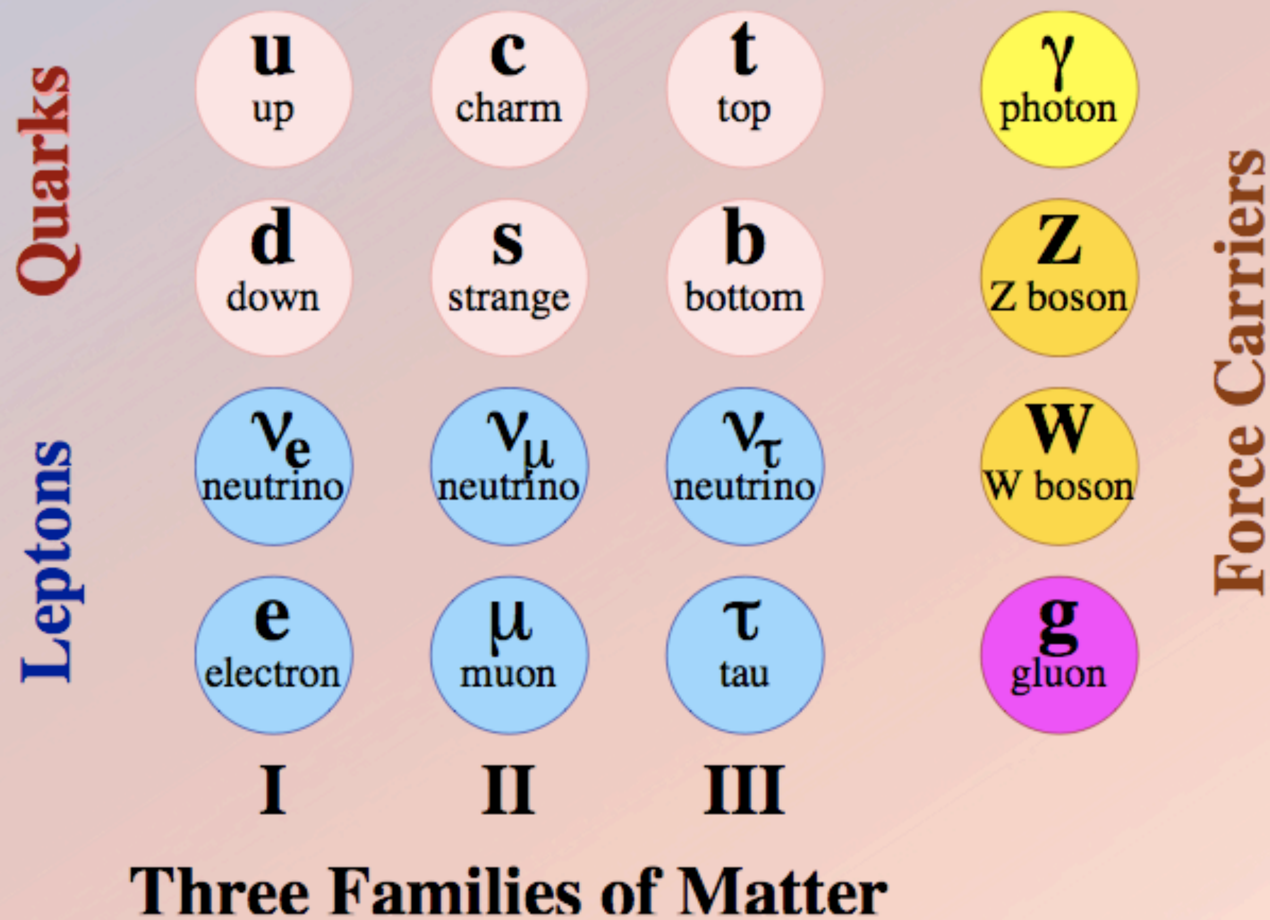
Insensitive Unification of Gauge Couplings

Radovan Dermisek
Indiana University, Bloomington

arXiv:1204.6533 [hep-ph]

Known Elementary Particles



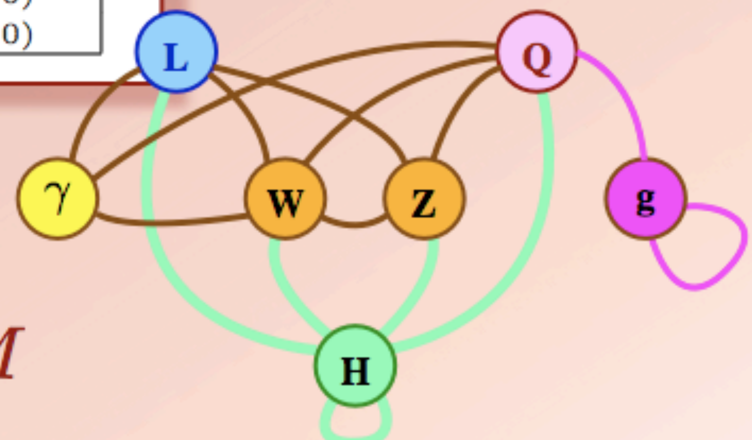
Standard Model

$$SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

Names	Content			SM
Quarks and Leptons	$i=1$	2	3	
Q_i	$\begin{pmatrix} u \\ d \end{pmatrix}_\alpha$	$\begin{pmatrix} c \\ s \end{pmatrix}_\alpha$	$\begin{pmatrix} t \\ b \end{pmatrix}_\alpha$	$(\mathbf{3}, \mathbf{2}, 1/3)$
$\bar{u}_i$	$\bar{u}_\alpha$	$\bar{c}_\alpha$	$\bar{t}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, -4/3)$
$\bar{d}_i$	$\bar{d}_\alpha$	$\bar{s}_\alpha$	$\bar{b}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, 2/3)$
L_i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$
$\bar{e}_i$	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$	$(\mathbf{1}, \mathbf{1}, 2)$
Higgs H		$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$		$(\mathbf{1}, \mathbf{2}, 1)$
Gauge bosons				
gluon		g_α		$(\mathbf{8}, \mathbf{1}, 0)$
W bosons		$W^\pm,\ W^0$		$(\mathbf{1}, \mathbf{3}, 0)$
B boson		B^0		$(\mathbf{1}, \mathbf{1}, 0)$

$$\alpha_3(M_Z)_{exp} = 0.1184$$
$$\alpha_2(M_Z)_{exp} = 0.03380$$
$$\alpha_1(M_Z)_{exp} = 0.01695$$
$$\alpha_{EM}(M_Z) = 1/127.916$$
$$\sin^2\theta_W = 0.2313$$

$$\sin^2\theta_W \equiv \frac{\alpha'}{\alpha_2 + \alpha'}$$
$$\alpha_{EM} = \alpha_2 \sin^2\theta_W$$
$$\alpha' = \frac{3}{5}\alpha_1,$$



Spontaneous symmetry breaking:

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

Electroweak Symmetry Breaking

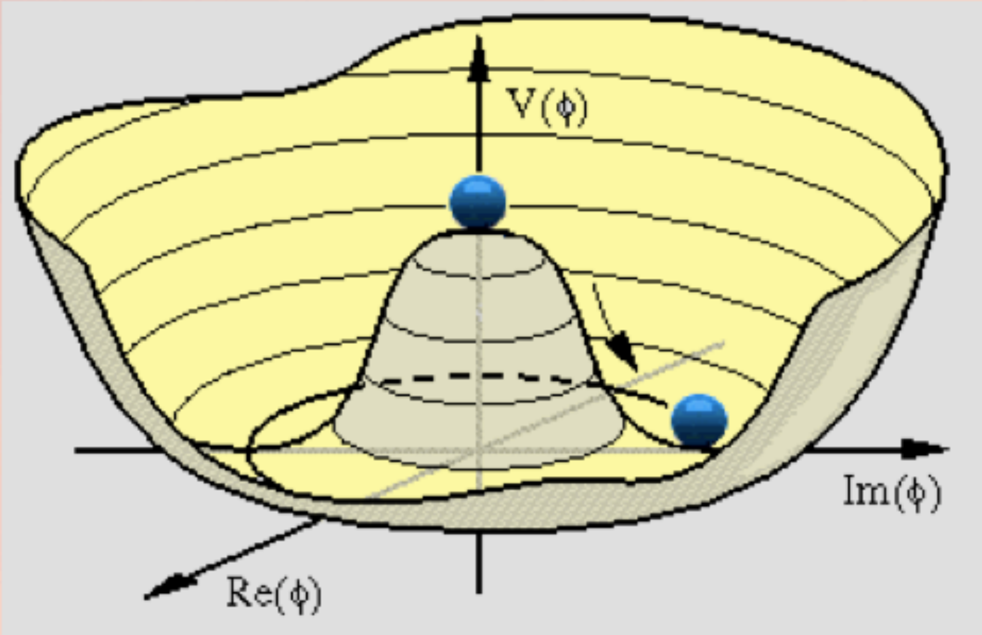
Spontaneous symmetry breaking:

$$V_{higgs} = -m^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$v = m/\sqrt{\lambda}$

$$M^2_{Z^0} = v^2(g^2 + g'^2)/4$$
$$M^2_{W^\pm} = v^2 g^2/4$$

$$m_h^2 = 2\lambda v^2$$



Seeking Explanations

- **Numerology**

$$\alpha^{-1} = \frac{4\pi\epsilon_0\hbar c}{e^2} = 137.035\ 999\ 11(46)$$

must be an integer

Eddington (1930)

$$\alpha^{-1} = 2^4 3^3/\pi$$

Heisenberg

$$\alpha^{-1} = (8\pi^4/9)(2^4 5!/\pi^5)^{1/4}$$

Wyler (1969)

$$\alpha^{-1} = 108\pi(8/1843)^{1/6}$$

Aspden and Eagles (1972)

$$\alpha^{-1} = 2^{-19/4} 3^{10/3} 5^{17/4} \pi^{-2}$$

Robertson (1971)

$$\alpha^{-1} = (137^2 + \pi^2)^{1/2}$$

Burger (1978)

frequencies of the hydrogen spectral lines:

$$\nu = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

Balmer (1885)

Seeking Explanations

- **Numerology**
- **Symmetry: GUTs, SUSY, ...**

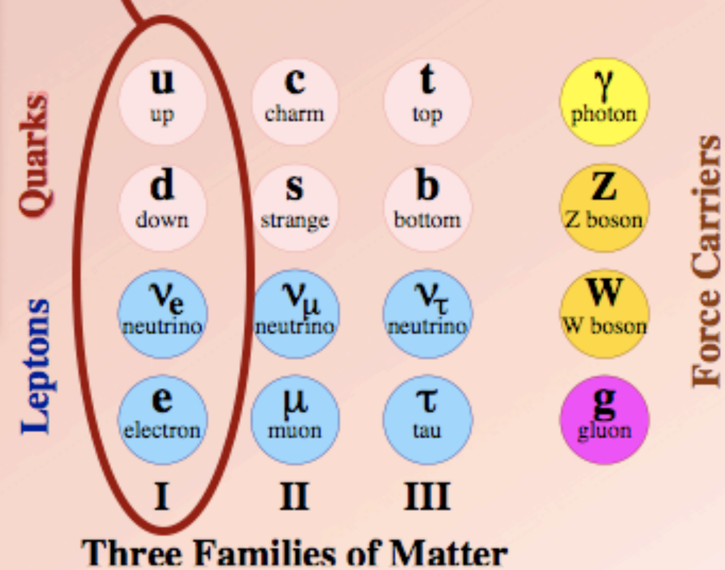
Grand Unification, SUSY, ...

- Understanding of quantum numbers of SM particles

$$SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

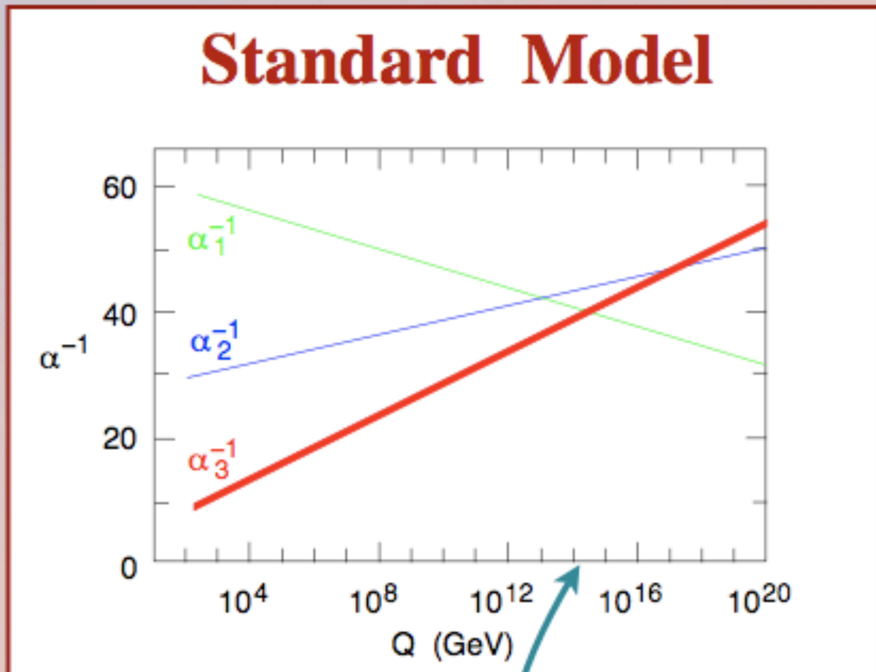
Names	Content			SM
Quarks and Leptons	$i = 1$	2	3	
Q_i	$\begin{pmatrix} u \\ d \end{pmatrix}_\alpha$	$\begin{pmatrix} c \\ s \end{pmatrix}_\alpha$	$\begin{pmatrix} t \\ b \end{pmatrix}_\alpha$	$(\mathbf{3}, \mathbf{2}, 1/3)$
$\bar{u}_i$	$\bar{u}_\alpha$	$\bar{c}_\alpha$	$\bar{t}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, -4/3)$
$\bar{d}_i$	$\bar{d}_\alpha$	$\bar{s}_\alpha$	$\bar{b}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, 2/3)$
L_i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$
$\bar{e}_i$	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$	$(\mathbf{1}, \mathbf{1}, 2)$
Higgs H		$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$		$(\mathbf{1}, \mathbf{2}, 1)$
Gauge bosons				
gluon		g_α		$(\mathbf{8}, \mathbf{1}, 0)$
W bosons		$W^\pm$, W^0		$(\mathbf{1}, \mathbf{3}, 0)$
B boson		B^0		$(\mathbf{1}, \mathbf{1}, 0)$

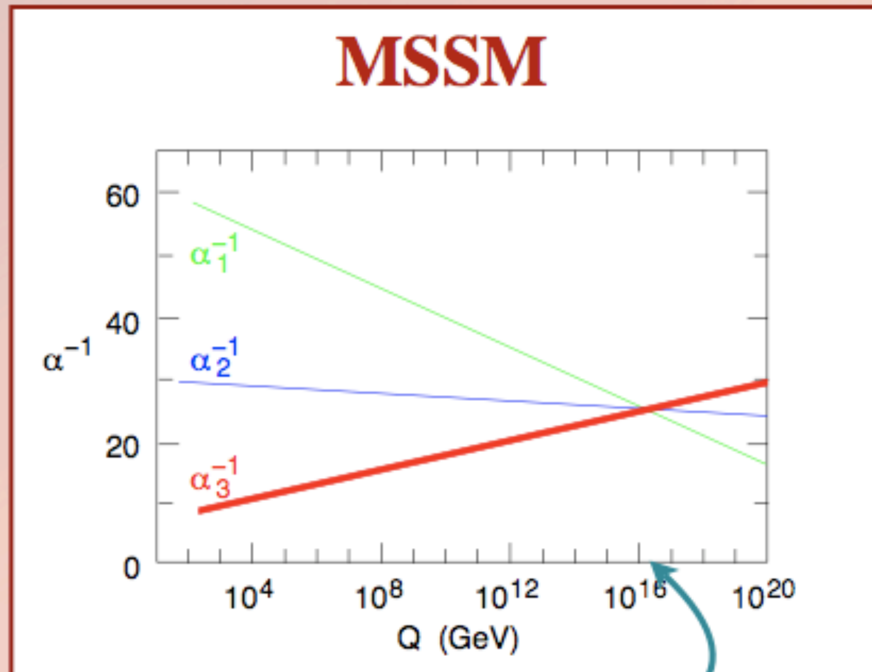
16 of SO(10)



Grand Unification, SUSY, ...

- Understanding of quantum numbers of SM particles
- Gauge coupling unification





GUT scale too small

$\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}\text{yrs}$

large enough

Particle content of the minimal supersymmetric model:

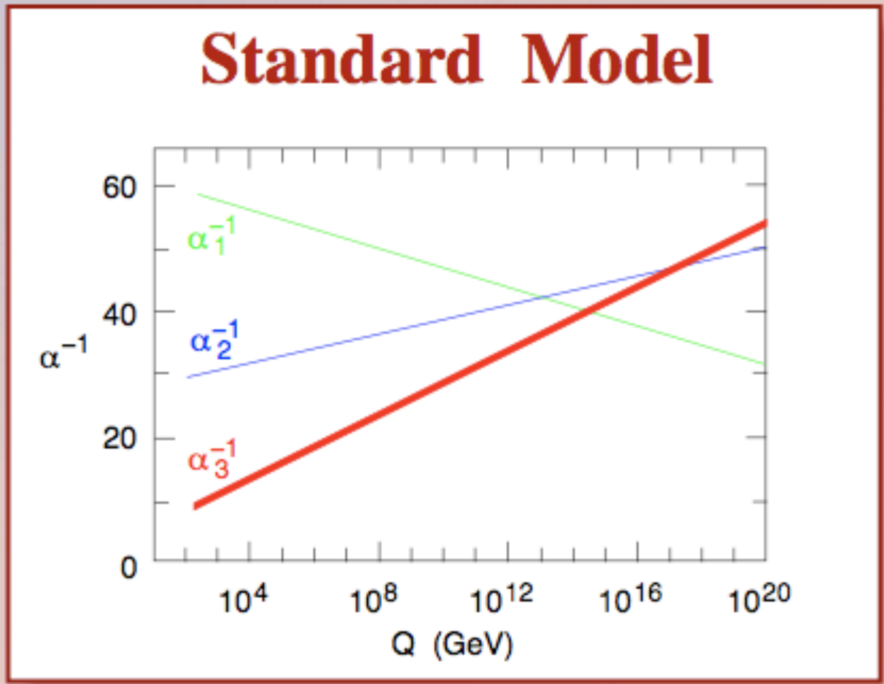
MSSM

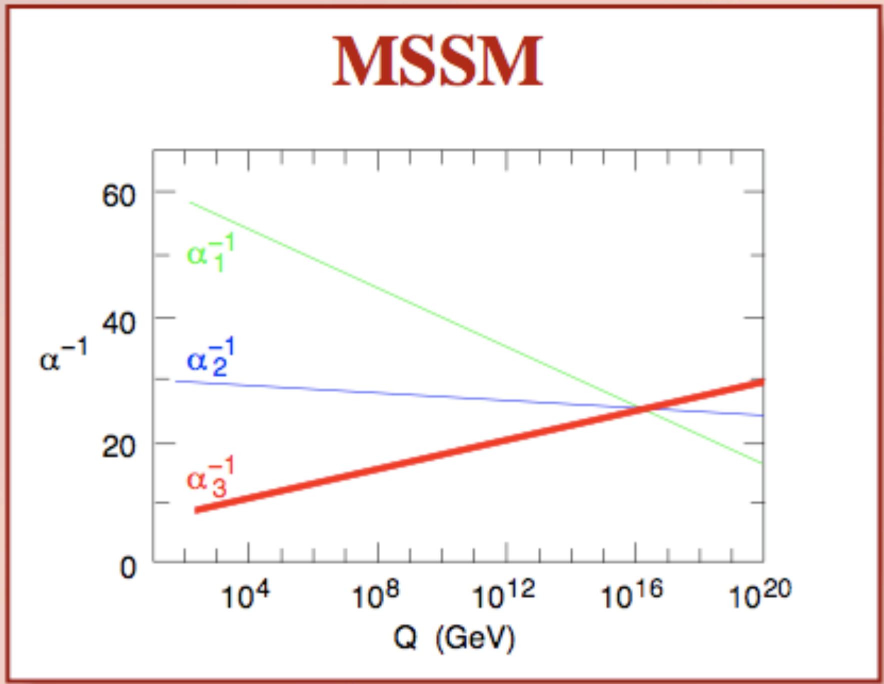
Names	Content		SM
Chiral supermultiplets	spin 0	spin 1/2	
(S)Quarks and (S)Leptons			
Q	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_\alpha$	$\begin{pmatrix} u \\ d \end{pmatrix}_\alpha$	$(\mathbf{3}, \mathbf{2}, 1/3)$
$\bar{u}$	$\tilde{\bar{u}}_\alpha$	$\bar{u}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, -4/3)$
$\bar{d}$	$\tilde{\bar{d}}_\alpha$	$\bar{d}_\alpha$	$(\bar{\mathbf{3}}, \mathbf{1}, 2/3)$
L	$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{e} \end{pmatrix}$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$
$\bar{e}$	$\tilde{\bar{e}}$	$\bar{e}$	$(\mathbf{1}, \mathbf{1}, 2)$
Higgs, higgsino			
H_u	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, 1)$
H_d	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -1)$
Gauge supermultiplets	spin 1/2	spin 1	
gluino, gluon	$\tilde{g}_\alpha$	g_α	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\tilde{W}^\pm, \tilde{W}^0$	$W^\pm, W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\tilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

$$SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

Grand Unification, SUSY, ...

- Understanding of quantum numbers of SM particles
- Gauge coupling unification

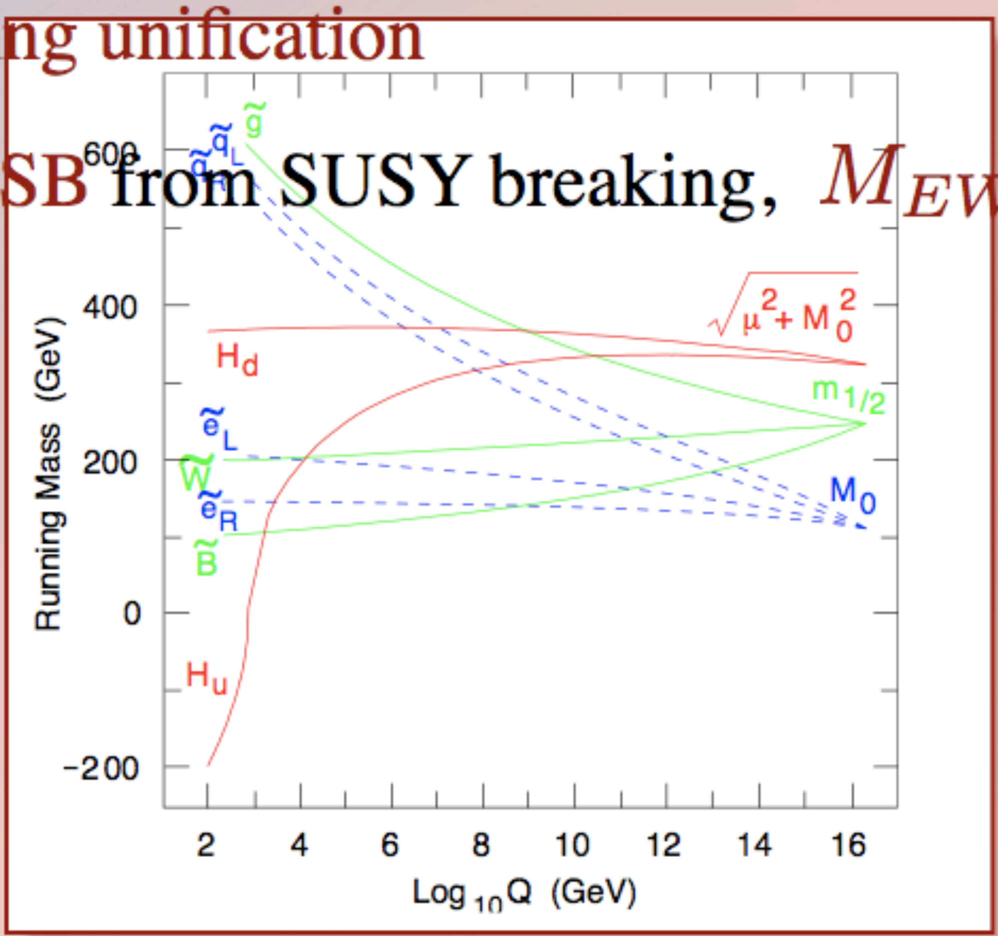




α_3 off by ~ 10%

Supersymmetric Grand Unification

- Understanding of quantum numbers of SM particles
- Gauge coupling unification
- radiative EWSB from SUSY breaking, $M_{EW} \sim M_{SUSY}$



Supersymmetric Grand Unification

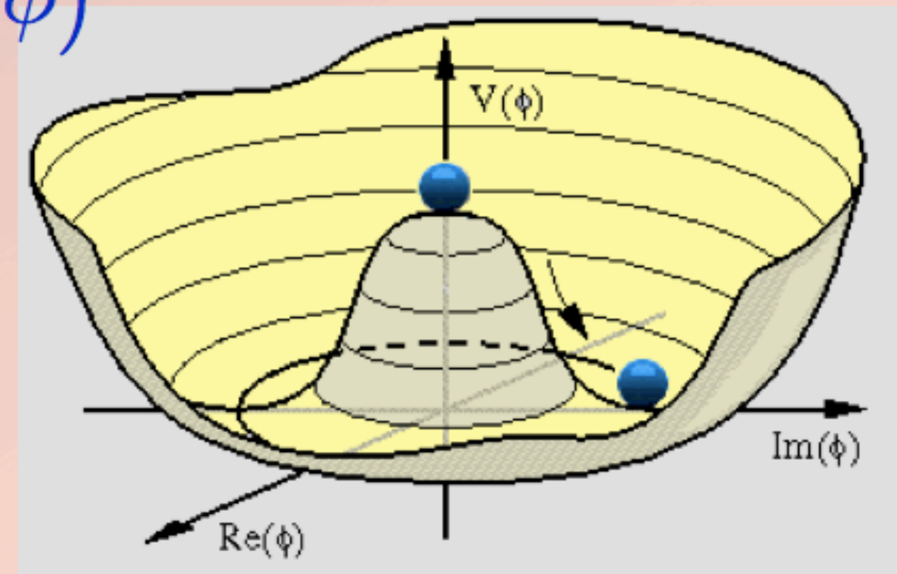
- Understanding of quantum numbers of SM particles
- Gauge coupling unification
- radiative EWSB from SUSY breaking, $M_{EW} \sim M_{SUSY}$

$$V_{higgs} = -m^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$v = m/\sqrt{\lambda}$$

$$M_{Z^0}^2 = v^2(g^2 + g'^2)/4$$
$$M_{W^\pm}^2 = v^2 g^2/4$$

$$m_h^2 = 2\lambda v^2$$



Supersymmetric Grand Unification

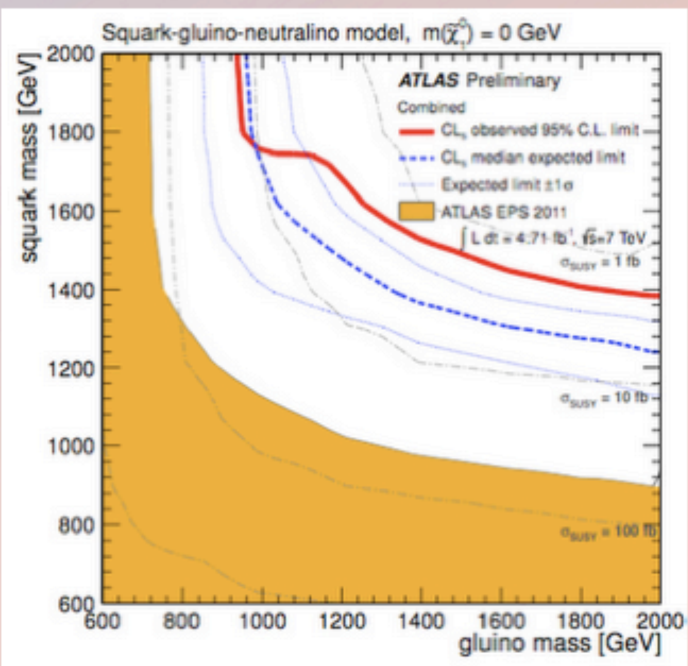
- Understanding of quantum numbers of SM particles
- Gauge coupling unification
- radiative EWSB from SUSY breaking, $M_{EW} \sim M_{SUSY}$
- M_{EW}/M_{GUT} stable
- prediction for the mass of the Higgs boson

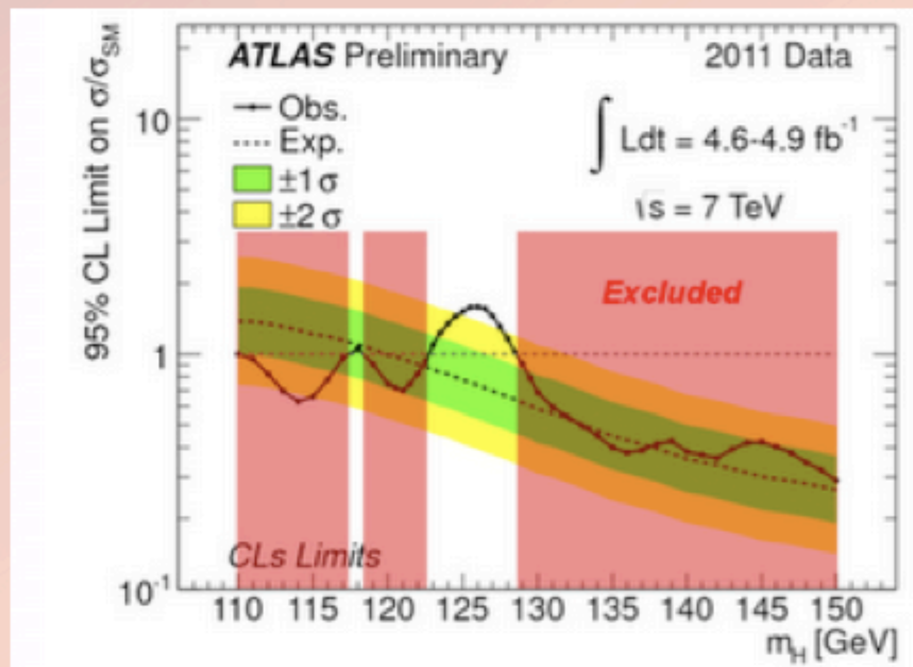
(Higgs quartic coupling given by gauge couplings)

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + 1 - loop$$

Grand Unification, SUSY, ...

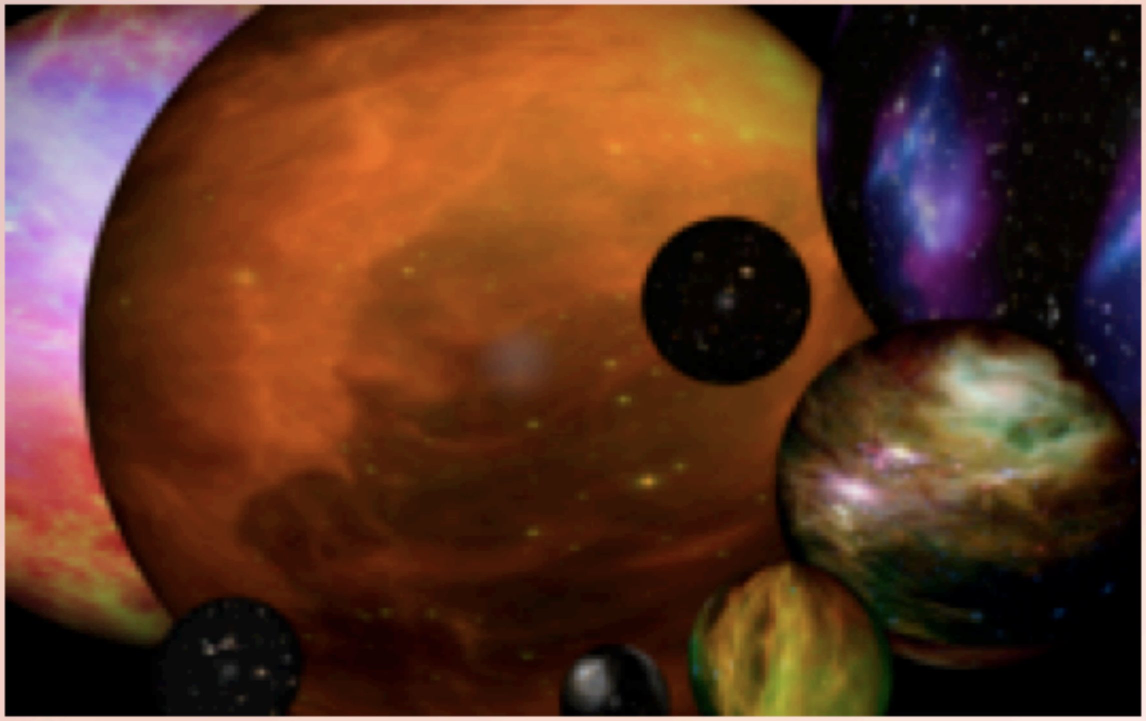
- Understanding of quantum numbers of SM particles
- Gauge coupling unification
- SUSY seems to be hiding from us





Seeking Explanations

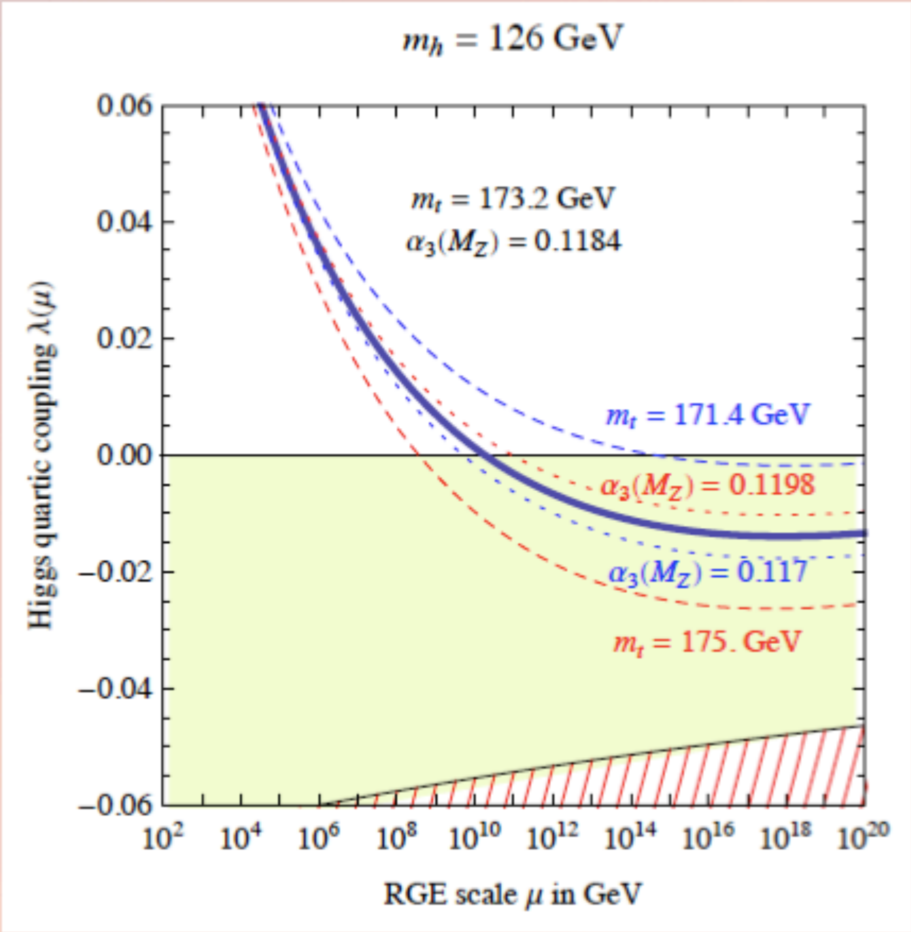
- Numerology
- Symmetry: GUTs, SUSY, ...
- Anthropic, multiverse, ...



Seeking Explanations

- **Numerology**
- **Symmetry: GUTs, SUSY, ...**
- **Anthropic, multiverse, ...**

Just the standard model?



Elias-Miro et al., arXiv:1112.3022 [hep-ph]

Seeking Explanations

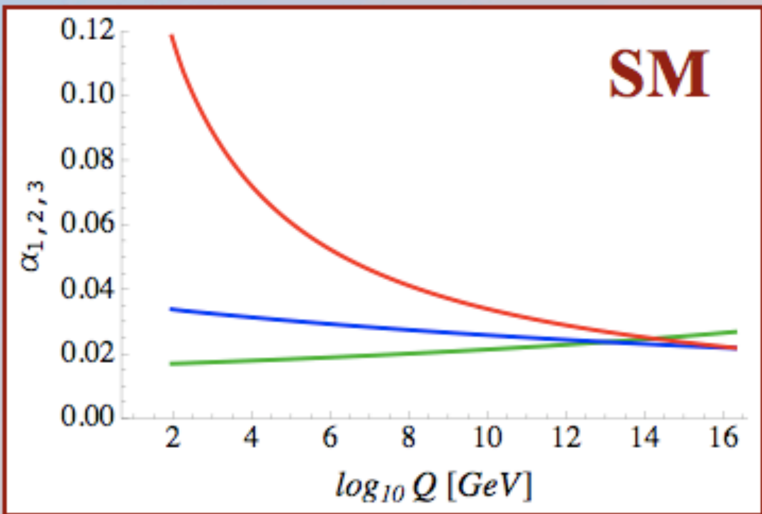
- **Numerology**
- **Symmetry: GUTs, SUSY, ...**
- **Anthropic, multiverse, ...**

- **Self-organization**

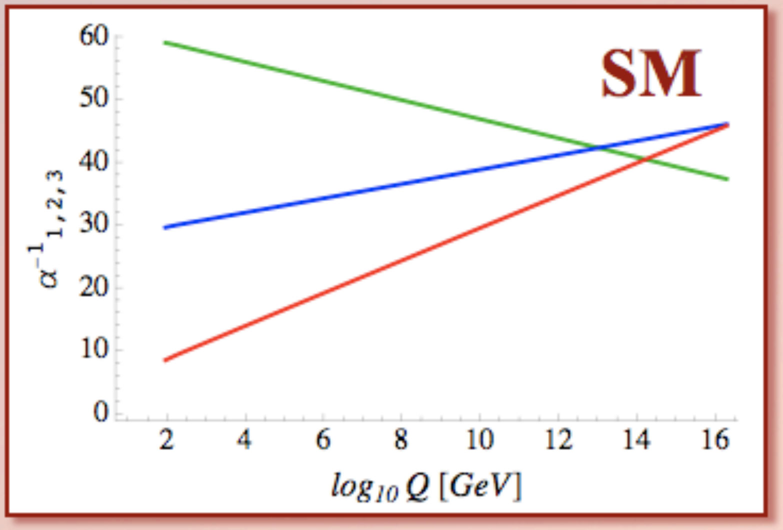
  particle content determines strength of interactions (at least basis features)

Insensitive Unification of Couplings

Gauge couplings in the standard model



$\alpha_3(M_Z)_{exp} = 0.1184$
$\alpha_2(M_Z)_{exp} = 0.03380$
$\alpha_1(M_Z)_{exp} = 0.01695$
$\alpha_{EM}(M_Z) = 1/127.916$
$\sin^2\theta_W = 0.2313$



RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$t = \ln Q/Q_0$

$$b_i = \left(\frac{1}{10} + \frac{4}{3}n_g,\ -\frac{43}{6} + \frac{4}{3}n_g,\ -11 + \frac{4}{3}n_g\right)$$

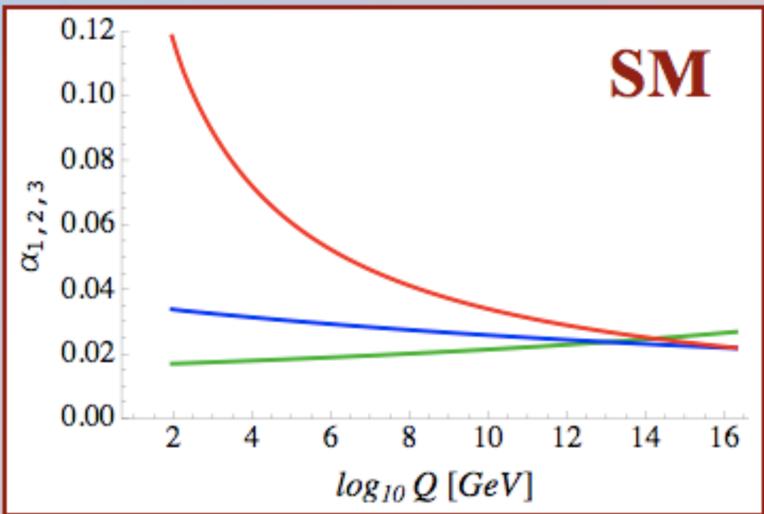
$$b_i = (41/10,\ -19/6,\ -7)$$

solution:

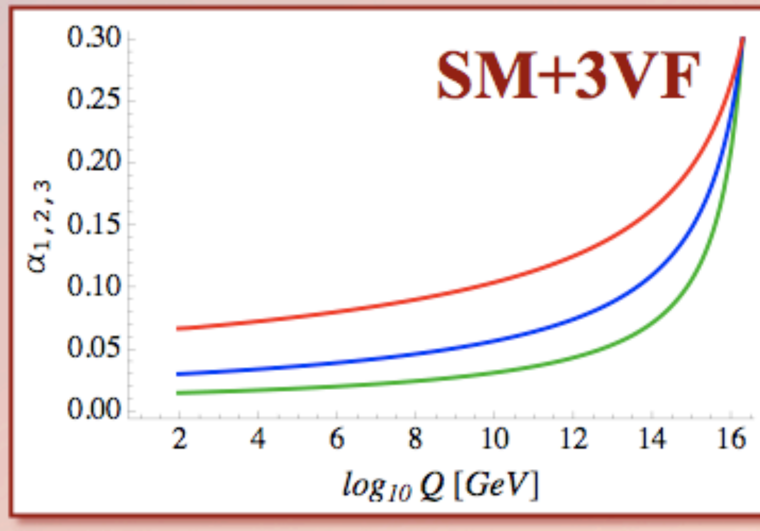
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln\frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)$$

SM

SM+3VF



$\alpha_3(M_Z)_{exp} = 0.1184$
$\alpha_2(M_Z)_{exp} = 0.03380$
$\alpha_1(M_Z)_{exp} = 0.01695$
$\alpha_{EM}(M_Z) = 1/127.916$
$\sin^2\theta_W = 0.2313$



RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$t = \ln Q/Q_0$$

$$b_i = \left(\frac{1}{10} + \frac{4}{3}n_g,\ -\frac{43}{6} + \frac{4}{3}n_g,\ -11 + \frac{4}{3}n_g\right)$$

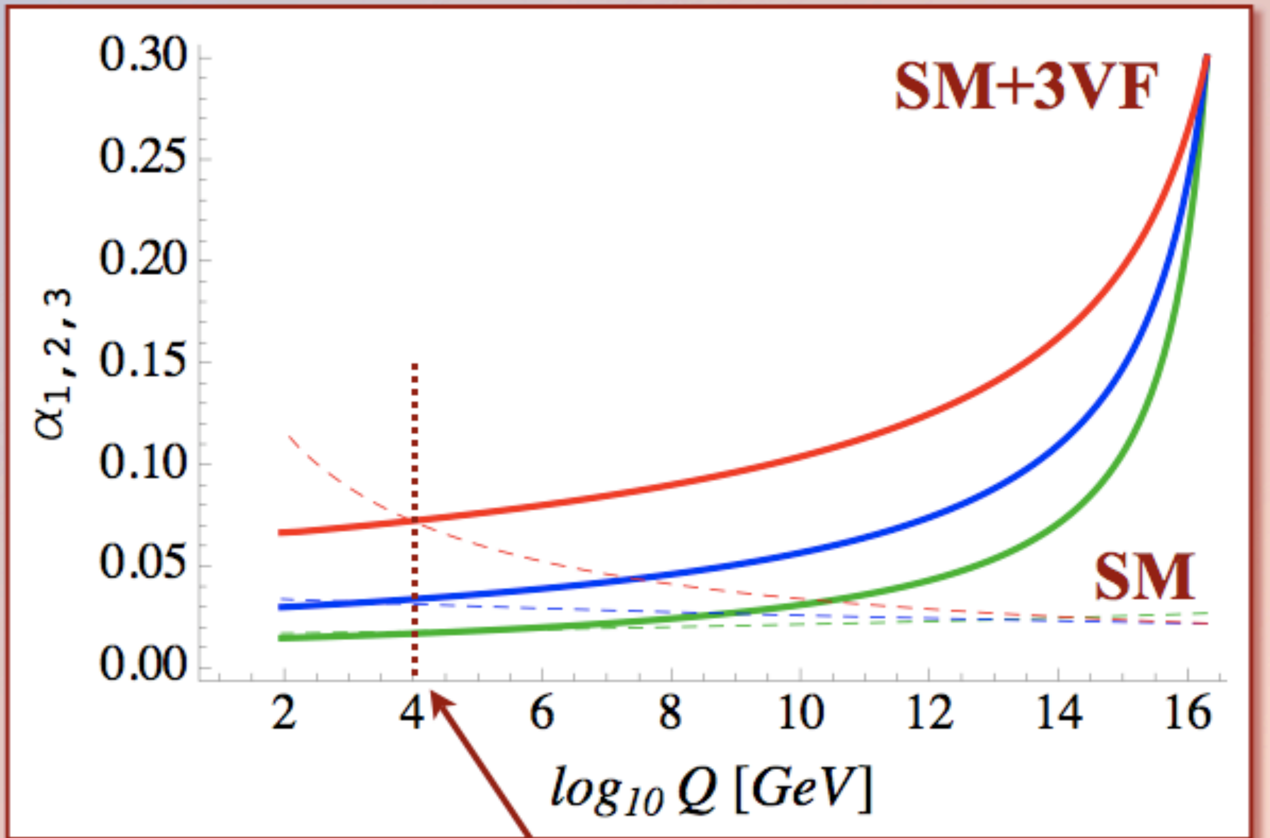
$$b_i = (41/10,\ -19/6,\ -7)$$

$$b_i = (121/10,\ 29/6,\ +1)$$

solution:

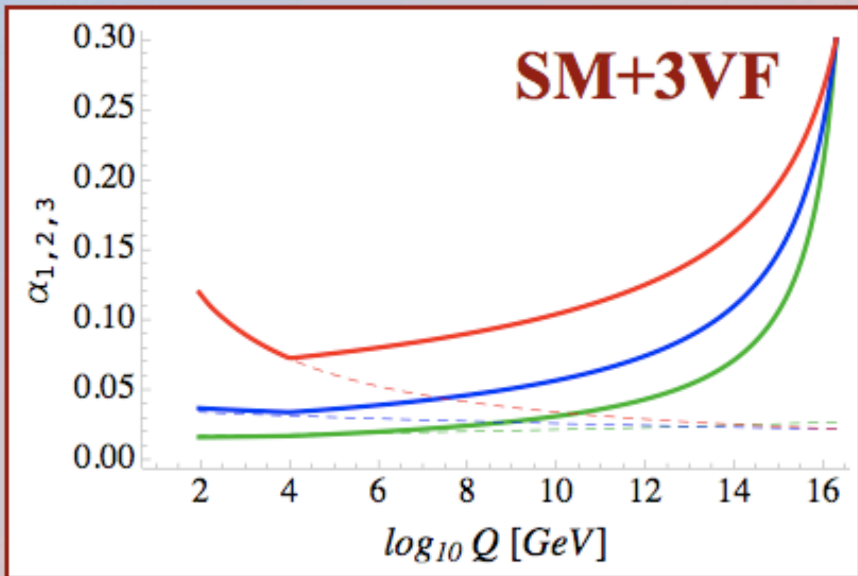
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)$$

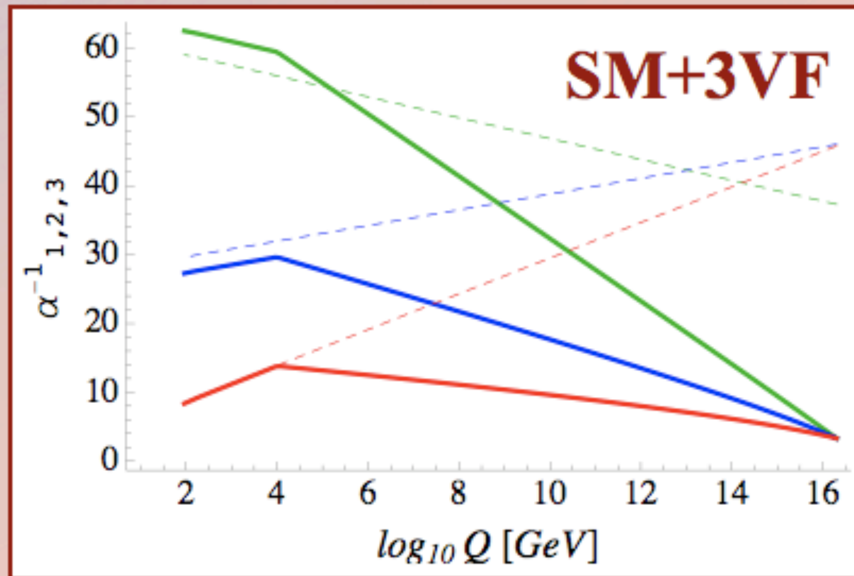
SM + 3 vectorlike families



Is this a threshold effect?

SM + 3 vectorlike families at 10 TeV





Exp. values of gauge couplings reproduced within 8%

the only relevant parameters are M_G and M_{VF}

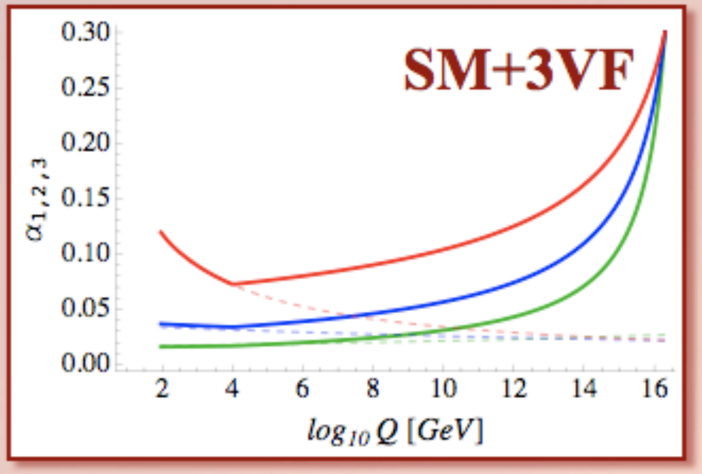
predictive, comparable to MSSM unification

Predictions and Sensitivity

RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$b_i = (121/10,\ 29/6,\ +1)$$



solution at 1-loop **(good approximation for i=1,2):**

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_G^{-1} - T_i$$

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

~ 60, 30 ~ 3 ~ 6 (for common mass 10 TeV)

neglecting threshold effect and α_G :

$$\frac{\alpha_i(M_Z)}{\alpha_j(M_Z)} \simeq \frac{b_j}{b_i}$$

$$\alpha' = \frac{3}{5}\alpha_1, \quad b' = \frac{5}{3} b_1$$

Parameter free prediction:

$$\sin^2\theta_W \equiv \frac{\alpha'}{\alpha_2 + \alpha'} = \frac{b_2}{b_2 + b'} = 0.193$$

$$\alpha_{EM} = \alpha_2 \sin^2\theta_W$$

Maiani, Parisi, and Petronzio (1978)

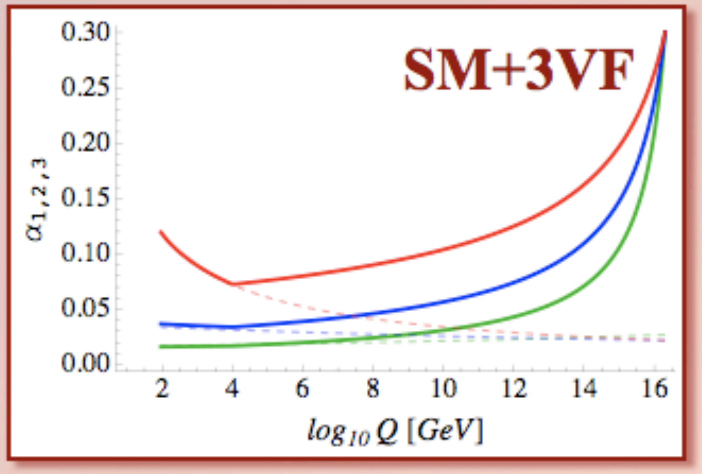
Predictions and Sensitivity

RGEs:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i + \frac{\alpha_i^3}{8\pi^2} B_i + \dots$$

$$b_i = (121/10,\ 29/6,\ +1)$$

$$B_3 = -102 + (76/3) n_g = 126$$



neglecting 1-loop **(good approximation for i=3):**

$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2} \ln\frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln\frac{M_f}{M_Z}$$

~ 8 ~ 100 ~ 9 ~ 6 (for common mass 10 TeV)

Parameter free prediction:

(neglecting threshold effect and α_G)

$$\frac{\alpha_3^2(M_Z)}{\alpha_{EM}(M_Z)} \simeq 2\pi \frac{b_2 + b'}{B_3}$$

predicts $\alpha_3 = 0.099$

1-loop contribution can be added:

$$\alpha_3(M_Z) \to \frac{\alpha_3(M_Z)}{1 + \frac{1}{3}\frac{\epsilon}{\alpha_3(M_Z)} - \frac{1}{12}\left(\frac{\epsilon}{\alpha_3(M_Z)}\right)^2 + \dots}$$

$$\epsilon = 4\pi b_3 / B_3$$

predicts $\alpha_3 = 0.073$

Sensitivity to M_G, α_G, and M_{VF}

approximate solutions:

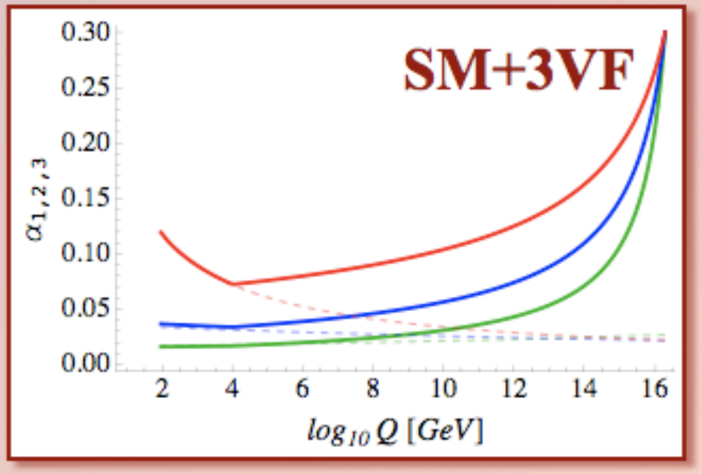
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_G^{-1} - T_i \quad (i=1,2)$$

$\alpha_i^{-1}(M_Z)$: ~ 60, 30; α_G^{-1}: ~ 3; T_i: ~ 6

$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2} \ln \frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

$\alpha_3^{-1}(M_Z)$: ~ 8; $\frac{B_3}{4\pi^2} \ln \frac{M_G}{M_Z}$: ~ 100; α_G^{-2}: ~ 9; T_i: ~ 6

~ 14



$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

1-loop contribution can be added:

$$\alpha_3(M_Z) \to \frac{\alpha_3(M_Z)}{1 + \frac{1}{3}\frac{\epsilon}{\alpha_3(M_Z)} - \frac{1}{12}\left(\frac{\epsilon}{\alpha_3(M_Z)}\right)^2 + \ldots}$$

Changing any of the fundamental parameters by a factor of 2 does not modify predicted values of gauge couplings by more than ~10%.

Realistic example

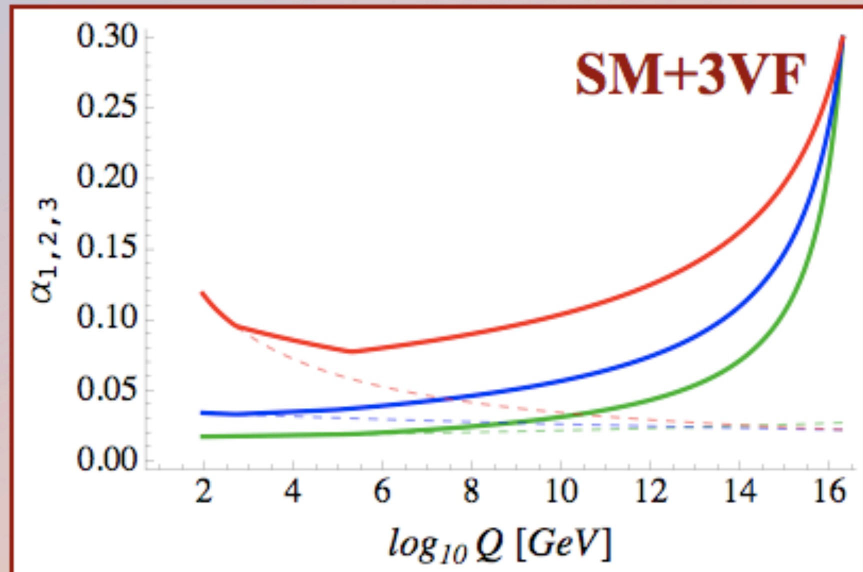
$\alpha_3(M_Z)_{exp} = 0.1184$
$\alpha_2(M_Z)_{exp} = 0.03380$
$\alpha_1(M_Z)_{exp} = 0.01695$
$\alpha_{EM}(M_Z) = 1/127.916$
$\sin^2\theta_W = 0.2313$

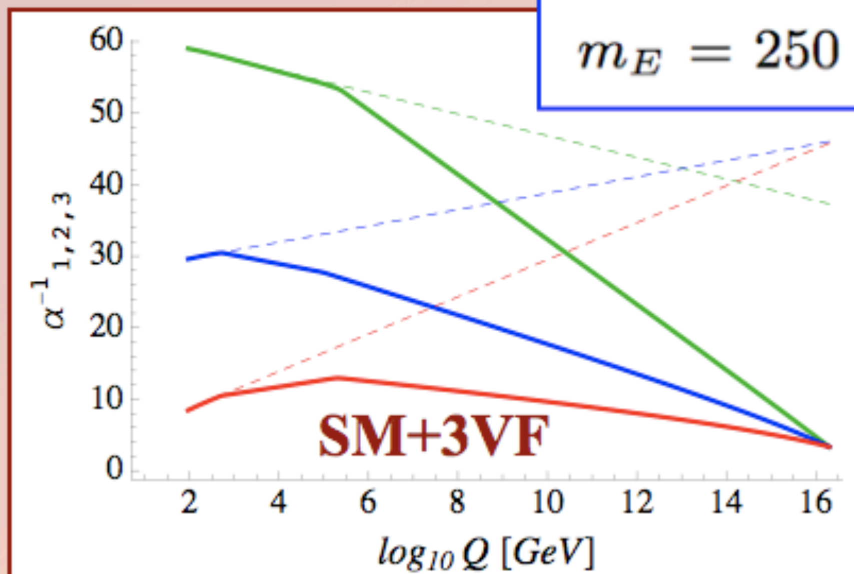
Gauge couplings reproduced (within fractions of exp. uncertainties) for :

4 sig. figures

2 sig. figures

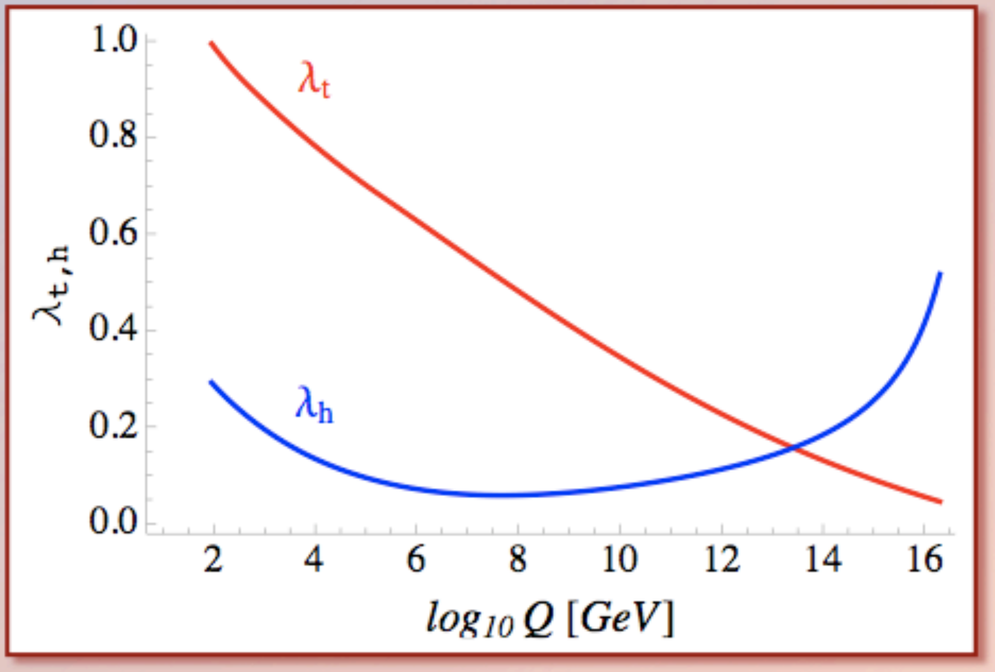
$$
\begin{aligned}
m_Q &= 500 \ \text{GeV} \\
m_L &= 95 \ \text{TeV} \\
m_U &= 220 \ \text{TeV} \\
m_D &= 180 \ \text{TeV} \\
m_E &= 250 \ \text{TeV}
\end{aligned}
$$





Many possible solutions!

Top Yukawa and Higgs quartic couplings



$m_H = 125$ GeV

- reduced sensitivity of Yukawa and Higgs quartic couplings to GUT scale boundary conditions

  (different textures for fermion masses compared to usual GUTs)

- Electroweak minimum is stable!

What else are extra VFs good for?

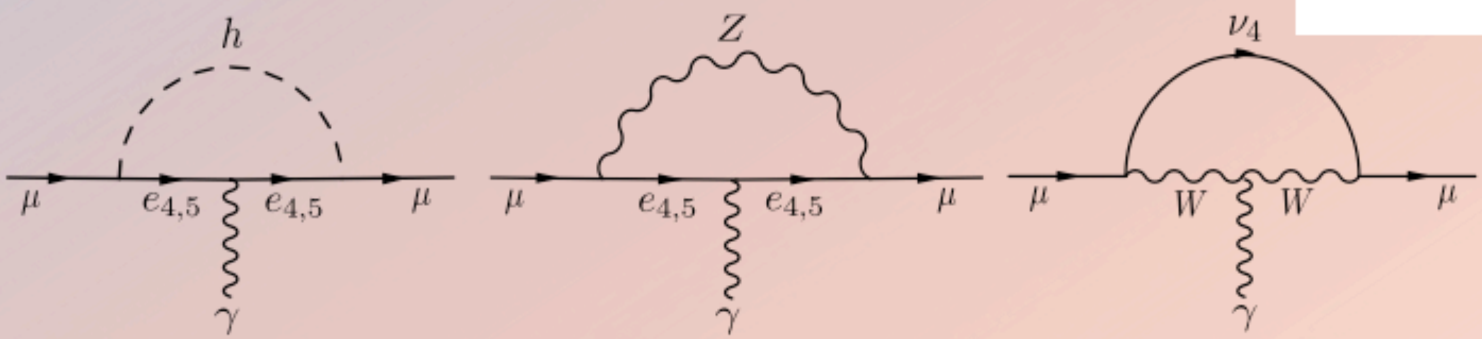
- **muon g-2**

K. Kannike, M. Raidal, D.M. Straub and A. Strumia, 1111.2551 [hep-ph]

R.D. and A. Raval, in progress

$$\mathcal{L} \supset -\bar{l}_{Li} y_{ij} e_{Rj} H - \bar{l}_{Li} \lambda_i^E E_R H - \bar{L}_L \lambda_j^L e_{Rj} H - \lambda \bar{L}_L E_R H - \bar{\lambda} H^\dagger \bar{E}_L L_R$$
$$-\mu_L \bar{L}_L L_R - \mu_E \bar{E}_L E_R + h.c.,$$

$$(\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_{ij} v & 0 & \lambda_i^E v \\ \lambda_j^L v & \mu_L & \lambda v \\ 0 & \bar{\lambda} v & \mu_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix}$$



- **anomalies in Z-pole observables A_{FB}^b and A_e**

D. Choudhury, T.M.P. Tait and C.E.M. Wagner, hep-ph/0109097

R.D., S.G. Kim and A. Raval, 1105.0773 [hep-ph], 1201.0315 [hep-ph]

Conclusions

- 3 (or more) pairs of vectorlike families allow for insensitive unification of gauge couplings

  predictive, comparable to SUSY unification

- resurrects simple non-supersymmetric GUTs (proton decay)

  the GUT scale is adjustable and could be identified with the string or Planck scale

- the electroweak minimum is stable all the way to the GUT scale

- some of the extra fermions might be within the reach of the LHC

  and modify phenomenology of the SM: small flavor violation from mixing through Yukawa couplings, contributions in loops, ...