

PyR@TE: Python Renormalization Group Equations @ Two-loop

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PyR@TE-Motivations



Description

- Python RGEs @ Two-loop
- Goal: Generate the **Renormalization Group Equations** for non-supersymmetric theories @ 2-loop

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Why ?

- No evidence of SUSY so far
- Systematic studies of non-SUSY models require the RGEs
- One possible application: constraining non-SUSY BSM models via the stability bound

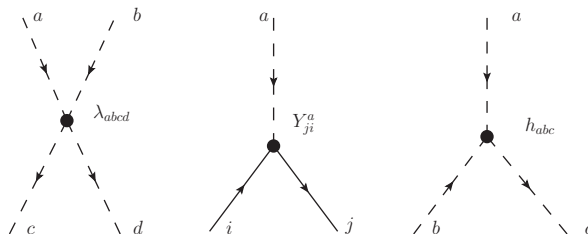
Renormalization Group Equations

- Renormalization scale μ

$$\Rightarrow g_{10}, \alpha_{S0}, \lambda_0 \cdots \Rightarrow \tilde{g}_1(\mu), \tilde{\alpha}_S(\mu), \tilde{\lambda}(\mu).$$

- RGEs : ensure the invariance of the observables.

► e.g. : $\mu \frac{d}{d\mu} \tilde{\alpha}_S(\mu) = \beta_{\alpha_S}$



- β functions depend on the theory i.e. **particles and gauge groups**.
- Can be calculated in perturbation theory.

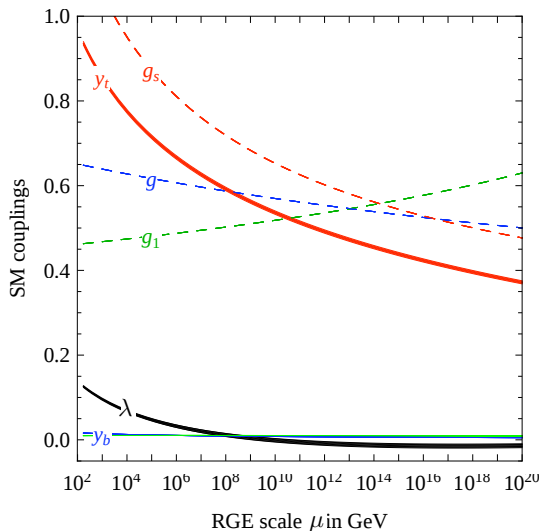


Fig: Higgs mass and vacuum stability in the SM at NNLO, *G. Degrandi et al.*

- RGEs for general gauge theories known for a long time:
 - ▶ *M. Machacek and M. T. Vaughn, 1983 Nuc.Phys.B222*
 - ▶ *M. Luo et al. Phys.Rev. D67 (2003) 065019*
- Calculation of beta functions "by hand" is time consuming and prone to error \Rightarrow Difficult to use in practice.
- Full set of 2-loop RGEs known only for few specific cases:
 - ▶ SM + Neutrinos
from *A. Wingerter Phys.Rev. D84 (2011) 095012*
 - ▶ SM + chiral fourth generation
from *C. Cheung et al. JHEP 1207 (2012) 105*
 - ▶ SM + real singlet scalar
 - ▶ SM + real triplet scalar
 - ▶ SM + complex doublet scalar
 - ▶ ...

Definition

$G_1 \times G_2 \times \cdots \times G_n$ direct product of simple groups

$$\begin{aligned} \mathcal{L} \supset \quad & - N_a Y_{jk}^a \psi_j \xi \psi_k \phi_a + h.c. \Rightarrow \beta_{jk}^a \\ & - N_\lambda \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d \Rightarrow \beta_{abcd} \\ & - N_{mf} (mf)_{jk} \psi_j \xi \psi_k + h.c. \Rightarrow (\beta_{mf})_{jk} \\ & - N_{mab} m_{ab}^2 \phi_a \phi_b \Rightarrow \beta_{ab} \\ & - N_h \phi_a \phi_b \phi_c \Rightarrow \beta_{abc}, \end{aligned}$$

\Rightarrow 6 types of beta functions to calculate:

- ▶ $\beta(g) \Rightarrow$ gauge couplings
- ▶ $\beta_{jk}^a \Rightarrow$ yukawas
- ▶ $\beta_{abcd} \Rightarrow$ quartic couplings
- ▶ $\beta_{ab} \Rightarrow$ scalar mass
- ▶ $(\beta_{mf})_{jk} \Rightarrow$ fermion mass
- ▶ $\beta_{abc} \Rightarrow$ trilinear couplings

e.g. SM

- $SU(3) \times SU(2) \times U(1) \Rightarrow \beta_{g_1}, \beta_{g_2}, \beta_{g_3}$
- $Y_u \bar{Q} H^c u_R \Rightarrow \beta_{Y_u}(\beta_{Y_d}, \beta_{Y_l})$
- $\lambda(H^\dagger H)^2 \Rightarrow \beta_\lambda$
- $\mu(H^\dagger H) \Rightarrow \beta_\mu$

The Quartic Terms

The diagram illustrates the expansion of a quartic vertex (represented by a grey circle) into various terms, showing the relationship between different types of interactions and their corresponding mathematical expressions.

The expansion is shown as follows:

$$\begin{aligned}
 & \text{Quartic Vertex} = \text{Exchange Diagrams} + \text{Wavy Line Diagrams} + \text{Multi-loop Diagrams} + \dots \\
 & \sim \sum_{perm} \lambda_{abef} \lambda_{efcd} \sim \sum_{perms, k, l} g^{2k} g^{2l} \{\theta^A, \theta^B\}_{ab} \{\theta^A, \theta^B\}_{cd} \\
 & \sim \sum_{perms} \sum_{i, j, k, l} Y_{ij}^a Y_{jk}^{b\dagger} Y_{kl}^c Y_{li}^{d\dagger} \\
 & \sim \sum_{perm} g^2 C_2^{fg}(S) \lambda_{abef} \lambda_{cdeg}
 \end{aligned}$$

The diagrams include:

- Exchange diagrams with vertices λ_{abef} and λ_{efcd} .
- Wavy line diagrams with vertices $\theta_{ae}^A, \theta_{bf}^A, \theta_{ce}^B, \theta_{fd}^B$.
- Multi-loop diagrams with vertices $Y_{ij}^a, Y_{jk}^{b\dagger}, Y_{kl}^c, Y_{li}^{d\dagger}$.
- Diagrams involving $C_2(S)^{fg}$ and $\lambda_{abef} \lambda_{cdeg}$.

Goal

- Public code for any non-SUSY theories at 2-loop
- Any gauge group (with single $U(1)$ factor)

Status

- beta version working with SM gauge groups
- LateX output ready
 - SM + one real scalar field \Rightarrow Trilinear terms
 - SM + t' vector like quark \Rightarrow Fermion mass terms
- Collaborator F. Staub implemented same RGEs in SARAH 4 (unpublished) \Rightarrow independent cross check.

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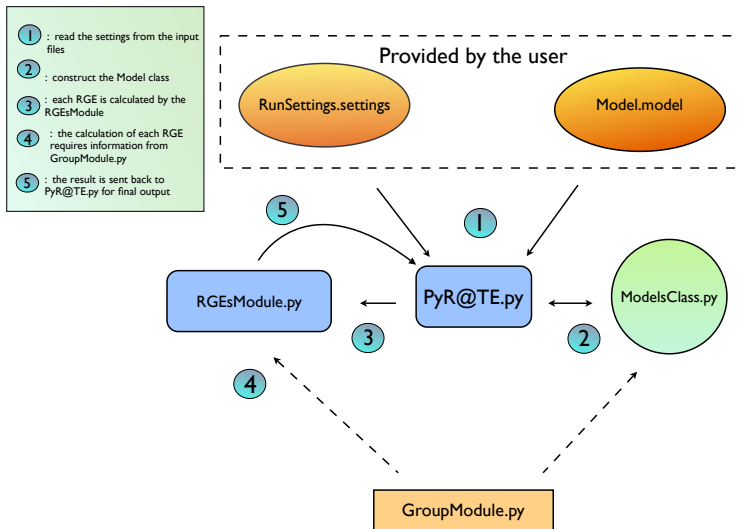
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TODO :

- Extend the group part i.e. get the CGCs as well as the matrix representation (following R. Fonseca, hep-ph/1106.5016)
- Interface to C++,...

Structure of PyR@TE



Input files

- *.model* and *.settings* files are required to run.

.model

- we are using text files from the input (YAML)
- keys :
 - ▶ Author Date Name
 - e.g. Name : SMtp

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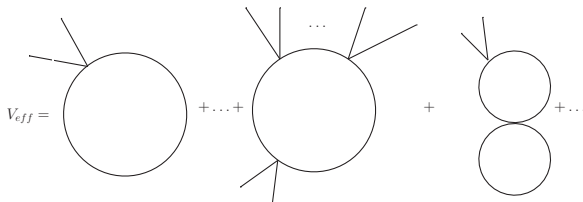
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 - ▶ **Potential** \Rightarrow is given in a similar way.
e.g. $Y_t : \{ \textit{Fields} : [tpR, Q, H], \textit{Norm} : 1, \textit{latex} : Y_t \}$

Higgs Potential

- At the classical level $\Rightarrow V_{SM}(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$
 - ▶ "Higgs Miracle" \Rightarrow allow us to give mass to the vector bosons
 - ▶ Include all the terms that are gauge invariant and renormalizable
 - ▶ Potential gives the energy density of the field.
- Equivalent of $V(\phi)$ which takes into account all the radiative processes $\Rightarrow V_{eff}(\phi_c)$
 - ▶ The effective potential will give the energy density of $\phi_c = \langle\phi\rangle$ **including the quantum effects**
 - ▶ Needs to take into account all the possible processes !

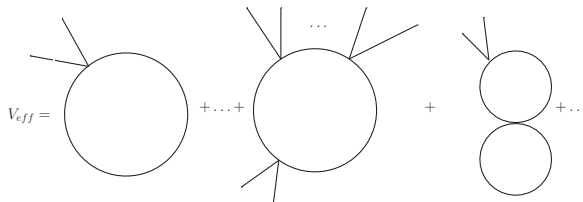
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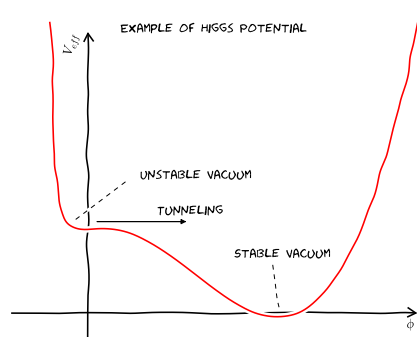
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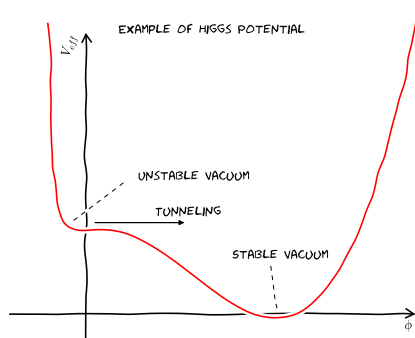
Stability bound

- Our existence demands that the minimum of the EW potential be stable !
 - ▶ Stable : Only one minimum
 - ▶ Potential Stable up to scale $\Lambda \Leftrightarrow \lambda(\Lambda) > 0$

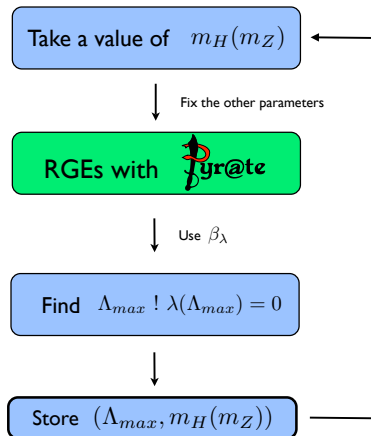


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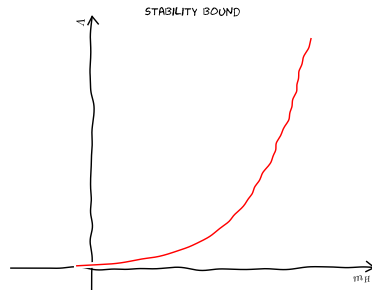
- Our existence demands that the minimum of the EW potential be stable !
 - ▶ Stable : Only one minimum
 - ▶ Potential Stable up to scale $\Lambda \Leftrightarrow \lambda(\Lambda) > 0$
- $\lambda(\mu)$ calculated from β_λ , depends on m_H, m_t, \dots
- Stability bound :
 - ▶ Curve $\Lambda(m_H) \Rightarrow$ gives $m_{H_{max}}$ so that the theory be valid up to Λ



Calculating the Stability bound



Iterate



Stability bound SM

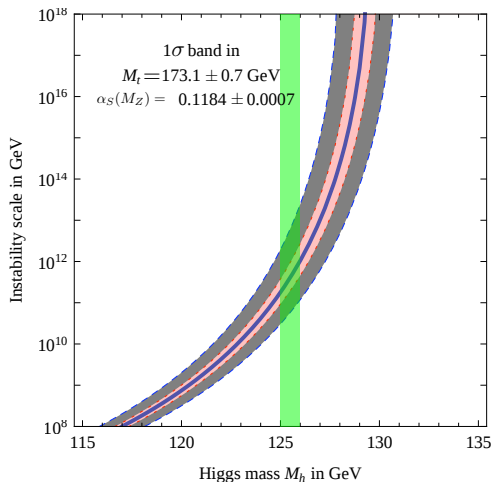


Fig: Higgs mass and vacuum stability in the SM at NNLO, *G. Degrandi et al.*

Vector like t' model

Vector like quarks

L, R components \Rightarrow same transformation

One of the simplest extension of the SM

- One vector like $t' \sim (3, 1)_{4/3} \Rightarrow$ vector like mass.

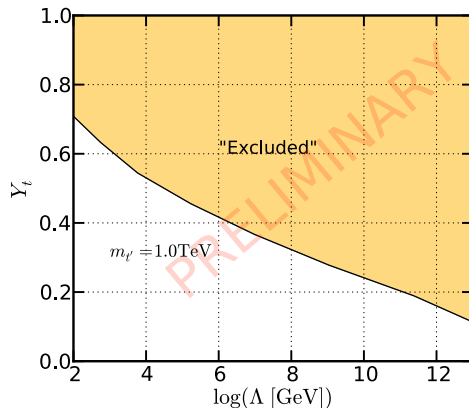
Lagrangian

$$\mathcal{L} \supset \mathcal{L}_{SM} - \underbrace{m_t t_L'^{\dagger} t_R'}_{\beta_{m_t}} - \underbrace{Y_t^i \bar{Q}^i H t_R'}_{\beta_{Y_t}}$$

- t' modifies the RGEs $\Rightarrow Y_t$ enters β_{λ} at 1-loop.
- Constrains from Wtb and T parameter by G. Cacciapaglia et al. *JHEP11(2010)159*
for $m_{t'} \simeq 500$ GeV $\Rightarrow Y_t \sim 0.8$ excluded

Stability Bound

- For a given $Y_t(m_{t'})$
 - ▶ Estimate the stab. bound.
 - ▶ Intersection with $m_H = 125 \text{ GeV} \Rightarrow (\Lambda, Y_t(m_{t'}))$
- Possibility of extracting constraints in the plane (Y_t, Λ) .
- Very small dependence on $m'_{t'}$



Conclusion

- For a more systematic study of non SUSY models RGEs are needed.
- We are working on a tool that generates RGEs @2-loop
⇒ PyR@TE
- Group Theory part has to be finalized
- Have fun !



Other Projects

- Heavy new resonances with Auger
 - ▶ Calculate the event rate of downward-going neutrinos @PAO.
 - ▶ $SM +$ extensions of the SM with W', Z' .
- $Z/Z' \rightarrow t\bar{t}$ (Tomas) and $W/W' \rightarrow t\bar{b}$ @ NLO