Exact solution for the stirring of a one-dimensional interacting Bose gas on a ring trap

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Outline

MOTIVATION I

What about superfluidity in 1D?

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- Landau's criterion for superfluidity

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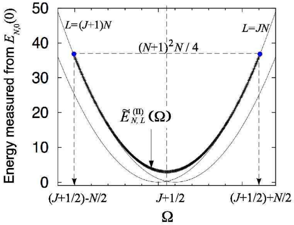
- Angular momentum enters in quanta of $\hbar/m$ via the formation of a soliton into 1D Bose gas

  [R. Kanamoto, L. D. Carr, and M. Ueda, Phys. Rev. A **81** 023625 (2010)]



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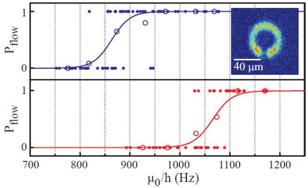

[A. Ramanathan et al, Phys. Rev. Lett. **106** 130401 (2011)]

- Persistant currents in a toroidal 3D BEC
- Decay of superflow due to a weak link in the BEC

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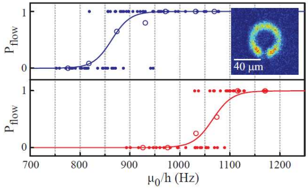

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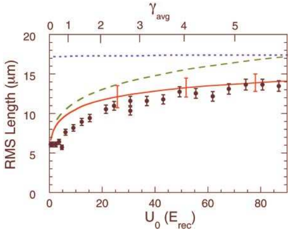

[T. Kinoshita, T. Wenger, and D. S. Weiss, Science **305** 5687 (2004)]

- Realization of a strongly interacting Bose-gas (Tonks-Girardeau gas)
- For very strong interactions fermionic properties become visible

$$\gamma = \frac{V_{int}}{E_{kin}} = \frac{g_{1D}\, n_{1D}}{\hbar^2 n_{1D}^2/m}$$

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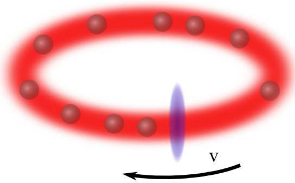

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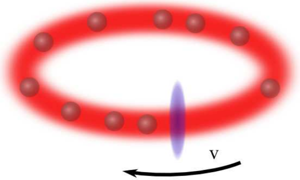

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- The Hamiltonian

$$H = H_{LL} + V_{ext}(t)$$

$$H_{LL} = \sum_{l=1}^{N} \underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_l^2}}_{E_{kin}} + \underbrace{g\sum_{j<l}\delta(x_j - x_l)}_{V_{int}} \qquad \text{and} \qquad V_{ext}(t) = U_0\ \delta(x_l - vt)$$

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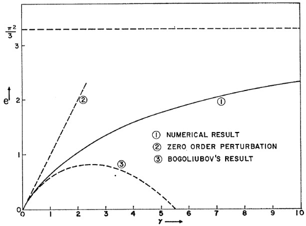

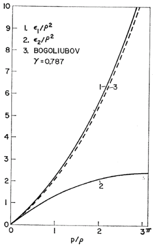

The External Potential

- The time-dependent external Potential

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- PBCs for the many-body wavefunction at $x = \pm\frac{L}{2}$

The Many-Body Wavefunction

- Wavefunction for **ideal bosons** is a product of "lowest" single particle wavefunction $\psi_1(x_\ell, t)$ (factorizable)

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$$\left(\frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_l}\right)\Psi\Big|_{x_j = x_l^+} - \left(\frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_l}\right)\Psi\Big|_{x_j = x_l^-} = 2\frac{gm}{\hbar^2}\Psi\Big|_{x_j = x_l}$$

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Interactions $\equiv$ Pauli-principle

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- Wavefunction for **impenetrable bosons** obtained from time-dependent Fermi Bose mapping theorem

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$$\Psi_{TG}(x_1 \ldots x_N, t) = A(x_1, \ldots, x_N, t)\Psi_F(x_1, \ldots, x_N, t)$$

$$\text{with} \quad \begin{cases} A(x_1, \ldots, x_N, t) = \text{Mapping Function} \\ \Psi_F(x_1, \ldots, x_N, t) = \frac{1}{\sqrt{N!}}\det[\psi_l(x_k, t)] \end{cases}$$

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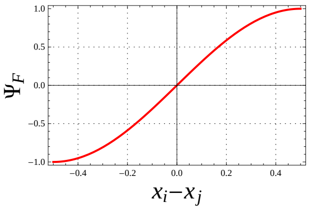

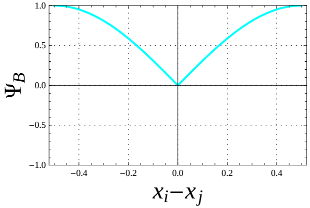

THE ROTATING FRAME

- Move to the rotating frame via two unitary transformations (gauge transformations)

$$U_1 = e^{-\frac{i}{\hbar}\,\hat{p}\,v\,t} \qquad \text{translation by} \quad vt$$
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The Wavevector Landscape

- Single particle wavevectors *vs* stirring momentum ($q = \frac{mv}{\hbar}$)

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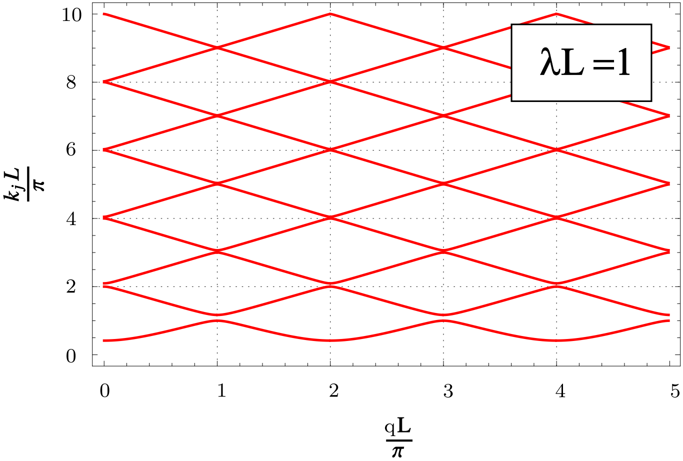

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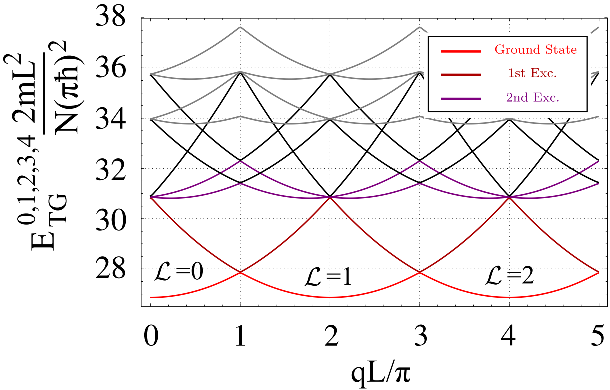

- The parabolas represent states of different angular momentum

The Reduced Single Particle Density Matrix

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in terms of single particle wavefunctions

[R. Pezer and H. Buljan, Phys. Rev. Lett. 98 240403 (2007)]

$$\rho_B(x,y,t) = \sum_{i,j=1}^{N} \psi_i^*(x,t)A_{ij}(x,y,t)\psi_j(y,t)$$

$$\mathbf{A}(x,y,t) = \left(\mathbf{P}^{-1}(x,y,t)\right)^T \det\left(\mathbf{P}(x,y,t)\right)$$

$$P_{ij}(x,y,t) = \delta_{i,j} - 2\int_x^y dx'\ \psi_i^*(x',t)\psi_j(x',t)$$

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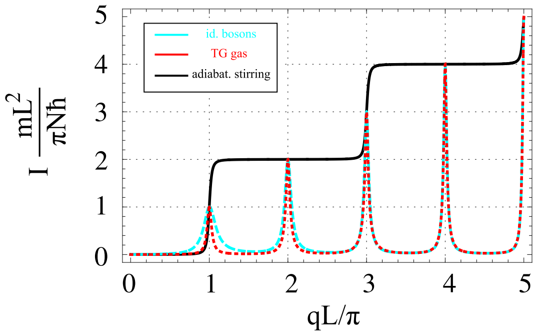

[C. Schenke, M. Minguzzi and F.W.J. Hekking, Phys. Rev. A 85 053627 (2012)]

The current-current fluctuations

- The current-current correlation function

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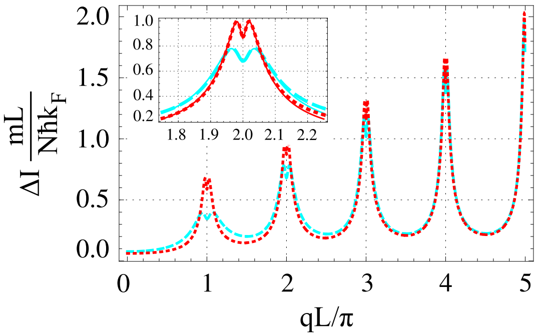

THE DRAG FORCE

- The classical force on the particles (Ehrenfest theorem)

$$\begin{aligned} F_{class} &= \left\langle \frac{\partial p}{\partial t} \right\rangle = \frac{i}{\hbar}\langle [H,p] \rangle = -\frac{i}{\hbar}\langle p V_{ext} \rangle = -\langle \psi_l^*(x,t)| \left(\frac{\partial}{\partial x} V_{ext}(x,t) \right) |\psi_l(x,t)\rangle \\ &= -\int_{-L/2}^{L/2} dx\, \rho(x,x,t)\partial_x V_{ext}(x,t) = U_0 \partial_x \rho(x,x,t)\Big|_{x=0} \end{aligned}$$

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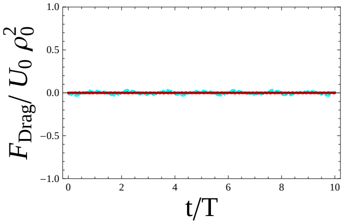

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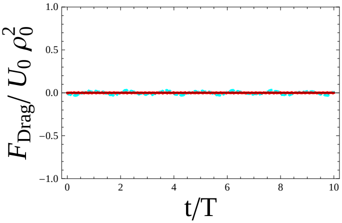

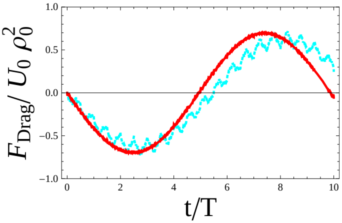

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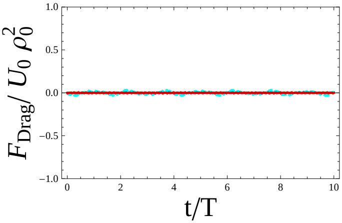

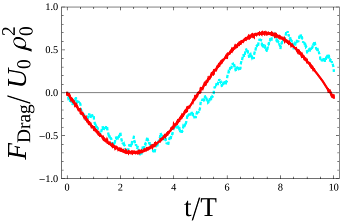

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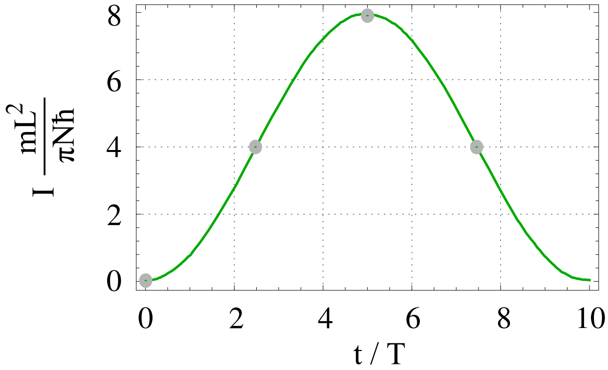
THE TIME-DEPENDENCE

- Current vs time at $q = 4\frac{\pi}{L}$



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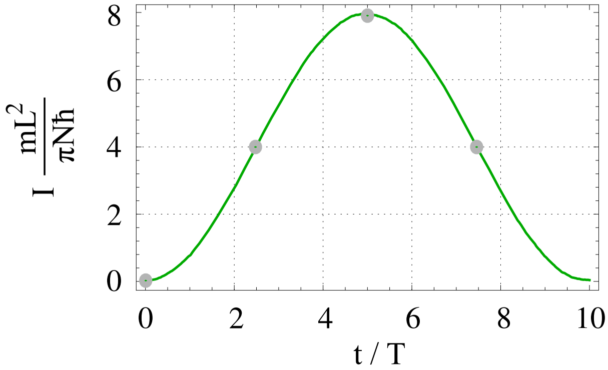

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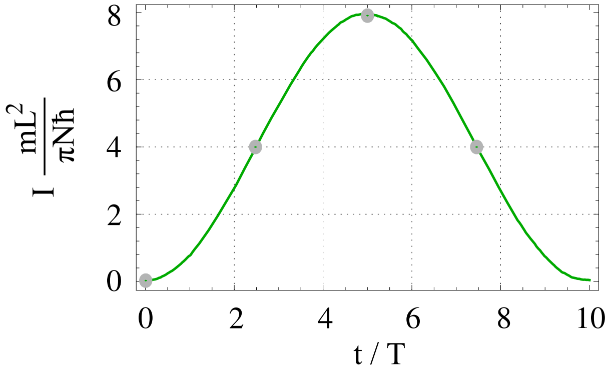

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  $\Rightarrow$ oscillation frequency $\simeq$ highest occupied avoided level crossing

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- Momentum Dist.: $n_B(k,q,t) = \int dx \int dy\, e^{ik(x-y)} \rho_B(x,y,q,t)$

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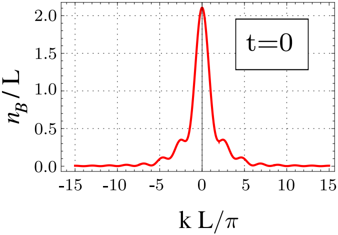

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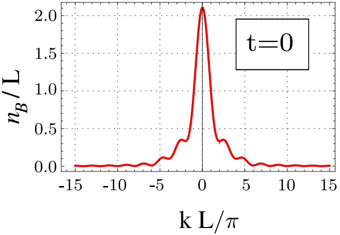

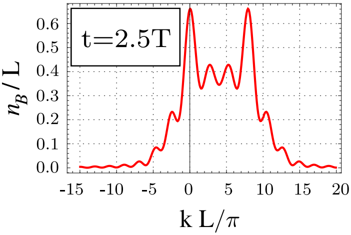

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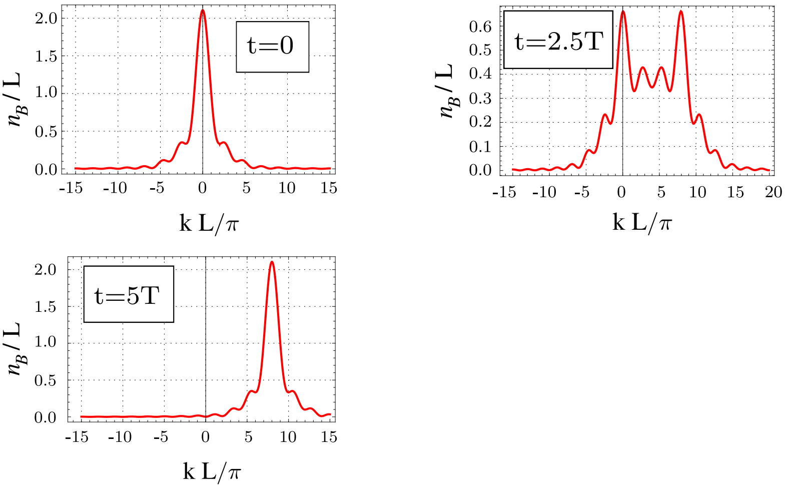

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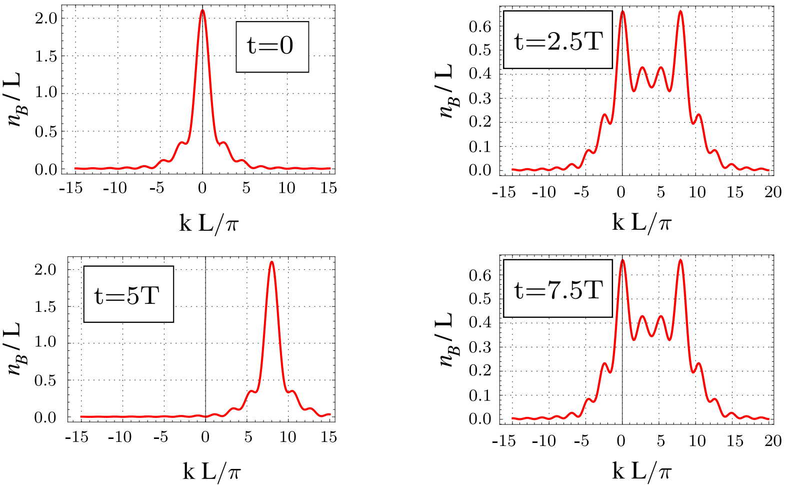

The State

What kind of state are we dealing with?

Superpositions of Particle States

What about Superpositions and Entanglement?

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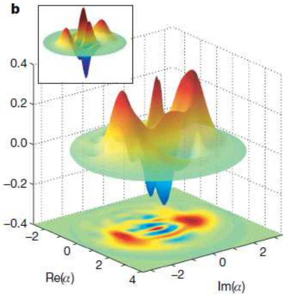

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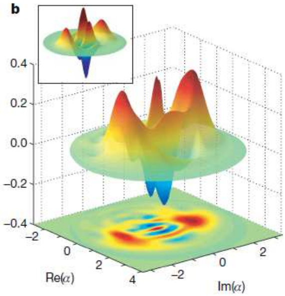

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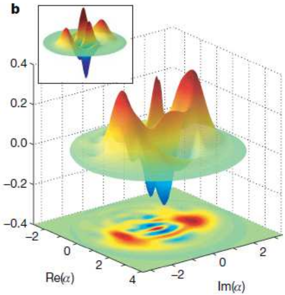

- Theoretical predictions for ultracold atomic gases
  $\longrightarrow$ in Bose Josephson Junctions (superposition of coherent states)
  [G. Ferrini, A. Minguzzi, F.W. Hekking, PRA 78, 023606 (2008); PRA 80, 043628 (2009)]

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Have been analyzed for weak/intermediate interaction strength

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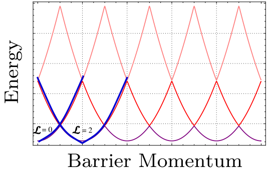

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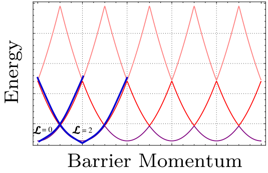

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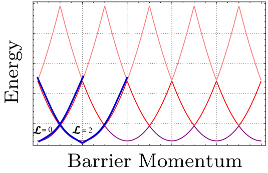

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$$|NOON\rangle \propto [(b_0^\dagger)^N + (b_{2q_0}^\dagger)^N|vac\rangle$$

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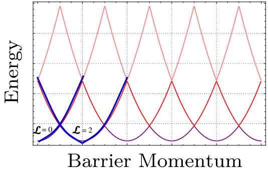

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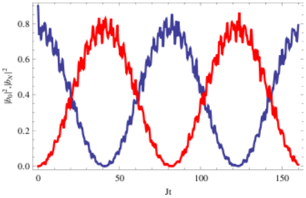

- interactions $\longrightarrow$ fidelity $\simeq 0.8$

  $\Longrightarrow$ no "NOON"-state

MACROSCOPIC SUPERPOSITION OF STRONGLY INTERACTING BOSONS (TONKS-GIRARDEAU GAS)

Why are strongly interacting Bosons favorable?

Macroscopic superposition of strongly interacting Bosons (Tonks-Girardeau gas)

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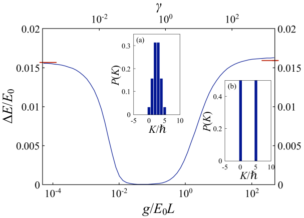

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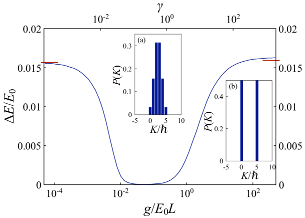

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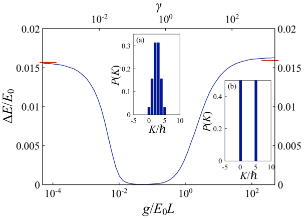


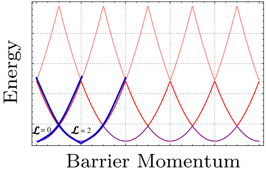
Moreover: fermionization prevents two- and three-body losses

- Ideal candidate for robust superpositions
  $\Longrightarrow$ **Tonks Girardeau gas**

Superpositions in the TG-Limit

- typical momentum $q = q_F \gg \frac{\pi}{L}$

  $\Longrightarrow$ adiabatic coupling of states is not resolvable!



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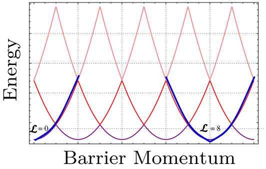

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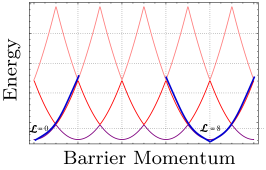

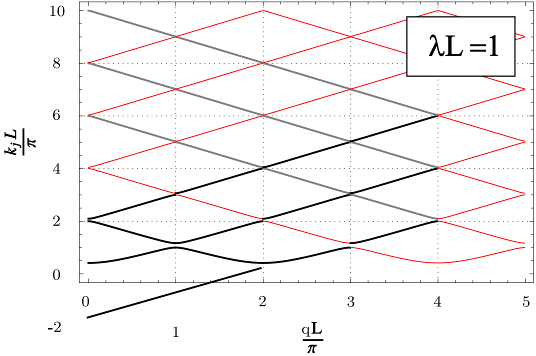

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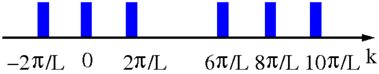

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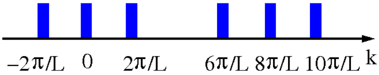

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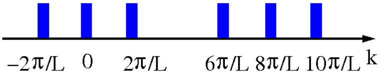

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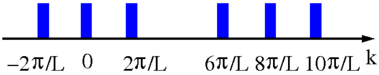

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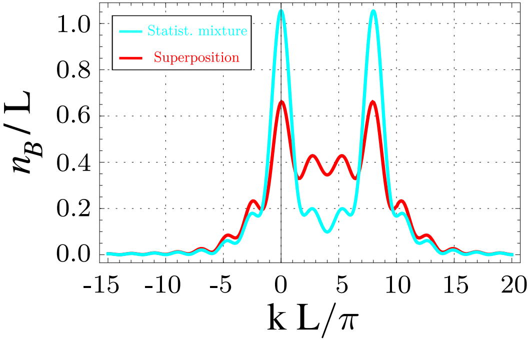

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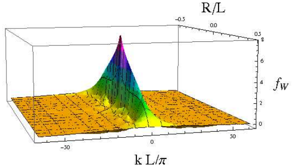

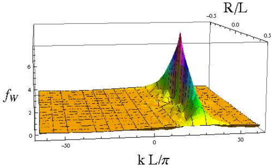

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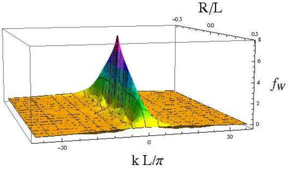
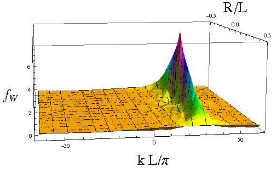
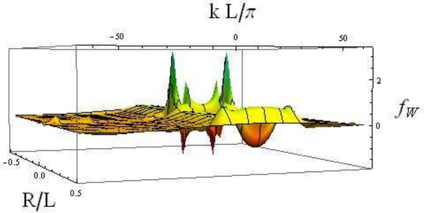




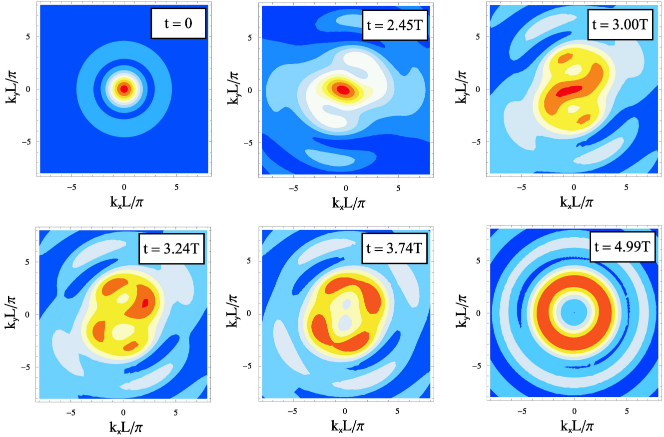


[C. Schenke, M. Minguzzi and F.W.J. Hekking, Phys. Rev. A 84 053636 (2011)]

Time of Flight

- Dynamics of the transfer of angular momentum

TOF: $$n_{TOF}(\mathbf{k}) = \int d^3x \int d^3y \, \rho_B^{ring}(\mathbf{x},\mathbf{y},t) e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}$$

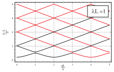


Summary and Perspectives

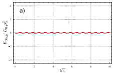
TG gas on a ring

Summary

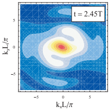
- Exact solution for sudden stirring



- Mesoscopic superfluidity



- Quantum interferences can be seen in the time of flight images



Perspectives

- Out-of-equilibrium description beyond TG limit?

THE PHYSICAL OBSERVABLES

- Density Profile: $\rho_B(x,x,q,t)$
- Current: $I_l(t) = \int dx \, \frac{\hbar}{m} \mathrm{Im}\left\{ \psi_l^*(x,t) \frac{\partial}{\partial x} \psi_l(x,t) \right\}$
- Drag-Force: $F_d = \langle \psi_l^*(x,t) | \left(\frac{\partial}{\partial x} V_{ext}(x,t) \right) | \psi_l(x,t) \rangle$
- Momentum Dist.: $n_B(k,q,t) = \int dx \int dy \, e^{ik(x-y)} \rho_B(x,y,q,t)$
- Wigner Function: $f_w(k,R,t) = \int dr \, \rho_B(R+r/2, R-r/2, q, t) e^{ikr}$
- TOF: $n_{TOF}(\mathbf{k}) = \int d^3x \int d^3y \, \rho_B^{ring}(\mathbf{x},\mathbf{y},t) e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}$
- Note:

$\Rightarrow$ $\rho_B^{ring}(r,\theta,z;r',\theta',z';t) = \delta(r-R)\delta(r'-R)\delta(z)\delta(z')\rho_B(R\theta,R\theta',t)$

$\Rightarrow$ Angular Momentum: $\mathfrak{L} = \frac{L}{2\pi} I$

$\Rightarrow$ Center of Mass Coord: $r = x - y$ and $R = (x+y)/2$

THE SINGLE-PARTICLE WAVEFUNCTION

- The time-dependent single-particle wavefunction

$$\psi_l(x,t) \quad = \quad e^{iqx}\, e^{-\frac{i}{2m}q^2 t} \sum_j c_{jl}\, e^{-i\tilde{E}_j t}\, \tilde{\varphi}_j(x-vt)$$

- Coefficients determined through the initial condition $\psi_l(x,t=0)=\varphi_l(x)$

$$c_{jl} = \int_0^L dx\, e^{-iqx}\, \tilde{\varphi}_j^*(x)\, \varphi_l(x)$$

where

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U_0\, \delta(x)\right)\varphi_l(x) = \varepsilon_l\, \varphi_l(x)$$

The Single-Particle Orbitals

- Ansatz

$$\tilde{\varphi}_j(x) = \begin{cases} \tilde{\varphi}_j^-(x) = \frac{1}{N^-}\left(e^{ik_jx} + \gamma_j^- \, e^{-ik_jx}\right) & x \in [-\frac{L}{2}, 0] \\ \tilde{\varphi}_j^+(x) = \frac{1}{N^+}\left(e^{ik_jx} + \gamma_j^+ \, e^{-ik_jx}\right) & x \in [0, \frac{L}{2}] \end{cases}$$

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- Unknowns $k_j, \tilde{N}_j^+, \tilde{N}_j^-, \gamma_j^+, \gamma_j^-$ determined by the matching conditions

1. Normalization
2. TBCs
3. Cusp Conditions

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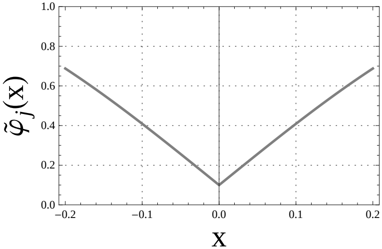

The Single-Particle Orbitals II

- The complete set of orbitals for $q \neq \frac{\pi}{L} n \qquad n = 1, 2, 3\ldots$

$$\tilde{\varphi}_j(x) = \begin{cases} \tilde{\varphi}_j^-(x) = \frac{1}{\tilde{N}_j} e^{iq\frac{L}{2}} \left(e^{ik_j(x+\frac{L}{2})} + A(k_j, q)\, e^{-ik_j(x+\frac{L}{2})} \right) & x \in [-\frac{L}{2}, 0] \\ \tilde{\varphi}_j^+(x) = \frac{1}{\tilde{N}_j} e^{-iq\frac{L}{2}} \left(e^{ik_j(x-\frac{L}{2})} + A(k_j, q)\, e^{-ik_j(x-\frac{L}{2})} \right) & x \in [0, \frac{L}{2}] \end{cases}$$

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- with

$$\tilde{N}_j = \sqrt{L \left(1 + A^2(k_j,q) + 2A(k_j,q) \frac{\sin(k_j L)}{k_j L} \right)}$$

$$A(k_j,q) = \frac{\sin\left((k_j + q)\frac{L}{2}\right)}{\sin\left((k_j - q)\frac{L}{2}\right)}$$

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$$k_j = \lambda \frac{\sin(k_j L)}{\cos(qL) - \cos(k_j L)} \quad \text{transcendental equation!}$$

The Transcendental Equation

Solutions for the transcendental equation $k_j = \lambda \frac{\sin(k_j L)}{\cos(qL) - \cos(k_j L)}$ with $\lambda = \frac{m\, U_0}{\hbar^2}$

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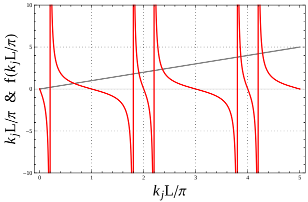
- For $q = 0.2 \frac{\pi}{L}$



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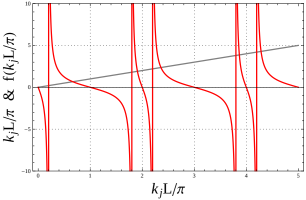
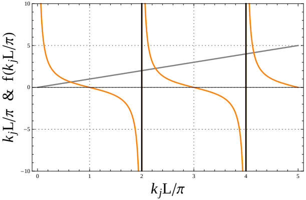
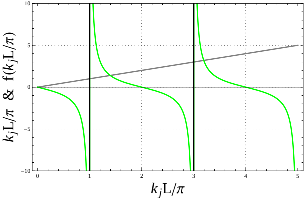


- For $q = \frac{\pi}{L} n \qquad n = 1, 2, 3 \ldots$ this reduces to





The Single-Particle Orbitals III

Need to distinguish between **even** and **odd** states for $q = \frac{\pi}{L} n \qquad n = 1, 2, 3...$

- $q = \frac{2\pi n}{L}$

$$\tilde{\varphi}_j^e(x) = \frac{2}{N_j} \cos\left(k_j(|x| - \frac{L}{2})\right)$$

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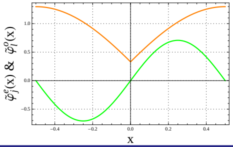

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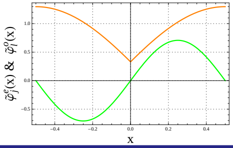

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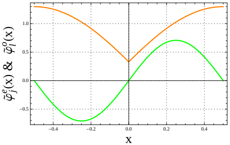

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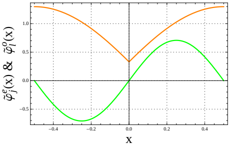

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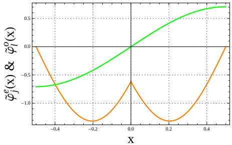
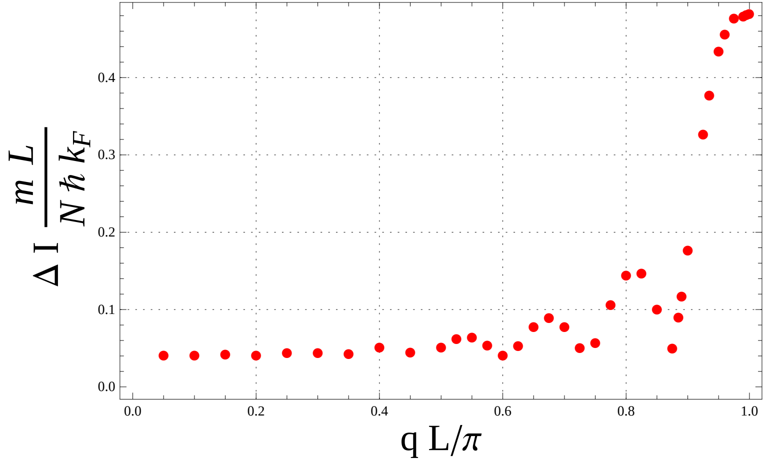


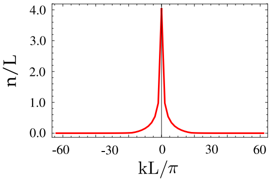
Current Fluctuations

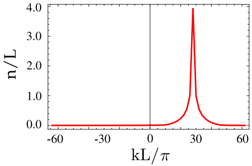
- The current fluctuations at time $t = 2.5T$

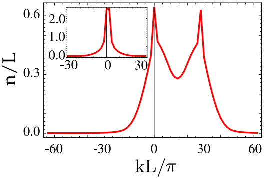


MOMENTUM DISTRIBUTION

- The momentum distribution

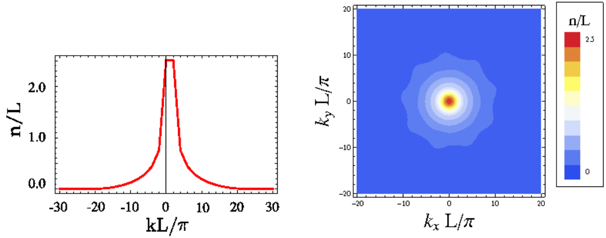






Resolving the components

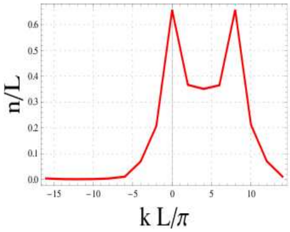
- momentum distribution and TOF images for a small velocity $v = \pi\hbar/mL$

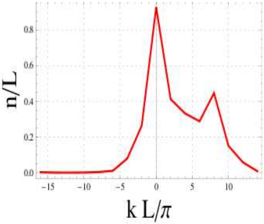


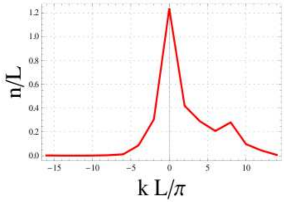
- the components are not well resolved at $v \ll v_F$
(the Fermi spheres largely overlap)

VELOCITY FLUCTUATIONS

- what if $v \neq n\pi\hbar/mL$?







"mesoscopic superfluid":
difficult to transfer angular momentum to the gas

- velocity fluctuations are less important for a larger barrier

TOF

- evidence of interference effects