Physics Beyond the Standard Model

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Outline

- The Standard Model of particle physics
- The Large Hadron Collider (LHC) at CERN
- Need for physics beyond the Standard Model
- Some popular ideas for physics beyond the Standard Model
- How our activities at LPSC fit into the search for new physics
- Disclaimer: I will concentrate on work done at LPSC.
  As such, my talk is unbalanced.
  A general overview is beyond the scope of my talk!

The Standard Model of Particle Physics

Gauge group

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

Particle content

Q	$(\mathbf{3},\mathbf{2})_{1/3}$	L	$(\mathbf{1},\mathbf{2})_{-1}$	H	$(\mathbf{1},\mathbf{2})_{1}$	A	$(\mathbf{1},\mathbf{1})_0$
u_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{-4/3}$	e_R^c	$(\mathbf{1},\mathbf{1})_{2}$			W	$(\mathbf{1},\mathbf{3})_0$
d_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{2/3}$	ν_R^c	$(\mathbf{1},\mathbf{1})_{0}$			G	$(\mathbf{8},\mathbf{1})_0$

Lagrangian (Lorentz + gauge + renormalizable)

$$\mathcal{L} = -\frac{1}{4} G^{\alpha}_{\mu\nu} G^{\alpha\mu\nu} + \ldots \overline{Q}_k \not{D} Q_k + \ldots (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{4!}(H^\dagger H)^2 + \ldots Y_{k\ell} \overline{Q}_k H (u_R)_\ell$$

Spontaneous symmetry breaking

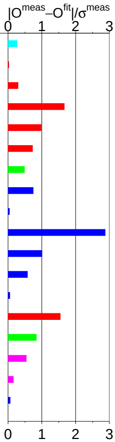
- $H \to H' + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$
- $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_Q$
- $A, W^3 \to \gamma, Z^0$ and $W^1_\mu, W^2_\mu \to W^+, W^-$
- Fermions acquire mass through Yukawa couplings to Higgs

The Standard Model of Particle Physics

Excellent agreement between theory and data

The LEP Electroweak Working Group, Plots for Winter 2012

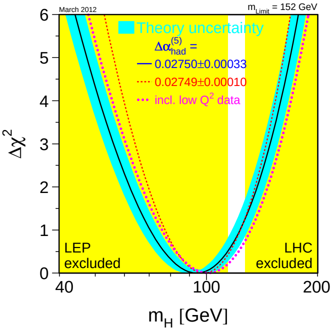
	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^0 [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26



March 2012

The Standard Model of Particle Physics

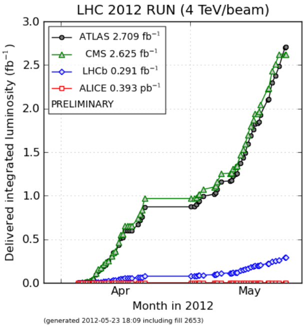
➢ Lower bound from LEP: $m_H > 114.4$ GeV
Preferred $m_H = 94$ GeV, upper limit $m_H < 171$ GeV (95% C.L.)
The LEP Electroweak Working Group, Plots for Winter 2012



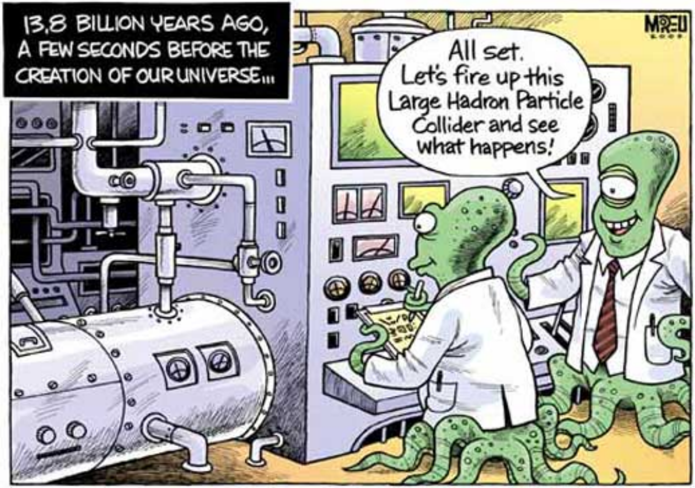
➢ LHC has new exclusion limits & hints of a Higgs boson ~ 125 GeV
$\rightarrow$ More on that later in this talk!

Physics at the Large Hadron Collider

- LEP (1989 - 2000) has tested the SM up to $\mathcal{O}(100)$ GeV
- Commissioning of LHC on March 30, 2010
- Collected ~ 5 fb^{-1} of data @7 TeV till end of 2011
- First 8 TeV collisions on March 30, 2012
- Eventually $\sqrt{s} = 14$ TeV (after shutdown in 2013/14)



Physics at the Large Hadron Collider



Physics at the Large Hadron Collider

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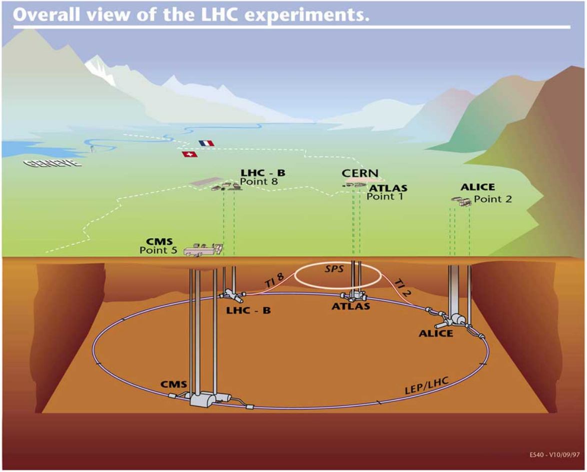
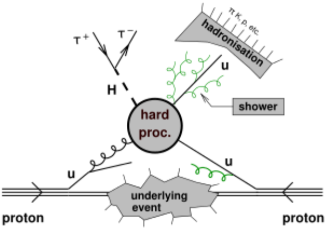


Physics at the Large Hadron Collider

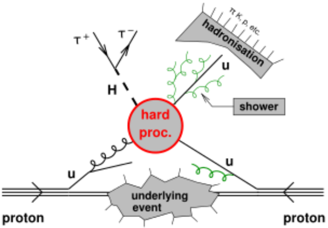
Gavin Salam, RPP2012



- High momentum transfer ⇝ Amenable to perturbation theory (→ Tzvetalina's & François' talk)
- Parton showering (correspond to "jets" under ideal circumstances)
- Hadronization (free quarks & gluons not observed: "Confinement")
- Junk from the proton remnants
- Higgs boson: That's the event we hope to observe

Physics at the Large Hadron Collider

Gavin Salam, RPP2012



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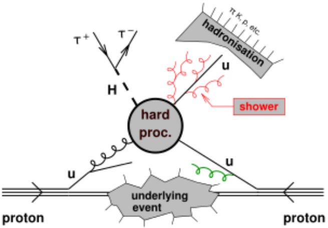

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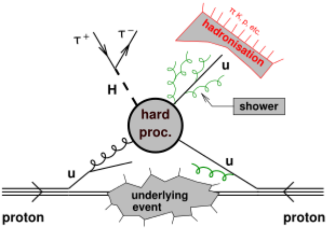

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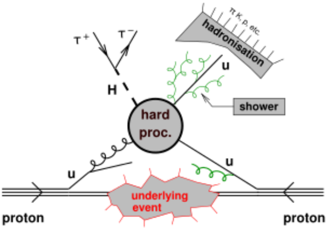

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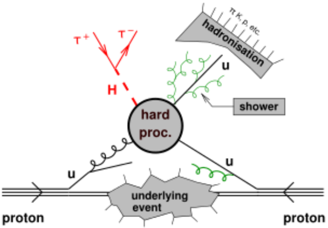

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Problems of the Standard Model

Why do we expect new physics beyond the Standard Model?

A word of warning:

- There is no concensus on the list of problems that will follow
- Some of the problems are more serious/imminent than others
- Many are purely "aestetic"

Problems of the Standard Model

❶ Too many free parameters

Gauge sector: 3 couplings g', g, g_3	3
Quark sector: 6 masses, 3 mixing angles, 1 CP phase	10
Lepton sector: 6 masses, 3 mixing angles and 1+2 phases	10+2
Higgs sector: Quartic coupling λ and vev v	2
θ parameter of QCD	1
	26+2

Problems of the Standard Model

❷ Structure of gauge symmetry

Why the product structure $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$?

Why 3 different coupling constants g', g, g_3?

❸ Structure of family multiplets

One family is

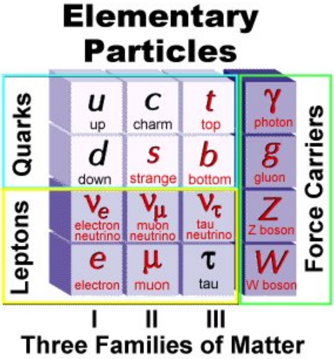
$$\underset{Q}{(\mathbf{3},\mathbf{2})_{1/3}} + \underset{\bar{u}}{(\bar{\mathbf{3}},\mathbf{1})_{-4/3}} + \underset{\bar{e}}{(\mathbf{1},\mathbf{1})_{-2}} + \underset{\bar{d}}{(\bar{\mathbf{3}},\mathbf{1})_{2/3}} + \underset{L}{(\mathbf{1},\mathbf{2})_{-1}} + \underset{\bar{\nu}}{(\mathbf{1},\mathbf{1})_{0}}$$

Can the particles be reorganized in a single representation?

Problems of the Standard Model

❹ Repetition of families

Why is this pattern for 1 generation replicated 3 times?



Problems of the Standard Model

❺ Mass hierarchies and texture of Yukawa couplings

up-quark mass $\sim 2 \times 10^{-3}$ GeV $\leftrightarrow$ top-quark mass ~ 172.3 GeV

Yukawa coupling of top ~ 1, but why are the other quarks so light?

Minimal mixing in quark sector

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.04 \\ 0.00 & 0.04 & 0.99 \end{pmatrix}$$

Problems of the Standard Model

❻ Light neutrinos and texture of Yukawa couplings

Why are neutrinos so light?

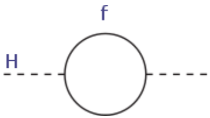
$$\Delta m_\nu^2 \sim 10^{-2} - 10^{-5} \text{ eV}, \quad \sum m_\nu \lesssim 2 \text{ eV}$$

Maximal mixing in lepton sector

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \simeq \begin{pmatrix} 0.8 & 0.5 & 0.0 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & -0.6 & 0.7 \end{pmatrix}$$

Problems of the Standard Model

❼ Hierarchy problem



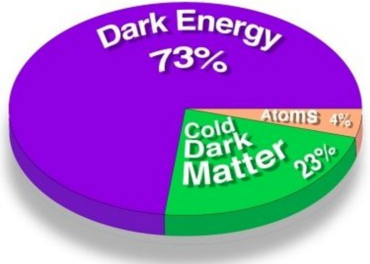
$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\rm UV}^2 + \dots$$

- Higgs mass is quadratically divergent
- Standard Model is renormalizable and infinities can be absorbed into a finite number of physical parameters
- Hierarchy problem arises if one goes beyond renormalizability $\rightsquigarrow$ Cut-off Λ_{UV} acquires physical meaning
- Higgs mass is dragged to cut-off scale e.g. $\Lambda_{UV} \sim M_{\rm Planck}$
- However, we need a light Higgs $\mathcal{O}(100)$ GeV
- Analogous problems arise from presence of any heavy particle

Problems of the Standard Model

❽ Dark Matter and Dark Energy

23% of our universe is made up of dark matter and the Standard Model offers no candidate particle . . .



73% of our universe is made up of dark energy and the cosmological constant as calculated from QFT is the worst-predicted quantity in particle physics

Problems of the Standard Model

❾ Gravity

- Scales relevant in everyday life $\rightsquigarrow$ Newton's theory
- Satellites, solar system, etc. $\rightsquigarrow$ Still Newton's theory
- Cosmological scales $\rightsquigarrow$ Einstein's theory of GR
- Very small scales $\rightsquigarrow$ Need quantum theory of gravitation
- Don't know how to quantize gravity and how to unify with SM

  $\longrightarrow$ String theory (A.W.) or loop quantum gravity (Aurelien Barrau)

❿ Many other problems

Baryon asymmetry in the universe, charge quantization, ...

$\longrightarrow$ Will not be addressed in this talk!

Problems of the Standard Model

Are there any hints

at physics beyond the Standard Model?

Supersymmetry & GUTs

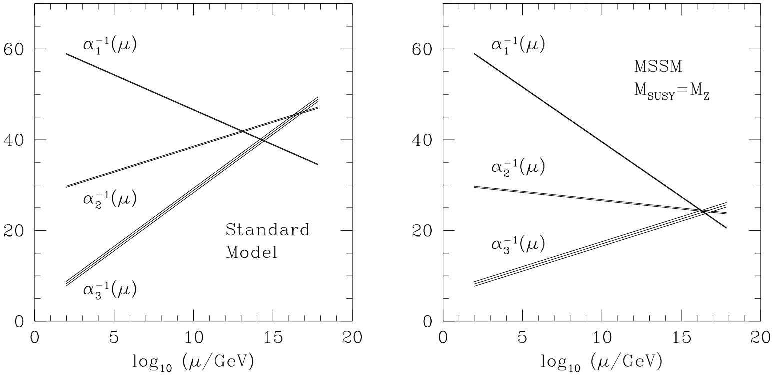
- Running gauge couplings seem to meet at $\sim 2 \times 10^{16}$ GeV

  H. Georgi and S. L. Glashow, "Unity of all elementary particle forces," *Phys. Rev. Lett.* **32** (1974) 438–441

  S. Dimopoulos, S. A. Raby, and F. Wilczek, "Unification of couplings," *Phys. Today* **44N10** (1991) 25–33

- Points towards supersymmetry (→ Suchita's talk) & unification

figures courtesy of K. Dienes



Supersymmetry & GUTs

➢ Remember gauge symmetry & particle content of SM

$$\mathrm{SO}(10) \longrightarrow \mathrm{SU}(5)\times\mathrm{U}(1)_X \longrightarrow \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_{\text{B-L}}$$

$$\mathbf{16} \longrightarrow \mathbf{10}+\overline{\mathbf{5}}+\mathbf{1}$$

$$\longrightarrow \underset{Q}{(\mathbf{3},\mathbf{2})_{1/3,1/3}} + \underset{u_R^c}{(\overline{\mathbf{3}},\mathbf{1})_{-4/3,-1/3}} + \underset{e_R^c}{(\mathbf{1},\mathbf{1})_{-2,-1}} + \underset{d_R^c}{(\overline{\mathbf{3}},\mathbf{1})_{2/3,-1/3}} + \underset{L}{(\mathbf{1},\mathbf{2})_{-1,1}} + \underset{\nu_R^c}{(\mathbf{1},\mathbf{1})_{0,-1}}$$

➢ $\mathrm{U}(1)_{\text{B-L}}$ can serve as R-parity ☺

➢ Partial solution to ❶ "Too many parameters":

- g', g, g_3 reduced to g_{GUT}
  B. Dundee, S. Raby, and A. Wingerter, "Reconciling Grand Unification with Strings by Anisotropic Compactifications," *Phys. Rev.* **D78** (2008) 066006, 0805.4186
- Relates quark & lepton masses ("good prediction" only for 3rd generation)
- Not-so-wrong prediction for $\sin^2\theta_w$ (only SU(5))
  S. Raby and A. Wingerter, "Can String Theory Predict the Weinberg Angle?," *Phys. Rev.* **D76** (2007) 086006, 0706.0217

Supersymmetry & GUTs

- Partial solution to ❷ "Structure of gauge symmetry"

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6 \subset \mathrm{E}_8 \times \mathrm{E}_8$$

O. Lebedev *et al.*, "A mini-landscape of exact MSSM spectra in heterotic orbifolds," *Phys. Lett.* **B645** (2007) 88–94, hep-th/0611095

- Elegant solution to ❸ "Structure of family multiplets"

$$(\mathbf{3},\mathbf{2})_{1/3}+(\overline{\mathbf{3}},\mathbf{1})_{-4/3}+(\mathbf{1},\mathbf{1})_{-2}+(\overline{\mathbf{3}},\mathbf{1})_{2/3}+(\mathbf{1},\mathbf{2})_{-1}+(\mathbf{1},\mathbf{1})_{0} = \mathbf{16} \subset \mathbf{27} \subset \mathbf{248}$$

- Elegant solution to ❻ "Light neutrinos & ..."

$$\mathcal{L} = \ldots + y_\nu \overline{L} H^c \nu_R + M \overline{\nu}_R \nu_R + \ldots \quad \text{(Majorana spinors)}$$

$$M = \begin{array}{c c} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \begin{array}{c} \nu_L \\ \nu_R \end{array} & \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \end{array} \rightsquigarrow m_1 \simeq M, \quad m_2 \simeq \frac{m^2}{M} \sim \frac{(100\ \text{GeV})^2}{10^{16}\ \text{GeV}} \sim 10^{-3}\text{eV}$$

Supersymmetry & GUTs

➢ Technical solution to ❼ Hierarchy problem

For fermions : $\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\mathrm{UV}}^2 + \ldots$

For scalars : $\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\mathrm{UV}}^2 - 2m_S^2 \ln(\Lambda_{\mathrm{UV}}/m_S) + \ldots\right]$

➢ May have an answer for ❽ Dark matter & . . .

R-parity $\rightsquigarrow$ lightest supersymmetric particle (LSP) is stable

LSP (if colorless & neutral) may constitute (part of) dark matter

Supersymmetry & GUTs

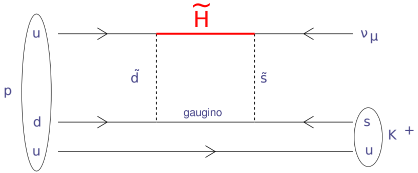
Some problems with GUTs

- Wrong prediction of m_d/m_e and m_s/m_μ

  Need more (and large) representations, i.e. more particles to fix that

- Higgs boson has color-triplet partner $\rightsquigarrow$ proton decay

  S. Forste, H. P. Nilles, P. K. S. Vaudrevange, and A. Wingerter, "Heterotic brane world," *Phys. Rev.* **D70** (2004) 106008, hep-th/0406208



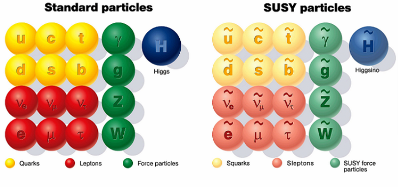
- Need large representations (i.e. more particles) to break $\mathrm{SU}(5) \to \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

  O. Lebedev *et al.*, "A mini-landscape of exact MSSM spectra in heterotic orbifolds," *Phys. Lett.* **B645** (2007) 88–94, hep-th/0611095

Supersymmetry & GUTs

Some problems with supersymmetry

- SUSY doubles the particle spectrum



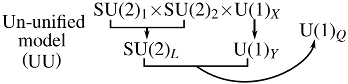
- SUSY predicts equal masses for particle and superpartner, e.g. $m_e = m_{\tilde{e}}$
- SUSY breaking introduces (in minimal model) 105 new parameters
  Origin of SUSY breaking not understood
  O. Lebedev *et al.*, "Low Energy Supersymmetry from the Heterotic Landscape," *Phys. Rev. Lett.* **98** (2007) 181602, hep-th/0611203
- cMSSM (SUSY+naive GUT) close to be excluded at LHC
- Proton decay in renormalizable superpotential $\rightsquigarrow$ need R-parity
  O. Lebedev *et al.*, "The Heterotic Road to the MSSM with R parity," *Phys. Rev.* **D77** (2008) 046013, 0708.2691

Extended Gauge Symmetries

T. Iežo, M. Klasen, I. Schienbein, arXiv:1203.5314v1 [hep-ph]

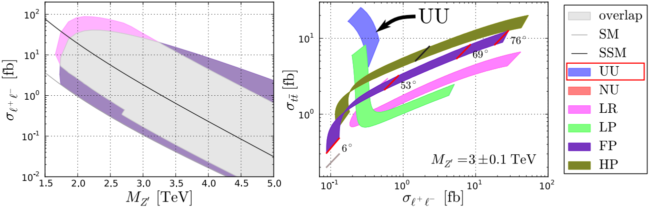
Gauge group

$SU(3)_c \times SU(2)_1 \times SU(2)_2 \times U(1)_X$



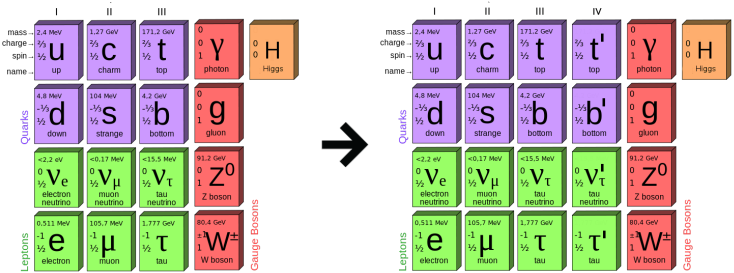
Particle content (Un-unified model or UU)

Q	$(\mathbf{3},\mathbf{2},\mathbf{1})_{1/3}$	L	$(\mathbf{1},\mathbf{1},\mathbf{2})_{-1}$	ϕ	$(\mathbf{1},\mathbf{2},\overline{\mathbf{2}})_0$	A	$(\mathbf{1},\mathbf{1},\mathbf{1})_0$
u_R^c	$(\mathbf{3},\mathbf{1},\mathbf{1})_{-4/3}$	e_R^c	$(\mathbf{1},\mathbf{1},\mathbf{1})_2$	H	$(\mathbf{1},\mathbf{2},\mathbf{1})_1$	W_1	$(\mathbf{1},\mathbf{3},\mathbf{1})_0$
d_R^c	$(\mathbf{3},\mathbf{1},\mathbf{1})_{2/3}$	ν_R^c	$(\mathbf{1},\mathbf{1},\mathbf{1})_0$			W_2	$(\mathbf{1},\mathbf{1},\mathbf{3})_0$
						G	$(\mathbf{8},\mathbf{1},\mathbf{1})_0$



A Fourth Generation of Chiral Fermions

- A straightforward extension of the SM
- 4th generation is exact copy of 3rd one *except* for masses
- Gauge and Higgs sectors unchanged



A Fourth Generation of Chiral Fermions

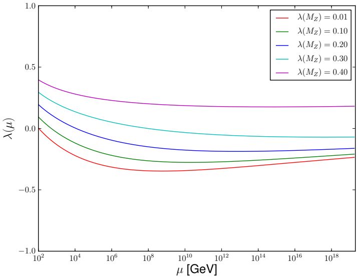
Lower bound on Higgs mass from stability of electroweak vacuum

➢ Require that Higgs coupling λ not become negative anywhere

N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, "Bounds on the Fermions and Higgs Boson Masses in Grand Unified Theories," *Nucl.Phys.* **B158** (1979) 295–305

G. Altarelli and G. Isidori, "Lower limit on the Higgs mass in the standard model: An Update," *Phys.Lett.* **B337** (1994) 141–144

J. A. Casas, J. R. Espinosa, and M. Quiros, "Standard Model stability bounds for new physics within LHC reach," *Phys. Lett.* **B382** (1996) 374–382, hep-ph/9603227

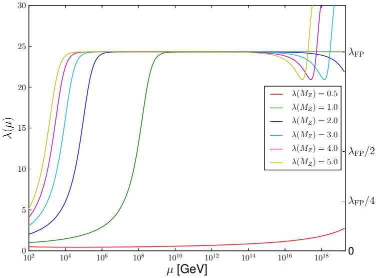


A Fourth Generation of Chiral Fermions

Upper bound on Higgs mass from triviality of theory

M. Lindner, "Implications of Triviality for the Standard Model," *Z.Phys.* **C31** (1986) 295

- At 1-loop: Higgs coupling $\lambda \to \infty$ for renormalization scale $\mu \to \infty$
- At 2-loop: Higgs coupling $\lambda \to \lambda_{\mathrm{FP}}$ (fixed point)
- Criterion for perturbativity: $\lambda < \lambda_{\mathrm{FP}}/4$ (tight) or $\lambda < \lambda_{\mathrm{FP}}/2$ (loose)



A Fourth Generation of Chiral Fermions

A. Wingerter, "Implications of the Stability and Triviality Bounds on the Standard Model with Three and Four Chiral Generations," *Phys.Rev.* **D84** (2011) 095012, 1109.5140

- Before LHC Era: $m_H < 114$ GeV (LEP), $158 < m_H < 175$ GeV (Tevatron), $m_H > 186$ GeV (EWPM)
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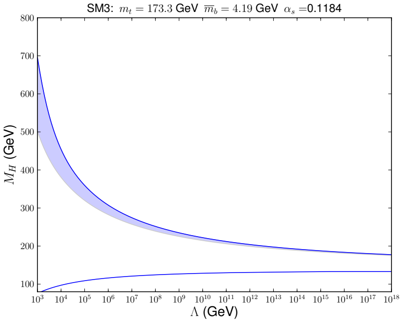

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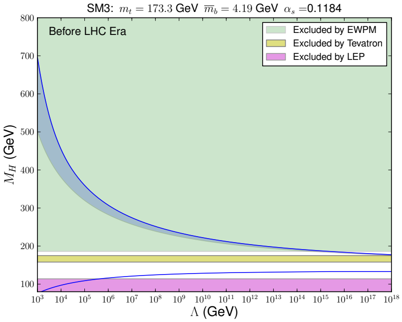

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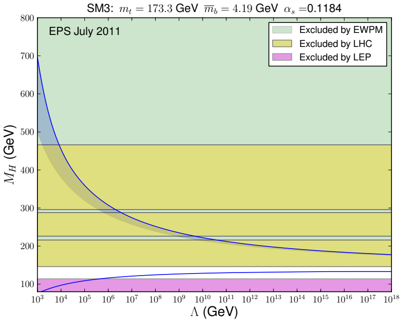

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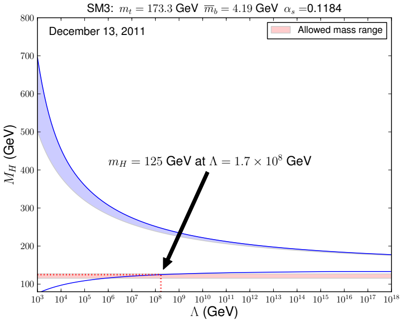

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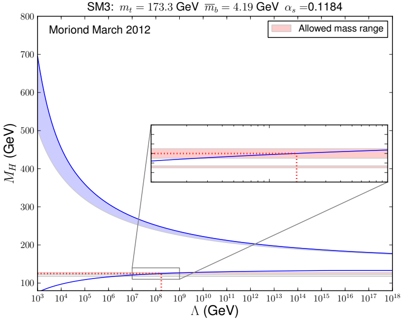

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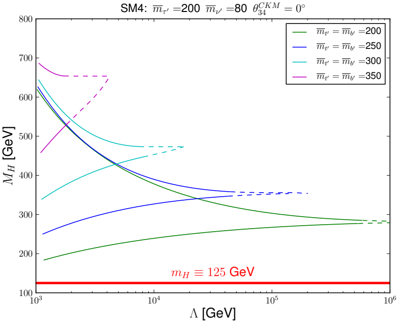

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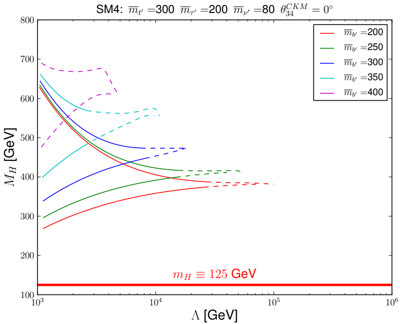

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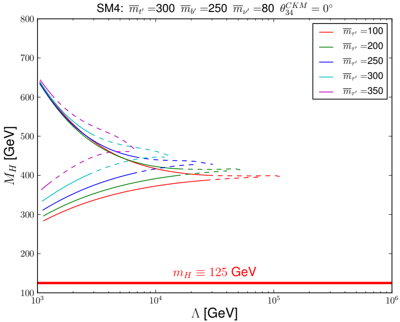

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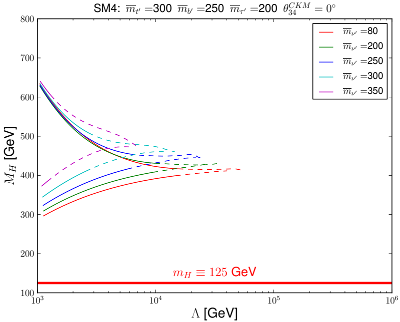

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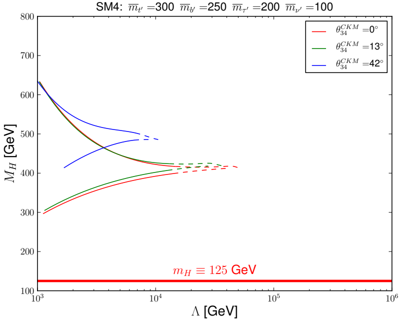
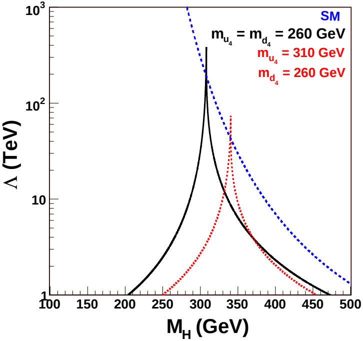
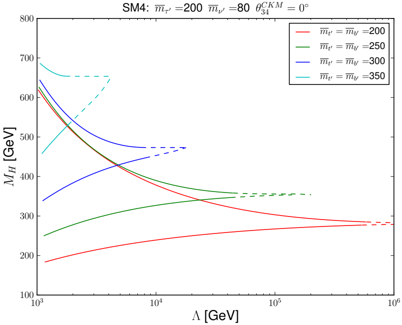


A Fourth Generation of Chiral Fermions

➢ 2-loop RGEs and matching corrections are relevant:
$m_{t'} = m_{b'} \simeq 250$ GeV: $\Lambda \simeq 300$ TeV $\leftrightarrow$ $\Lambda \simeq 40$ TeV

Kribs et al. arXiv:0706.3718





A Fourth Generation of Chiral Fermions

$$\lambda(\mu) = \frac{m_H^2}{v^2}\left(1+\delta_H(\mu)\right)\Big|_{\mu=m_H}, \quad y_t(\mu) = \frac{\sqrt{2}m_t}{v}\left(1+\delta_t(\mu)\right)\Big|_{\mu=m_t}$$

$$-\frac{12\log\left(c^2\right)c^4}{\xi}+\frac{12\log\left(\frac{\mu^2}{m_Z^2}\right)c^4}{\xi}+\frac{3\log\left(\frac{c^2m_Z^2}{\mu^2}\right)c^4}{c^2-\xi}-\frac{12Z\left(\frac{c^2}{\xi}\right)c^4}{\xi}+\frac{16c^4}{\xi}-\frac{14\log\left(c^2\right)c^2}{s^2}+28\log\left(c^2\right)c^2-\frac{3\xi\log\left(\frac{m_H^2}{\mu^2}\right)c^2}{c^2-\xi}-20\log\left(\frac{\mu^2}{m_Z^2}\right)c^2+4Z\left(\frac{c^2}{\xi}\right)c^2$$

$$-32c^2-17s^2-\frac{3m_{b'}^4V_{tb'}^2}{\left(m_t^2-m_{b'}^2\right)m_Z^2}+\frac{3m_t^4V_{tb'}^2}{\left(m_t^2-m_{b'}^2\right)m_Z^2}-\frac{3m_{b'}^4V_{t'b'}^2}{\left(m_{t'}^2-m_{b'}^2\right)m_Z^2}+\frac{3m_{t'}^4V_{t'b'}^2}{\left(m_{t'}^2-m_{b'}^2\right)m_Z^2}-\frac{3m_b^4V_{tb}^2}{\left(m_t^2-m_b^2\right)m_Z^2}+\frac{3m_t^4V_{tb}^2}{\left(m_t^2-m_b^2\right)m_Z^2}-\frac{3m_b^4V_{t'b}^2}{\left(m_{t'}^2-m_b^2\right)m_Z^2}+\frac{3m_{t'}^4V_{t'b}^2}{\left(m_{t'}^2-m_b^2\right)m_Z^2}$$

$$-\frac{m_{\tau'}^4U_{\tau'\nu'}^2}{\left(m_{\nu'}^2-m_{\tau'}^2\right)m_Z^2}+\frac{m_{\nu'}^4U_{\tau'\nu'}^2}{\left(m_{\nu'}^2-m_{\tau'}^2\right)m_Z^2}-\frac{3}{2}\sqrt{3}\pi\xi+\frac{25\xi}{2}+8s^2\log\left(c^2\right)-\xi\log\left(c^2\right)+\frac{17\log\left(c^2\right)}{s^2}-29\log\left(c^2\right)+\frac{6m_b^4V_{tb}^2\log\left(\frac{m_b^2}{\mu^2}\right)}{\left(m_t^2-m_b^2\right)m_Z^2}+\frac{6m_b^4V_{t'b}^2\log\left(\frac{m_b^2}{\mu^2}\right)}{\left(m_{t'}^2-m_b^2\right)m_Z^2}+\frac{6m_{b'}^4V_{tb'}^2\log\left(\frac{m_{b'}^2}{\mu^2}\right)}{\left(m_t^2-m_{b'}^2\right)m_Z^2}$$

$$+\frac{6m_{b'}^4V_{t'b'}^2\log\left(\frac{m_{b'}^2}{\mu^2}\right)}{\left(m_{t'}^2-m_{b'}^2\right)m_Z^2}+\frac{2m_{\tau'}^4U_{\tau'\nu'}^2\log\left(\frac{m_{\tau'}^2}{\mu^2}\right)}{\left(m_{\nu'}^2-m_{\tau'}^2\right)m_Z^2}+6\xi\log\left(\frac{\mu^2}{m_H^2}\right)-\frac{6m_t^4V_{tb'}^2\log\left(\frac{m_t^2}{\mu^2}\right)}{\left(m_t^2-m_{b'}^2\right)m_Z^2}-\frac{6m_t^4V_{tb}^2\log\left(\frac{m_t^2}{\mu^2}\right)}{\left(m_t^2-m_b^2\right)m_Z^2}-\frac{6m_{t'}^4V_{t'b'}^2\log\left(\frac{m_{t'}^2}{\mu^2}\right)}{\left(m_{t'}^2-m_{b'}^2\right)m_Z^2}-\frac{6m_{t'}^4V_{t'b}^2\log\left(\frac{m_{t'}^2}{\mu^2}\right)}{\left(m_{t'}^2-m_b^2\right)m_Z^2}$$

$$-\frac{2m_{\nu'}^4U_{\tau'\nu'}^2\log\left(\frac{m_{\nu'}^2}{\mu^2}\right)}{\left(m_{\nu'}^2-m_{\tau'}^2\right)m_Z^2}-\frac{24m_{b'}^4\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{8m_{\tau'}^4\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{24m_t^4\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{24m_{t'}^4\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{8m_{\nu'}^4\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^4\xi}+\frac{6\log\left(\frac{\mu^2}{m_Z^2}\right)}{\xi}+\frac{6m_{b'}^2\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^2}$$

$$+\frac{2m_{\tau'}^2\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^2}+\frac{6m_t^2\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^2}+\frac{6m_{t'}^2\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^2}+\frac{2m_{\nu'}^2\log\left(\frac{\mu^2}{m_Z^2}\right)}{m_Z^2}-2\log\left(\frac{\mu^2}{m_Z^2}\right)+\frac{24m_{b'}^4\log\left(\frac{m_{b'}^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{6m_{b'}^2\log\left(\frac{m_{b'}^2}{m_Z^2}\right)}{m_Z^2}+\frac{8m_{\tau'}^4\log\left(\frac{m_{\tau'}^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{2m_{\tau'}^2\log\left(\frac{m_{\tau'}^2}{m_Z^2}\right)}{m_Z^2}$$

$$+\frac{24m_t^4\log\left(\frac{m_t^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{6m_t^2\log\left(\frac{m_t^2}{m_Z^2}\right)}{m_Z^2}+\frac{24m_{t'}^4\log\left(\frac{m_{t'}^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{6m_{t'}^2\log\left(\frac{m_{t'}^2}{m_Z^2}\right)}{m_Z^2}+\frac{8m_{\nu'}^4\log\left(\frac{m_{\nu'}^2}{m_Z^2}\right)}{m_Z^4\xi}-\frac{2m_{\nu'}^2\log\left(\frac{m_{\nu'}^2}{m_Z^2}\right)}{m_Z^2}+11s^2\log\left(\frac{c^2m_Z^2}{\mu^2}\right)-7\log\left(\frac{c^2m_Z^2}{\mu^2}\right)+\frac{3}{2}\xi\log(\xi)$$

$$-\frac{1}{2}\xi Z\left(\frac{1}{\xi}\right)-\frac{6Z\left(\frac{1}{\xi}\right)}{\xi}+2Z\left(\frac{1}{\xi}\right)-\xi Z\left(\frac{c^2}{\xi}\right)+\frac{24m_{b'}^4Z\left(\frac{m_{b'}^2}{m_Z^2\xi}\right)}{m_Z^4\xi}-\frac{6m_{b'}^2Z\left(\frac{m_{b'}^2}{m_Z^2\xi}\right)}{m_Z^2}+\frac{8m_{\tau'}^4Z\left(\frac{m_{\tau'}^2}{m_Z^2\xi}\right)}{m_Z^4\xi}-\frac{2m_{\tau'}^2Z\left(\frac{m_{\tau'}^2}{m_Z^2\xi}\right)}{m_Z^2}+\frac{24m_t^4Z\left(\frac{m_t^2}{m_Z^2\xi}\right)}{m_Z^4\xi}-\frac{6m_t^2Z\left(\frac{m_t^2}{m_Z^2\xi}\right)}{m_Z^2}$$

$$+\frac{24m_{t'}^4Z\left(\frac{m_{t'}^2}{m_Z^2\xi}\right)}{m_Z^4\xi}-\frac{6m_{t'}^2Z\left(\frac{m_{t'}^2}{m_Z^2\xi}\right)}{m_Z^2}+\frac{8m_{\nu'}^4Z\left(\frac{m_{\nu'}^2}{m_Z^2\xi}\right)}{m_Z^4\xi}-\frac{2m_{\nu'}^2Z\left(\frac{m_{\nu'}^2}{m_Z^2\xi}\right)}{m_Z^2}-\frac{48m_{b'}^4}{m_Z^4\xi}-\frac{16m_{\tau'}^4}{m_Z^4\xi}-\frac{48m_t^4}{m_Z^4\xi}-\frac{48m_{t'}^4}{m_Z^4\xi}-\frac{16m_{\nu'}^4}{m_Z^4\xi}+\frac{8}{\xi}+\frac{12m_{b'}^2}{m_Z^2}+\frac{4m_{\tau'}^2}{m_Z^2}+\frac{12m_t^2}{m_Z^2}+\frac{12m_{t'}^2}{m_Z^2}+\frac{4m_{\nu'}^2}{m_Z^2}+\frac{19}{2}$$

Family Symmetries

- Neutrinos have mass and the different flavors can mix
- Charged lepton and neutrino mass matrices cannot be simultaneously diagonalized
$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$
- Pontecorvo-Maki-Nakagawa-Sakata matrix
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \underbrace{D_L U_L^\dagger}_{U_{\mathrm{PMNS}}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$
- Parametrization just as CKM matrix (Majorana phases not indicated)
$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & e^{-i\delta} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta} s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{13} s_{23} c_{12} e^{i\delta} & c_{23} c_{12} - s_{13} s_{23} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - s_{13} c_{23} c_{12} e^{i\delta} & -s_{23} c_{12} - s_{13} c_{23} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

Family Symmetries

Attempt at explaning ❻ "...neutrinos & texture of Yukawa couplings"

- Until recently, our best guess was tribimaximal mixing (TBM)

P. F. Harrison, D. H. Perkins, and W. G. Scott, "Tri-bimaximal mixing and the neutrino oscillation data," *Phys. Lett.* **B530** (2002) 167, hep-ph/0202074

$$U_{\rm PMNS} \stackrel{?}{=} U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$\hookrightarrow \quad \theta_{12} = 35.26^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ$

- Agreement still quite good for θ_{12}, θ_{23}, but $\theta_{13} = 0^\circ$ excluded @5σ

Schwetz et al, 1108.1376, **DAYA-BAY** Collaboration, 1203.1669, **RENO** Collaboration, 1204.0626

Parameter	Tribimaximal	Global fit 1σ	Daya Bay	Reno	
θ_{12}	35.26°	$33.02^\circ - 35.0^\circ$	-	-	✓
θ_{23}	45.00°	$42.13^\circ - 49.6^\circ$	-	-	✓
θ_{13}	0.00°	$5.13^\circ - 8.13^\circ$	8.8°	9.8°	✗

Family Symmetries

➢ Regular patter $U_{\rm PMNS}$ is suggestive of a family symmetry

$$U_{\rm PMNS} \stackrel{?}{=} U_{\rm HPS} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

➢ Situation is much less clear after Daya bay and Reno

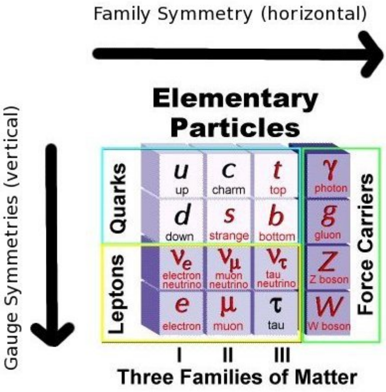
Daya bay and Reno rule out tribimaximal Mixing

➢ What are our options?

- Give up family symmetries!
- Look for groups that give $\theta_{13} \neq 0^\circ$
  R. d. A. Toorop, F. Feruglio, and C. Hagedorn, "Discrete Flavour Symmetries in Light of T2K," *Phys.Lett.* **B703** (2011) 447–451, 1107.3486
- Keep TBM and calculate higher corrections → This talk!

Family Symmetries

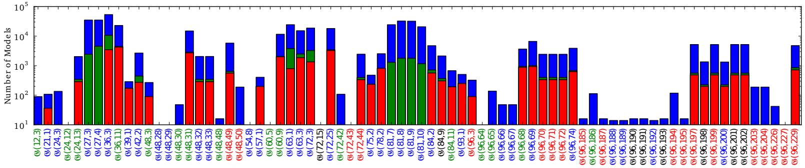
- Introduce relations between families of quarks and leptons
- But which discrete group do we take for the family symmetry?



Family Symmetries

K. M. Parattu and A. Wingerter, "Tribimaximal Mixing From Small Groups," *Phys.Rev.* **D84** (2011) 013011, 1012.2842

- In a recent publication we had scanned 76 discrete groups
- Most papers concentrate on $A_4 \times \mathbb{Z}_n$, $n \geq 3$
- $\Delta(96)$ gives $\theta_{13} \neq 0^\circ$ but is large
- We identified smallest group with TBM → Higher order corrections!



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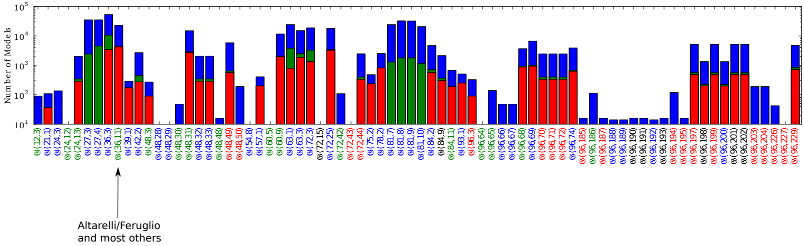

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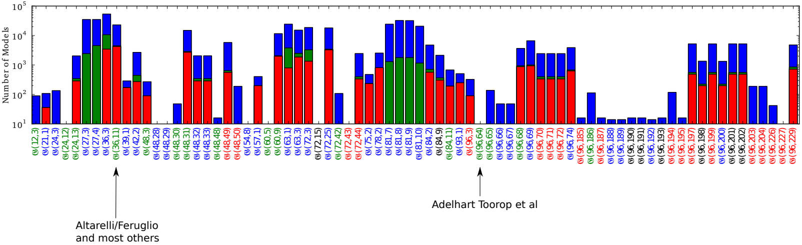

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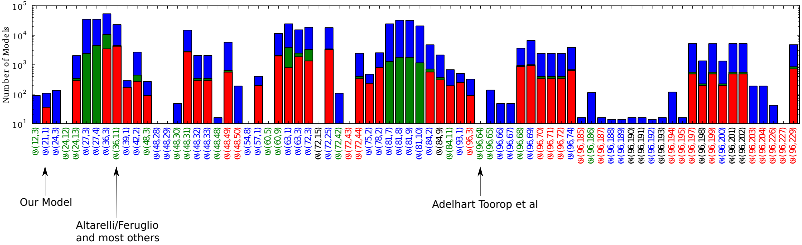

Family Symmetries

C. Luhn, K. M. Parattu, A. Wingerter, *work in progress*

❶ Symmetries of the model

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times T_7 \times \mathrm{U}(1)_R$$

❷ Particle content and charges

Field	$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$	T_7	$\mathrm{U}(1)_R$
L	$(\mathbf{2}, \text{-}1)$	$\mathbf{3}$	1
e	$(\mathbf{1}, 2)$	$\mathbf{1}$	1
μ	$(\mathbf{1}, 2)$	$\mathbf{1'}$	1
τ	$(\mathbf{1}, 2)$	$\mathbf{1''}$	1
h_u	$(\mathbf{2}, 1)$	$\mathbf{1}$	0
h_d	$(\mathbf{2}, \text{-}1)$	$\mathbf{1}$	0
φ	$(\mathbf{1}, 0)$	$\mathbf{3}$	0
$\widetilde{\varphi}$	$(\mathbf{1}, 0)$	$\mathbf{3'}$	0

❸ Breaking the family symmetry

$$\langle \varphi \rangle = (v_\varphi, v_\varphi, v_\varphi), \quad \langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$$

All you need to know about T_7

The GAP Group, "GAP – Groups, Algorithms, and Programming." http://www.gap-system.org, 2008

A. W. , Neutrino model calculator in Python, to be published

Tensor Products

$\mathbf{1} \times \mathbf{1} = \mathbf{1}$

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$\mathbf{3'} \times \mathbf{3'} = 2 \times \mathbf{3} + \mathbf{3'}$

Contractions

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{3}x_1y_1 + \frac{1}{3}\sqrt{3}x_2y_3 + \frac{1}{3}\sqrt{3}x_3y_2 \\ x_1y_2\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_2y_1\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) + x_3y_3\left(-\frac{1}{6}\sqrt{3} + \frac{1}{2}\imath\right) \\ x_1y_3\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_2y_2\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) + x_3y_1\left(-\frac{1}{6}\sqrt{3} - \frac{1}{2}\imath\right) \end{pmatrix}$$

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{6}x_1y_1 - \frac{1}{6}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 + \frac{1}{3}\sqrt{6}x_2y_2 - \frac{1}{6}\sqrt{6}x_3y_1 \\ -\frac{1}{6}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 + \frac{1}{3}\sqrt{6}x_3y_3 \end{pmatrix}$$

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} -\frac{1}{6}\sqrt{6}x_1y_1 + \frac{1}{3}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 - \frac{1}{6}\sqrt{6}x_2y_2 + \frac{1}{3}\sqrt{6}x_3y_1 \\ \frac{1}{3}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 - \frac{1}{6}\sqrt{6}x_3y_3 \end{pmatrix}$$

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$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{6}x_1y_1 - \frac{1}{6}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 + \frac{1}{3}\sqrt{6}x_2y_2 - \frac{1}{6}\sqrt{6}x_3y_1 \\ -\frac{1}{6}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 + \frac{1}{3}\sqrt{6}x_3y_3 \end{pmatrix}$$

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} -\frac{1}{6}\sqrt{6}x_1y_1 + \frac{1}{3}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 - \frac{1}{6}\sqrt{6}x_2y_2 + \frac{1}{3}\sqrt{6}x_3y_1 \\ \frac{1}{3}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 - \frac{1}{6}\sqrt{6}x_3y_3 \end{pmatrix}$$

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$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} -\frac{1}{6}\sqrt{6}x_1y_1 + \frac{1}{3}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 - \frac{1}{6}\sqrt{6}x_2y_2 + \frac{1}{3}\sqrt{6}x_3y_1 \\ \frac{1}{3}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 - \frac{1}{6}\sqrt{6}x_3y_3 \end{pmatrix}$$

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$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} \frac{1}{3}\sqrt{6}x_1y_1 - \frac{1}{6}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 + \frac{1}{3}\sqrt{6}x_2y_2 - \frac{1}{6}\sqrt{6}x_3y_1 \\ -\frac{1}{6}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 + \frac{1}{3}\sqrt{6}x_3y_3 \end{pmatrix}$$

$x \sim \mathbf{3}, \quad y \sim \mathbf{3}, \quad z \sim \mathbf{3'},$

$$z = \begin{pmatrix} -\frac{1}{6}\sqrt{6}x_1y_1 + \frac{1}{3}\sqrt{6}x_2y_3 - \frac{1}{6}\sqrt{6}x_3y_2 \\ -\frac{1}{6}\sqrt{6}x_1y_3 - \frac{1}{6}\sqrt{6}x_2y_2 + \frac{1}{3}\sqrt{6}x_3y_1 \\ \frac{1}{3}\sqrt{6}x_1y_2 - \frac{1}{6}\sqrt{6}x_2y_1 - \frac{1}{6}\sqrt{6}x_3y_3 \end{pmatrix}$$

Family Symmetries

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$\begin{aligned} & y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3 \\ & + y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2 \\ & + y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1 \end{aligned}$$

➢ Contract $\mathrm{SU}(2)_L$ indices and substitute *vevs* $\langle \widetilde{\varphi} \rangle = (v_{\widetilde{\varphi}}, 0, 0)$, etc:

$$y_e \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_1^{(2)} e + y_\mu \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_2^{(2)} \mu + y_\tau \frac{1}{3}\sqrt{3} v_d v_{\widetilde{\varphi}} L_3^{(2)} \tau$$

Family Symmetries

➢ Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:

$$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$

$$\mathbf{3'} \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}) \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3} + \mathbf{3'}$$

➢ Contract family indices (need to know Clebsch-Gordan coefficients):

$$y_e \frac{1}{3}\sqrt{3} L_1 e h_d \widetilde{\varphi}_1 + y_e \frac{1}{3}\sqrt{3} L_2 e h_d \widetilde{\varphi}_2 + y_e \frac{1}{3}\sqrt{3} L_3 e h_d \widetilde{\varphi}_3$$
$$+ y_\mu \frac{1}{3}\sqrt{3} L_1 \mu h_d \widetilde{\varphi}_3 + y_\mu \frac{1}{3}\sqrt{3} L_2 \mu h_d \widetilde{\varphi}_1 + y_\mu \frac{1}{3}\sqrt{3} L_3 \mu h_d \widetilde{\varphi}_2$$
$$+ y_\tau \frac{1}{3}\sqrt{3} L_1 \tau h_d \widetilde{\varphi}_2 + y_\tau \frac{1}{3}\sqrt{3} L_2 \tau h_d \widetilde{\varphi}_3 + y_\tau \frac{1}{3}\sqrt{3} L_3 \tau h_d \widetilde{\varphi}_1$$

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Family Symmetries

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\widetilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\widetilde{\varphi}} & \sqrt{2}y_2 v_\varphi \end{pmatrix} \end{array}$$

➢ Singular value decomposition

$$\hat{M}_{\ell^+} = D_L M_{\ell^+} D_R^\dagger, \quad \hat{M}_\nu = U_L M_\nu U_R^\dagger$$

➢ Neutrino mixing matrix

$$U_{\mathrm{PMNS}} = D_{\mathrm{L}} U_{\mathrm{L}}^\dagger = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \overset{e,\mu,\tau \to \tau,\mu,e}{=} \begin{pmatrix} -2/\sqrt{6} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & -1/\sqrt{3} & -1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Formdiagonalizable! Mixing matrix does not depend on A, B, masses do!

➢ Mixing angles: $\theta_{12} = 35.26°$, $\theta_{23} = 45°$, $\theta_{13} = 0°$ Tribimaximal ✔

Family Symmetries

➢ Mass matrices

$$M_{\ell^+} = -\frac{v_d v_{\tilde{\varphi}}}{\sqrt{6}\Lambda} \times \begin{array}{c} \\ L_1^{(2)} \\ L_2^{(2)} \\ L_3^{(2)} \end{array} \begin{array}{c} \begin{array}{ccc} e & \mu & \tau \end{array} \\ \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \end{array}, \qquad M_\nu = \frac{v_u^2}{12\Lambda^2} \times \begin{array}{c} \\ L_1^{(1)} \\ L_2^{(1)} \\ L_3^{(1)} \end{array} \begin{array}{c} \begin{array}{ccc} L_1^{(1)} & L_2^{(1)} & L_3^{(1)} \end{array} \\ \begin{pmatrix} \sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} & -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & \sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} \\ -\frac{1}{2}\sqrt{2}y_2 v_\varphi & -\frac{1}{2}\sqrt{2}y_2 v_\varphi + 2y_1 v_{\tilde{\varphi}} & \sqrt{2}y_2 v_\varphi \end{pmatrix} \end{array}$$

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Family Symmetries

- Remember leading-order superpotential:

> - Terms that are invariant, have 2 leptons and mass dimension ≤ 5 or 6:
>
> $$W = y_e \frac{\widetilde{\varphi}}{\Lambda} L e h_d + y_\mu \frac{\widetilde{\varphi}}{\Lambda} L \mu h_d + y_\tau \frac{\widetilde{\varphi}}{\Lambda} L \tau h_d + y_1 \frac{\widetilde{\varphi}}{\Lambda^2} L L h_u h_u + y_2 \frac{\varphi}{\Lambda^2} L L h_u h_u$$
>
> $$\mathbf{3}' \otimes \mathbf{3} \otimes \mathbf{1} \otimes \mathbf{1} = (\mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}') \otimes \mathbf{1} \otimes \mathbf{1} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3} + \mathbf{3}'$$

- Superpotential with two flavon fields (massdim $\leq$6 or 7)

$$y_e\, L\, e\, h_d\, \widetilde{\varphi} + C_7\, L\, e\, h_d\, \varphi\, \varphi + C_8\, L\, e\, h_d\, \varphi\, \widetilde{\varphi} + C_9\, L\, e\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{10}\, L\, e\, h_u\, h_d\, h_d\, \widetilde{\varphi}$$

$$y_\mu\, L\, \mu\, h_d\, \widetilde{\varphi} + C_{12}\, L\, \mu\, h_d\, \varphi\, \varphi + C_{13}\, L\, \mu\, h_d\, \varphi\, \widetilde{\varphi} + C_{14}\, L\, \mu\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{15}\, L\, \mu\, h_u\, h_d\, h_d\, \widetilde{\varphi}$$

$$y_\tau\, L\, \tau\, h_d\, \widetilde{\varphi} + C_{17}\, L\, \tau\, h_d\, \varphi\, \varphi + C_{18}\, L\, \tau\, h_d\, \varphi\, \widetilde{\varphi} + C_{19}\, L\, \tau\, h_d\, \widetilde{\varphi}\, \widetilde{\varphi} + C_{20}\, L\, \tau\, h_u\, h_d\, h_d\, \widetilde{\varphi}$$

$$y_2\, L\, L\, h_u\, h_u\, \varphi + y_1\, L\, L\, h_u\, h_u\, \widetilde{\varphi} + \underbrace{C_3\, L\, L\, h_u\, h_u\, \varphi\, \varphi} + \underbrace{C_4\, L\, L\, h_u\, h_u\, \varphi\, \widetilde{\varphi}} + \underbrace{C_5\, L\, L\, h_u\, h_u\, \widetilde{\varphi}\, \widetilde{\varphi}}$$

- Achieved $\theta_{13} \simeq 3.5^\circ$, pushing for higher values
- Pick only contributions that change θ_{13}
  $\rightarrow$ Introduce N_R and/or Δ for renormalizable UV completion

Conclusions

- We are living in exciting times!
- LHC is showing an excellence performance
- The Higgs boson will soon be discovered or excluded
- New physics is probably just around the corner
  (Unfortunately so far no sign of it yet!)
- Presented here some of the "crazy" ideas for new physics
- Beware Michael Turner's words:
  "Not every crazy idea is a solution to a profound problem. Some of them are just crazy ideas."

Future Research

BACKUP

A Fourth Generation of Chiral Fermions

- Limit on t' mass does not consider $t' \to Wb'$

CDF Collaboration, T. Aaltonen *et al.*, "Search for a Heavy Top-Like Quark in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV," *Phys.Rev.Lett.* (2011) 1107.3875

- Limit on b' mass assumes $\mathrm{Br}(b' \to Wt) = 100\%$

CDF Collaboration, T. Aaltonen *et al.*, "Search for heavy bottom-like quarks decaying to an electron or muon and jets in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV," *Phys. Rev. Lett.* **106** (2011) 141803, 1101.5728

M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Ludwig, *et al.*, "Updated Status of the Global Electroweak Fit and Constraints on New Physics," 1107.0975