Kaon physics from the RBC-UKQCD collaborations

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Motivation

How to find new physics?

- In many cases theoretical uncertainties dominate
- LHC is a proton-proton collider

$\Rightarrow$ have to understand the QCD "background"

$\Rightarrow$ have to reduce hadronic uncertainties

What can we do with lattice QCD ?

- Test the Standard Model in some sectors where not much is known theoretically
- Compute hadronic matrix elements to constrain the free parameters of the Standard Model
- Can we quantify the new-physics effects in a model-independent way ?

Goal

$\Rightarrow$ Understand of the theory at the quantitative level

$\Rightarrow$ Confront theory (SM or beyond) with experimental results

Outline

The RBC-UKQCD collaborations have a broad kaon physics program:

- $K \to (\pi\pi)$ amplitudes
  - The $\Delta I = 3/2$ channel
  - The $\Delta I = 1/2$ channel
- Neutral kaon mixing
  - B_K
  - Beyond the Standard Model contributions

Not covered in this talk

- Kaon semi-leptonic form factors
- $K_L - K_S$ mass difference
- Rare kaon decays

The setup

Use numerical simulations of lattice QCD with the Domain-Wall formulation $\longrightarrow$ details tomorrow.

Relevant points:

- Chiral-Flavour symmetry (almost) exact at finite lattice spacing
- Dynamical $n_f = 2 + 1$ flavours
- Various quark masses: lightest pion mass is 170 MeV (partially quenched 140 MeV)

$\Rightarrow$ Realistic setup

Drawback: numerically expensive

More details

- Several lattice spacing: $a \sim 0.08, 0.1, 0.14$ fm
- Space extent $L \sim 3 - 4.6$ fm
- Two different lattice actions: Iwasaki and IDSDR

$K \to \pi\pi$ amplitudes

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)
- Noble prize in 1980 (Cronin and Fitch)
- Very nice measurements of both direct and indirect CP violation [PDG '10]

$$\begin{cases} Re\left(\frac{\varepsilon'}{\varepsilon}\right) & = (1.65 \pm 0.26) \times 10^{-3} \\ |\varepsilon| & = (2.228 \pm 0.011) \times 10^{-3} \end{cases}$$

- Still lacking a quantitative theoretical description
- Theoretically:

  Relate indirect CP violation parameter (ϵ) to neutral kaon mixing (B_K)

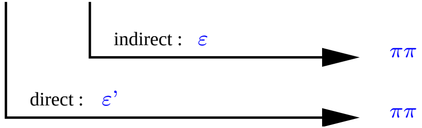
  Still lacking a quantitative description of direct CP violation (ε')

Flavour eigenstates $\begin{pmatrix} K^0 = \bar{s}\gamma_5 d \\ \overline{K}^0 = \bar{d}\gamma_5 s \end{pmatrix} \neq$ CP eigenstates $|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\overline{K}^0\rangle\}$

They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle & \sim & |K^0_-\rangle + \bar{\varepsilon}|K^0_+\rangle \\ |K_S\rangle & \sim & |K^0_+\rangle + \bar{\varepsilon}|K^0_-\rangle \end{cases}$

Direct and indirect CP violation in $K \to \pi\pi$

$$|K_L\rangle \propto |K^0_-\rangle + \bar{\varepsilon}|K^0_+\rangle$$



Experimentally [PDG '10] $\begin{cases} Re\left(\frac{\varepsilon'}{\varepsilon}\right) & = (1.65 \pm 0.26) \times 10^{-3} \\ |\varepsilon| & = (2.228 \pm 0.011) \times 10^{-3} \end{cases}$

$K \to \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \to (\pi\pi)_{I=0,2}$$

Corresponding amplitudes defined as

$$A[K \to (\pi\pi)_{\rm I}] = A_{\rm I} \exp(i\delta_{\rm I}) \qquad /\text{w } {\rm I} = 0, 2 \qquad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{{\rm Re} A_2}{{\rm Re} A_o} \sim 1/22 \qquad \text{(experimental number)}$$

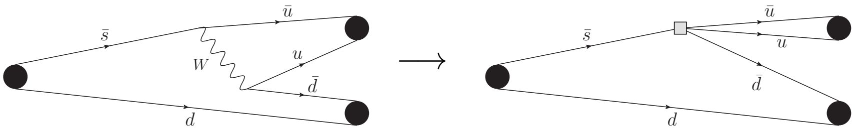
Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via

$$Re\left[\frac{\epsilon'}{\epsilon}\right] = \frac{\omega}{\sqrt{2}|\epsilon|}\left[\frac{{\rm Im}(A_2)}{{\rm Re}(A_2)} - \frac{{\rm Im}(A_0)}{{\rm Re}(A_0)}\right]$$

$$\epsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}}\left[\frac{{\rm Im}(A_2)}{{\rm Re} A_2} - \frac{{\rm Im} A_0}{{\rm Re} A_0}\right]$$

Overview of the computation

Operator Product expansion



Describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1}^{10} \left(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \right) Q_i(\mu) \right\}$$

Short distance effects factorized in the Wilson coefficients y_i, z_i

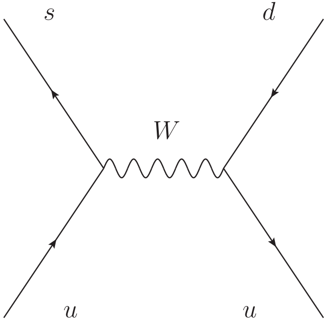
Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i | K \rangle \longrightarrow \text{Lattice}$$

See eg [Norman Christ @ Kaon'09] for an overview of different strategies.

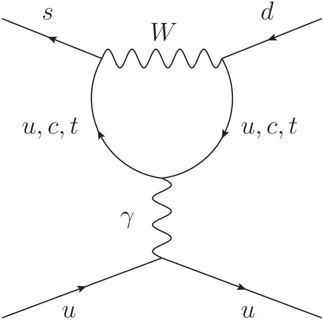
4-quark operators

Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $\qquad$ $Q_2 =$ color mixed

Electroweak penguins

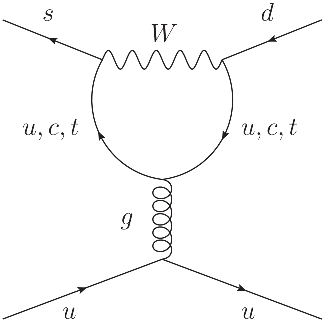


$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q(\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

4-quark operators

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$\begin{aligned} \overline{3} \otimes 3 &= 8 + 1 \\ \overline{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1 \end{aligned}$$

Decomposition of the 4-quark operators gives

$$\begin{aligned}
Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\
Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\
Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\
Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\
Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2}
\end{aligned}$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

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Only 7 are independent: one $(27,1)$ four $(8,1)$, and two $(8,8)$, $\Rightarrow$ we called them Q'

$$\begin{aligned}
(27,1) \quad Q'_1 &= Q_1'^{(27,1),\Delta I=3/2} + Q_1'^{(27,1),\Delta I=1/2} \\
(8,1) \quad Q'_2 &= Q_2'^{(8,1),\Delta I=1/2} \\
Q'_3 &= Q_3'^{(8,1),\Delta I=1/2} \\
Q'_5 &= Q_5'^{(8,1),\Delta I=1/2} \\
Q'_6 &= Q_6'^{(8,1),\Delta I=1/2} \\
(8,8) \quad Q'_7 &= Q_7'^{(8,8),\Delta I=3/2} + Q_7'^{(8,8),\Delta I=1/2} \\
Q'_8 &= Q_8'^{(8,8),\Delta I=3/2} + Q_8'^{(8,8),\Delta I=1/2}
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$$\begin{aligned} (27,1) \quad Q'_1 &= {Q'_1}^{(27,1),\Delta I=3/2} + {Q'_1}^{(27,1),\Delta I=1/2} \\ (8,1) \quad Q'_2 &= {Q'_2}^{(8,1),\Delta I=1/2} \\ Q'_3 &= {Q'_3}^{(8,1),\Delta I=1/2} \\ Q'_5 &= {Q'_5}^{(8,1),\Delta I=1/2} \\ Q'_6 &= {Q'_6}^{(8,1),\Delta I=1/2} \\ (8,8) \quad Q'_7 &= {Q'_7}^{(8,8),\Delta I=3/2} + {Q'_7}^{(8,8),\Delta I=1/2} \\ Q'_8 &= {Q'_8}^{(8,8),\Delta I=3/2} + {Q'_8}^{(8,8),\Delta I=1/2} \end{aligned}$$

A challenge !

The full computation presents many obstacles and remains a challenge

- Two-pion final state
- Many operators that mix under renormalisation
- Require the evaluation of disconnected graphs

Moreover, using a chiral disctretisation of the Dirac operator is probably unavoidable.

Plus the usual difficulties: light dynamical quarks, large volume, ...

Although highly non-trivial, the $\Delta I = 3/2$ channel is much easier

- Only 3 of these operators contribute to the $\Delta I = 3/2$ channel
  - A tree-level operator
  - 2 electroweak penguins
- No disconnect graphs contribute to the $\Delta I = 3/2$ channel
- Can use the Wigner-Eckart theorem to simplify the extraction of the physical amplitude

$K \to \pi\pi$ amplitudes: a short summary

The $\Delta I = 3/2$ channel

- Use the Lellouch-Lüscher method to get the physical amplitude [Lellouch Lüscher '00]
- Use the Wigner-Eckart theorem to simplify the extraction of the physical amplitude
- We compute the bare matrix elements and the phase shift at the physical point
- We have introduced a new non-perturbative renormalisation method [Boyle, Arthur, N.G. , Kelly, Lytle, PRD'11]

  $\Rightarrow$ NP renormalisation at low-energy ($\sim 1\ \mathrm{GeV}$) and NP-running to 3 GeV

  $\Rightarrow$ One-loop perturbative matching to $\overline{\mathrm{MS}}$ at 3 GeV

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We found a value of $\mathrm{Re}A_2$ compatible with the experimental value and gave a prediction for $\mathrm{Im}A_2$

$$\begin{aligned}\mathrm{Re}\,A_2 &= (1.436 \pm 0.063_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}}) \times 10^{-8}\,\mathrm{GeV},\\ \mathrm{Im}\,A_2 &= -(6.29 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}}) \times 10^{-13}\,\mathrm{GeV}\,.\end{aligned}$$

and

$$\mathrm{Im}A_2/\mathrm{Re}A_2 = \big(-4.76 \pm 0.37_{\mathrm{stat}} \pm 0.81_{\mathrm{syst}}\big) \times 10^{-5}$$

[Blum, Boyle, Christ, N.G., Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawhinney, Sachrajda, Soni, Sturm, PRL'12, PRD'12]

Received the Ken Wilson lattice award 2012

$K \to \pi\pi$ amplitudes: a short summary

The $\Delta I = 1/2$ channel

Pilot computation [T. Blum, Boyle, Christ, N.G., Goode, Izubuchi, Lehner, Liu, Mawhinney, Sachrajda, Soni, Sturm, Yin, Zhou, PRD'11].

- Unphysical: Small volume and non-physical kinematics ($m_\pi \sim 300\ \mathrm{MeV}$, pions at rest)
- All the contractions are computed
- Renormalisation done non-perturbatively

$K \to \pi\pi$ amplitudes: a short summary

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With this unphysical kinematics, we found

- $\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1(2.1)$ for $m_K = 878$ MeV $m_\pi = 422$ MeV
- $\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0(1.7)$ for $m_K = 662$ *MeV* $m_\pi = 329$ MeV

We observed a pattern which could explain the $\Delta I = 1/2$ enhancement

[Boyle, Christ, N.G., Goode, Izubuchi, Janowski, Lehner, Liu, Lytle, Sachrajda, Soni, Zhang, '12]

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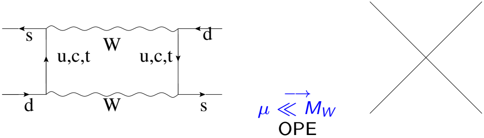
We believe that the first physical computation will be possible with the new generation of supercomputer

Generation of new ensembles has started

Neutral kaon mixing

Neutral kaon mixing

In the Standard Model, $K^0 - \bar{K}^0$ mixing dominated by box diagrams with W exchange, e.g.

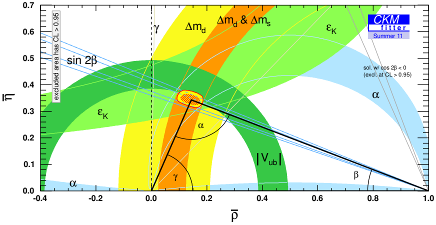


Factorise the non-perturbative contribution into

$$\langle \bar{K}^0 | \mathcal{O}_{LL}^{\Delta S=2}(\mu) | K^0 \rangle = \frac{8}{3} F_K^2 M_K^2 B_K(\mu) \qquad \text{w/ } \mathcal{O}_{LL}^{\Delta S=2} = (\bar{s}\gamma_\mu(1-\gamma_5)d)(\bar{s}\gamma^\mu(1-\gamma_5)d)$$

Related to ε via CKM parameters, schematically

$$\varepsilon \sim \text{ known factors} \times V_{\rm CKM} \times C(\mu) \times B_K(\mu)$$



[CKMfitter'11]

See [F. Gabbiani et al '96]

In the SM, neutral kaon mixing occurs through W-exchanges $\rightarrow$ $(V-A)\times(V-A)$

$$O_1^{\Delta s=2} = (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta \gamma_\mu (1-\gamma_5) d_\beta)\,,$$

Beyond the SM, other Dirac structure appear in the generic Hamiltonian $H^{\Delta s=2} = \sum_{i=1}^{5} C_i(\mu)\, O_i^{\Delta s=2}(\mu)\,.$

SUSY basis

$$\begin{aligned}
O_2^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1-\gamma_5) d_\beta) \\
O_3^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1-\gamma_5) d_\alpha) \\
O_4^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)\,(\bar{s}_\beta (1+\gamma_5) d_\beta) \\
O_5^{\Delta s=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)\,(\bar{s}_\beta (1+\gamma_5) d_\alpha)
\end{aligned}$$

This work: study of $\langle \bar{K}^0 | O_i^{\Delta s=2} | K^0 \rangle$

Remarks

- Only the parity even part O_i^+ contribute to $\langle \bar{P}|O_i|P\rangle$
- They can be expressed in the renormalization basis

$$O_1^+ \leftrightarrow Q_1 = (\gamma_\mu \times \gamma_\mu + \gamma_\mu \times \gamma_5)_{\text{unmixed}} \Rightarrow (27,1) \rightarrow m_P^2$$

$$(O_4^+, O_5^+) \leftrightarrow \begin{pmatrix} Q_2 & = & \gamma_\mu \times \gamma_\mu - \gamma_\mu \times \gamma_5 \\ Q_3 & = & 1 \times 1 - \gamma_5 \times \gamma_5 \end{pmatrix}_{\text{unmixed}} \Rightarrow (8,8) \rightarrow \text{Cst}$$

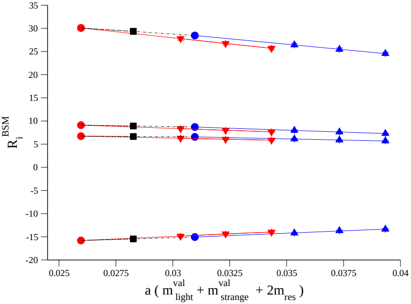
$$(O_2^+, O_3^+) \leftrightarrow \begin{pmatrix} Q_4 & = & 1 \times 1 + \gamma_5 \times \gamma_5 \\ Q_5 & = & \sigma_{\mu\nu} \times \sigma_{\mu\nu} \end{pmatrix}_{\text{unmixed}} \Rightarrow (6,\bar{6}) \rightarrow \text{Cst}$$

- O_2 and O_3 mix under renormalization, so do O_4 and O_5
- In the chiral limit $O_1 \rightarrow m_P^2$ and $O_{i\geq 2} \rightarrow \text{Cst}$

Extrapolation to the physical point

Normalisation proposed in [Babich, N.G., Hoelbling, Howard, Lellouch, Rebbi '07]

$$R_i^{\mathrm{BSM}}(m_P) = \left[\frac{f_K^2}{m_K^2}\right]_{\mathrm{expt}} \left[\frac{m_P^2}{f_P^2}\,\frac{\langle \bar{P}|O_i|P\rangle}{\langle \bar{P}|O_1|P\rangle}\right]_{\mathrm{latt}}$$



Keep $m^{val}_{\mathrm{strange}} = m^{sea}_{\mathrm{strange}} = 0.03$ fixed, and extrapolate light quark masses to physical value

Keep $m^{val}_{\mathrm{strange}} = 0.025$ fixed, and extrapolate the light quark masses to physical value

Interpolate to the physical strange $m^{val}_{\mathrm{strange}} = 0.0273(7)$

Results in $\overline{\mathrm{MS}}$ at 3 GeV and error budget (in %)

[Boyle, N.G., Hudspith, PRD'12]

i	R_i^{BSM}	B_i	stat.	discr.	extr.	NPR	PT	total
2	$-15.3(1.7)$	0.43 (5)	1.3	1.5	4.0	9.4	4.7	11.3
3	5.4(0.6)	0.75 (11)	2.0	1.5	3.9	7.8	7.6	12.0
4	29.3(2.9)	0.69 (7)	1.3	1.5	4.1	8.2	8.2	9.8
5	6.6(0.9)	0.47 (6)	2.1	1.5	3.8	12.6	12.6	13.8

(N.B. Discretisation errors computed only for B_K)

Compatible to the results found by [ETMC '12]

Results can be used to obtain bounds on the scale of new physics Λ_{NP} or constraints on a specific model.

See e.g. [Mescia and Virto '12] for constraints on natural SUSY

And for the Standard Model bag parameter, [Arthur et al, RBC-UKQCD '12]

$$B_K = 0.525(8)_{\mathrm{stat}}(1.1)_{\mathrm{syst}}$$

Conclusions and outlook

- First realistic *ab initio* computation of hadrdonic decay $K \to (\pi\pi)_{I=2}$
  - Technical improvements (eg IDSDR)
  - New theoretical tools are being developed, e.g. for the renormalization
- "Complete" but unphysical computation of both $K \to (\pi\pi)$ amplitudes
  - Possible explanation for the $\Delta I = 1/2$
  - Aim for a physical computation of the $\Delta I = 1/2$ part (BlueGene Q, G-parity)
- BSM contributions to $K - \bar{K}^0$ mixing: first realistic study
  - Confirm previous quenched study where large ratios non-SM/SM were found
  - Errors $\sim$ 10%
  - Two dominant sources of errors come from the NPR with exceptional scheme
  - Plan to compute the matching factors and add another lattice spacing
  - Expect to reduce the error below $\sim$ 5%

Conclusions and outlook

Future plans

- Going toward the physical point: implementing different strategies
  - coarse lattice (IDSDR) and physical pion mass
  - fine lattice (IW) and not-quite-physical pion mass
- Will reduce the error on A_2
- We are also implementing G-parity $\Rightarrow$ computation of A_0
- Adding a lighter quarks for the kaon semi-leponic form factor f_0
- Adding another lattice spacing for the BSM $K^0 - \bar{K}^0$ oscillations
- working on the BSM contributions to $K \to \pi\pi$
- and on the implementation of a renormalisation scheme with an active charm

Many thanks to the organisers of RPP'13