

Effective potential for
quantum scalar fields
in de Sitter space

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Motivations

QFT on curved background / geometry

Understanding loop effects

→ Radiative corrections to inflationary dynamics (power spectra, nongaussianities, decoherence, reheating ...)

→ (Analog) Black Hole radiation

→ Curvature-induced phase transitions

...

→ Mathematical consistency of QFT on non-trivial geometries

Motivations

A test scalar field in de Sitter

↑ (Inflation)

Expanding geometry \equiv nonequilibrium

⇒ perturbation theory is secular

[Tsamis, Woodard ('95); Weinberg ('05); ...]

Massless, minimally coupled field

⇒ perturbation theory is IR divergent

[e.g. Van der Meulen, Smit ('07) ...]

⇒ Alternative schemes
(re-summations)

e.g.
: 2PI

[Berges, J.S., SEWM04]

See also : [Ramsey, Hu ('97); Riotto, Sloth ('08); Tranberg ('08);
Garbrecht, Rigopoulos ('11)]

DRG → [Burgess, Leblond, Holman, Shandrea ('10)]

Model and approximation

$$D = d + 1$$

$$S = - \int d^D x \sqrt{-g} \left\{ \varphi_a (\square + \mu^2) \varphi_a + \frac{\lambda}{4! N} (\varphi_a \varphi_a)^2 \right\}$$

$$ds^2 = dt^2 - e^{2Ht} dx^2$$

(flat spatial sections)

$$\square = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$$

$$\mu^2 = m^2 + 3R \quad \text{and} \quad R = d(d+1)H^2$$

(2PI) \hbar -expansion at L.O.

$$\Gamma_2[\phi, G] = \text{diagram 1} + \text{diagram 2}$$

Diagram 1: A circle with two external lines labeled ϕ .
Diagram 2: A figure-eight diagram with two external lines labeled G .

$$[\square + M^2(x)] \phi(x) = 0$$

$$[\square + M^2(x)] G(x, y) = -i \delta^{(D)}(x, y)$$

$$M^2(x) = \mu^2 + \frac{\lambda}{6} \phi^2(x) + \frac{\lambda}{6} G(x, x)$$

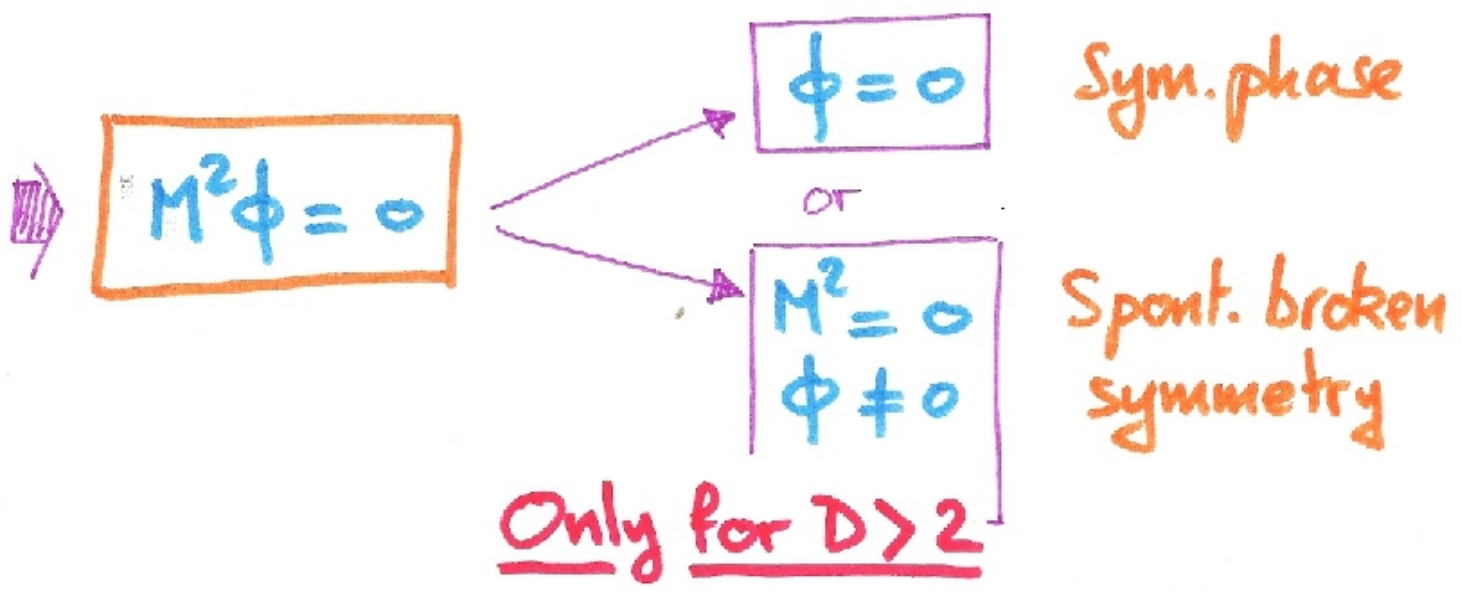
Reminder: flat space-time (ST)
($H \rightarrow 0$)

III ST translation invariant state (e.g. vacuum)

$\hookrightarrow \phi(x) = \phi = \text{const.}$ and $G(x, y) \equiv G(x - y)$

$\hookrightarrow G(x, x) = \text{const.}$ and $M^2(x) = \text{const.}$

$\Rightarrow M^2 = m^2 + \frac{\lambda}{6} \phi^2 + \frac{\lambda}{6} \int \frac{d^D k}{(2\pi)^D} \frac{i}{k^2 - M^2}$
(+ renormalization)

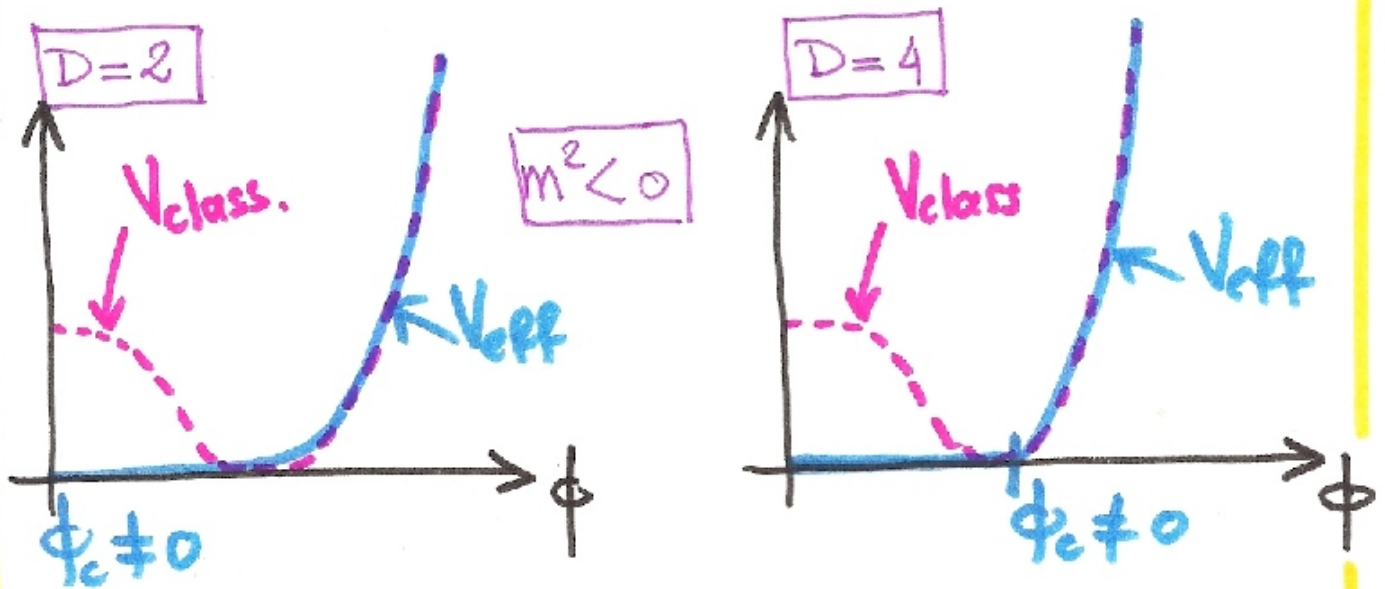


Infra-Red (IR) effects prevent SSB in $D = 2$

Reminder: flat ST

Effective potential

$$V_{\text{eff}}(\phi) = \frac{1}{2} \int_0^\phi d\varphi^2 M^2(\varphi^2) + \text{const.}$$



Finite temperature

➔ Phase transition at $T_c \neq 0$ for $D > 2$

➔ Thermal mass generation

e.g. high T : $M^2 \sim \lambda T^2$

de Sitter space-time ($H \neq 0$)

dS-invariant state

$\hookrightarrow \phi(x) = \phi = \text{const.} ; G(x, x') = G(\underline{z})$

$$z(x, x') = \text{ch } H(t-t') - \frac{|\vec{x} - \vec{x}'|^2}{2} e^{H(t+t')}$$

$\hookrightarrow G(x, x) = \text{const. and } M^2(x) = \text{const.}$

$\Rightarrow M^2 = \mu^2 + \frac{\Lambda}{6} \phi^2 + \frac{\Lambda}{6} \kappa \int \frac{d^d z}{(2\pi)^d} |H_\nu(z)|^2$
 (+renormalization)

$$\nu = \sqrt{\frac{d^2}{4} - \frac{M^2}{H^2}}$$

Hankel
Function

IR issues in dS.

$$\bullet \quad |H_\nu(z)|^2 \underset{z \rightarrow 0}{\sim} z^{-2\nu} \underset{(M^2 \rightarrow 0)}{\sim} z^{-d}$$

Strongly enhanced IR fluctuations
(Compare to z^{-1} in Minkowsky ST)

⇒ Perturbative expansion is ill-defined
in any dimension (for $\mu^2 = 0$)

● Large- N resums IR div. in a well-defined expression

$$\text{Requires } \nu < \frac{d}{2} \Leftrightarrow M^2 > 0$$

⇒ IR effects prevent the existence of a dS-inv. broken phase in any d !!

Solving the gap-equation

Case $\frac{M^2}{H^2} \gg 1 \longrightarrow v \approx i \frac{M}{H}$

$$M^2(\phi^2) = \mu^2(\phi^2) + \frac{\lambda}{6} \int \frac{d^d z}{(2\pi)^d} \frac{1}{2|z^2 + M^2} + \mathcal{O}(e^{-M/H})$$

$$\mu^2 + \frac{\lambda}{6} \phi^2$$

Minkowsky limit ✓

Case $\frac{M^2}{H^2} \ll 1 \longrightarrow v = \frac{3}{2} - \frac{M^2}{8H^2} \quad (d=3)$

$$M^2(\phi^2) = \mu^2(\phi^2) + \frac{\lambda H^2}{48\pi^2} \left(\frac{3H^2}{M^2(\phi^2)} + c \right)$$

$$M^2(\phi^2) = a(\phi^2) + \sqrt{a^2(\phi^2) + b}$$

$$a(\phi^2) = \frac{1}{2} \mu^2(\phi^2) + \frac{cdH^2}{96\pi^2}$$

$$b = \frac{\lambda H^4}{16\pi^2}$$

see also [Garbrecht, Rigopoulos (14)]

Mass generation

Minimally coupled massless field

$$\mu^2 = m^2 + \xi R = 0$$

Loop diagrams are IR divergent

Self-interactions generate a non-zero mass (at $\phi=0$)

$$\frac{M^2}{H^2} = \frac{c\lambda}{96\pi^2} + \sqrt{\frac{\lambda}{16\pi^2} + \left(\frac{c\lambda}{96\pi^2}\right)^2}$$

$$M^2 \sim \sqrt{\lambda} H^2$$

[Starobinsky, Yokoyama ('94)]

[Compare to $M^2 \sim \lambda T^2$ at finite T]

Enhancement of IR-sensitive obs.

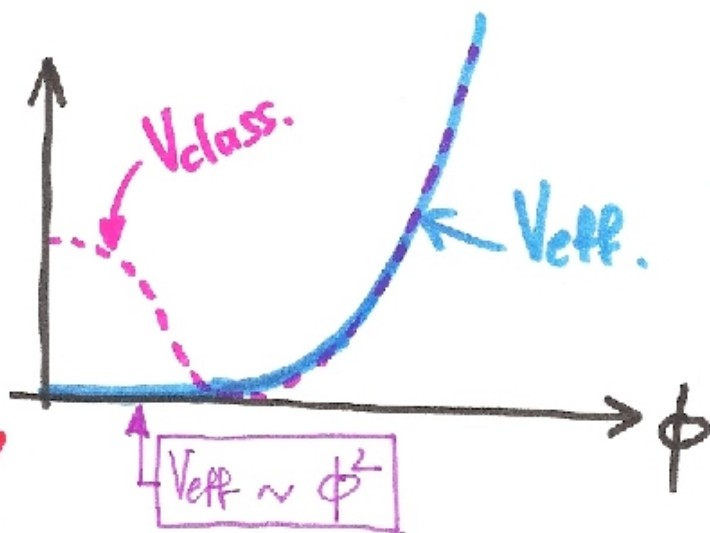
(e.g. non-gaussianities) [Riotto, Sloth ('08)]

Curvature-induced symmetry restoration

SSB at classical level: $\mu^2 < 0$

→ The effective potential gets lifted

$$V_{\text{eff}} \approx \frac{3}{2\lambda} M^4(\phi^2) + \frac{3H^4}{16\pi^2} \ln \frac{M^2(\phi^2)}{M^2(0)} + \text{const.}$$



$$M^2(0) \approx \frac{\lambda H^4}{16\pi |\mu^2|} \sim \frac{H^4}{\phi_c^2}$$

Stronger enhancement of IR sensitive observables?

Conclusions

- ➔ A new, exact result of QFT on dS.
(Large- N limit: exact solution)
- ➔ \nexists dS-invariant state for a massless minimally coupled interacting scalar field
- ➔ dS IR effects prevent the existence of (dS-inv.) spontaneously broken phase in any d and for arbitrary weak curvature
(No curvature-induced phase transition)
- ➔ Gravitational lifting of Minkowsky results (modes $P_H \sim e^{-3H^2/2M^2}$)

PROSPECTS

▨ Non gaussian correlators in the large- N limit

IR enhancement at tree level

[Riotto, Sloth ('08)]



Large- N :  + ...

[Serreau, Parentani ; in preparation]

▨ $1/N$ corrections

NLO : nonvanishing self-energies

$$\Sigma_F, \Sigma_P$$

↳ Unequal time correlators

[Gautier, Serreau ; in preparation]