{-2})$$

More systematic study needed for final answer!

The exercise just carried out does not account for any conceptual uncertainties!

Outlook & Conclusion

Conclusion:

→ Current top mass measurements from the Tevatron refer to the top mass parameter in Pythia m_t^{Pythia}.

$m_t^J(2\ \text{GeV})$

→ For a high energy Linear Collider we have a factorization theorem to do MC independent short-distance Lagrangian top mass measurements (jet mass)

→ The analogy between MC generators and factorization theorem indicates that the m_t^{Pythia} is a short-distance mass like the jet mass (and not the pole mass).

m_t^{Pythia}	$m_t^J(2\ \text{GeV})$		
	1-loop	2-loop	3-loop
160.00			
165.00			
170.00			

Plans:

→ "Measure" the m_t^{Pythia} in terms of the Jet mass $m_t^J(2\ \text{GeV})$ using thrust and other event shapes

→ Derivation of eventshape-like factorization theorems for Tevatron/LHC

→ "Measure" m_t^{Pythia} for LHC-Pythia