Improved treatment of kinematics for gluon saturation in QCD at high energy

Guillaume Beuf

Universidade de Santiago de Compostela

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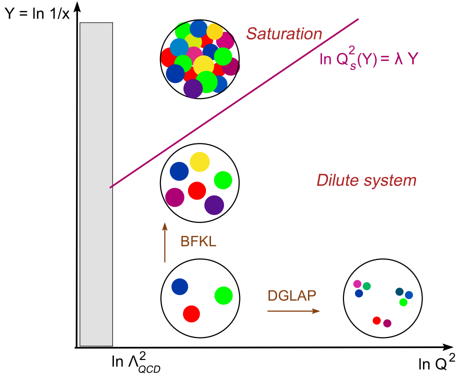
Kinematical regimes of QCD (ex: DIS)



At large Q^2: hard scattering described with collinear factorization. Leading logs $(\alpha_s \log(Q^2/\Lambda^2_{\mathrm{QCD}}))^n$ are resummed with the DGLAP equations.

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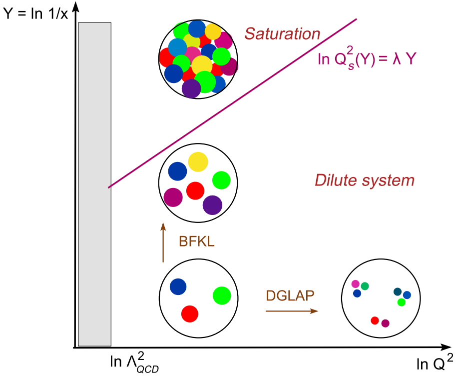
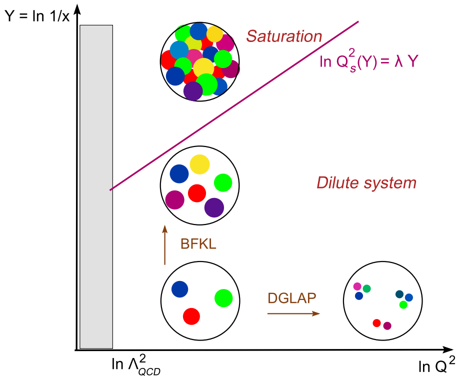


At small x_{Bj}: high-energy scattering.
When the target is dilute, leading logs $(\alpha_s \log(1/x_{Bj}))^n$ are resummed with the BFKL equation.

Kinematical regimes of QCD (ex: DIS)



BFKL increases the partonic density in the target.
$\Rightarrow$ At low enough x_{Bj}, gluon saturation occurs, and BFKL is replaced by the JIMWLK or BK evolution equations.

Motivations to study the gluon saturation regime

- Interesting by itself:
  regime of non-perturbatively strong color fields
  - accessible experimentally:
    LHC (pp, pA and AA), RHIC (dA, AA), HERA.
  - many observables are calculable systematically (CGC effective theory, High-energy OPE, ...) because the coupling is typically weak.

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- Gluon saturation is one of the dominant phenomena at work in the *underlying event* of a hard process. $\Rightarrow$ Unavoidable ingredient for a future *theory* of the *underlying event*.

Gluon saturation phenomenology in DIS

General method:

1. Model the dipole-target elastic amplitude at not too high energy (not too low x_{Bj}).
2. Evolve numerically that amplitude with the BK equation.
3. Plug the result into the dipole factorization formulae for DIS structure functions or other observables.
4. Compare with the data and fit the parameters of the model for the initial condition.

Up-to-date studies: with the leading log (LL) BK equation improved by running coupling effects,
and with dipole factorization formula at LO.
Albacete, Armesto, Milhano, Quiroga, Salgado (2011)
Kuokkanen, Rummukainen, Weigert (2012)

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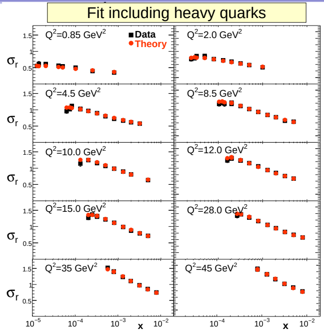

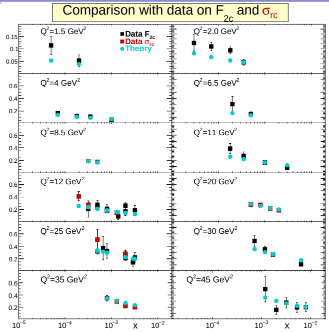

Very good fit in the end, but at the price of some peculiar tunig of parameters.

Albacete, Armesto, Milhano, Quiroga, Salgado (2011)

The need for such tuning can be explained qualitatively by the lack of higher order corrections to the BK equation.

Kuokkanen, Rummukainen, Weigert (2012)

Other observables

- Other inclusive observables: The dipole-target amplitude fitted in DIS can be used for them, like the single inclusive particle production at RHIC and at the LHC, both in pp and pA or dA. $\rightarrow$ Good results, but not perfect yet for the y dependence in pA at the LHC.

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- More exclusive observables, like multi-particle production: They require using the JIMWLK evolution equation, far more complicated than BK. $\rightarrow$ Theory less under control. However: predict qualitatively new phenomena, indeed observed experimentally.
  - Azimuthal decorrelation of two forward particles produced in pA or dA collisions (STAR and PHENIX)
  - Two-particle *ridge* correlation, at long range in rapidity, seen in pp (CMS) and in pA (CMS, ALICE, ATLAS).

Gluon saturation at higher orders

Going to higher orders is necessary for precision studies.
For the simpler, inclusive observables, the calculation of higher order corrections has started:

- NLL corrections to the BK equation
  Balitsky, Chirilli (2008)
- NLO corrections to DIS structure functions
  Balitsky, Chirilli (2011)
  G.B. (2012)
- NLO corrections to forward single inclusive particle production in pA or pp
  Chirilli, Xiao, Yuan (2012)

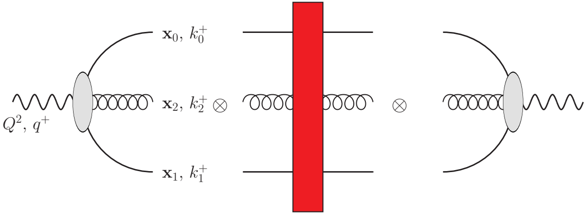
Need for further resummations

However, besides running coupling effects, pathologically large corrections of two types plague higher order results and have to be resummed to obtain reliable results from BK at NLL.

- Kinematical corrections: due to a too naive treatment of the high-energy limit.
  $\rightarrow$ Main topic of the rest of this talk.
- Dynamical corrections: induced from DGLAP evolution, due to the duality between low x_{Bj} and high Q^2 evolutions.
  $\rightarrow$ Left for further studies.

The same problems appears in the linear regime for the BFKL equation, and the corresponding resummations have been fully performed.
Ciafaloni, Colferai, Salam, (Staśto) (1998-2007)
Altarelli, Ball, Forte (1999-2008)

DIS at high energy at NLO



$$\sigma^{\gamma}_{T,L}(Q^2,x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \Bigg\{ \mathcal{I}^{LO}_{T,L}(x_{01},z_1,Q^2)\ \Big[1-\langle\mathcal{S}_{01}\rangle\Big]$$

$$+\bar{\alpha}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\int_0^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\ \mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\ \langle\mathcal{S}_{01}-\mathcal{S}_{02}\,\mathcal{S}_{21}\rangle \Bigg\}$$

with $z_n = k_n^+/q^+$

G.B. (2012)

Subtraction of LL from NLO

In the soft gluon limit $z_2 \to 0$, the NLO impact factor verifies

$$\mathcal{I}^{NLO}_{T,L}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2 = 0, Q^2) = \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \mathcal{I}^{LO}_{T,L}(x_{01}, z_1, Q^2)$$

$\Rightarrow$ log divergence of the integral over z_2, which can be absorbed by renormalizing the bare dipole-target amplitude $\langle \mathcal{S}_{01} \rangle$ in the LO term.

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The LL of $1/x_{Bj}$ can then be removed from $\mathcal{I}^{NLO}_{T,L}$ and absorbed into $\langle \mathcal{S}_{01} \rangle_{Y^+}$ using the BK evolution

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2 \mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \langle \mathcal{S}_{02}\mathcal{S}_{21} - \mathcal{S}_{01} \rangle_{Y^+}$$

up to a scale k_f^+ close enough to the photon momentum q^+.

Formation time physics in the impact factors

The DIS impact factors $\mathcal{I}_{T,L}^{NLO}$ contain a factor $\mathrm{K}_{0,1}^2(QX_n)$ dependent on the variable

$$X_2^2 = z_1\, z_0\, x_{10}^2 \quad (\text{with } z_0 + z_1 = 1)$$

for the LO ones and

$$X_3^2 = z_1\, z_0\, x_{10}^2 + z_2\, z_0\, x_{20}^2 + z_2\, z_1\, x_{21}^2 \quad (\text{with } z_0 + z_1 + z_2 = 1)$$

for the NLO ones.

$2q^+X_2^2$ and $2q^+X_3^2$ are the formation time of the $q\bar{q}$ and $q\bar{q}g$ Fock states in the photon wave-function.

The $\mathrm{K}_{0,1}^2(QX_n)$ prefactors then suppress exponentially the Fock states whose formation time is larger than the virtual photon lifetime $2q^+/Q^2$.

Problems with standard LL subtraction with BK

At small $z_2 \neq 0$: $X_3^2 \simeq z_1(1-z_1)x_{01}^2 = X_2^2$ and

$$\mathcal{I}_{T,L}^{NLO}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2) \sim \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \mathcal{I}_{T,L}^{LO}(x_{01}, z_1, Q^2)$$

in most of the available range for $\mathbf{x}_2$.

But not valid when $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{12}^2$, where $X_3^2 \simeq z_2 x_{02}^2 \simeq z_2 x_{12}^2$.

$\Rightarrow$ There is no LL contribution for emission of a soft gluon at very large transverse distance: too long formation time!

The standard BK equation misses that effect and over-subtracts LL from the NLO impact factor. $\rightarrow$ Leads after LL subtraction to large unphysical NLO corrections in the collinear regime, spoiling any matching with DGLAP physics.

Kinematical constraint

In the case of the BFKL equation in momentum space, kinematical large higher order corrections are resummed by including a kinematical constraint in the BFKL kernel.
Ciafaloni (1988)
Kwieciński, Martin, Sutton (1996)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)

Analog for BK in mixed space :
Multiply the real soft gluon emission probability by
$\theta\left(z_1(1-z_1)\, x_{01}^2 - z_2\ \min(x_{02}^2, x_{21}^2)\right)$
in the *integral* version of the BK equation.

An equivalent constraint for the real terms in mixed space has been proposed in : Motyka, Staśto (2009)

However: inappropriate treatment of virtual corrections there.

Kinematically constrained BK equation

Natural prescription: use unitarity to calculate modified virtual corrections from modified real gluon emission kernel.

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Rewriting the new evolution equation as a differential equation:

$$\partial_{Y^+} \langle \mathcal{S}_{01} \rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y^+} \right\}$$

with the notation:

$$\Delta_{012} = \max\left\{0,\, \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\}$$

G.B., *in preparation*

Kinematically constrained BK equation

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Typical behavior of the shift:

$$\begin{aligned} \Delta_{012} &= 0 \qquad \text{for} \quad x_{02}^2 \lesssim x_{01}^2 \quad \text{or} \quad x_{21}^2 \lesssim x_{01}^2 \\ \Delta_{012} &\sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \qquad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2 \end{aligned}$$

Only gluon emission at large transverse distance is modified, and regime of very large transverse distances completely removed.

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Each of the two modifications should slow down BK evolution:

- Restriction of phase space by the theta function
- Shift of the Y^+ argument of the dipole amplitude in the real term but not in the virtual term.

Kinematically constrained BK equation

$$\partial_{Y_f^+} \langle \mathcal{S}_{01} \rangle_{Y_f^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y_f^+ - \Delta_{012})$$
$$\times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y_f^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \langle \mathcal{S}_{01} \rangle_{Y_f^+} \right\}$$

That modification of the LL BK equation resums precisely the largest and most pathological corrections appearing in the known NLL BK equation.
$\Rightarrow$ Necessary step towards a stable and reliable version of the NLL BK equation.

When regularizing the NLO DIS impact factors and removing the LL contribution using that modified BK equation:
fully correct subtraction the LL contributions, with no mismatch in the collinear regime, by contrast to the standard LL BK case.

Conclusions

Implementing the kinematical constraint in BK at LL:

- resums the largest highest order corrections to the BK equation and to impact factors
- restores the correct collinear double leading log regime for the BK equation and impact factors
- restores the physics of the formation time of fluctuations.

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Resummation of dynamical pathological higher order corrections still to do next:
$\simeq$ resumming the full DGLAP evolution into a modified BK equation!