# Fragmentation function and the Bjorken parent-child relation 

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## Motivation

As originally indicated by Bjorken in [1] (see also [2] and [3]) the particle $p_{T}$ distribution has quite similar shape as the jet cross section.
This is known as a "Parent-Child Relationship" (PCR).
It follows from assumptions:

- The inclusive $p_{\mathrm{T}}$-distribution can be quite accurately approximated by a power law function.
- Fragmentation function is a universal function independent of the parton momentum.
[1] J. D. Bjorken, Phys. Rev. D8, 4098 (1973).
[2] S. S. Adler et al., Phys. Rev. D74, 072002 (2006).
[3] M. J. Tannenbaum, PoS CFRNC2006, 001 (2006).


## Particle versus jet invariant cross section by ALICE




ALICE

- ARXIV:1307.1093
- Phys.Lett., 2013, B722, 262-272


## Parent-Child Relationship

The joint probability of detecting a hadron with $p_{\mathrm{T}}=z \hat{p}_{\mathrm{T}}$ originating from a parton of $\hat{p}_{T}$ can be written as

$$
\frac{d^{2} P_{q \rightarrow h}}{d \hat{p}_{\mathrm{T}} d z}=\frac{d P_{q}}{d \hat{p}_{\mathrm{T}}} \times D_{\mathrm{u}, \mathrm{~d}}^{\mathrm{h} \pm}(z)
$$

where the final state parton spectrum $d P_{q} / d \hat{p}_{\mathrm{T}} \propto \hat{p}_{\mathrm{T}}^{1-n}$ and $D_{\mathrm{u}, \mathrm{d}}^{\mathrm{h} \pm}$ is the "fragmentation function". The $p_{\mathrm{T}}$ spectrum may be found by integrating over all values of $\hat{p}_{\mathrm{T}} \geq p_{\mathrm{T}}$ to $\hat{p}_{\mathrm{T} \text { max }}=\sqrt{s} / 2$, which corresponds to values of $z$ from $x_{T}=2 p_{T} / \sqrt{s}$ to 1 .

$$
\frac{1}{p_{\mathrm{T}}} \frac{d \sigma_{h}}{d p_{\mathrm{T}}}=\int_{x_{\mathrm{T}}}^{1}\left(\frac{p_{\mathrm{T}}}{z}\right)^{-n} D_{\mathrm{u}, \mathrm{~d}}^{\mathrm{h} \pm}(z) \frac{d z}{z^{2}}=p_{\mathrm{T}}^{-n} \int_{x_{\mathrm{T}}}^{1} D_{\mathrm{u}, \mathrm{~d}}^{\mathrm{h} \pm}(z) z^{n-2} d z \propto p_{\mathrm{T}}^{-n}
$$

Last integral $\sim$ const $\rightarrow$

$$
\text { hadronic } \frac{1}{p_{\mathrm{T}}} \frac{d \sigma_{h}}{d p_{\mathrm{T}}} \propto \frac{1}{P_{q}} \frac{d P_{q}}{d \hat{p}_{\mathrm{T}}} \text { partonic }
$$

## Parent-Child Relationship

So according PCR knowing the inclusive particle cross section one can calculate the jet invariant cross section WITHOUT a jet reconstruction (obviously only the low- $p_{T}$ reach). One needs to evaluate only factor:

$$
\int_{x_{\mathrm{T}}}^{1} D_{\mathrm{u}, \mathrm{~d}}^{\mathrm{h} \pm}(z) z^{n-2} d z
$$

However, $D_{\mathrm{u}, \mathrm{d}}^{\mathrm{h}}(z)=d N / d z$ is not a probability distribution of finding a particle of momentum fraction $z$ ! The joint probability of detecting a hadron with $p_{\mathrm{T}}=z \hat{p}_{\mathrm{T}}$ originating from a parton of $\hat{p}_{\mathrm{T}}$ CANNOT be written as

$$
\frac{d^{2} P_{q \rightarrow h}}{d \hat{p}_{\mathrm{T}} d z}=\frac{d P_{q}}{d \hat{p}_{\mathrm{T}}} \times D_{\mathrm{u}, \mathrm{~d}}^{\mathrm{h} \pm}(z)
$$

when $D(z)$ represents the $d N / d z$ in the traditional way.

## 1. assumption power law

$$
n \neq n\left(x_{T}\right)
$$

not so bad.

See W. Horowitz, M Gyulassy,
"The Surprising Transparency of the sQGP at LHC",

Nucl.Phys., 2011, A872, 265-285


## 2. assumption Fragmentation Function

The total momentum fraction taken away by all fragments is equal to unity and thus an integral over the fragmentation function is equal to the average particle multiplicity

$$
\int_{0}^{1} z D_{q}^{\mathrm{h} \pm}(z) d z=1 \rightarrow \quad \int_{0}^{1} D_{q}^{\mathrm{h} \pm}(z) d z=\left\langle m_{\mathrm{f}}\right\rangle
$$



$$
D_{q}^{\mathrm{h} \pm}(z)=\mathcal{N} z^{-p_{0}}(1-z)^{p_{1}}(1+z)^{-p_{2}}
$$

|  | quark | gluon |
| :---: | :---: | ---: |
| $\mathcal{N}$ | 24.54 | 119.35 |
| $p_{0}$ | 0.49 | 0.16 |
| $p_{1}$ | 0.57 | 0.88 |
| $p_{2}$ | 8.00 | 13.29 |
| $\langle z\rangle$ | 0.068 | 0.066 |
| $\left\langle m_{\mathrm{f}}\right\rangle$ | 14.7 | 15.2 |

## Possible solution - Leading particle Fragmentation Function

Thorsten Renk and Kari Eskola realized this problem and suggested to use Leading particle Fragmentation Function. Since there is only one LP in the jets $\left\langle m_{\mathrm{LP}}\right\rangle=1$ then it has a property of probability distribution function.


FIG. 1: (Color online) Comparison of the KKP [25] fragmentation function $D(z, \mu)$ for u-quarks and gluons into charged hadrons ( $h^{+}+h^{-}$) at two hadronic scales with the leading hadron momentum fraction probability density $A_{1}\left(z_{1}, p_{T}\right)$ at partonic $p_{T}=25 \mathrm{GeV}$ as extracted from shower simulations in PYTHIA [24].

## $d N / d z$ associated probability distribution

However, my supervisor believed that there must be a way how to extract the probability distribution function to given $d N / d z$. In the simplified picture (no evolution/violation $\left.D_{\mathrm{u}, \mathrm{d}}^{\mathrm{h} \pm}\left(z, Q^{2}\right)=D_{\mathrm{u}, \mathrm{d}}^{\mathrm{h} \pm}(z)\right)$ the fragmentation process can be $\widehat{\mathrm{p}}_{\mathrm{T}}=10.0 \mathrm{GeV}$

seen as a "cascading"

## Example



## Feynman knew how to solve already back in 70ties

Field, R. D. and Feynman, R. P. "A parametrization of the properties of quark jets" Nucl. Phys., 1978, B136, 1.
The probability that we have a hierarchy sequence of primary mesons with the $k$ th having momentum $\xi_{k}$ in $d \xi_{k}$ is

$$
\operatorname{Prob}\left(\xi_{1} \xi_{2} \ldots \xi_{k}\right)=\prod_{l=1}^{\infty} f\left(\eta_{i}\right) d \eta_{i}
$$

ORIGINAL QUARK
ORIGINAL QUARK
OF FLAVOR "a"
OF FLAVOR "a"

Fig. 1. Illustration of the "hierarchy" structure of the final mesons produced when a quark of type " a " fragments into hadrons. New quark pairs $\mathrm{b} \overline{\mathrm{b}}, \mathrm{c} \bar{c}$, etc., are produced and "primary" mesons are formed. The "primary" meson ba that contains the original quark is said to have "rank" one and primary meson cb rank two, etc. Finally, some of the primary mesons decay and we assign all the decay products to have the rank of the parent. The order in "hierarchy" is not the same as order in momentum or rapidity.

## Feynman knew how to solve already back in 70ties

He derived the integral equation

$$
D(z)=P D F(1-z)+\int_{1}^{z} P D F(\eta) D(z / \eta) d \eta
$$

where $\eta=1-z$. Feynman mentioned that this equation can be solved by help of Fourier transform but he showed the solution only two particular solution taking the momenta in $z\left(M(r)=\int_{0}^{1} z^{r} D(z) d z\right)$. If someone know how to solve this eq. by Fourier please let us know.
Petja Paakkinen (CERN summer student) tested the solutions by use of monte carlo. He showed then if one uses the KKP parameterization

$$
D_{q}^{\mathrm{h} \pm}(z)=\mathcal{N} z^{-p_{0}}(1-z)^{p_{1}}(1+z)^{-p_{2}}
$$

and for the PDF

$$
P D F(1-z)=N(1-z)^{\nu} e^{-\lambda z}
$$

## Numerical solution




## Summary

- We learned from 1978 Feynman paper how to derive a probability distribution function for measured $d N / d z$
- However, Feynman showed only some solution of his integral equation. Any advice on general solution of

$$
D(z)=P D F(1-z)+\int_{1}^{z} P D F(\eta) D(z / \eta) d \eta
$$

would be welcome.

- This approach can be used for simplified corrections of trigger bias as done e.g. by PHENIX
Jet properties from dihadron correlations in p+p collisions at $\sqrt{s}=200 \mathrm{GeV}$, Phys. Rev., 2006, D74, 072002)

