Langevin Description of Non Equilibrium Quantum Field Dynamics

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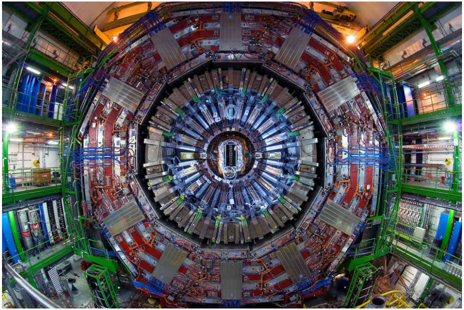
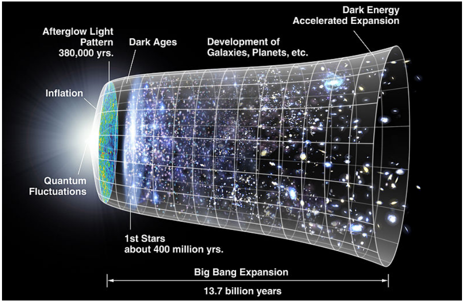


Ion Collisions

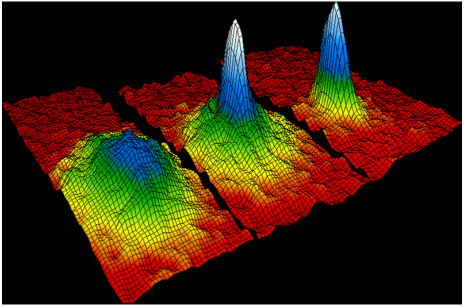
- RHIC,
- LHC, ALICE, CMS,
- ...

Early Universe

- (P-)reheating after inflation,
- Baryogenesis,
- ...



- Condensed Matter,
- Cold atoms,
- ...



Why Langevin ?

- Major recent breakthroughs :
  - 2-PI For reviews see [Berges,Serreau (03)],[Berges (05)] and references therein
  - Numerical fermions [Aarts,Smit(99)], [Borsanyi,Hindmarsh (08)], [Saffin,Tranberg(11)],
  - faster computers, ...
- Still difficult to implement in certain physically interesting situations :
  - Large number of fields (eg .Supersymmetric models)
  - inhomogeneous and/or expanding backgrounds (Baryogenesis, (p-)reheating).
- Adequate effective descriptions such as kinetic equations. Eg. [Herranen, Kainulainen, Rahkila(08)]
- Langevin-like equations with various additive and/or multiplicative noises. Eg. [Rischke(98)]

But

- Assuming a clear separation of scales between the "memory time"Eg. [Anisimov, Buchmuller,Drewes, Mendizabal(09)] , and the "relaxation time" , which characterize the dynamics of the relevant degrees of freedom, allows one to do further drastic simplifications, namely the assumption of effective local damping and Markovian noise.

- But existing calculations show that memory kernels and noise correlators typically decay as power law in time, thereby questioning the possibility of a local limit. [Boyanovsky, de Vega, Holman, Kumar,Pisarski (98)]

Outline

1. *Strategy*
   - General settings
   - Approximation strategy
   - But

2. *Exactly solvable example*
   - Exact solution
   - Breit-Wigner approximation
   - Markovian dynamics
   - Consistency check

3. *Conclusion and prospects*

Strategy

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General settings - 1

- We consider the dynamics of a $\mathbb{Z}_2$-symmetric scalar field in the symmetric phase
- In-In Formalism : $\langle \ldots \rangle = \langle \text{in} | \ldots | \text{in} \rangle$
- For gaussian initial conditions, dynamics require two independent two point functions :

$$\rho(x,y) = i\langle [\Phi(x), \Phi(y)] \rangle \tag{1}$$

$$F(x,y) = \frac{1}{2}\langle \{\Phi(x), \Phi(y)\} \rangle \tag{2}$$

- It can be useful to introduce advanced and retarded propagators

$$G^R(x,y) = \theta(x_0 - y_0)\rho(x,y) \tag{3}$$

$$G^A(x,y) = -\theta(y_0 - x_0)\rho(x,y) \tag{4}$$

General settings - Equivalent Langevin process

- They obey the exact Kadanoff-Baym equations of motion :

$$\begin{aligned}\left[\Box_x + M^2(x)\right] F(x,y) &= -\int_{-\infty}^{x^0} \mathrm{d}^4 z\, \Sigma_\rho(x,z) F(z,y) \\ &\quad + \int_{-\infty}^{y^0} \mathrm{d}^4 z\, \Sigma_F(x,z)\rho(z,y), \qquad (5)\\ \left[\Box_x + M^2(x)\right] \rho(x,y) &= -\int_{y^0}^{x^0} \mathrm{d}^4 z\, \Sigma_\rho(x,z)\rho(z,y). \qquad (6)\end{aligned}$$

- Equivalent to a fictious Langevin process with

$$\left[\Box_x + M^2(x)\right] \varphi^\xi(x) + \int \mathrm{d}^4 z\, \Sigma_R(x,z)\varphi^\xi(z) = \xi(x) \qquad (7)$$

- Where one has to set

$$\begin{aligned}\Sigma_F(x,y) &\equiv -\overline{\xi(x)\xi(y)} \\ F(x,y) &\equiv \overline{\varphi^\xi(x)\varphi^\xi(y)}\end{aligned}$$

Approximation strategy -1

- Spatially homogeneous and isotropic solutions

- What are the conditions under which the memory integrals can be approximated by a local damping term ?

$$\int \mathrm{d}u\, \Sigma_p^R(t,u) F_p(u,t') \sim \gamma_p \partial_t F_p(t,t') \tag{8}$$

Approximation strategy -2

- We assume
  - a clear separation of scales,
  - that the self energies decay "fast enough"
- Harmonic approximation

$$\begin{aligned} F_p(u,t') &\approx F_p(t,t')\cos\epsilon_p(t)(u-t) \\ &- \frac{\partial_t F_p(t,t')}{\epsilon_p(t)}\sin\epsilon_p(t)(u-t) \end{aligned} \tag{9}$$

$$\int du\,\Sigma_p^R(t,u)F_p(u,t') \approx \delta\epsilon_p^2(t)F_p(t,t') + 2\bar{\gamma}_p(t)\partial_t F_p(t,t') \tag{10}$$

with

$$\delta\epsilon_p^2(t) \equiv \mathrm{Re}\tilde{\Sigma}_p^R(t;\epsilon_p(t)) \tag{11}$$

and

$$\gamma_p(t) \equiv -\frac{\mathrm{Im}\tilde{\Sigma}_p^R(t;\epsilon_p(t))}{2\epsilon_p(t)} \tag{12}$$

- Given the mixed time-frequency representation

$$\Sigma_p^R(t,t') = \int \frac{d\omega}{2\pi}\, e^{-i\omega(t-t')}\tilde{\Sigma}_p^R(t;\omega) \tag{13}$$

Approximation strategy -3

- the equations for F_p and G_p^R thus read

$$\left[\partial_t^2 + 2\gamma_p(t)\partial_t + \bar{\epsilon}_p^2(t)\right] F_p(t,t') = -\int_{-\infty}^{+\infty} \mathrm{d}u\, \Sigma_p^F(t,u) G_p^A(u,t') \tag{14}$$

and

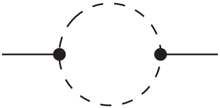
$$\left[\partial_t^2 + 2\gamma_p(t)\partial_t + \bar{\epsilon}_p^2(t)\right] G_p^{R,A}(t,t') = \delta(t-t') \tag{15}$$

- where we defined $\bar{\epsilon}_p^2(t) = \omega_p^2(t) + \delta\epsilon_p^2(t)$.

Approximation strategy - But

If the above conditions are met, the memory kernel can be replaced by a local damping term, without any assumption concerning the noise kernel.

But in known examples ([Boyanovsky, de Vega, Holman, Kumar,Pisarski (98)],[[F.G, Serreau (12)]), the self-energy behavior questions the local condition.

$\Sigma_\varphi =$ 

$$\Sigma_R(t) \approx \sigma_R^{\mathrm{pow}}(t) = -g^2 a_R \frac{\cos(2mt + \pi/4)}{(mt)^{3/2}} \tag{16}$$

Exactly solvable example

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3. Conclusion and prospects

Exact solution

- To understand the domain of validity of the approximation made above we focus on a near stationary situation where $\Sigma_p^{F,\rho}(t,t') \equiv \Sigma_p^{F,\rho}(t-t')$, such that the spectral function is also time translation invariant $\rho(t,t') = \rho(t-t')$ [Anisimov, Buchmuller,Drewes, Mendizabal(09)]

- In this situation the retarded propagators $G_R(x,y) = \theta(t-t')\rho(t-t')$ reads

$$G_R^{-1}(t-t') = \left[\partial_t^2 + \omega_p^2\right]\delta(t-t') + \Sigma_R(t-t') \tag{17}$$

- Which reads in Fourier space :

$$G_p^R(\omega) = \frac{1}{-(\omega+i0^+)^2 + \omega_p^2 + \tilde{\Sigma}_p^R(\omega+i0^+)}\,, \tag{18}$$

Breit-Wigner approximation

- We decompose it into a Breit-Wigner part and a rest

$$\rho_p(t) \approx \frac{\sin \epsilon_p t}{\epsilon_p} e^{-\gamma_p |t|} + \rho_p^{\mathrm{pow}}(t) \tag{19}$$

$$F_p^{\mathrm{eq}}(t) \approx \kappa_p \left(\cos \epsilon_p t + \frac{\gamma_p}{\epsilon_p} \sin \epsilon_p |t| \right) e^{-\gamma_p |t|} + F_p^{\mathrm{pow}}(t) \tag{20}$$

Where

$$\kappa_p = \frac{1}{\epsilon_p} \left(n\left(\beta \epsilon_p\right) + \frac{1}{2} \right)$$

- The non Breit-Wigner terms are typically power laws, at long time, dictated by cuts and thresholds in the self energy.
- For time short enough $e^{-\gamma_p |t|}/\epsilon_p \gg \rho_{pow} \propto (\mu t)^{-\nu}$.
- Characteristic time scale

$$\gamma t^L \approx \ln \left(\frac{\mu}{\gamma} \right)^{\nu} \tag{21}$$

Markovian dynamics

- One recognizes the equilibrium solution of a damped oscillator driven by a Markovian noise , and the approach towards it, with

$$\left[\partial_t^2 + 2\gamma_p \partial_t + \epsilon_p^2\right] \varphi_p^\xi(t) = \xi_p(t) \tag{22}$$

- and

$$\overline{\xi_p(t)\xi_p(t)} \equiv -4\gamma_p \epsilon_p (n(\epsilon_p) + 1/2)\delta(t)$$

In the close to equilibrium case, locality is equivalent (in the time regime) to emphasizing pole contribution.

Consistency check

- Understand the above approximation (neglect of long time power laws) directly at the level of the equations of motion
- One plugs back the effective solution in the full damping term

$$\int_0^t d\tau\, \Sigma_p^\rho(t-\tau)\rho_p^{\mathrm{eff}}(\tau) =$$
$$= \rho_p^{\mathrm{eff}}(t)\mathrm{Re}\,\sigma_p^{i\epsilon_p+\gamma_p}(t) - \frac{\dot\rho_p^{\mathrm{eff}}(t)+\gamma_p\rho_p^{\mathrm{eff}}(t)}{\epsilon_p}\mathrm{Im}\,\sigma_p^{i\epsilon_p+\gamma_p}(t) \tag{23}$$

where,

$$\sigma_p^z(t) = \int_0^t d\tau\, \Sigma_p^\rho(\tau)e^{z\tau} = \int_{-\infty}^t d\tau\, \Sigma_p^R(\tau)e^{z\tau}$$

- A detailed analysis reveals

$$\mathrm{Re}\,\sigma_p^{i\epsilon_p+\gamma_p}(t) = \mathrm{Re}\,\tilde\Sigma_p^R(\epsilon_p)\left[1+\mathcal{O}\left(\frac{a_p^\rho}{\epsilon_p\delta\epsilon_p^2}\frac{e^{\gamma_p t}}{(\mu_p t)^\nu}\right)\right] \tag{24}$$

$$\mathrm{Im}\,\sigma_p^{i\epsilon_p+\gamma_p}(t) = \mathrm{Im}\,\tilde\Sigma_p^R(\epsilon_p)\left[1+\mathcal{O}\left(\frac{a_p^\rho}{\gamma_p\epsilon_p^2}\frac{e^{\gamma_p t}}{(\mu_p t)^\nu}\right)\right] \tag{25}$$

- For time smaller than Langevin time, the exponential corrections are negligible.

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Conclusion

- This work aimed to shed some light on the dynamics of out-of-equilibrium quantum scalar fields. We tried to understand the domain of validity of a local Langevin approximation.
- Despite the non local kernels and thus no true separation of scale, a local approximation is still valid in a certain time range, within which the dynamics is essentially local.
- This may be useful for Decoherence and entropy production in QFT (inflationnary models).
- But also to understand locality in other approaches (Boltzmann [Anisimov, Buchmuller,Drewes, Mendizabal(09)], cQPA [Herranen, Kainulainen, Rahkila(08)])
- And used in more involved non equilibrium situations :
    - Expanding and/or non homogeneous background, (Baryogenesis, (p-)reheating)
    - Backreaction,
    - ...