Band Gap for the Neutron in a multilayer due to non-Newtonian gravity-like force

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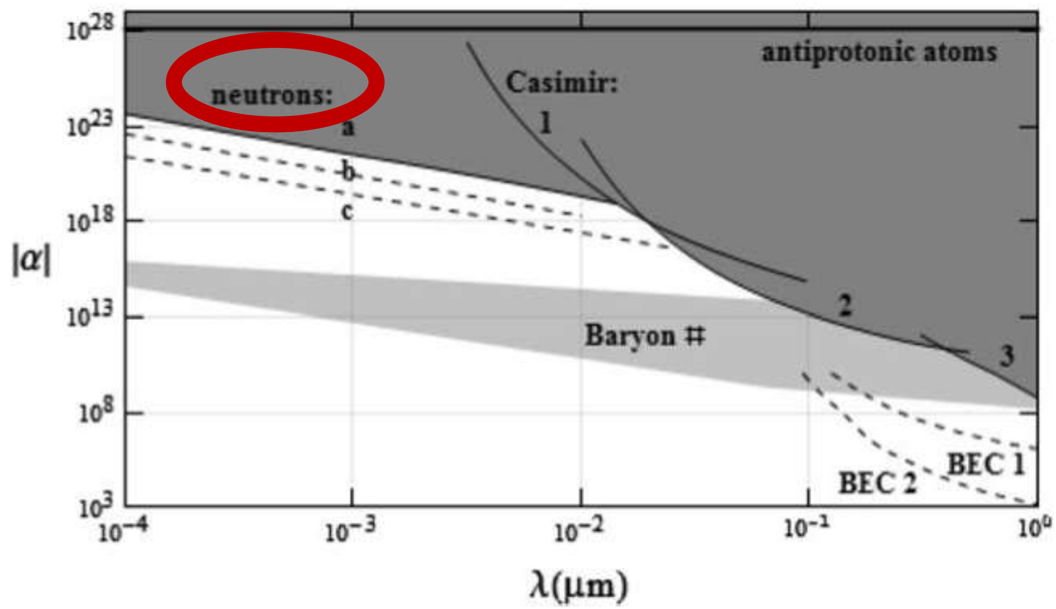
Outline of this study

- Searches for non-Newton gravity-like force
  - Neutron Experiments provide the strongest constraints in the range $\lesssim 10$ nm
- Neutron reflection probability off a bulk material
  - Neutron wave function for different interaction-range
- Reflection probability off a multilayer
  - Band gap prohibits for neutrons to go into the multilayer
- Band gap for the neutron in a multilayer
  - Multilayer designed not to reflect neutrons
- Summary

Searches for non-Newton gravity-like force

$$V(z) = -\frac{GMm}{r} \times \left(1 + \alpha e^{-r/\lambda}\right)$$

G: the gravitational constant
M: the first mass
m: the second mass
r: the distance between the masses



R. D. Newman et al., Space Sci. Rev. 148, 175 (2009)

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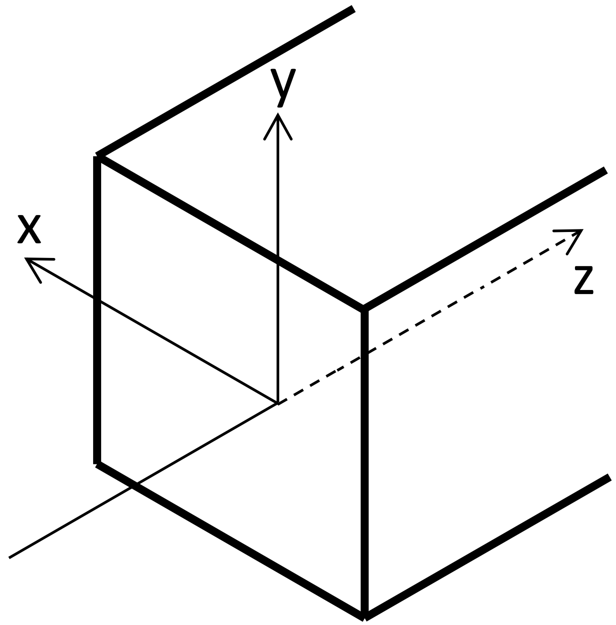
Potential for neutron



$$\begin{cases} 0 + V_{\mathrm{g}} \exp(2k_{\mathrm{g}}z) & z < 0 \\ V_{\mathrm{f}} + V_{\mathrm{g}}\left(2 - \exp\left(-2k_{\mathrm{g}}z\right)\right) & z > 0 \end{cases}$$

$$\mathrm{k_g} = 1/\lambda$$

$$V_g = \frac{-Gm\pi\rho\alpha_g}{2k_g^2}$$

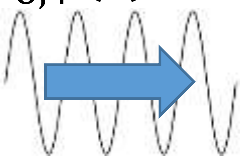
m: neutron mass

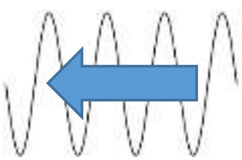
ρ: weight density of the bulk

V_{f}: Fermi potential of the bulk

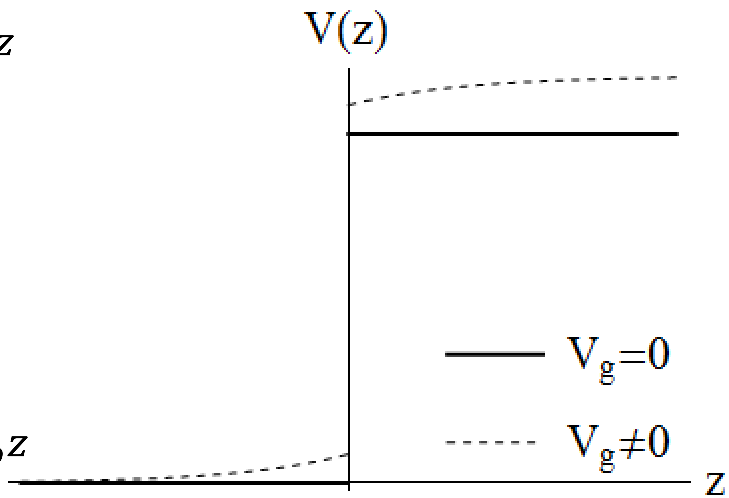
How to calculate the reflection amplitude

$\phi_{\mathrm{o},+}(z) \cong e^{ik_o z}$

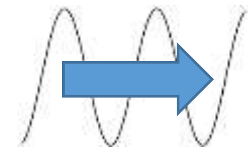




$\phi_{\mathrm{o},-}(z) \cong e^{-ik_o z}$



$\phi_{\mathrm{i},+}(z) \cong e^{ik_i z}$



$$R = -\frac{\phi_{o,+}(0)}{\phi_{o,-}(0)} \frac{\frac{d}{dz}\ln\phi_{i,+}(z) - \frac{d}{dz}\ln\phi_{\mathrm{o},+}(z)}{\frac{d}{dz}\ln\phi_{i,+}(z) - \frac{d}{dz}\ln\phi_{\mathrm{o},-}(z)}\Bigg|_{z=0}$$

Schrodinger equations

$$\begin{cases} \Psi''(z) + \left(k_{\mathrm{o}}^2 - \eta e^{+2k_{\mathrm{g}}z}\right)\Psi(z) = 0 & z < 0 \\ \Psi''(z) + \left(k_{\mathrm{i}}^2 + \eta e^{-2k_{\mathrm{g}}z}\right)\Psi(z) = 0 & z > 0, \end{cases}$$

Independent solutions

$$\phi_{\mathrm{o},\pm}(z) = \Gamma\left(1 \pm \mathrm{i}\frac{k_{\mathrm{o}}}{k_{\mathrm{g}}}\right)\left(\frac{\sqrt{\eta}}{2k_{\mathrm{g}}}\right)^{\mp \mathrm{i}\frac{k_{\mathrm{o}}}{k_{\mathrm{g}}}} I_{\pm \mathrm{i}\frac{k_{\mathrm{o}}}{k_{\mathrm{g}}}}\left(e^{+k_{\mathrm{g}}z}\frac{\sqrt{\eta}}{k_{\mathrm{g}}}\right) \rightarrow \quad e^{\pm i k_o z}$$

(as $z \rightarrow -\infty$)

$$\phi_{\mathrm{i},\pm}(z) = \Gamma\left(1 \mp \mathrm{i}\frac{k_{\mathrm{i}}}{k_{\mathrm{g}}}\right)\left(\frac{\sqrt{\eta}}{2k_{\mathrm{g}}}\right)^{\pm i\frac{k_{\mathrm{i}}}{k_{\mathrm{g}}}} J_{\pm \mathrm{i}\frac{k_{\mathrm{i}}}{k_{\mathrm{g}}}}\left(e^{-k_{\mathrm{g}}z}\frac{\sqrt{\eta}}{k_{\mathrm{g}}}\right) \rightarrow e^{\pm i k_i z}$$

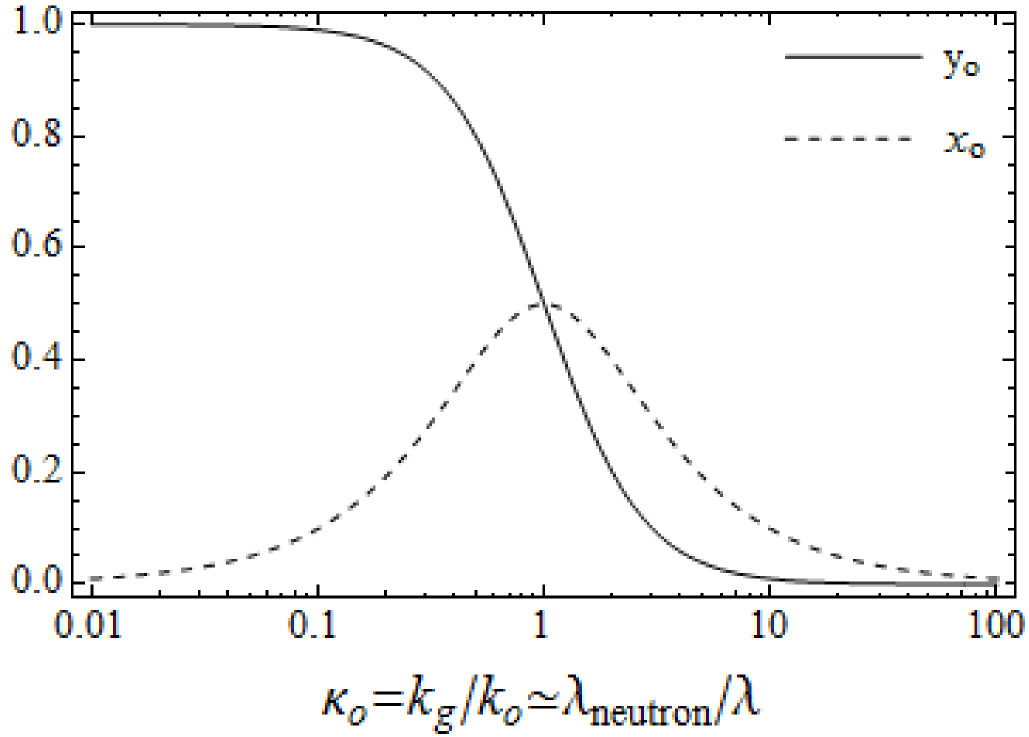
$(as\ z\ \rightarrow +\infty)$

Corrections on the boundary

$$\phi_{o,\pm}(0) \cong 1 + \theta(0)(-x_\mathrm{o} \pm \mathrm{i}\, y_\mathrm{o})$$

$$\frac{d}{dz} \ln \phi_{\mathrm{o},\pm}(z)\Big|_{z=0} \cong \pm\, \mathrm{i}\, k_\mathrm{o} + \frac{\theta'(0)}{2}(x_o \pm \mathrm{i} y_o)$$

$$\theta(z) = \int_{-\infty}^{z} \left(\sqrt{k_\mathrm{o}^2 - \eta e^{+2k_\mathrm{g} z'}} - k_\mathrm{o} \right) dz'$$



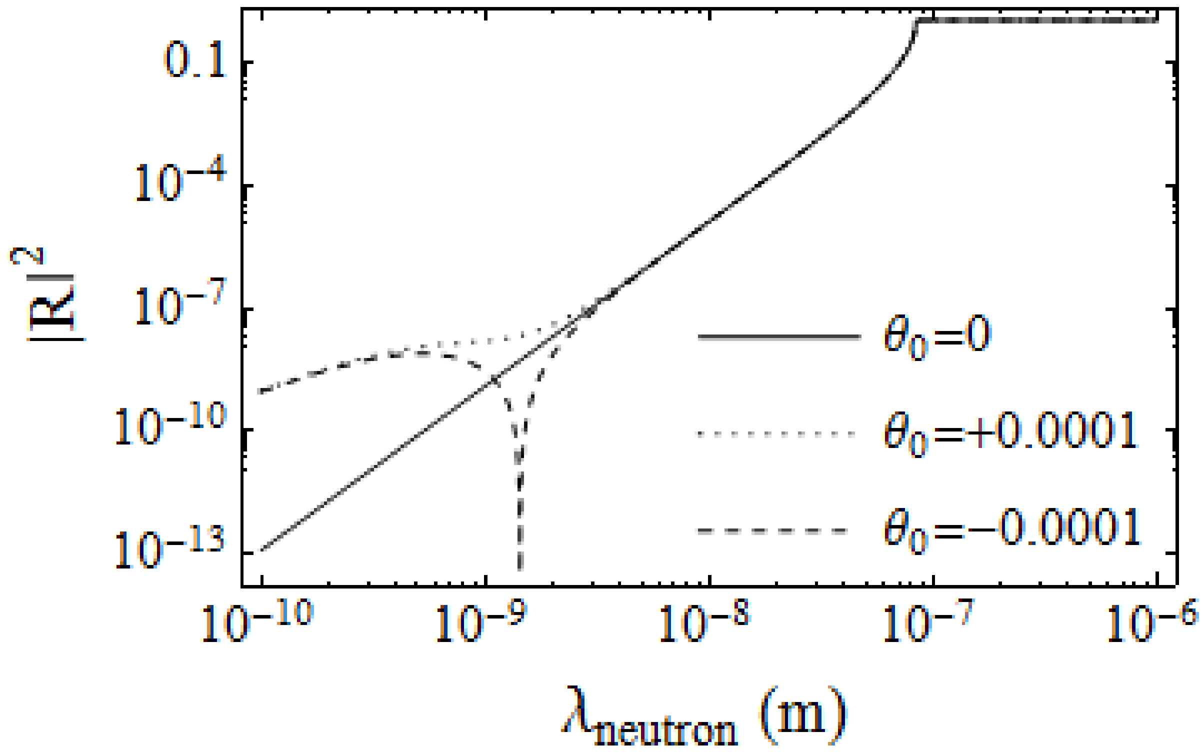
Reflection amplitude off gold

$\lambda_{\text{neutron}} \ll \lambda$

$$R \cong \frac{k_{\mathrm{i}} - k_{\mathrm{o}}}{k_{\mathrm{i}} + k_{\mathrm{o}}} + O(\theta(0)^2)$$

$\lambda_{\text{neutron}} \cong \lambda$

$$R \cong \frac{k_{\mathrm{i}} - k_{\mathrm{o}}}{k_{\mathrm{i}} + k_{\mathrm{o}}} + \frac{k_{\mathrm{o}}}{k_{\mathrm{i}} + k_{\mathrm{o}}}\theta(0) + O(\theta(0)^2)$$



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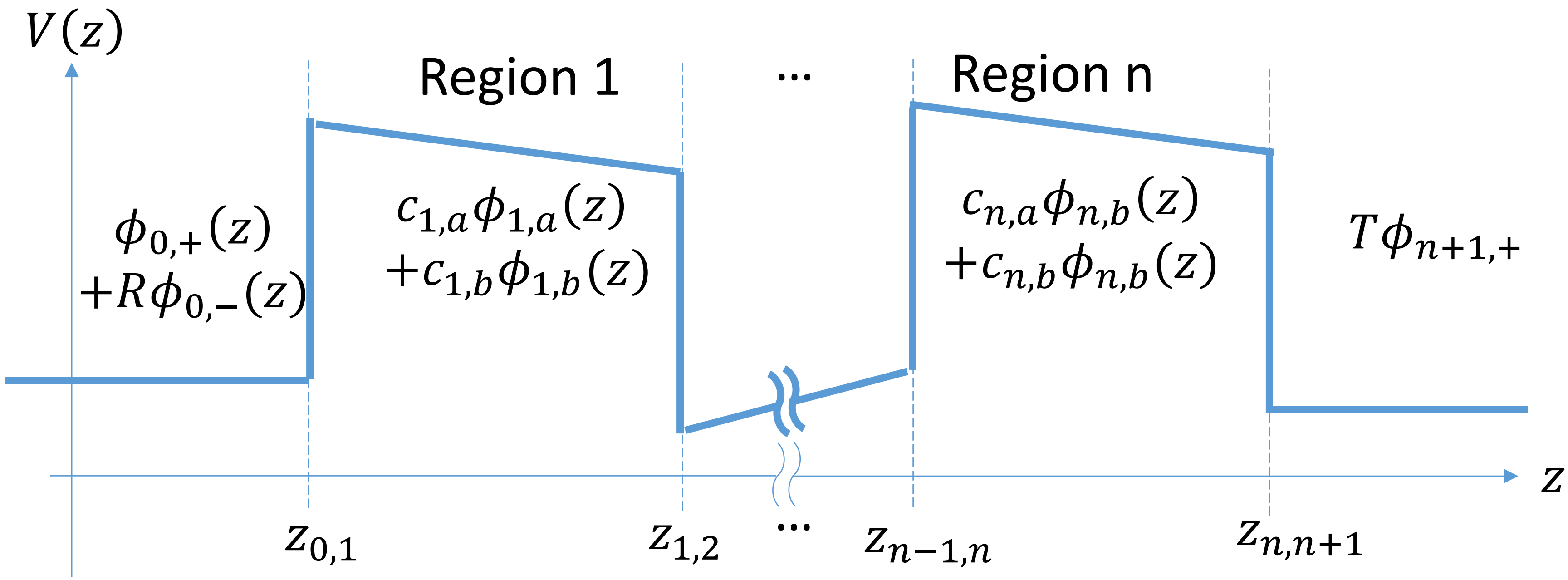
Reflection from multiple boundaries



$$\begin{pmatrix} \phi_{0,+} & \phi_{0,-} \\ \phi'_{0,+} & \phi'_{0,-} \end{pmatrix}\Bigg|_{z=z_{0,1}} \begin{pmatrix} 1 \\ R \end{pmatrix} = \begin{pmatrix} \phi_{1,a} & \phi_{1,b} \\ \phi'_{1,a} & \phi'_{1,b} \end{pmatrix}\Bigg|_{z=z_{0,1}} \begin{pmatrix} c_{1,a} \\ c_{1,b} \end{pmatrix}$$

$$\begin{pmatrix} \phi_{n,a} & \phi_{n,b} \\ \phi'_{n,a} & \phi'_{n,b} \end{pmatrix}\Bigg|_{z=z_{n,n+1}} \begin{pmatrix} c_{n,a} \\ c_{n,b} \end{pmatrix} = \begin{pmatrix} \phi_{n+1,+} & \phi_{n+1,-} \\ \phi'_{n+1,+} & \phi'_{n+1,-} \end{pmatrix}\Bigg|_{z=z_{n,n+1}} \begin{pmatrix} T \\ 0 \end{pmatrix}$$

General

$$\begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} 1 \\ R \end{pmatrix} = F_1 \dots F_n \times \begin{pmatrix} 1 \\ ik \end{pmatrix} T e^{ik(z_{n,n+1} - z_{0,1})}$$

$$F_m = \begin{pmatrix} \phi_{m,a} & \phi_{m,b} \\ \phi'_{m,a} & \phi'_{m,b} \end{pmatrix}\Bigg|_{z=z_{m-1,m}} \left(\begin{pmatrix} \phi_{m,a} & \phi_{m,b} \\ \phi'_{m,a} & \phi'_{m,b} \end{pmatrix}\Bigg|_{z=z_{m,m+1}} \right)^{-1}$$

Periodic

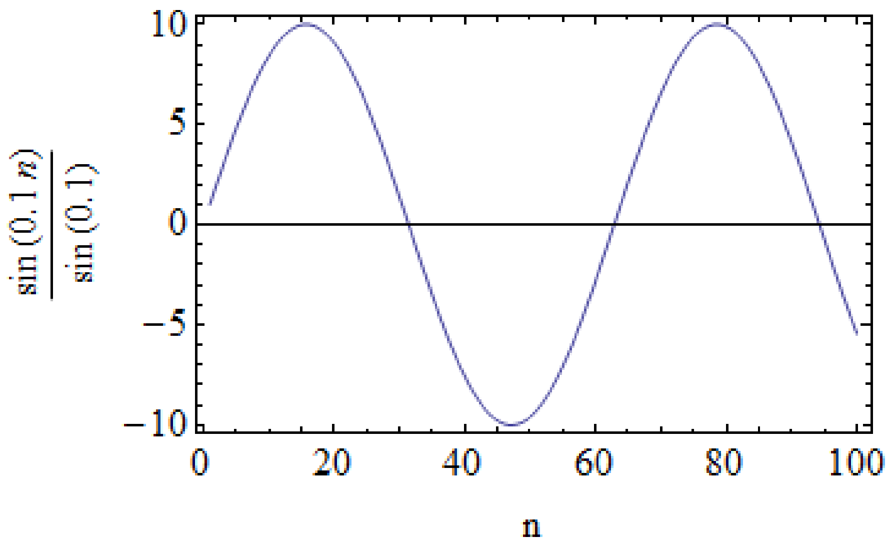
$$\begin{pmatrix} 1 & 1 \\ ik & -ik \end{pmatrix} \begin{pmatrix} 1 \\ R \end{pmatrix} = F^n \times \begin{pmatrix} 1 \\ ik \end{pmatrix} T e^{ik(z_{n,n+1} - z_{0,1})}$$

$$F := \begin{pmatrix} \cos\theta & b \\ c & \cos\theta \end{pmatrix}$$

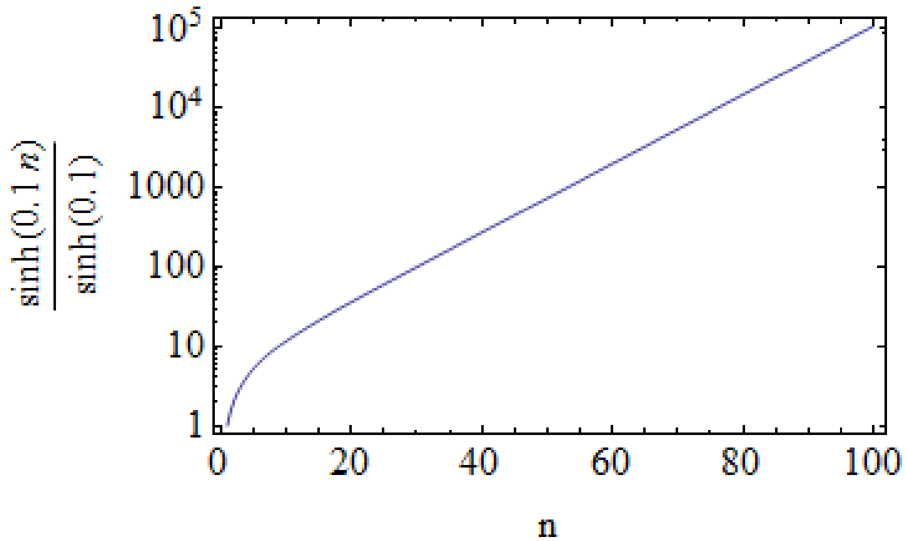
$$F^n = \begin{pmatrix} \cos n\,\theta & b\,\dfrac{\sin n\theta}{\sin\theta} \\ c\,\dfrac{\sin n\theta}{\sin\theta} & \cos n\,\theta \end{pmatrix}$$

$$|R|^2 = \frac{\beta^2}{4 + \beta^2} \qquad \beta = \left(b\,k + \frac{c}{k}\right)\frac{\sin(n\theta)}{\sin\theta}$$

$|\cos\theta| < 1$



$|\cos\theta| > 1$



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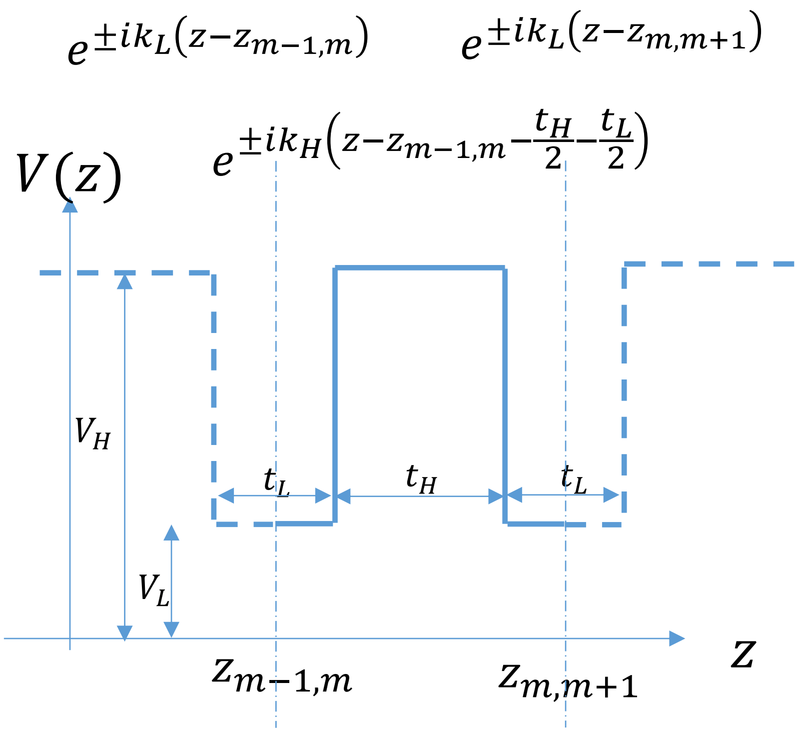
When there is no new force

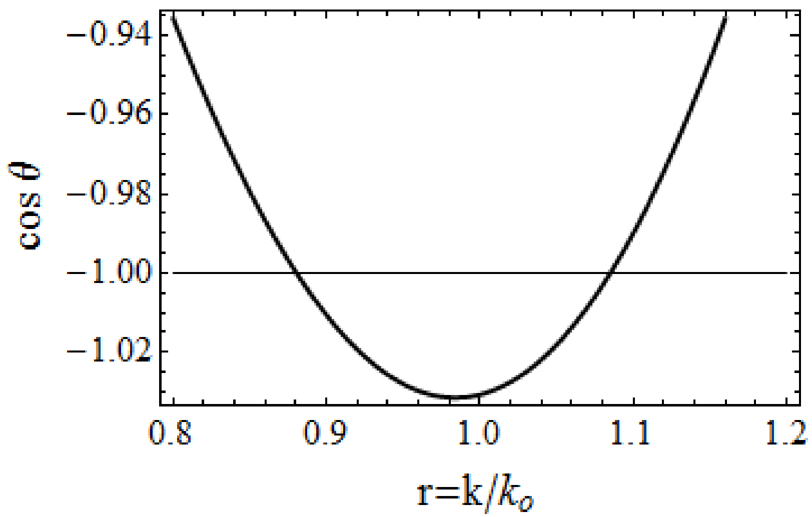


$$F = \begin{pmatrix} e^{\frac{ik_Lt_L}{2}} & e^{-\frac{ik_Lt_L}{2}} \\ ik_Le^{\frac{ik_Lt_L}{2}} & -ik_Le^{-\frac{ik_Lt_L}{2}} \end{pmatrix} \times \begin{pmatrix} e^{\frac{ik_Ht_H}{2}} & e^{-\frac{ik_Ht_H}{2}} \\ ik_He^{\frac{ik_Ht_H}{2}} & -ik_He^{-\frac{ik_Ht_H}{2}} \end{pmatrix} \times \begin{pmatrix} e^{-\frac{ik_Ht_H}{2}} & e^{\frac{ik_Ht_H}{2}} \\ ik_He^{\frac{ik_Ht_H}{2}} & -ik_He^{\frac{ik_Ht_H}{2}} \end{pmatrix}^{-1} \times \begin{pmatrix} e^{-\frac{ik_Lt_L}{2}} & e^{\frac{ik_Lt_L}{2}} \\ ik_Le^{-\frac{ik_Lt_l}{2}} & -ik_Le^{\frac{ik_Lt_l}{2}} \end{pmatrix}^{-1}$$

$$\cos\theta = \cos k_L t_L \cos k_H t_H - \frac{1}{2}\left(\frac{k_L}{k_H} + \frac{k_H}{k_L}\right)\sin k_L t_L \sin k_H t_H$$

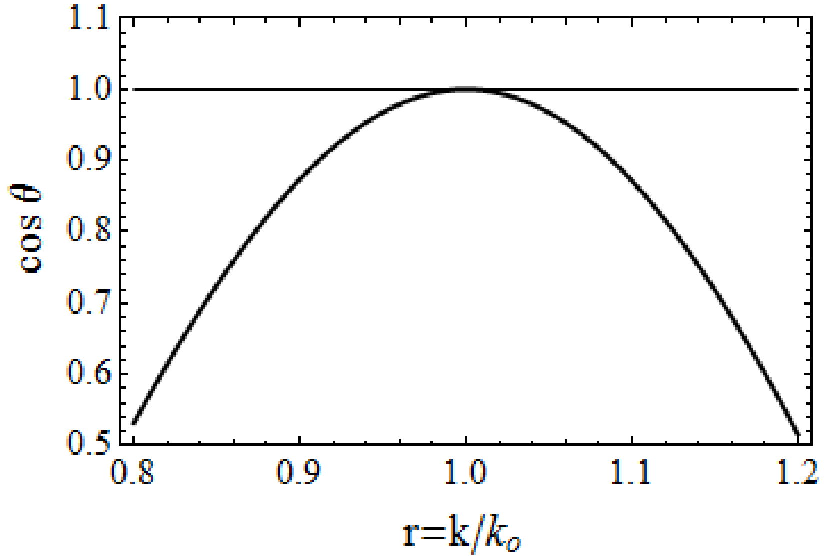
Designed to reflect neutrons

$$k_H t_H\Big|_{k=k_0} = k_L t_L\Big|_{k=k_0} = \pi/2$$



Designed **NOT** to reflect neutrons

$$k_H t_H\Big|_{k=k_0} = k_L t_L\Big|_{k=k_0} = \pi$$



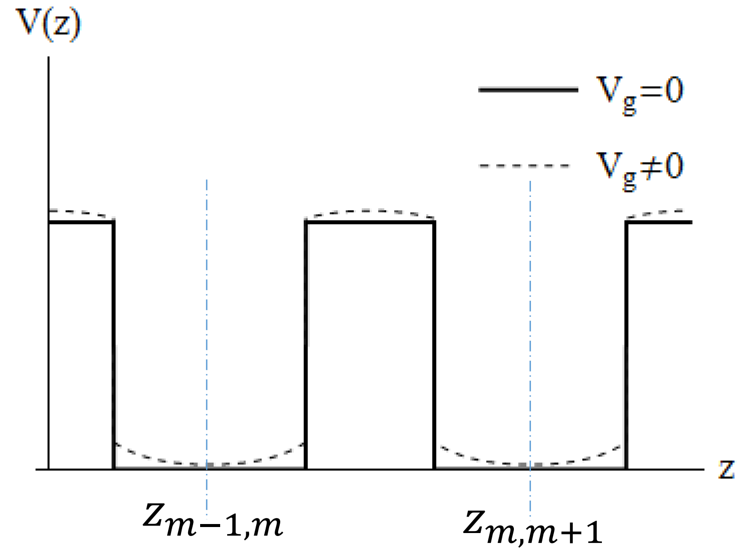
Effects of the new force

Potential for neutrons

$$\begin{cases} V_{f,L} + a_L + b_L \cosh\left(2k_g\left(z - z_{m,m+1}\right)\right) & z_{m,m+1} - \dfrac{t_L}{2} < z < z_{m,m+1} \\ V_{f,H} + a_H + b_H \cosh\left(2k_g\left(z - \left(z_{m,m+1} + z_{m-1,m}\right)/2\right)\right) & \dfrac{\left(z_{m,m+1} + z_{m-1,m}\right)}{2} - \dfrac{t_H}{2} < z < \dfrac{\left(z_{m,m+1} + z_{m-1,m}\right)}{2} + \dfrac{t_H}{2} \\ V_{f,L} + a_L + b_L \cosh\left(2k_g\left(z - z_{m-1,m}\right)\right) & z_{m-1,m} < z < z_{m-1,m} + \dfrac{t_L}{2} \end{cases}$$

Schrodinger equations

$$\Psi''(z) + \left(k_X^2 - \eta_X \cosh\left(2k_g\left(z - z_X\right)\right)\right)\Psi(z) = 0$$

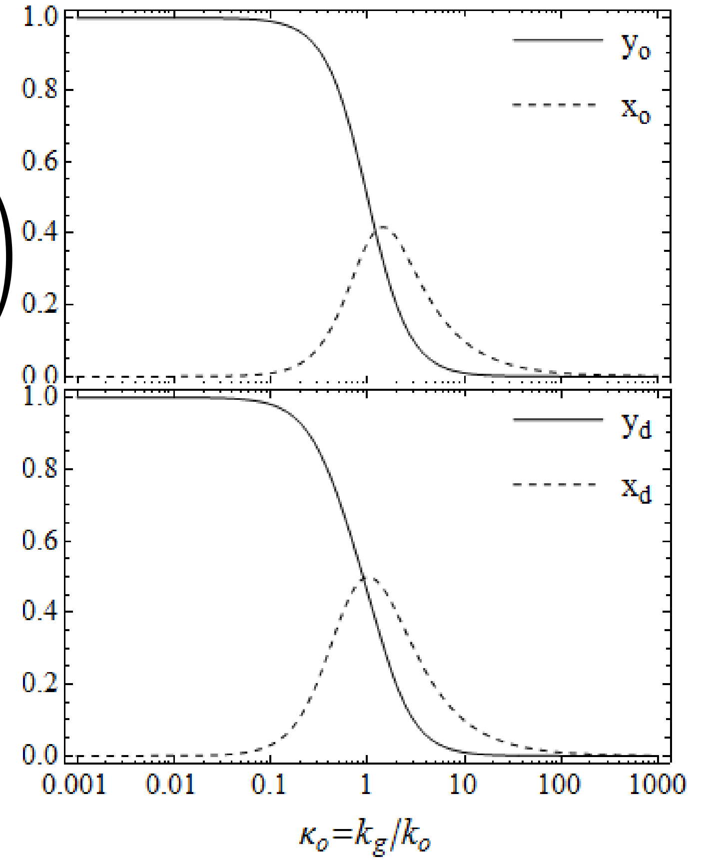


Corrections on the boundary

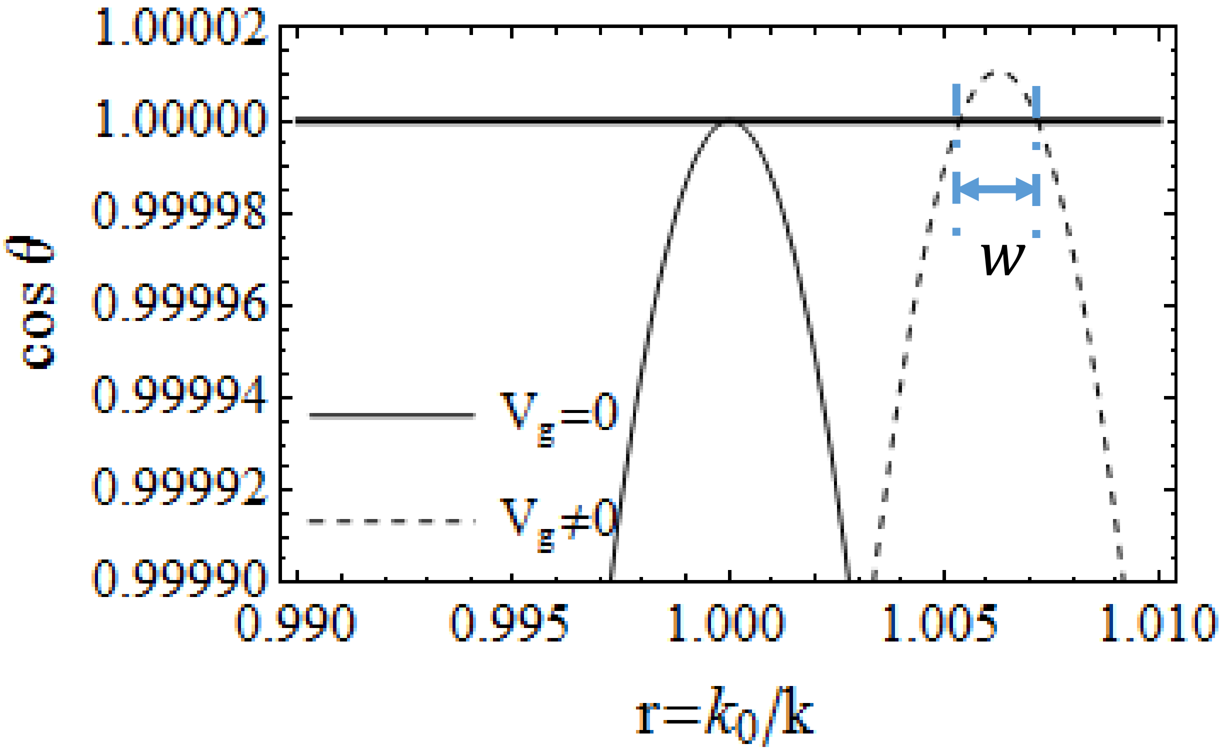
$$\phi_{X,n,\pm}(z_{X,n} + t_X/2) \cong e^{\pm i k_X \frac{t_X}{2}} \left(1 - \theta_1\left(\frac{t_X}{2}\right)(x_o \mp i y_o)\right)$$

$$\left.\frac{d}{dz} \ln \phi_{X,n,\pm}(z)\right|_{z=z_{X,n}+t_X/2} \cong \pm \mathrm{i} k_X - \theta_1'\left(\frac{t_X}{2}\right)(x_d \mp i y_d)$$

$$\theta_1(z) = \int_{z_X}^{z_X+z} \left(\sqrt{k_X^2 - \eta_X e^{+2k_g(z'-z_X)}} - k_X\right) dz'$$



Band structure for the neutron in Pt/C

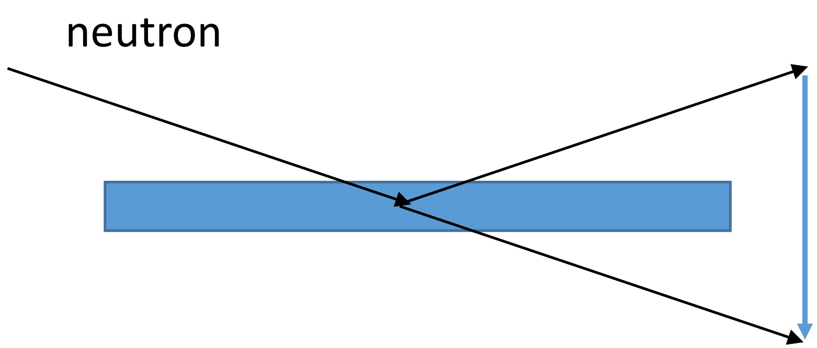


$k_0 = 0.1\ \text{nm}^{-1}$
$\lambda_g = 10\ \text{nm}$
$\alpha_g = 10^{20}$

$w \cong 0.2\%$

Reflectivity measurement

$$\left.\frac{\delta q}{q}\right|_{\text{present constraint}} = 0.2\ 10^{-2}$$



$$q = 0.02\ \text{Å}^{-1}$$

$\delta q = 10^{-7}\ \text{Å}^{-1}$?@ S18 ILL

G. Kroupa, et al., NIMA 440, 604 (2000)

$$\left.\frac{\delta q}{q}\right|_{\text{expected}} = 0.2\ 10^{-6}$$

$\frac{\delta q}{q} \propto \alpha_g \rightarrow 10^4$times stronger constraints?

Summary

- Reflection amplitude changes significantly when $\lambda_{\mathrm{neutron}} \cong \lambda$.
- Reflection probability significantly increases in a band gap.
- Band gap due to the new force exists.
- Reflectivity measurement may provide 10^4 times stronger constraint when λ is 10 nm.

Thank you for your attention