

# Carrier multiplication in quantum dots for more efficient solar cells

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## Introduction

- Semiconductor nanocrystal quantum dots (QDs)
- Efficiency of solar cells, energy losses
- Carrier multiplication (CM)

## Review of experimental results

- PbSe & PbS QDs
- Controversy
- Latest results

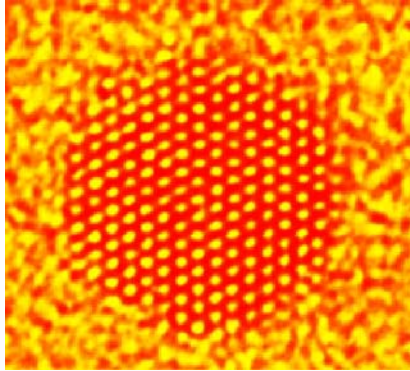
## Our theoretical works

- Theory of impact ionization & CM
- Efficiency of CM in PbS nanocrystals
- Possible strategy to improve CM
- Case of  $\alpha$ -Sn QDs

## On-going works

- Relaxation of carriers by emission of phonons

# Semiconductor nanocrystals



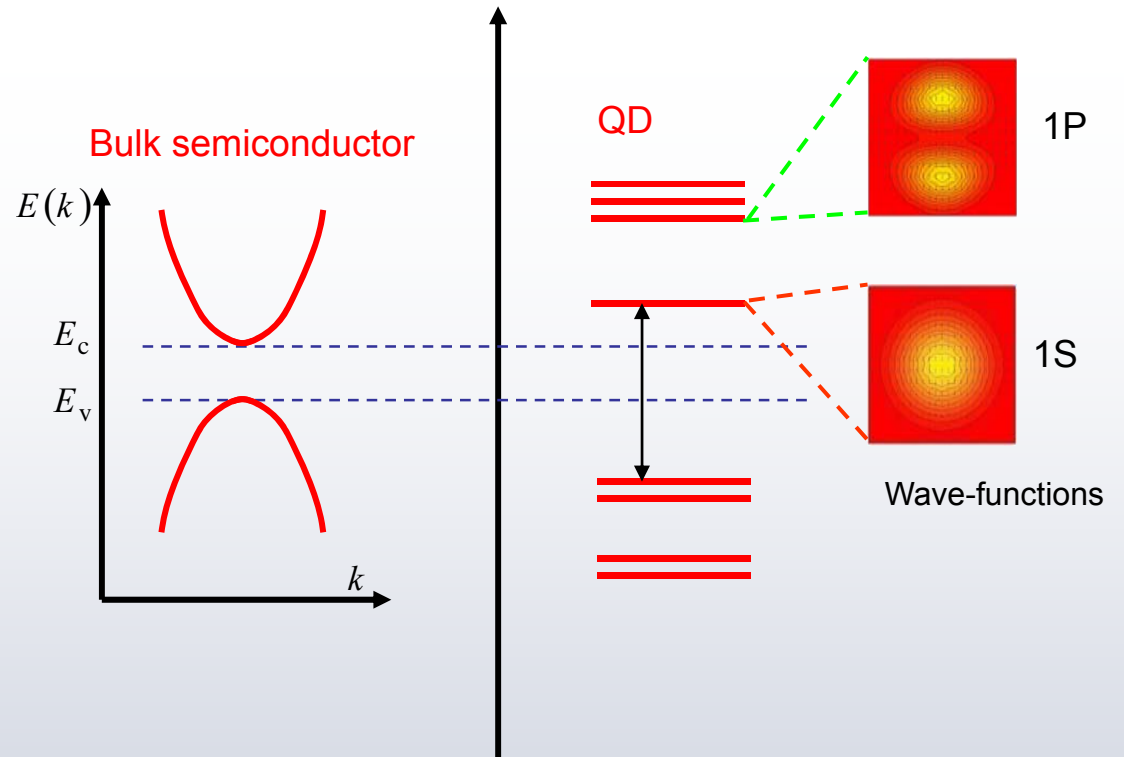
CdSe Nanocrystal



Increasing diameter



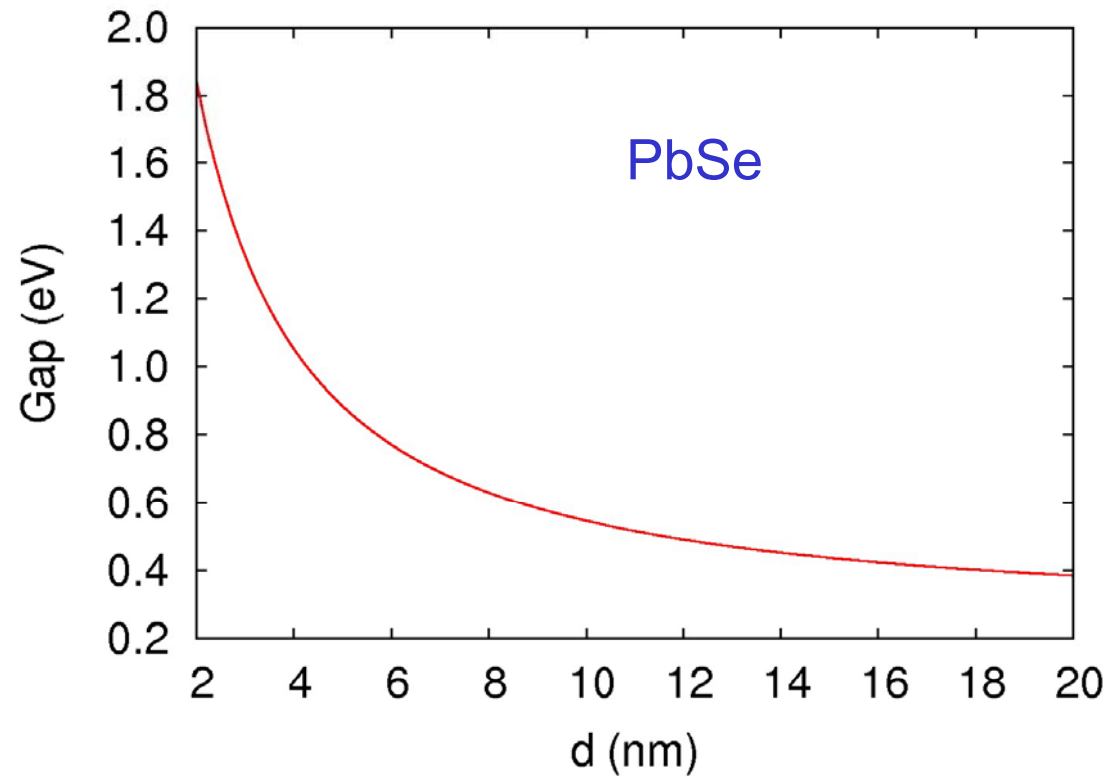
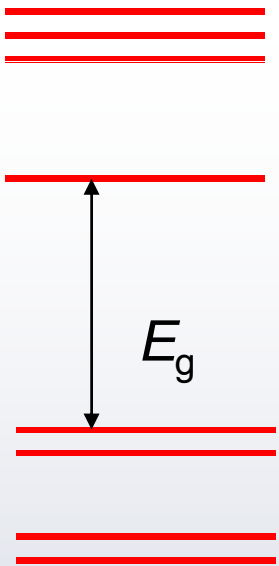
Quantum dot (QD) for electrons and holes



Size-dependent optical properties  
Attractive materials for new generation of PV

# Quantum confinement in PbSe

QD



Bulk bandgap: 0.28 eV at 300K

## Schockley-Queisser limit

Max theoretical efficiency of a single c-Si solar cell ~30%.

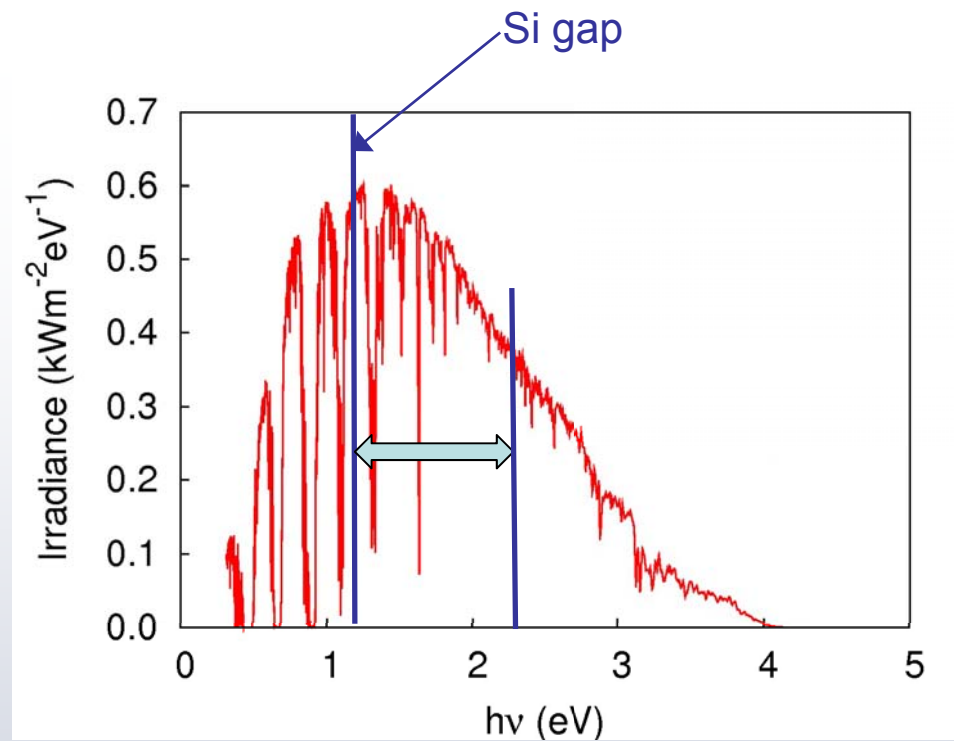
Actual c-Si solar cell efficiency : ~ 20 %

New strategies to improve PV efficiency

⇒ Carrier multiplication : fundamental physical process that could potentially improve PV efficiency

# PV efficiency

Main sources of energy loss in c-Si PV cells ?

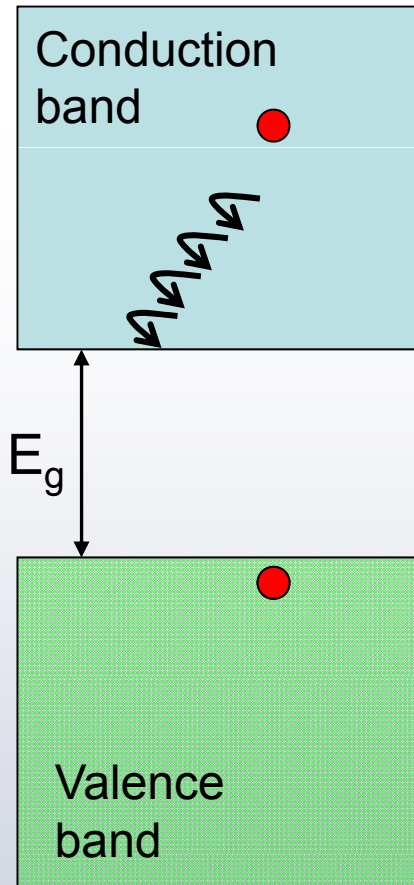


Excess energy  $\rightarrow$  heat



# Multi-phonon relaxation

Carrier cooling by sequential phonon emission



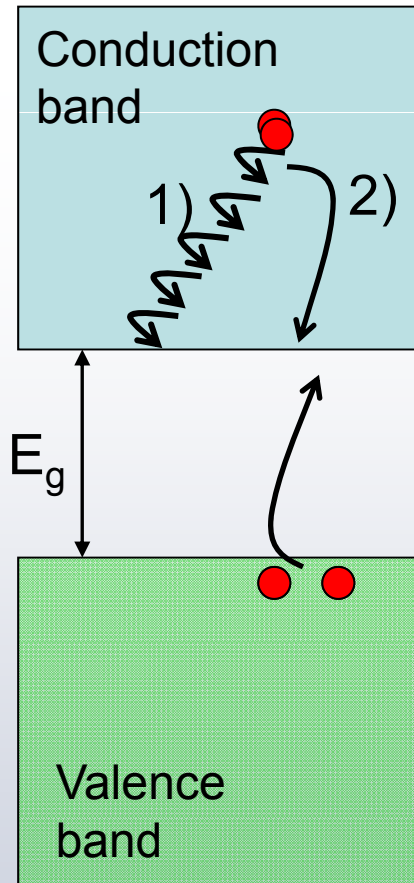
Relaxation pathways:

1) Multi-phonon  $\rightarrow$  heat

Relaxation pathways:

# Impact ionization → carrier multiplication

CM: generation of 2 or more e-h pairs after absorption of single photon



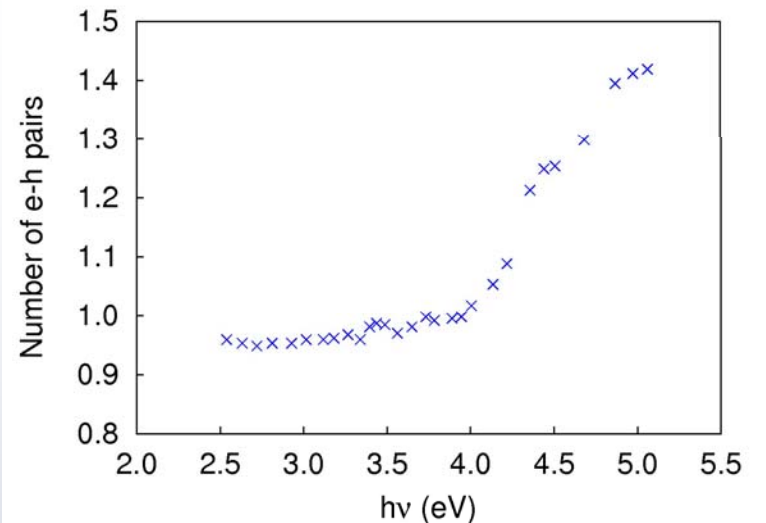
Relaxation pathways:

- 1) Multi-phonon → heat
- 2) CM → multiple e-h<sup>+</sup> per photon

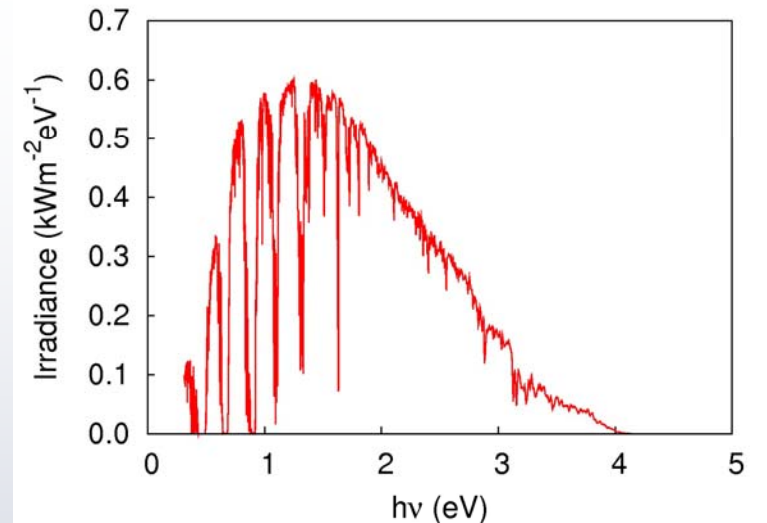


# CM in bulk Si

Bulk Si: CM efficient only above the threshold at  $\sim 3.8$  eV [ $\sim 3.5 \times E_g(\text{Si})$ ]



Number of photo-generated e-h pairs in bulk Si  
Wolf et al, J. Appl. Phys. 1998

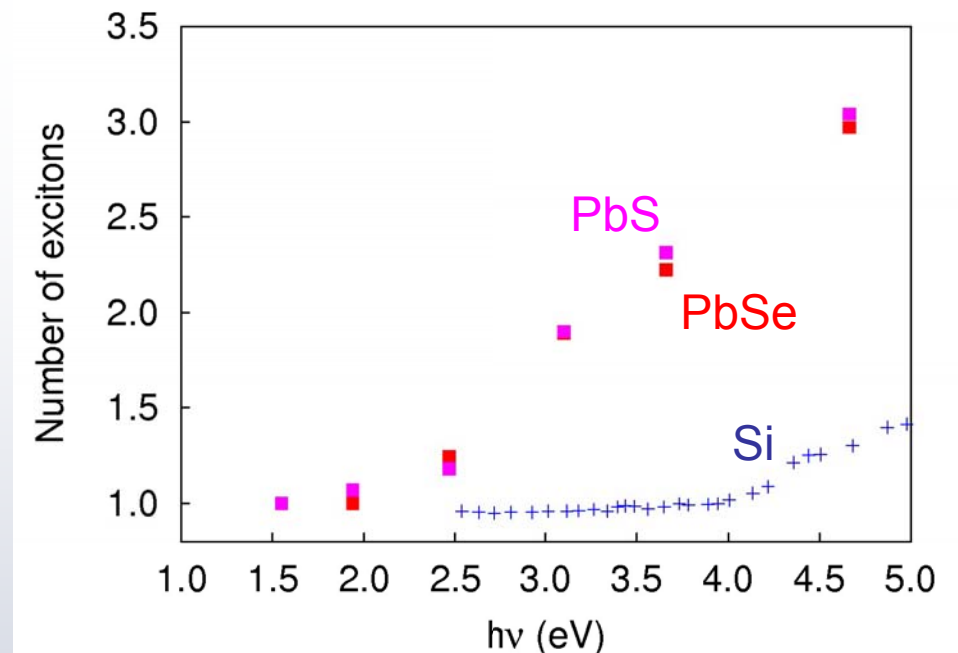


Solar spectrum

# CM in bulk PbSe & PbS

Bulk PbSe & PbS: CM efficient above  $\sim 2.3$  eV (much lower threshold)

but PbSe gap = 0.28 eV, PbS gap = 0.43 eV !



+ : Wolf et al, J. Appl. Phys. 1998

■ , ■ : J. J. H. Pijpers, R. Ulbricht, K.J. Tielrooij, A. Osherov, Y. Golan, C. Delerue, G. Allan & M. Bonn, Nature Physics 5, 811 (2009)

# CM: best situation for PV?

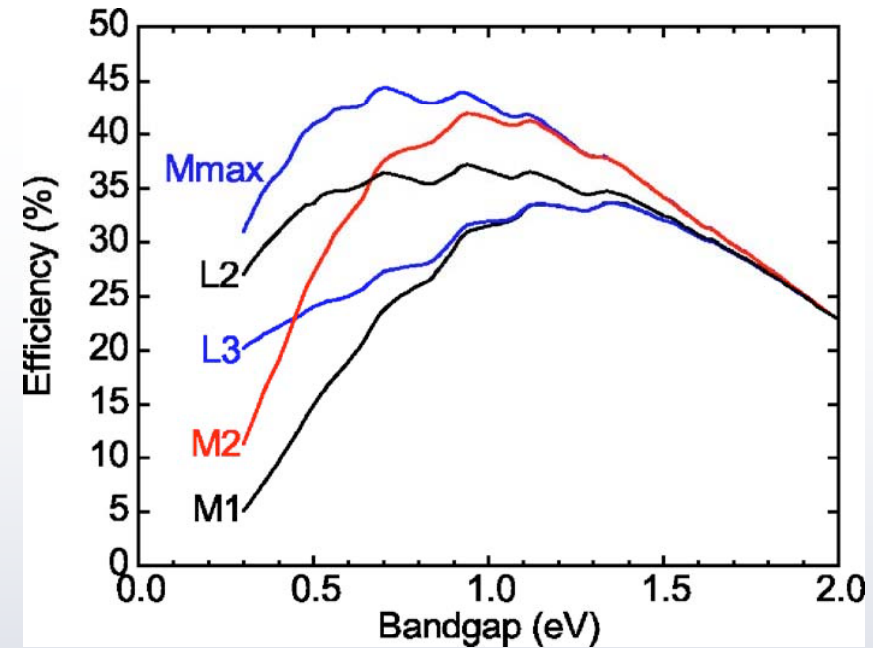
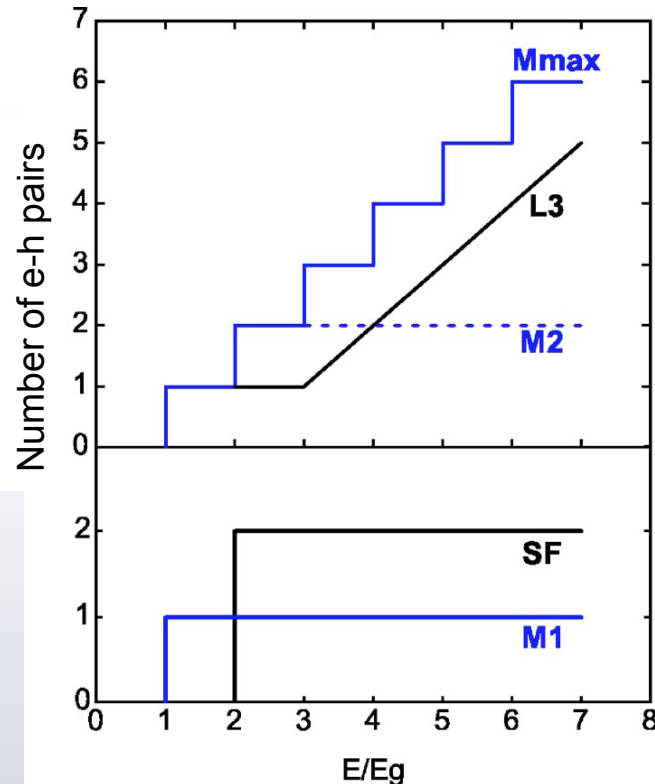
CM:

For PV, the goal (dream ?) is to find materials with:

- 1) The lowest threshold for CM
- 2) A sufficiently large gap  $E_g$  (determines the  $V_{oc}$  of the solar cell)

Constraint (energy conservation): threshold  $> 2 E_g$

# Potential impact of CM on PV efficiency ?



Maximum power conversion efficiency for single gap devices with various absorber types

Hanna & Nozik, J. Appl. Phys. 100, 074510 (2006)

# Impact ionization & CM in QDs

## 02 : A. Nozik: CM could be enhanced in QDs

Nozik, Physica E 14, 115–120 (2002)

- 1) strong e-e interactions favor impact ionization
- 2) confinement breaks  $k$  conservation rules
- 3) less efficient multi-phonon relaxation

## 04 : group of V. Klimov reported efficient CM in PbSe QDs

Schaller & Klimov, Phys. Rev. Lett. 92, 186601 (2004).

Since then: many contradictory reports

# CM in QDs: controversy

## • InAs QDs:

CM observed:

- Klimov, Nano Lett. 7, p3469, 2007
- Pijpers/Bonn, JPC, 111, p4146, 2007
- Pijpers/Bonn, JPC, 112, p4783, 2008
- Ben Lulu, Nano Lett. 8, p1207, 2008

No CM:

## • CdSe QDs:

CM observed:

- Klimov, APL 87, p253102, 2005
- Bawendi, PRB, 76, 081304, 2007

No CM:

## • PbS QDs:

CM observed:

- Nozik, Nano Lett, 5, p865, 2006
- Nootz, Sargent... Phys. Rev. B 83, 155302 (2011)
- Bawendi, PRB, 78, 125325, 2008

No CM:

## • PbSe QDs:

Efficient CM:

- Klimov: PRL(2004), Nano Lett. (2006), Nature Phys. (2005), APL (2005)

- Nozik: Nano Lett. (2006), JACS (2006)

Intermediate:

- Trinh et al, Nano Lett., 8, 1713 (2008)

- Klimov, Accounts Chem. Res. (2008)

- Trinh et al, Nano Lett 11, 1623 (2011)

Weak CM:

- Bawendi, PRB, 78, 125325 (2008)

No CM:

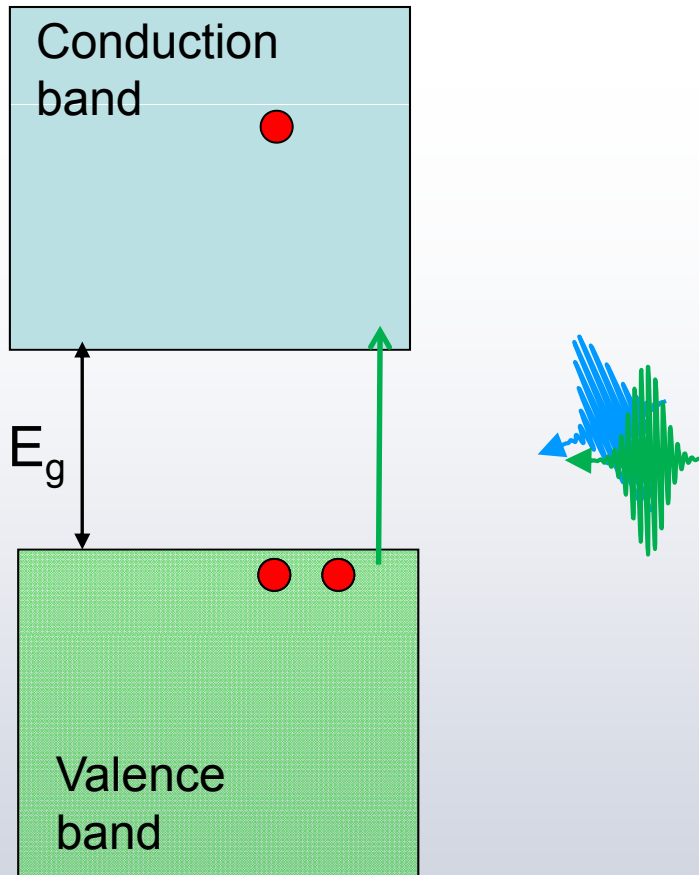
- Bawendi, ACS Nano (2012)

- Miaja-Avila et al, Nano Lett (2012)



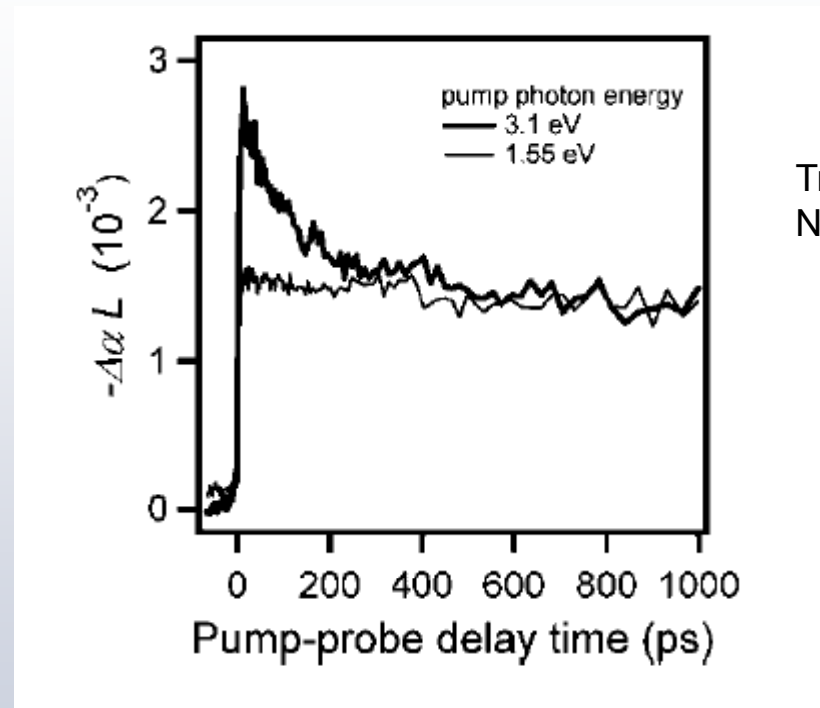
# Transient absorption experiments

## Pump-probe experiments



Pump-pulse-induced absorption change :  $\Delta\alpha$

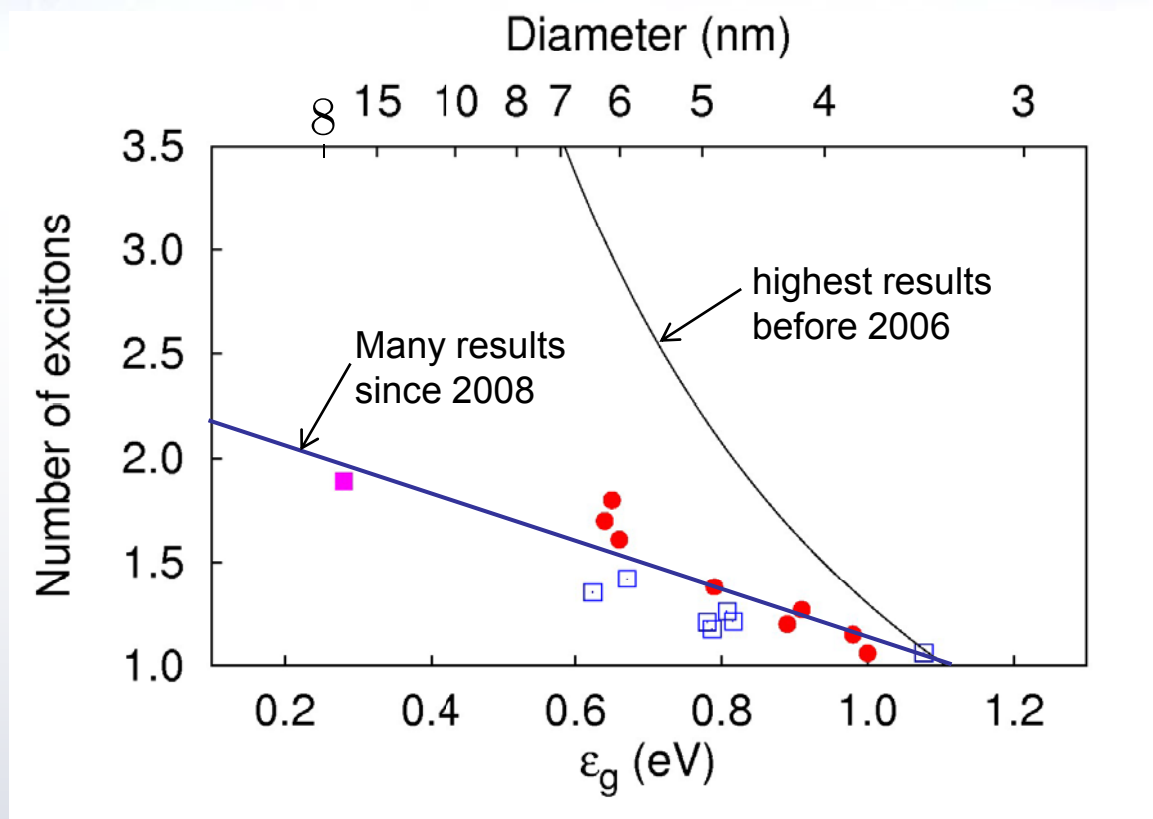
Lifetime of bi-excitons  $\ll$  mono-excitons



Trinh et al,  
Nano Lett 08

# Convergence of the latest results ?

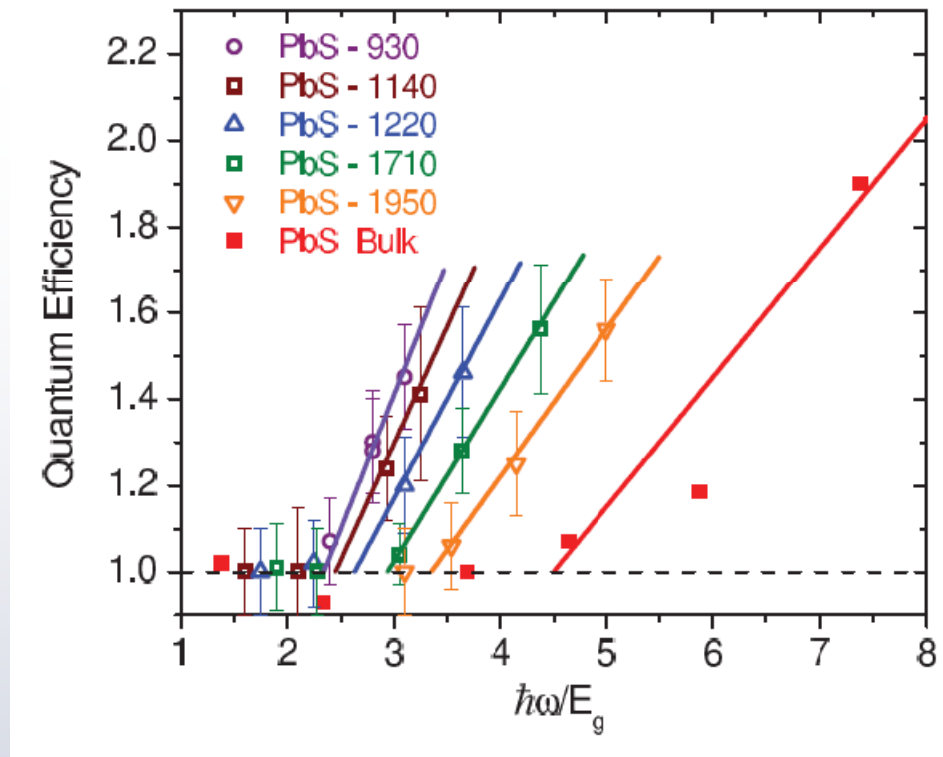
$h\nu = 3.1$  eV  
**PbSe** QDs  
 Variable size  $\rightarrow \epsilon_g$



- : J. J. H. Pijpers, R. Ulbricht, K.J. Tielrooij, A. Osherov, Y. Golan, C. Delerue, G. Allan & M. Bonn, Nature Physics (2009)
- : John A. McGuire, Jin Joo, Jeffrey M. Pietryga, Richard D. Schaller, and Victor I. Klimov, Acc. Chem. Res. (2008)
- : M. Aerts, C.S.S. Sandeep, Y. Gao, T.J. Savenije, J.M. Schins, A.J. Houtepen, S. Kinge, and L.D.A. Siebbeles, Nano Lett (2011)
- \_\_\_\_\_ : R.D. Schaller, V.M. Agranovitch, V.I. Klimov, Nature Physics (2005)

# CM in PbS QDs

Recent systematic studies  
CM in **PbS** QDs



G. Nootz et al  
Phys. Rev. B 83, 155302 (2011)

Interpretation of these results ?  
Impact on the power conversion efficiency ?

# Theoretical description of CM

## Possible approaches:

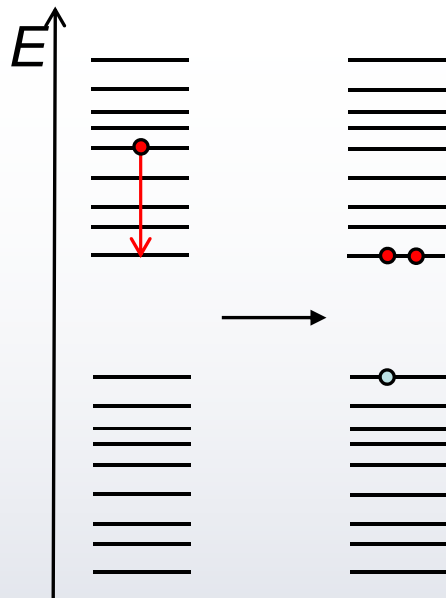
- Models (many free parameters)
- *Ab initio* Time-Domain Studies (e.g. group of O. Prezhdou, Rochester). Time-consuming, small clusters (10 atoms).

## Our approach: Two steps:

- 1) Step 1: Calculation of the impact ionization rates
- 2) Step 2 : Simulation of the relaxation of the carriers  $\Rightarrow$  CM

# Step 1: calculation of impact ionization rates

Calculation of the impact ionization rate  $W$  versus energy  $E$   
(lifetime  $\tau = 1/W$ )



$E = 0$  : top of the bulk valence band

## Methodology / calculations:

Electronic structure in tight-binding

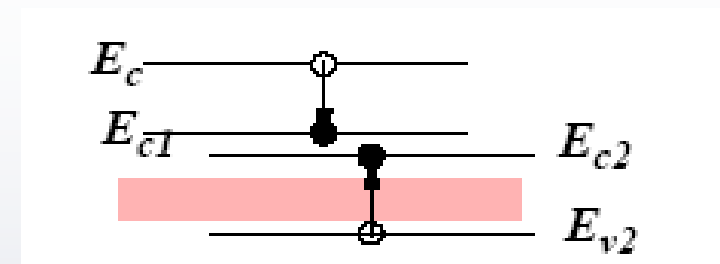
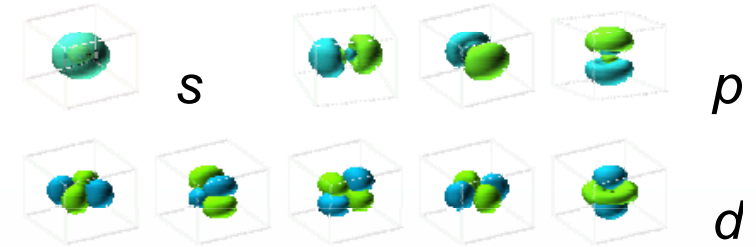
Dielectric matrix in RPA

Matrix elements of the Coulomb interaction

$W$  : Fermi golden rule

## Transition rate?

Electronic states calculated in tight binding  
 Calculation of the dielectric screening (RPA)  
 Calculation of the screened Coulomb interaction  
 Fermi golden rule → transition rate



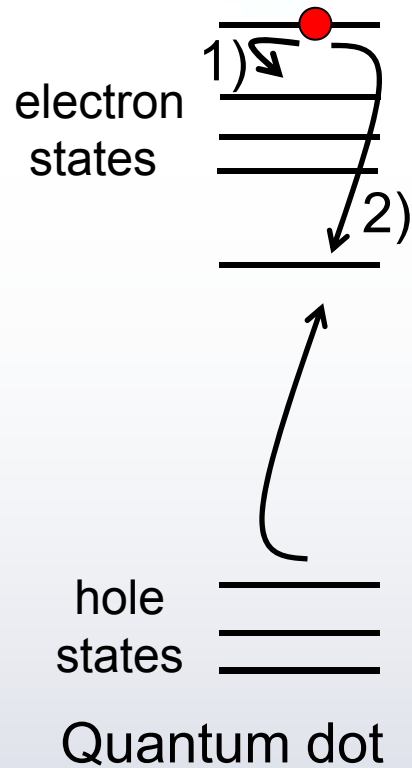
$$W = \frac{2\pi}{\hbar} \sum_{i,f} p(i) |\langle f | V | i \rangle|^2 \delta(E_f - E_i)$$

$|i\rangle, |f\rangle$  : Slater determinants built on tight binding wave-functions

$$V(r, r') = \int \varepsilon^{-1}(r, r'') \frac{e^2}{|r - r''|} d^3 r''$$



# Step 2: simulation of the relaxation of the carriers



## Calculation of CM efficiency

Competition between:

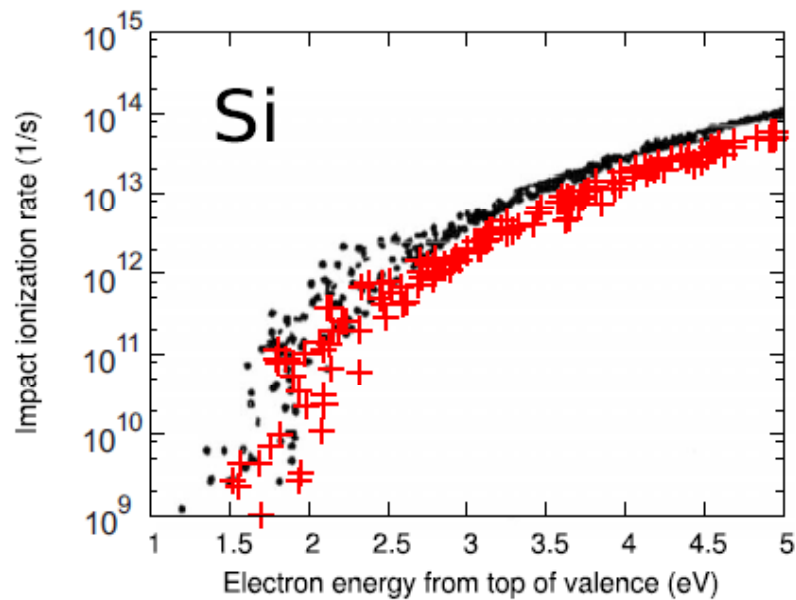
- 1) multi-phonon relaxation
- 2) impact ionization

### Methodology:

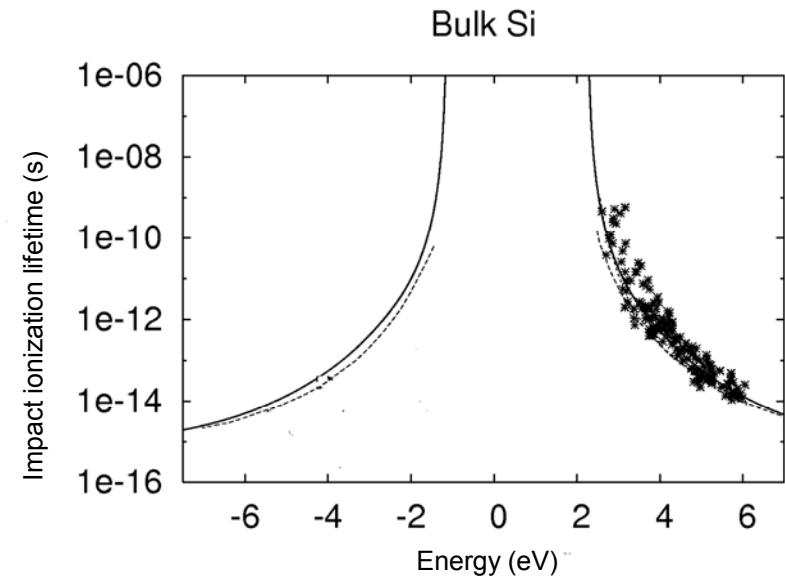
- 1) Calculation of oscillator strengths and  $\alpha(h\nu)$
- 2) Multi-phonon relaxation: assume an energy-independent relaxation time  $\tau_{ph}$
- 3) Impact ionization: using calculated lifetimes
- 4) Numerical simulation of the relaxation

# Impact ionization in bulk Si

Calculated impact ionization rates/lifetimes in bulk Si:



- + : Kotani et al, PRB 81, 125201 (2010)
- = Sano et al, JAP 75, 5102 (94)

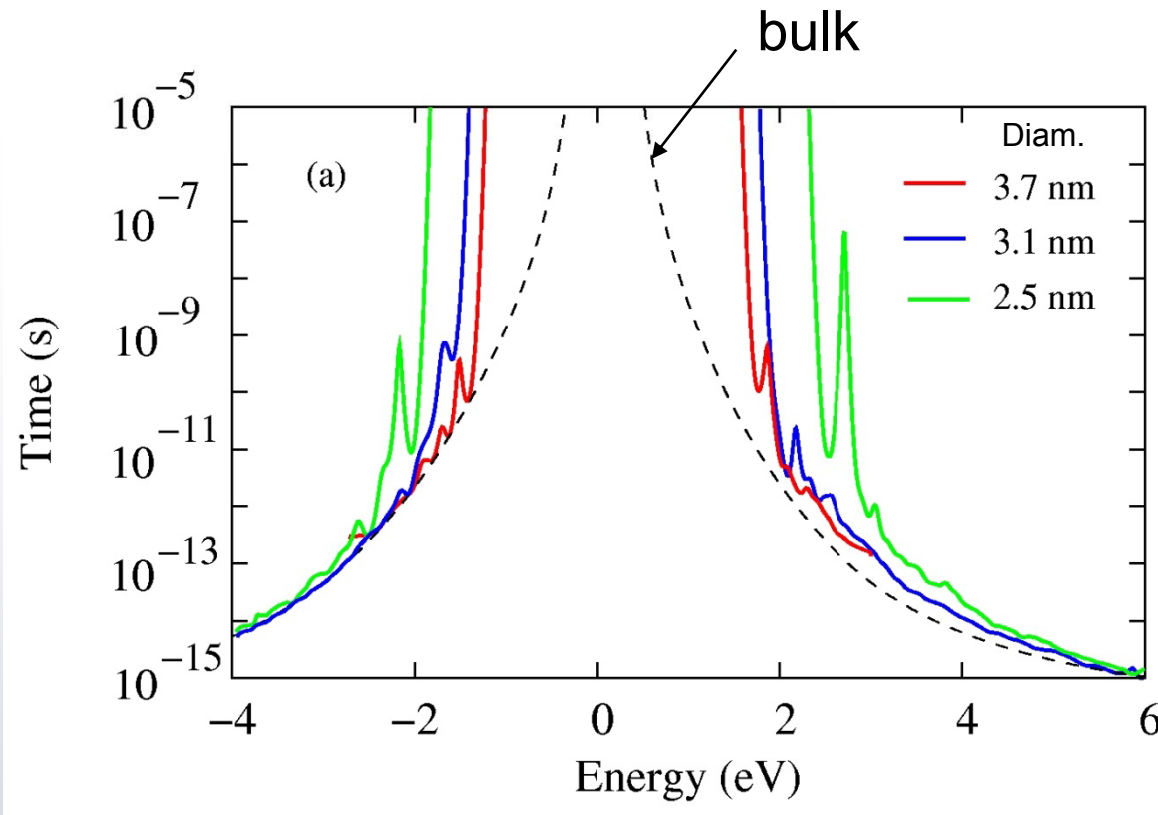


Continuous line : our calculations.  
Dashed line: Sano et al, JAP (1994).  
\* = Kunikiyo et al, 1993

$E > 0 : E_c$   
 $E < 0 : E_v$

# Impact ionization lifetime in PbSe QDs

$$\tau = 1/W$$



$$E > 0 : E_c$$

$$E < 0 : E_v$$

(0 = top of bulk valence band)

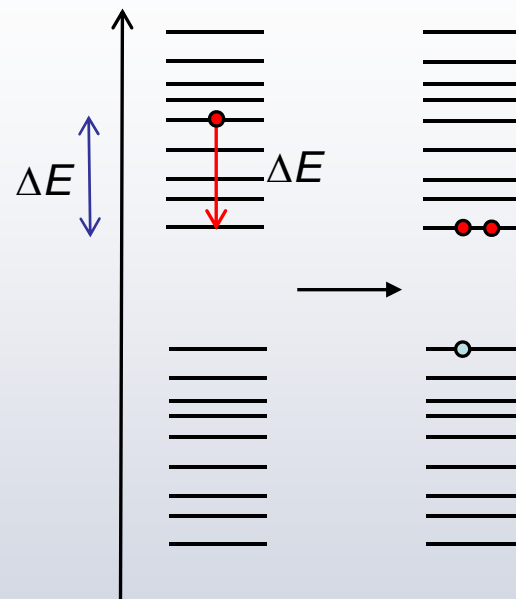
G. Allan & C. Delerue, PRB 73, 205423 (2006)

Very similar behavior in PbS, InAs, Si

# Origin of the energy dependence

The variations of the impact ionization rate are mainly due to the variations of the density of final states

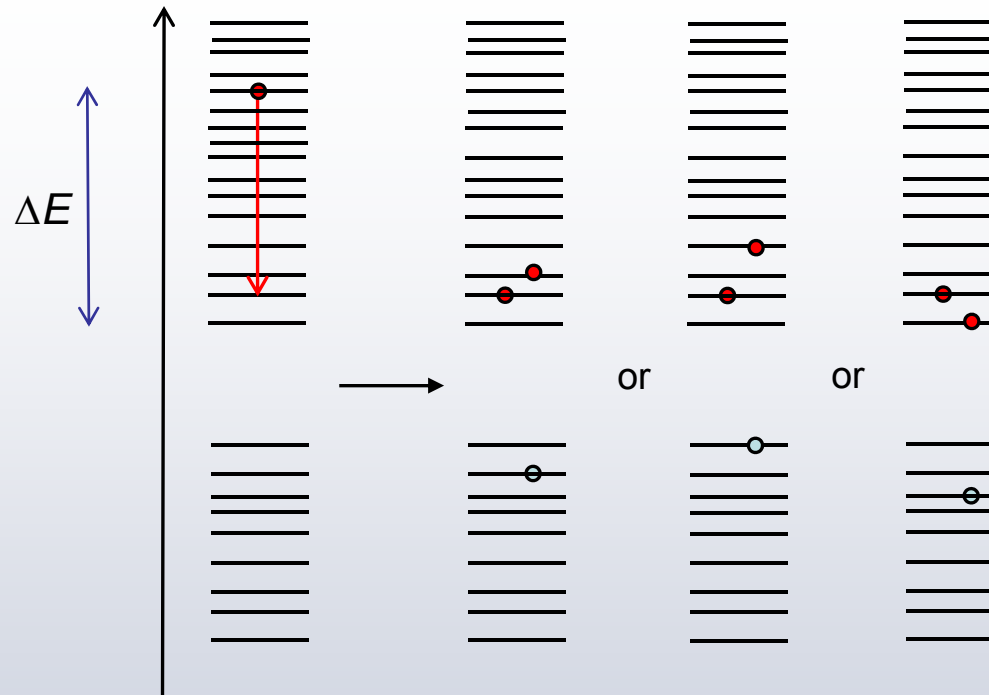
If  $\Delta E = E_g$ , only one final state



# Origin of the energy dependence

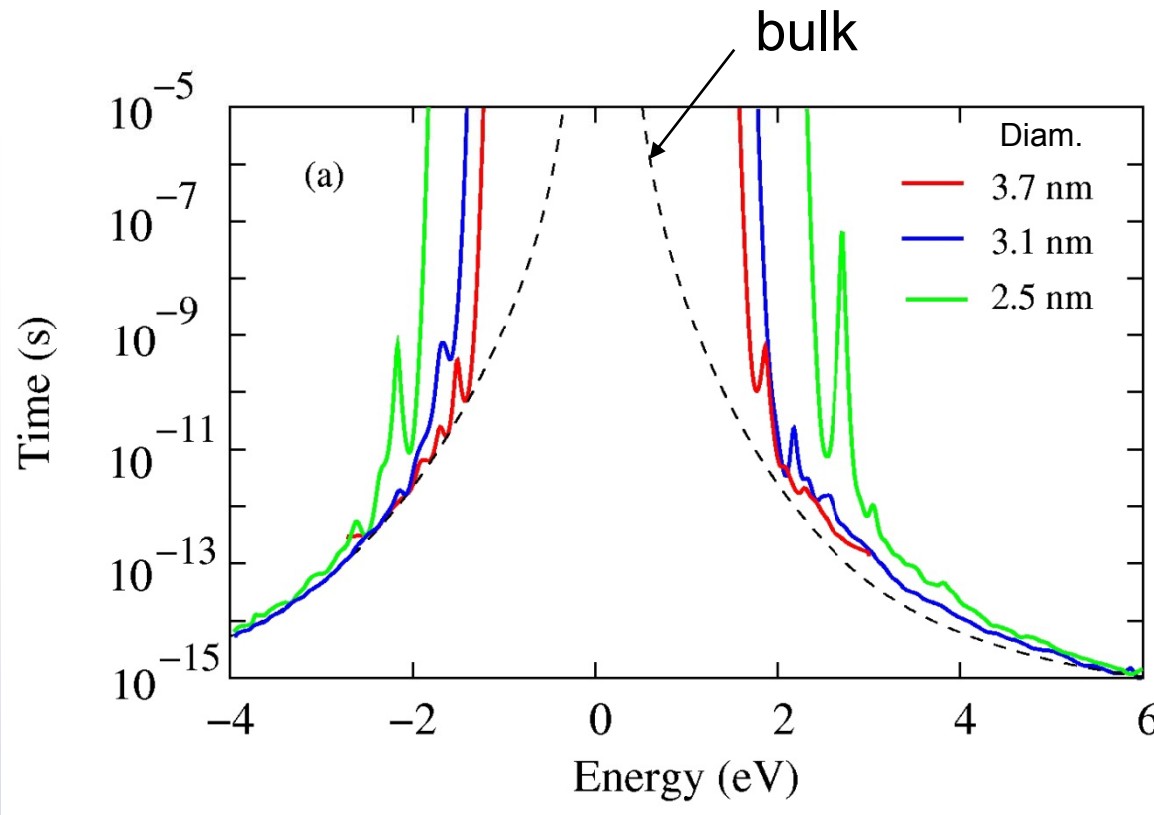
The variations of the impact ionization rate are mainly due to the variations of the density of final states

If  $\Delta E \gg E_g$ , many final states



# Impact ionization lifetime in PbSe QDs

$$\tau = 1/W$$



$$E > 0 : E_c$$

$$E < 0 : E_v$$

(0 = top of bulk valence band)

G. Allan & C. Delerue, PRB 73, 205423 (2006)

Very similar behavior in PbS, InAs, Si



# Impact ionization: QD / bulk

Impact ionization is less efficient in QDs than in bulk  
(same efficiency at high excess energy)

Reasons :

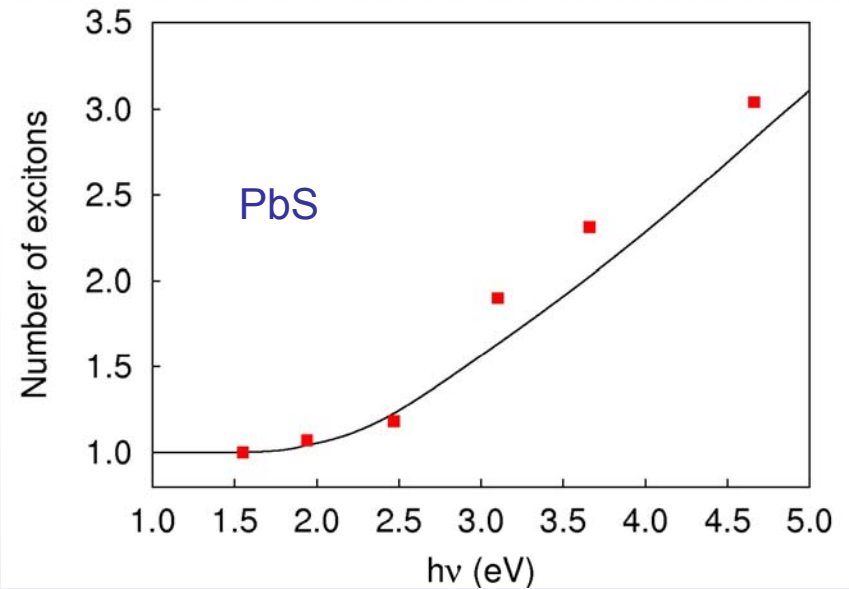
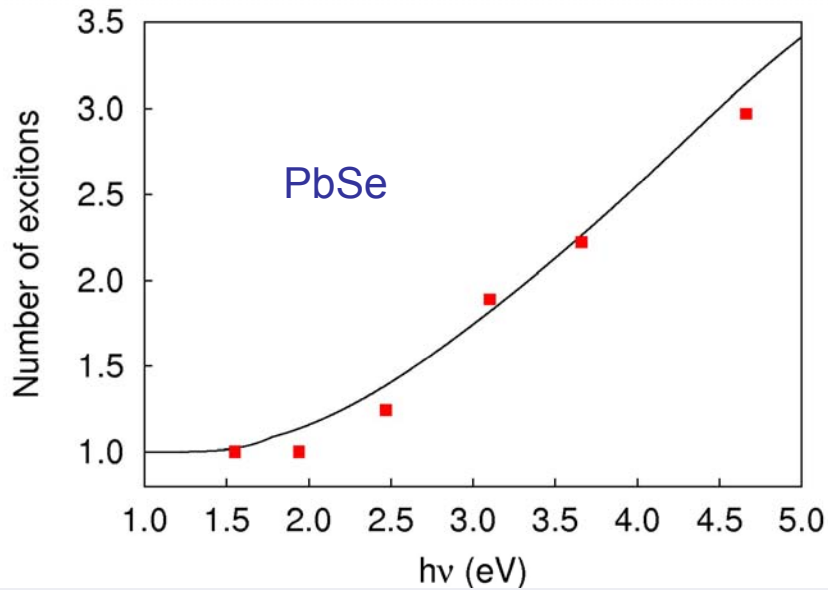
1) The momentum conservation rule plays no role at high excess energy.

In bulk,  $k$  conservation can be ignored.

E.O. Kane, Phys. Rev., 159 (1967)

2) The density of final states is smaller in QDs than in bulk

# Calculations: case of bulk PbSe & PbS

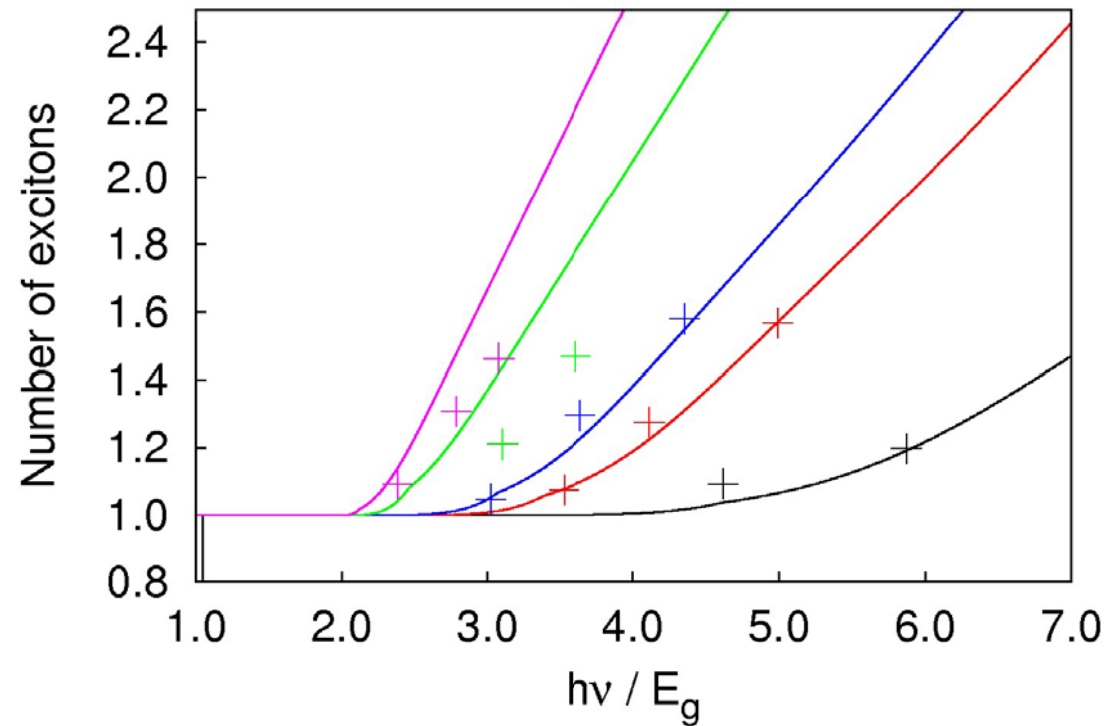


■ : experiments

\_\_\_\_\_ : theory ( $\tau_{ph} = 0.5$  ps)

J. J. H. Pijpers, R. Ulbricht, K.J. Tielrooij, A. Osherov, Y. Golan, C. Delerue, G. Allan & M. Bonn, Nature Physics (2009)

# Calculations: case of PbS QDs



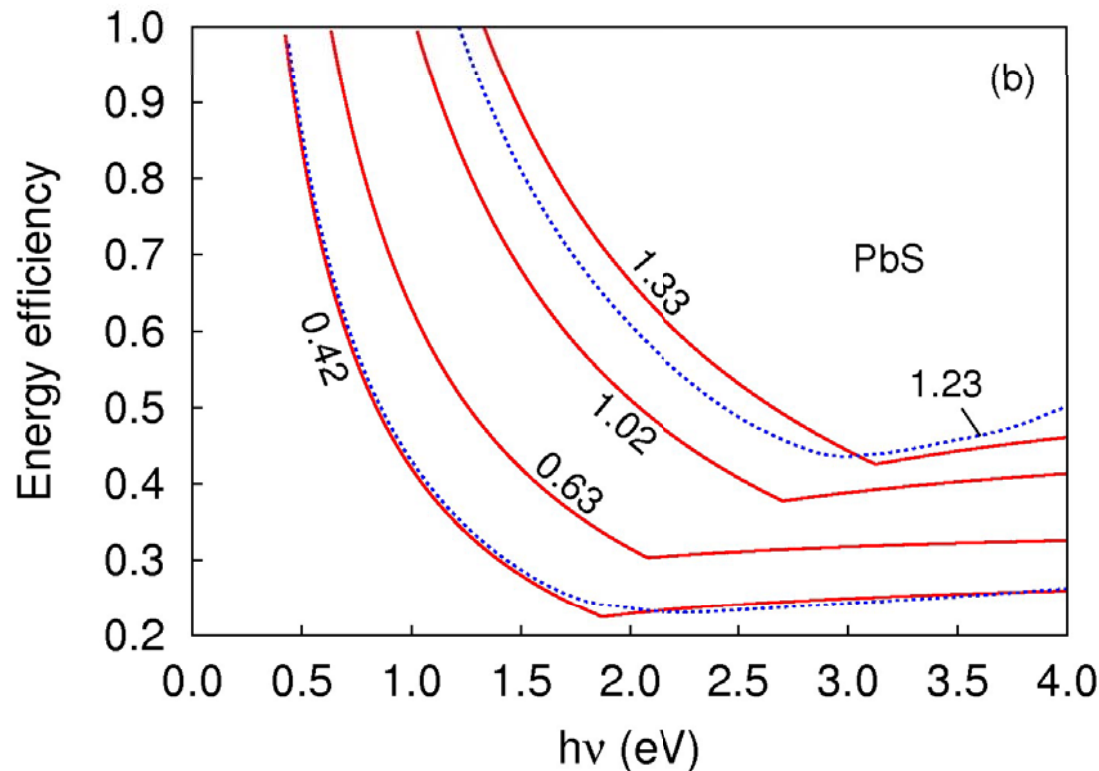
Experiments: G. Nootz et al, Phys. Rev. B 83, 155302 (2011)

+ :  $E_g = 0.42$  eV (bulk) + :  $E_g = 0.63$  eV  
+ :  $E_g = 0.72$  eV + :  $E_g = 1.02$  eV  
+ :  $E_g = 1.33$  eV

Lines : theory ( $\tau_{ph} = 0.5$  ps)

# PbS: energy efficiency of CM

$$\text{CM Energy efficiency} = \frac{\text{number of excitons} * \text{energy gap}}{\text{photon energy } h\nu}$$



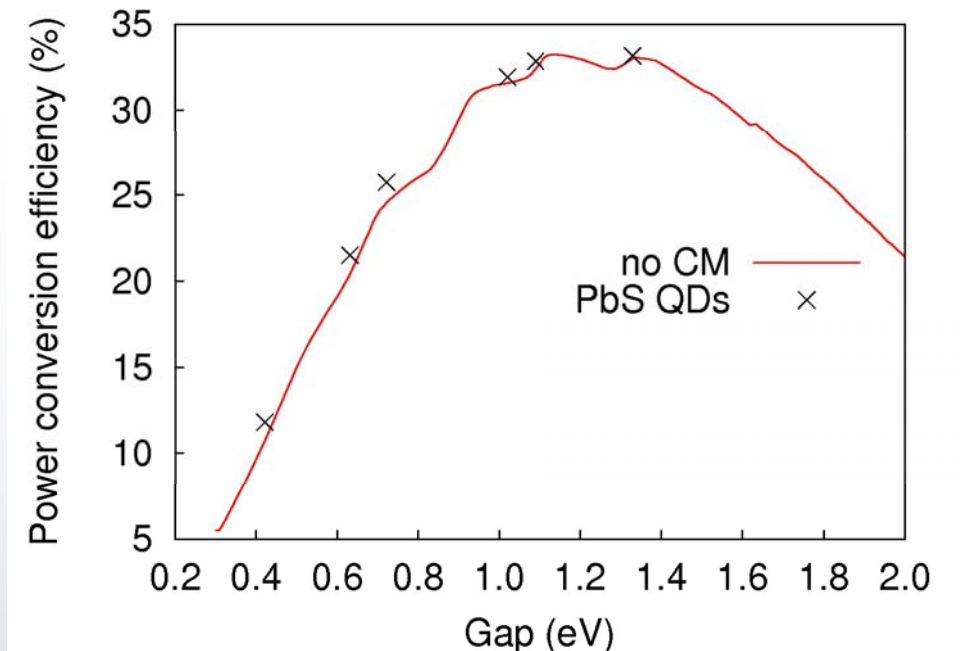
Energy efficiency versus  
hv for PbS QDs

\_\_\_\_\_ : Experiments: G. Nootz et al, Phys. Rev. B 83, 155302 (2011)

.....: theory ( $\tau_{\text{ph}} = 1$  ps)

# Max PV efficiency of PbS QD cell

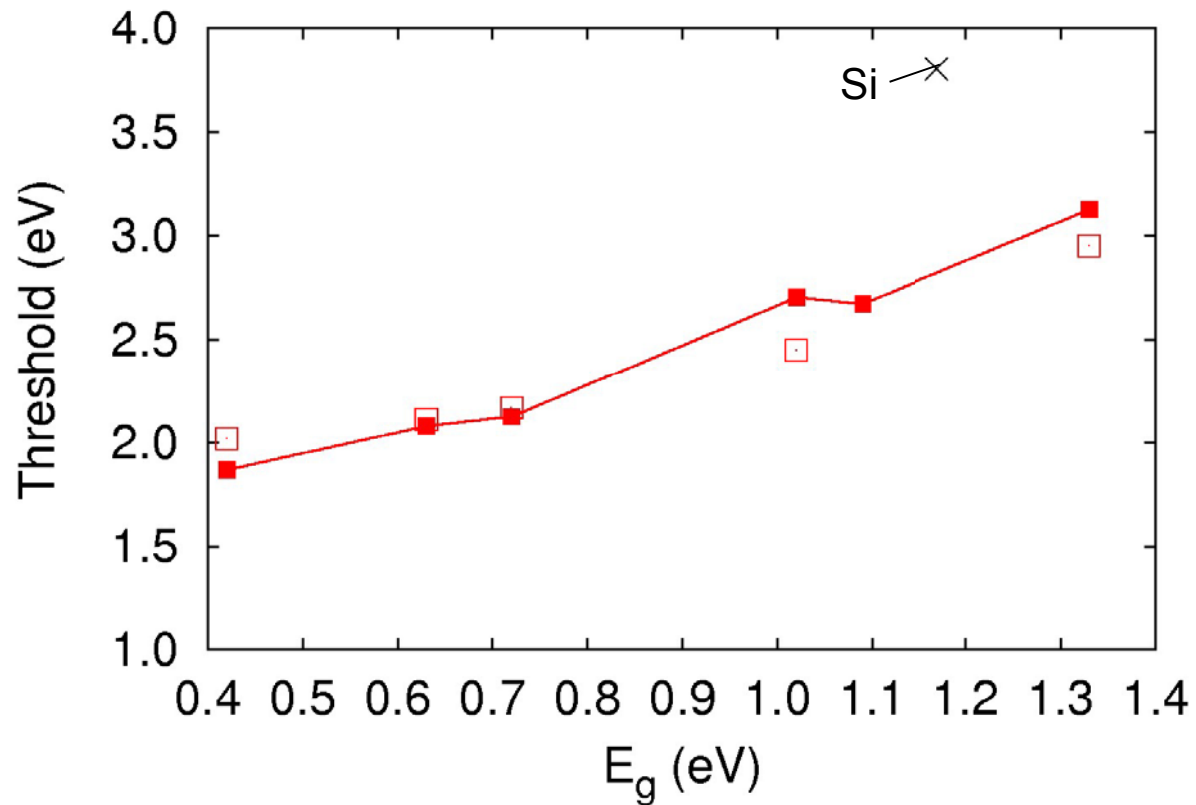
Maximum power conversion efficiency for single gap devices



Efficiency calculated from the experimental CM yields  
Similar results using calculated CM yields

⇒ Small impact of CM in PbS QD PV devices  
(could be higher using concentrated light)

# CM threshold in PbS QDs



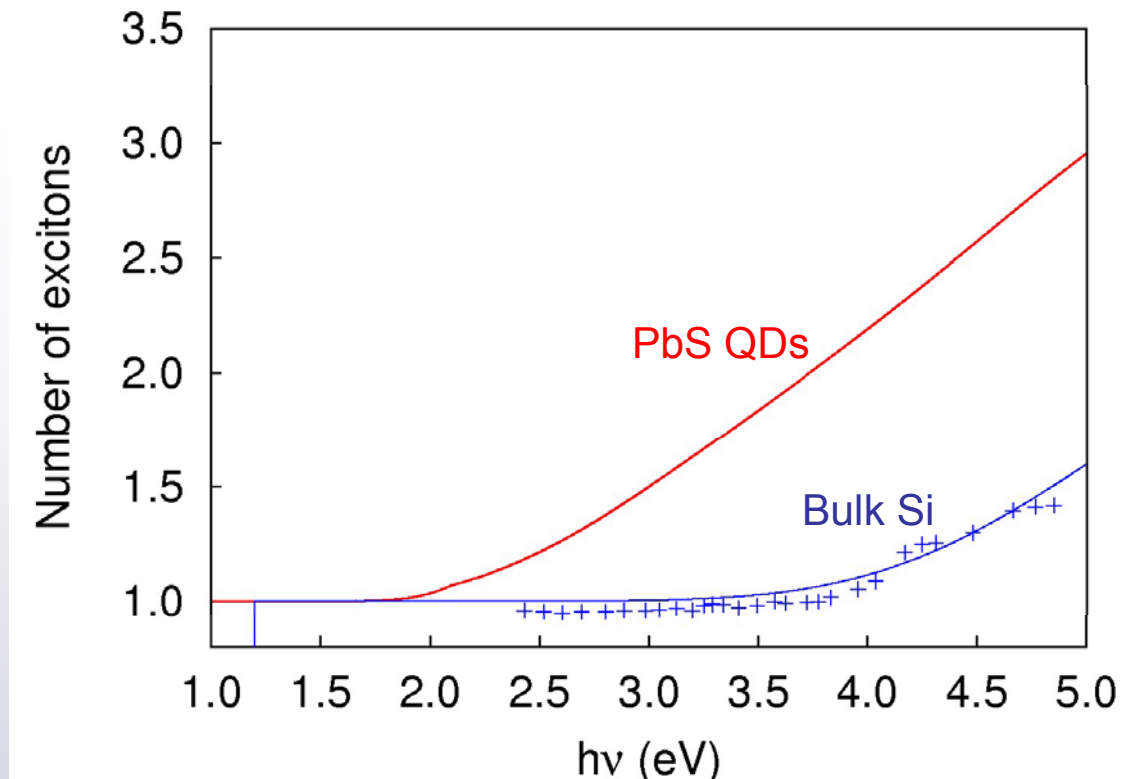
■ : experiments (Nootz et al, PRB 11)  
□ : theory ( $\tau_{ph} = 0.5$  ps)

CM threshold remains too high in energy even if it is considerably smaller than in Si



# CM yield: Si bulk / PbS QDs

Comparison : **PbS QDs / bulk Si** (gap = 1.2 eV)

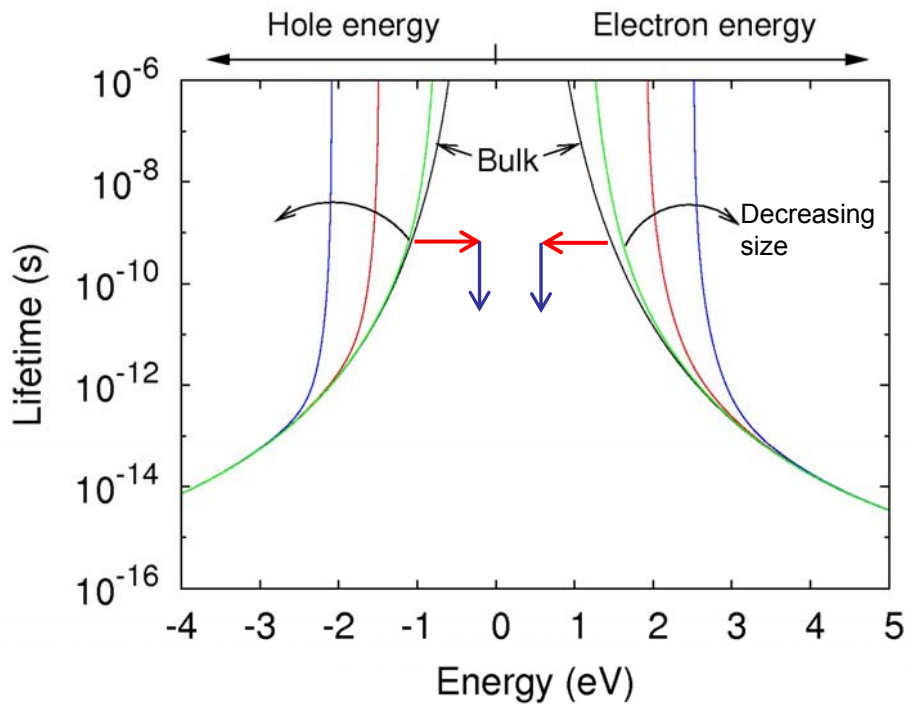


Lines : our calculations.  
 $\tau_{\text{ph}} = 0.5 \text{ ps}$

+ : Wolf, M.; Brendel, R.; Werner, J. H.; Queisser, H. J. J. Appl. Phys. 1998

# Strategy to get more efficient CM

General behavior of the impact ionization lifetime

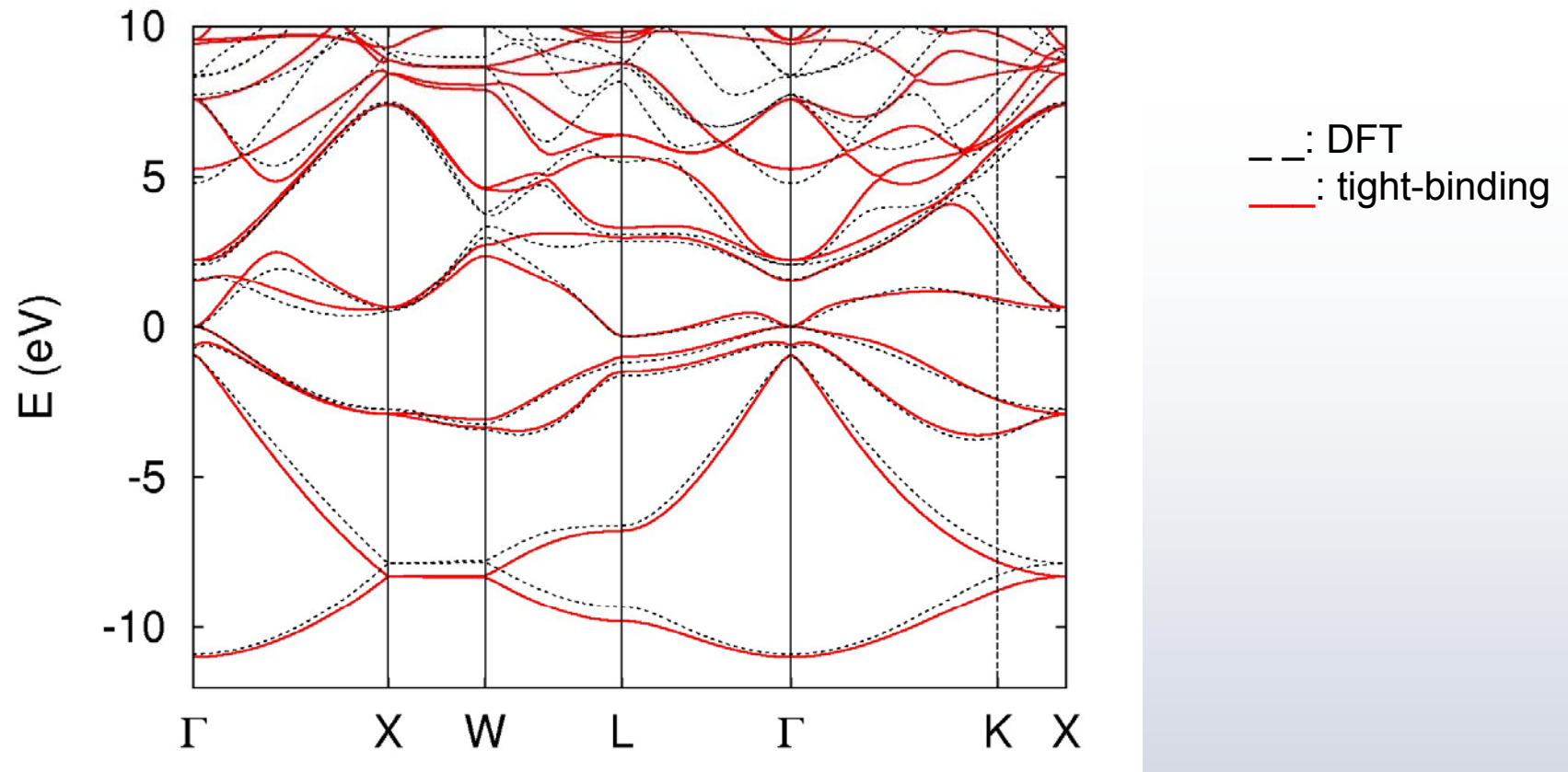


Possible strategy:

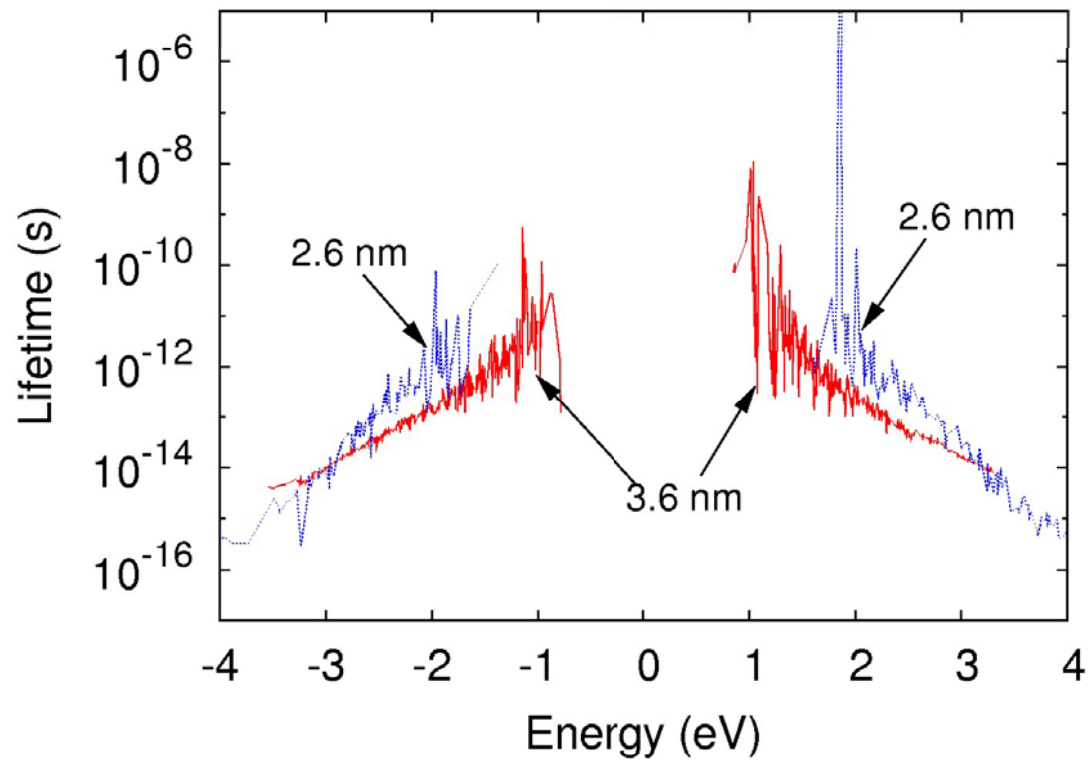
- 1) Horizontal shift : to small  $|E|$   
 $\Rightarrow$  small bandgap materials
- 2) Vertical shift : to high density of final states  
 $\Rightarrow$  materials with degenerate bands
- 3) To adjust the gap using quantum confinement

# Test of the strategy: $\alpha$ -Sn

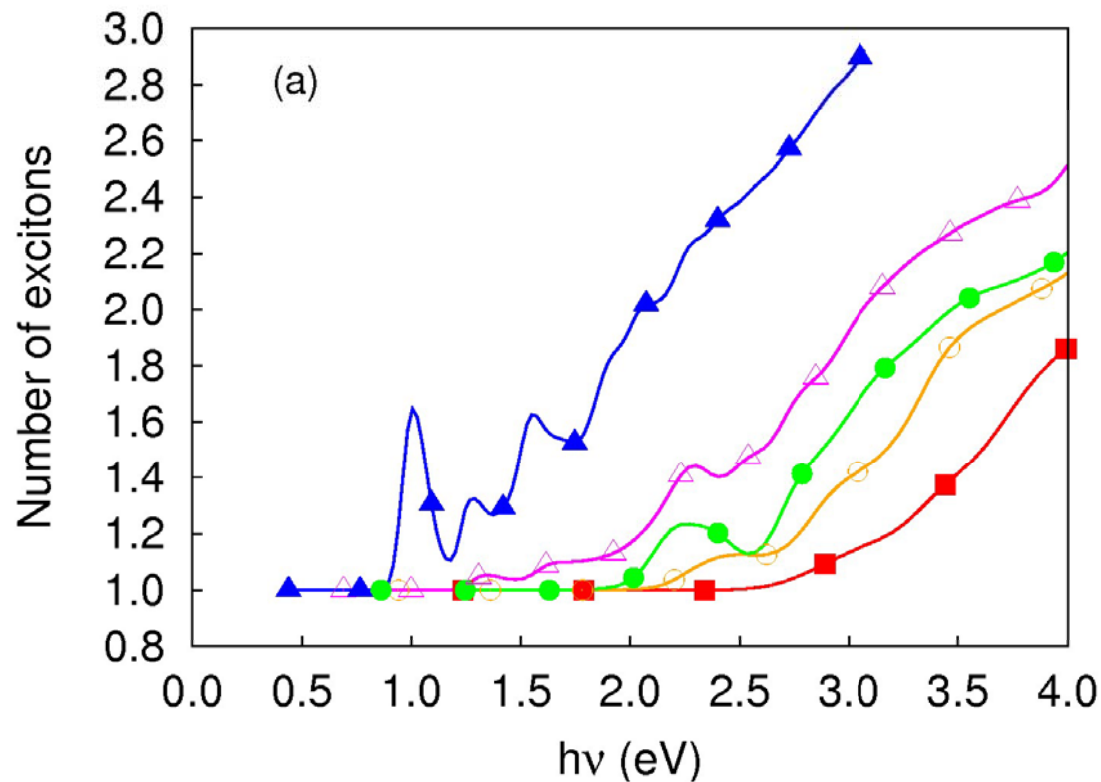
Calculated band structure of  $\alpha$ -Sn



# $\alpha$ -Sn: impact ionization lifetime



# $\alpha$ -Sn: CM yield

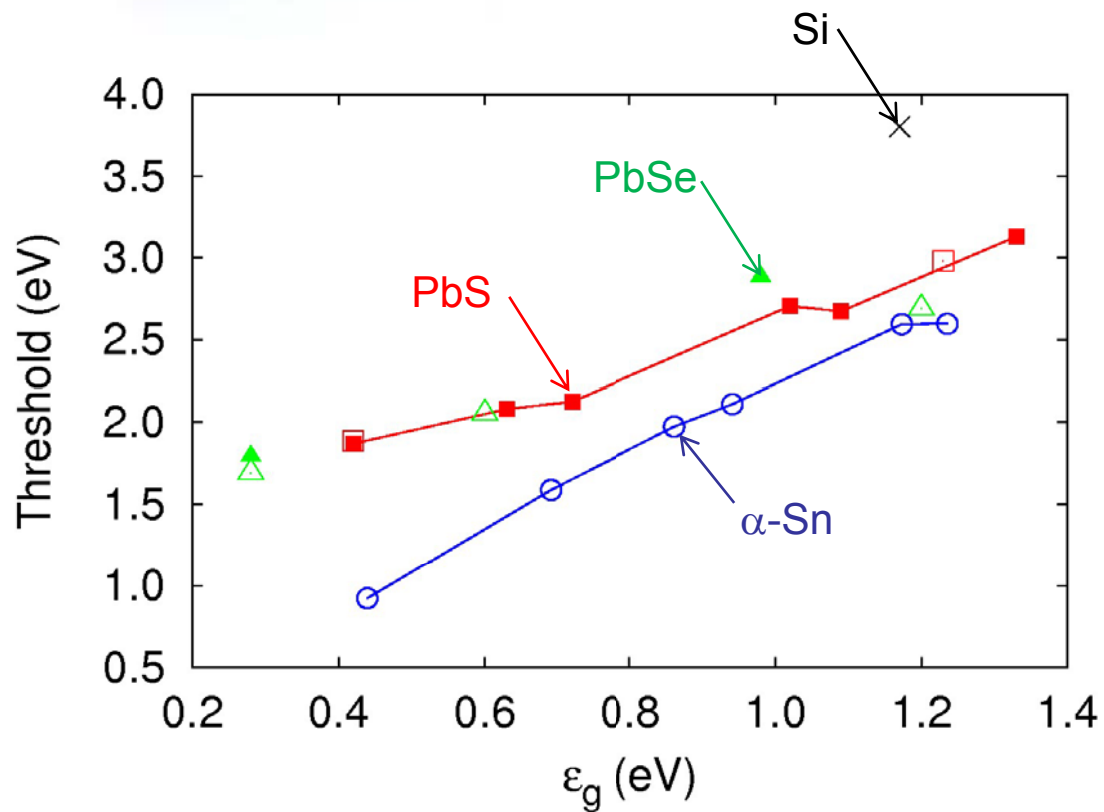


—:  $E_g = 0.44$  eV  
—:  $E_g = 0.69$  eV  
—:  $E_g = 0.86$  eV  
—:  $E_g = 0.94$  eV  
—:  $E_g = 1.24$  eV

$\tau_{ph} = 1$  ps



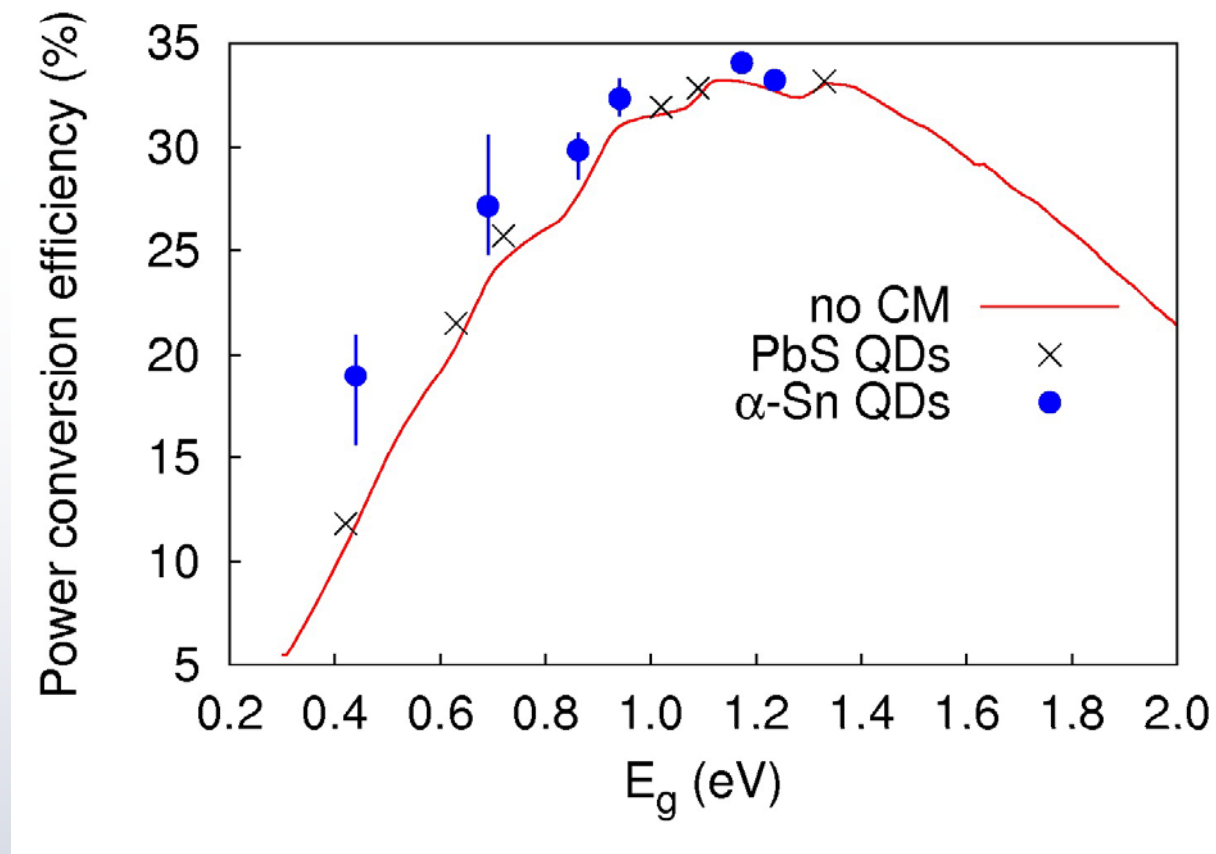
# Test of the strategy: $\alpha$ -Sn



G. Allan and C. Delerue, ACS Nano 5, 7318 (2011).



# Test of the strategy: $\alpha$ -Sn



G. Allan and C. Delerue, ACS Nano 5, 7318 (2011).

# On-going work

**Weak point of all these works:**

The phonon-assisted relaxation lifetime  $\tau_{\text{ph}}$  is a parameter

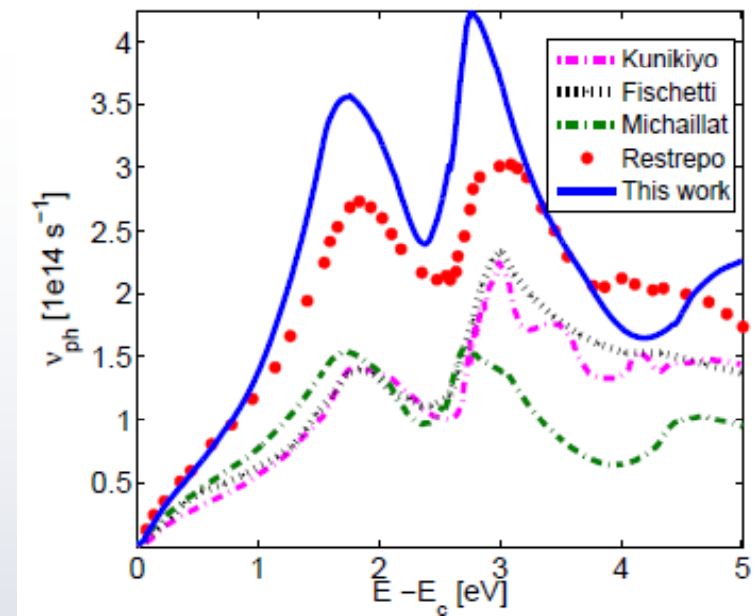
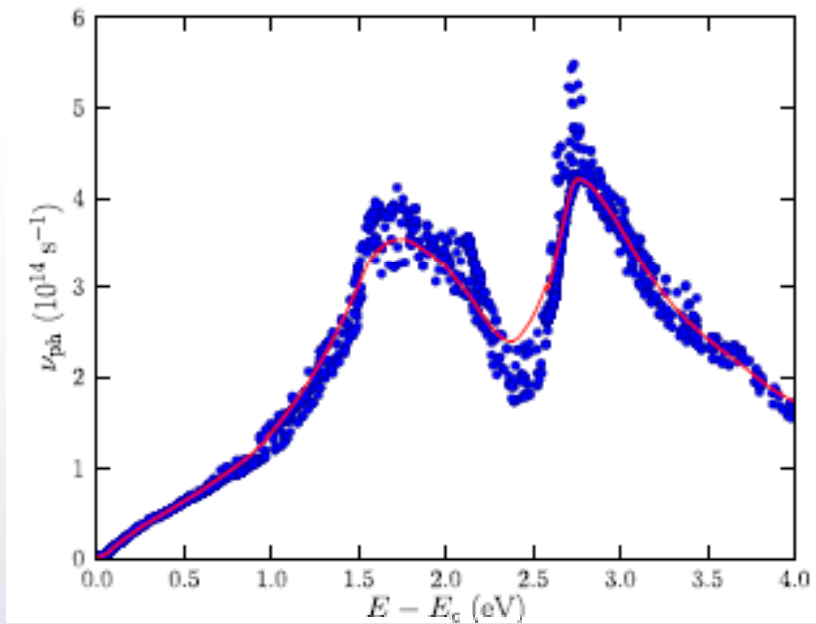
⇒ Calculation of scattering rates for the relaxation by emission of phonons

Our approach:

- 1) Electronic structure in TB
- 2) Phonons using a valence-force field model
- 3) Calculation of the e-ph coupling Hamiltonian (position-dependent TB)
- 4) Calculation of the couplings and scattering rates

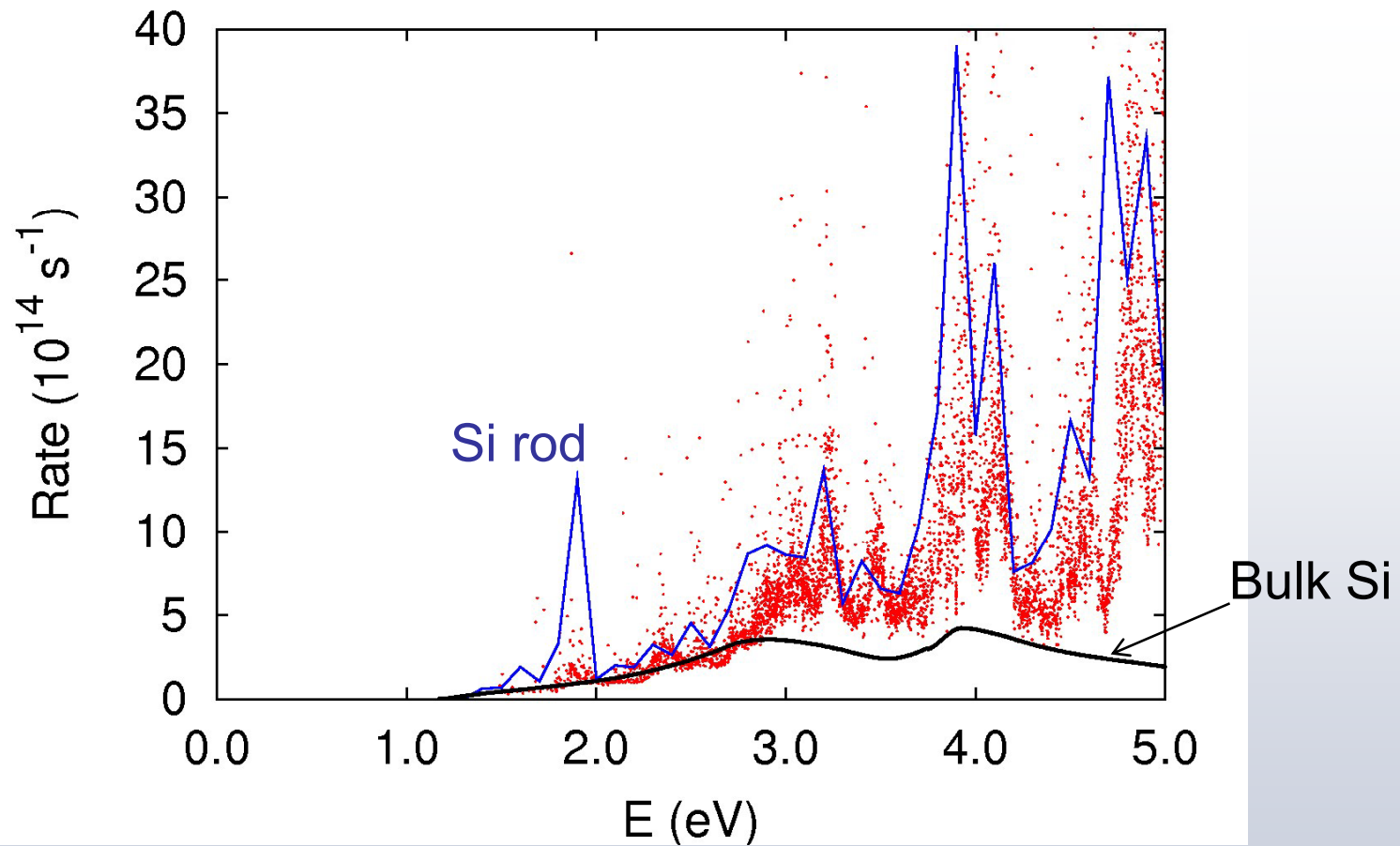
Collaboration: Y.M. Niquet, INAC/CEA

# Scattering rates: bulk Si



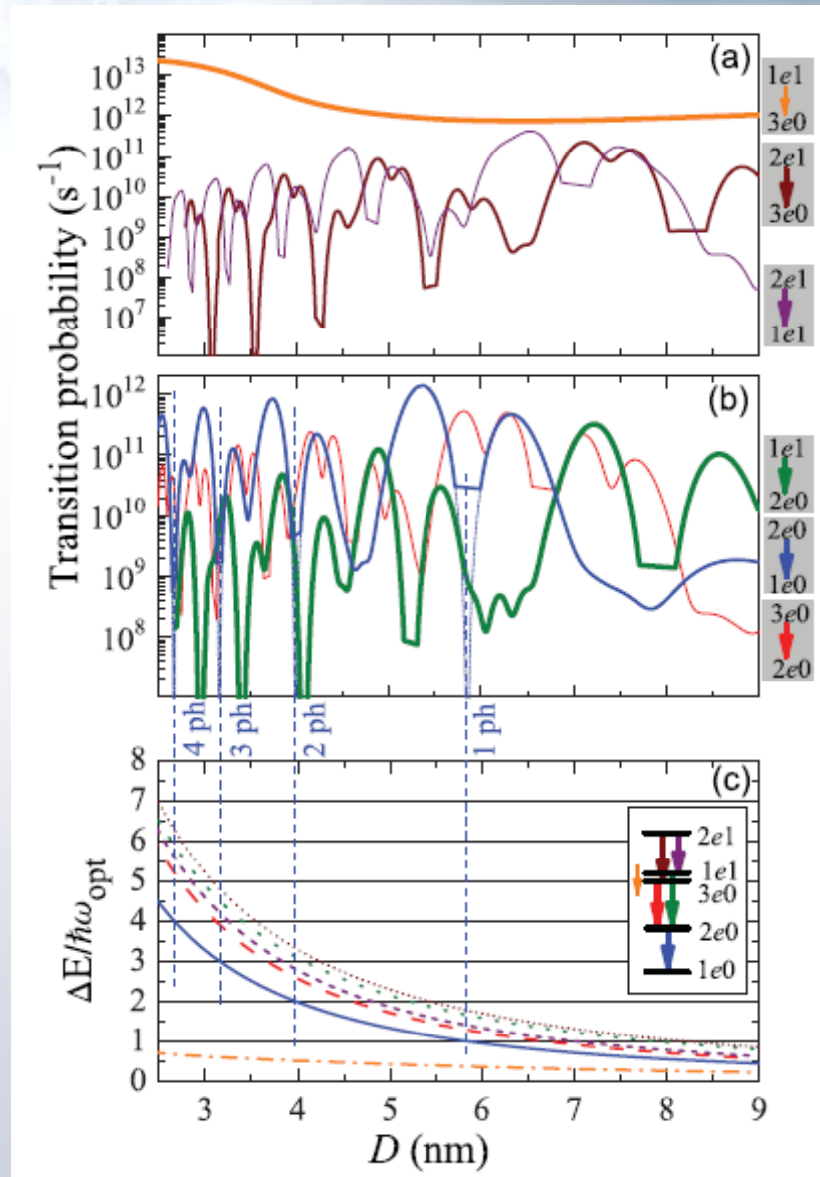
# Scattering rates: Si nanorods

Cylindrical rod: 77 nm x 2 nm



# Scattering rates: Si QDs

Spherical Si QD



Moskalenko et al, PRB 85, 085432 (2012)

# Future work

Next steps:

- 1) Calculation of the impact ionization rates
- 2) Full simulation of the CM process



# Conclusion

## CM in QDs:

- Controversies remain
- Origin of the discrepancies not really known
- Latest experimental results (with CM >0) explained by impact ionization
- Small influence of CM in PbS & PbSe QDs solar cells
- But still room for improvement  $\Rightarrow$  CM engineering
- We have proposed a possible strategy
- On-going calculations for phonon-assisted processes

G. Allan and C. Delerue, Phys. Rev. B 73, 205423 (2006)

G. Allan and C. Delerue, Phys. Rev. B 77, 125340 (2008)

G. Allan and C. Delerue, Phys. Rev. B 79, 195324 (2009).

J. J. H. Pijpers, R. Ulbricht, K.J. Tielrooij, A. Osherov, Y. Golan, C. Delerue, G. Allan & M. Bonn, Nature Physics 5, 811 (2009)

C. Delerue, G. Allan, J.J.H. Pijpers, and M. Bonn, Phys. Rev. B 81, 125306 (2010).

G. Allan and C. Delerue, ACS Nano 5, 7318 (2011).