Determination of the b-quark mass and B mesons decay constant from non-pertubatively renormalized HQET in two-flavor lattice QCD

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Rencontres de Physique des Particules 2013

LPT Orsay

Introduction

- Why do we compute m_b and f_B on the lattice ?

  - m_b if a free parameter of SM

  - f_B is needed to determined SM parameters (V_{ub})

- Simulating b-quark on the lattice is challenging because of the heavy mass
  $\longrightarrow$ the lattice spacing should be smaller than the compton wavelenght
  $\longrightarrow$ use an effective field theory (HQET)

- ALPHA collaboration
  (F. Bernardoni, B. Blossier, J. Bulava, M. Della Morte, P. Fritzsch, N. Garron, J. Heitger, G. von Hippel, H. Simma, R. Sommer)

CLS ensembles

Lattice discretization

- $N_f = 2$ $O(a)$ improved Wilson-Clover Fermions

Discretization effects

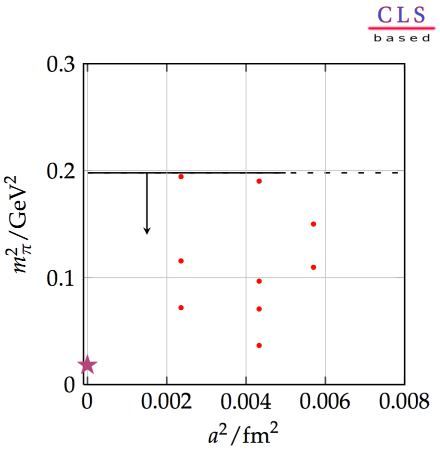
- 3 lattice spacings a : $(0.048, 0.065, 0.075 < 0.1)$fm

Volume effects

- $Lm_\pi > 4.0$

Light quark mass chiral extrapolation

- different pion masses (down to 190MeV)



$\Rightarrow$ **total of 9 ensembles**

Generalized Eigenvalue Problem (GEVP)

We want to compute masses and matrix elements $\rightarrow$ $f_B \sim \langle 0|A_0|B(p=0)\rangle$

- define 2-points correlation functions ($N \times N$ matrix) :

$$C_{ij}(t) = \sum_{\vec{x}} \langle A_{0i}(\vec{x},t) A_{0j}(\vec{0},0)\rangle \quad \text{with} \quad \underbrace{A_{0i}(x) = \bar{\Psi}_h(x)\gamma_0\gamma_5\Psi_l^{(i)}(x)}_{\text{B-meson interpolating field}}$$

- i : different level of smearing for the interpolating operator : $\Psi_l^{(i)}(x) = \left(1+\kappa_G a^2\Delta\right)^{R_i} \Psi_l(x)$
  $\rightarrow$ spread the hadronic operator in space to increase overlap with physical states

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With the following normalization of states $< n|m >= \delta_{nm}$:

$$C_{ij}(t) = \langle A_i(t) A_j^*(0)\rangle = \sum_n e^{-E_n t}\langle 0|A_i|n\rangle\langle n|A_j^*|0\rangle \xrightarrow[t\to\infty]{} \langle 0|A_i|B\rangle\langle B|A_j^*|0\rangle e^{-m_B t} + O\left(e^{-(E_2-E_1)t)}\right)$$

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- To improve the convergence, we solve GEVP :

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

[Michael, '85; Lüscher and Wolff,'90; Blossier et al, '09]

Eigenvalues $\lambda_n \longrightarrow E_n$

$$E_n^{\text{eff}}(t,t_0) = \log\frac{\lambda_n(t,t_0)}{\lambda_n(t+1,t_0)}$$

$$E_n^{\text{eff}}(t,t_0) = E_n + O\left(e^{-(E_{N+1}-E_n)t}\right)$$

Eigenvectors $v_n \longrightarrow$ matrix elements

$$\langle 0|A_0|n\rangle^{\text{eff}} = R_n(t)\ \left(v_n(t,t_0), C_A(t)\right)$$

$$\langle 0|A_0|n\rangle^{\text{eff}} = \langle 0|A_0|n\rangle + O\left(e^{-(E_{N+1}-E_n)t_0}\right)$$

Heavy Quark Effective Theory on the lattice [Eichten , '88; Eichten & Hill, '90]

- Problem : the b-quark is too heavy to be simulated on the lattice : $1/m_b \approx 0.04$ fm

- Solution : Use effective theory (*HQET* $\rightarrow$ expansion in $1/m_b$) :

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\Psi}(x)\,(i\gamma^\mu D_\mu - m_b)\,\Psi(x) \quad \text{with} \quad p_b^\mu = (m_b + k^0, \vec{k}) :$$

- isolate the heavy momentum part : $\Psi_h(x) = e^{im_b v\cdot x} P_+ \Psi(x)$ where $P_+ = \frac{1+\not{v}}{2}$
- P_+ is a projector on the large component of the spinor

$$\mathcal{L}_{\mathrm{HQET}} = \underbrace{\bar{\Psi}_h(x) D_0 \Psi_h(x)}_{\textit{static term}} - \underbrace{\omega_{\mathrm{kin}}\mathcal{O}_{\mathrm{kin}} - \omega_{\mathrm{spin}}\mathcal{O}_{\mathrm{spin}}}_{1/m \textit{ terms}} + \ldots \qquad \begin{aligned}\mathcal{O}_{\mathrm{kin}} &= \bar{\Psi}_h(x)\mathbf{D}^2\Psi_h(x)\\ \mathcal{O}_{\mathrm{spin}} &= \bar{\Psi}_h(x)\sigma\cdot\mathbf{B}\Psi_h(x)\end{aligned}$$

- The theory is renormalisable at each order in $1/m_b$ [Heitger and Sommer, '03]

Heavy-light Axial Current : $A_0^{\mathrm{HQET}} = Z_A^{\mathrm{HQET}}\left[A_0^{\mathrm{stat}} + c_A A^{(1)}\right]$

$$A_0^{\mathrm{stat}} = \bar{\Psi}_h(x)\gamma_0\gamma_5\Psi_l(x) \quad \textit{and} \quad A^{(1)} = \bar{\Psi}_l(x)\gamma_5\gamma_i\frac{1}{2}\left(\overrightarrow{\nabla}_i^S - \overleftarrow{\nabla}_i^S\right)\Psi_h(x)$$

- The 5 HQET parameters $\left(m_{\mathrm{bare}}, Z_A^{\mathrm{HQET}}, c_A, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}\right)$ have been determined non-pertubatively for different heavy quark masses [Blossier et al, '12]

Heavy Quark Effective Theory

$$\mathcal{L}_{\mathrm{HQET}} = \underbrace{\bar{\Psi}_h(x) D_0 \Psi_h(x)}_{\mathcal{L}_{\mathrm{stat}}} - \underbrace{\omega_{\mathrm{kin}} \mathcal{O}_{\mathrm{kin}} - \omega_{\mathrm{spin}} \mathcal{O}_{\mathrm{spin}}}_{\mathcal{L}_{\mathbf{1}/m}}$$

- Correlation function : $\mathcal{L}_{\mathbf{1}/m}$ is considered as operator insertions :

$$\begin{aligned}
\langle \mathcal{A} \rangle &= \frac{1}{Z} \int \mathcal{D}\phi \; \mathcal{A} \; e^{-a^{\mathbf{4}} \sum_x [\mathcal{L}_{\mathrm{light}}(x) + \mathcal{L}_{\mathrm{stat}}(x) + \mathcal{L}_{\mathbf{1}/m}(x)]} \\
&\approx \frac{1}{Z} \int \mathcal{D}\phi \; \mathcal{A} \left(1 - a^4 \sum_x \mathcal{L}_{1/m}(x) \right) e^{-a^{\mathbf{4}} \sum_x [\mathcal{L}_{\mathrm{light}}(x) + \mathcal{L}_{\mathrm{stat}}(x)]} \\
&\approx \langle \mathcal{A} \rangle_{\mathrm{stat}} + \omega_{\mathrm{kin}} a^4 \sum_x \langle \mathcal{A} \mathcal{O}_{\mathrm{kin}} \rangle_{\mathrm{stat}} + \omega_{\mathrm{spin}} a^4 \sum_x \langle \mathcal{A} \mathcal{O}_{\mathrm{spin}} \rangle_{\mathrm{stat}}
\end{aligned}$$

$\Rightarrow$ Expectation values are computed from static lagrangian

$$\langle \mathcal{A} \rangle = \langle \mathcal{A} \rangle_{\mathrm{stat}} + \langle \mathcal{A} \rangle_{\mathrm{kin}} + \langle \mathcal{A} \rangle_{\mathrm{spin}}$$

Explicit formula for m_B and f_B

GEVP : 1/m expansion $(1/m \in \{\mathrm{kin}, \mathrm{spin}\})$

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0) \qquad \begin{cases} C(t,t_0) = C^{\mathrm{stat}}(t,t_0) + C^{1/\mathrm{m}}(t,t_0) \\ \lambda_n(t,t_0) = \lambda_n^{\mathrm{stat}}(t,t_0) + \lambda_n^{1/\mathrm{m}}(t,t_0) \\ v_n(t,t_0) = v_n^{\mathrm{stat}}(t,t_0) + v_n^{1/\mathrm{m}}(t,t_0) \end{cases}$$

Mass of the B-meson

$$m_B^{1/\mathrm{m}} = m_{\mathrm{bare}} + E_{\mathrm{stat}} + \omega_{\mathrm{kin}} E_{\mathrm{kin}} + \omega_{\mathrm{spin}} E_{\mathrm{spin}}$$

- $E^{\mathrm{eff}}_{\mathrm{stat}}(t,t_0) = a^{-1}\log\dfrac{\lambda_n^{\mathrm{stat}}(t,t_0)}{\lambda_n^{\mathrm{stat}}(t+a,t_0)}$, $E_n^{\mathrm{eff},1/\mathrm{m}}(t,t_0) = \dfrac{\lambda_n^{1/\mathrm{m}}(t,t_0)}{\lambda_n^{\mathrm{stat}}(t,t_0)} - \dfrac{\lambda_n^{1/\mathrm{m}}(t+a,t_0)}{\lambda_n^{\mathrm{stat}}(t+a,t_0)}$
- m_{bare}, ω_{kin} and ω_{spin} : HQET parameters

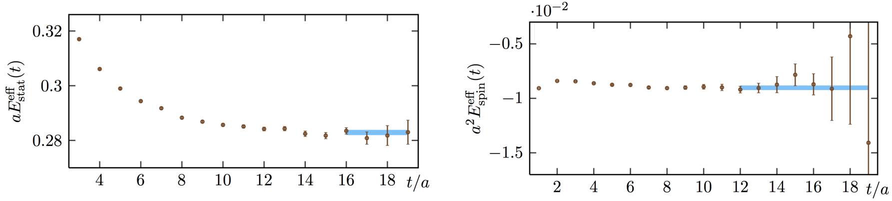
Decay constant

$$f_B\sqrt{m_B/2} = Z_A\left(1 + b_A^{\mathrm{stat}} a m_q\right)\left[f_B^{\mathrm{stat}}\left(1 + \omega_{\mathrm{spin}} f_B^{\mathrm{spin}} + \omega_{\mathrm{kin}} f_B^{\mathrm{kin}}\right) + c_A f_B^{\mathrm{da}}\right]$$

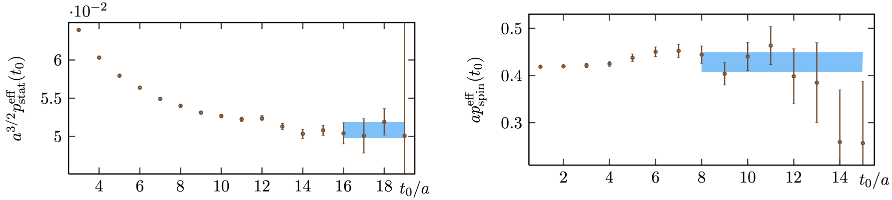
- $f_B^{\mathrm{stat}}(t,t_0) = R^{\mathrm{stat}}(t,t_0)\left(v_1^{\mathrm{stat}}(t,t_0), C_A^{\mathrm{stat}}(t)\right)$
- $f_B^{1/\mathrm{m}}(t,t_0) = \dfrac{R^{1/\mathrm{m}}(t,t_0)}{R^{\mathrm{stat}}(t,t_0)} + \dfrac{\left(v_1^{\mathrm{stat}}(t,t_0), C_A^{1/\mathrm{m}}(t)\right)}{\left(v_1^{\mathrm{stat}}(t,t_0), C_A^{\mathrm{stat}}(t)\right)} + \dfrac{\left(v_1^{1/\mathrm{m}}(t,t_0), C_A^{\mathrm{stat}}(t)\right)}{\left(v_1^{\mathrm{stat}}(t,t_0), C_A^{\mathrm{stat}}(t)\right)}$

Plateau fit

- Energy :



- Matrix elements :



m_B : Chiral and continuum extrapolation

$$m_B^{1/\mathrm{m}}(z, m_\pi, a) = m_{\mathrm{bare}}(z) + E_{\mathrm{stat}}(m_\pi, a) + \omega_{\mathrm{kin}}(z)\ E_{\mathrm{kin}}(m_\pi, a) + \omega_{\mathrm{spin}}(z)\ E_{\mathrm{spin}}(m_\pi, a)$$

- Each computation has been done at finite lattice spacing a and unphysical pion mass m_π
- HQET parameters are known for some values of the heavy quark mass $z = LM_b$ (M_b :RGI quark mass, L constant volume ~ 0.4 fm).

$$m_B(z, m_\pi, a) = \alpha(z) + \beta m_\pi^2 - \frac{3\hat{g}^2}{(16\pi f_\pi)^2} m_\pi^3 + \gamma a^2$$

- $\alpha(z)$: 2^{nd} order polynomial
- HMχPT

- We know m_B at the physical point ($a = 0$, $m_\pi \approx 135$ MeV) $\Rightarrow$ we can determine z_b

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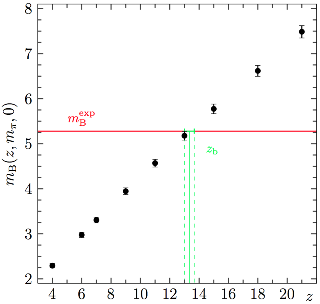
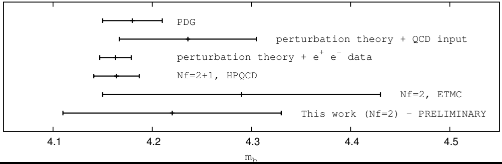


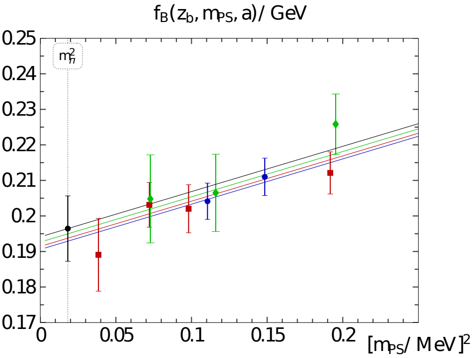
- $z_b = 13.34(33)(13)_z$
- $\Rightarrow \bar{m}_b(\bar{m}_b) = 4.22(10)(4)_{\text{QCD}}$ GeV



f_B : Chiral and continuum extrapolation

- We have computed $f(a, z, m_\pi)$ for each ensemble.
- From the previous result, we can use $z = z_b$.

$$f_B = \alpha + \beta m_\pi^2 + \gamma a^2 \quad (LO \quad HM\chi PT)$$



$$LO \; : \; f_B = 196(9) MeV$$

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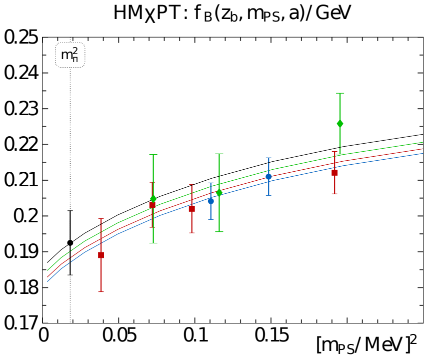
$$f_B = \alpha + \beta m_\pi^2 + \gamma a^2 \qquad (LO \quad HM\chi PT)$$

$$f_B = \tilde{\alpha}\left[1 - \frac{3}{4}\frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_\pi^2 \ln(m_\pi^2)\right] + \beta m_\pi^2 + \gamma a^2 \qquad (NLO \quad HM\chi PT)$$



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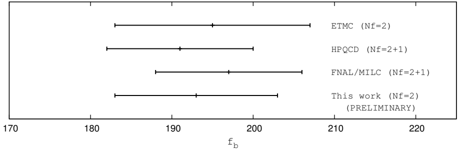
$$(HM\chi PT) : f_B = 193(9)(4)_\chi MeV$$



Conclusion

- Non perturbative computation of m_b and f_B
- We include $1/m$ corrections of HQET
- Continuum extrapolation and chiral extrapolation
- We obtain $\bar{m}_b(m_b) = 4.22(10)(4) GeV$ and $f_B = 193(9)(4) MeV$
- f_{B_s} was also computed ($f_{B_s} = 219(12)\text{MeV}$)
- Results are still preliminary !

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Thank you !